DESIGN OF REINFORCED CONCRETE STRUCTURES

First Edition-2008 **Volume 3**

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Features

- Reflects the very latest Egyptian Code provisions (ECP 203- 2007) and includes all major changes and additions.
- Numerous illustrations and figures for each topic.
- Good theoretical background for each topic with code provisions.
- Extensive examples in each chapter utilizing SI units.
- All examples are worked out step-by-step ranging from simple to advanced.
- Full reinforcement details for every example.
- Numerous design charts.

Volume 3 covers the following topics:

- Reinforced Concrete Frames, Arches and Arched Slabs
- Design of Deep Beams and Corbels
- Deflections of Reinforced Concrete Members
- Crack Control of Reinforced Concrete Members
- Design of Shallow Foundations and Pile Caps
- Design of Raft Foundations
- Strut-and-Tie Model for Reinforced Concrete Members
- Fundamentals of Prestressed Concrete
- Flexural Design of Prestressed Concrete Members
- Shear and Torsion in Prestressed Concrete
- Analysis of Continuous Prestressed Beams

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PREFACE

Teaching reinforced concrete design, carrying out research relevant to the behavior of reinforced concrete members, as well as designing concrete structures motivated the preparation of this book.

This volume is considered part of a series of books that covers the subject of *Reinforced Concrete Design*. The success and the positive feedback that we have received from our students and colleagues have provided the inspiration for us to proceed with volume three. Due to the numerous changes in the provisions of the 2007 edition of the Egyptian Code of Practice ECP 203, the publication of this volume became more of a necessity than mere addition.

The first volume covers the fundamentals of reinforced concrete design and the design of beams whereas the second volume focuses primarily on the design of slabs, columns and frames. This third volume covers the following topics:

- **Arches, Special Types of Frames and Vierendeel.**
- **Deep Beams and Corbels.**
- **Control of Deflections and Cracking.**
- **Design of Shallow and Deep Foundations.**
- **Strut-and-Tie Model.**
- **Prestressed Concrete.**

Numerous illustrative examples are given, the solution of which has been supplied so as to supplement the theoretical background and to familiarize the reader with the steps involved in actual design problem solving. To ensure the accuracy, all of the examples in this book are solved and verified using EXCEL spread sheet programs that were prepared exclusively for this book.

In writing the book, the authors are conscious of a debt to many sources, to friends, colleagues, and co-workers in the field. Finally, this is as good a place as any for the authors to express their indebtedness to their honorable professors of Egypt, Canada and the U.S.A. Their contributions in introducing the authors to the field will always be remembered with the deepest gratitude.

The book is aimed at two different groups. First, by treating the material in a logical and unified form, it is hoped that it can serve as a useful text for undergraduate and graduate student courses on reinforced concrete. Secondly, as a result of the continuing activity in the design and construction of reinforced concrete structures, it will be of value to practicing structural engineers. The authors strongly recommend that the Code be utilized as a companion publication to this book.

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1

ARCHES, SPECIAL TYPES OF FRAMES AND TRUSSES

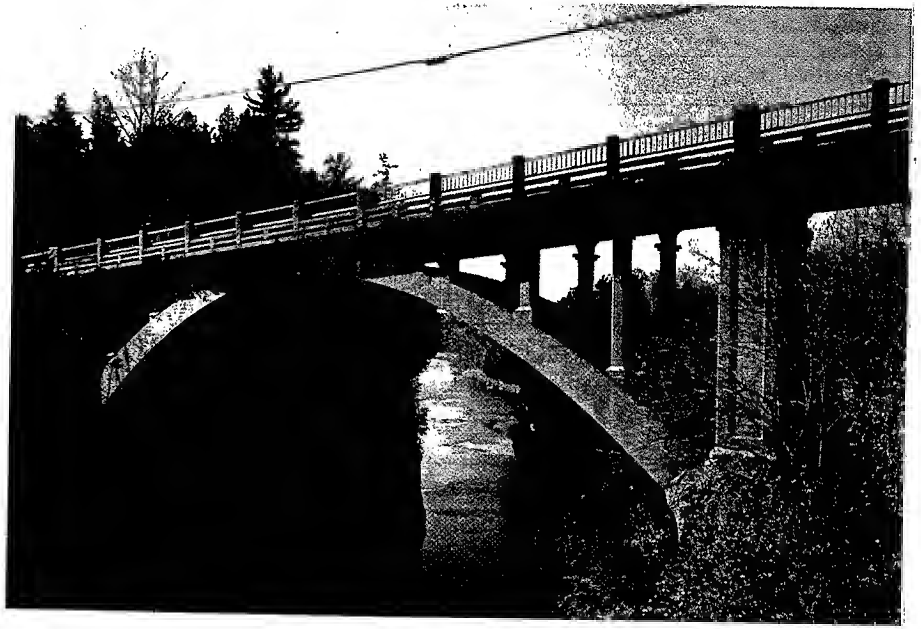


Photo 1.1 Arched reinforced concrete bridge.

1.1 Introduction

This chapter presents the use and design of reinforced concrete arches and trusses as supporting elements of systems that cover halls having relatively large spans. It covers also the design and the construction of the saw tooth roofs in which the light from the windows is directly reflected by the roof inside the hall giving a uniform distribution of natural light.

Choosing the most economical structural system depends on many factors such as the type of soil, the architectural features of the building, and most importantly the span that needs to be covered. Table 1.1 gives the suitable structural system according to the span of the hall (*short direction*). For example, simple girders are suitable for relatively short spans (7-10 m) while frames are appropriate for medium spans (12-25m). In contrast, arches and trusses are suitable to cover large spans.

Table 1.1 Choosing of structural system according to the span

Type of structure	**Span**
Simple girders	7→10 m
Frames	12→25 m
Arch with a tie	20→40 m
Trusses	20→40 m
Vierendeel Systems	30→40 m

1.2 Reinforced Concrete Arches

1.2.1 General

In simple beams, commonly one cross-section is subjected to the maximum design moment, and consequently only one cross-section of the beam is working at the maximum stress (see Fig. 1.1a).

Knowing that the mentioned maximum stresses act at the extreme fibers only and that all other fibers are less stressed, one can directly observe that the simple beam is not an efficient structural system. It should be mentioned that in a simple beam, significant parts of the concrete sections are not participating in resisting the applied straining actions and constitute additional weight as shown in Fig. 1.1b. If the unnecessary concrete is removed from a simple beam, an arch with a tie would result as shown in Fig. 1.1c.

Arched girders of convenient form are mainly subject to high compressive forces and low bending moments and shearing forces. Nearly all sections of the arch are approximately subjected to the same average compressive stress which means a high efficiency in the use of reinforced concrete as a building material. For this reasons, arched roofs give a convenient economic solution for long span roof structures without intermediate supports in cases where a flat roof surface is not necessary to meet the functional requirements of the structures. However, flat roofs or floors can occasionally be supported on arched girders.

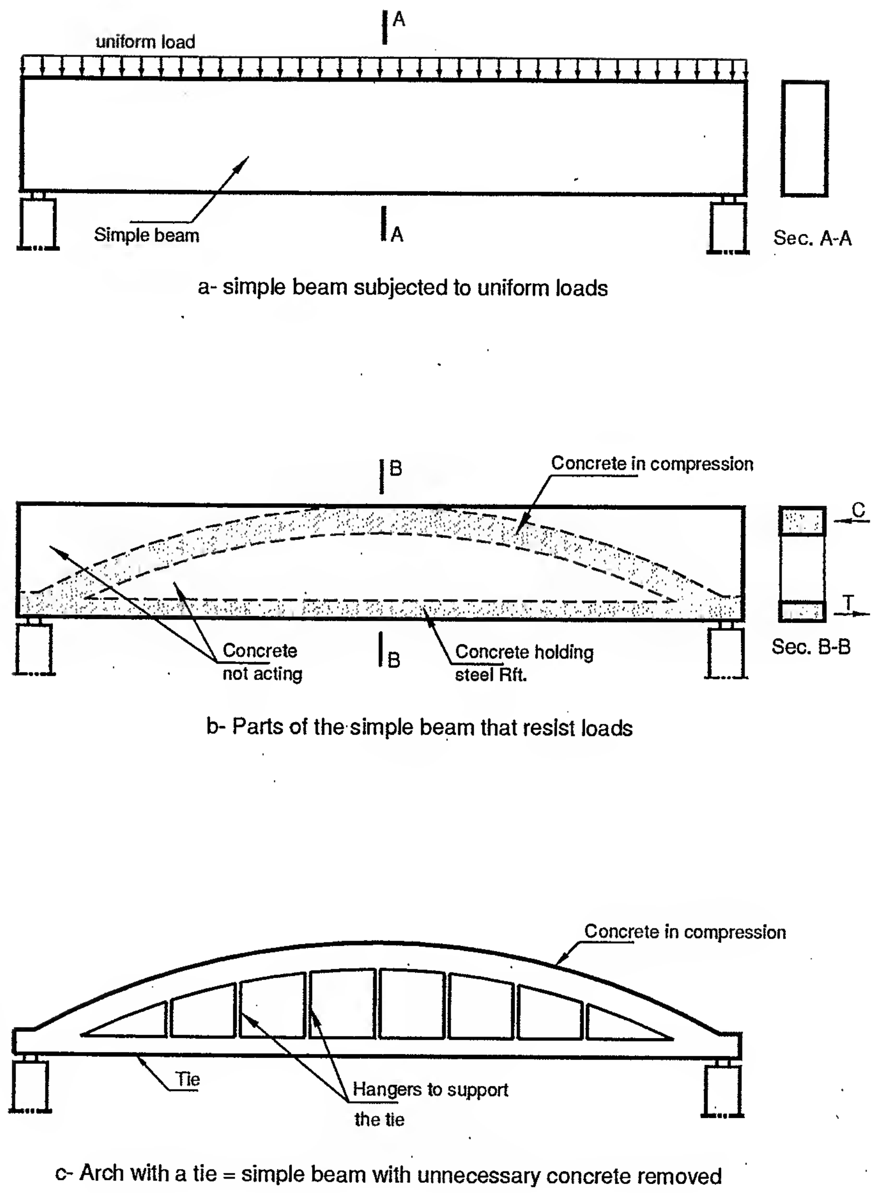


Fig. 1.1 The concept of R/C arches

1.2.2 Design of the Arch with a Tie

One of the most convenient systems for resisting uniform gravity loads is the two-hinged parabolic arch with a tie. This system is externally statically determinate and internally once statically indeterminate.
The equation of the axis of the arch according to Fig. (1.2) is given by:

$$y = \frac{4f\ x}{L^2}\ (L - x) \quad \text{.......................................} (1.1)$$

where

f = rise of the arch
L = span of the arch

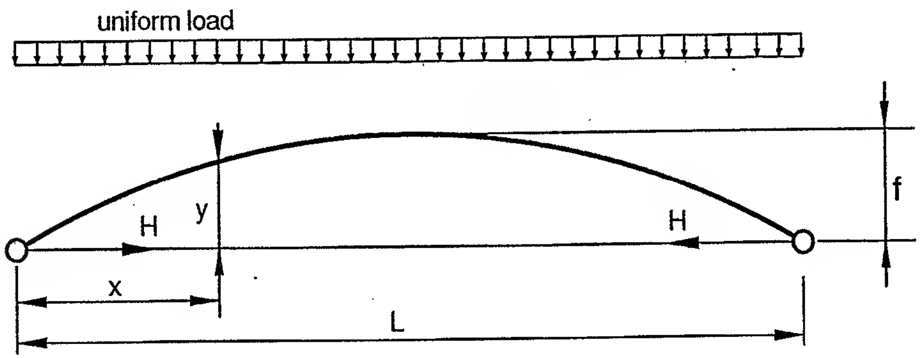


Fig. 1.2 Geometry of the arch with a tie

Structural analysis of the system can express the value of the horizontal thrust H as a ratio(λ) from the simple beam bending moment . Assuming that modular ratio between the steel used in the tie and concrete used in the arch is 10, one can obtain:

$$H = \lambda \frac{w\ L^2}{8f} \quad \text{.......................................} (1.2)$$

where

$$\lambda = \frac{1}{1 + \alpha_1 + \alpha_2} \quad \text{.......................................} (1.3)$$

$$\alpha_1 = 0.156 \frac{t^2}{f^2} \quad \text{.......................................} (1.4)$$

$$\alpha_2 = \frac{b\ t^3}{80\ A_{tie}\ f^2} \quad \text{.......................................} (1.5)$$

t = Total thickness of the arch.
b = Width of the arch.
I = Cross sectional moment of inertia.
A_{tie} = Cross sectional area steel in the tie.
f = The rise of the arch at the crown.

The amount of tension in the tie varies considerably according thickness of the arch, the steel in the tie, and the rise of arch f. However, it is customary to take the factor λ equals 0.95. Thus, the horizontal thrust = the force in the tie

$$H \cong 0.95 \frac{w \times L^2}{8f} \quad \text{.......................................} (1.6)$$

The bending moment at the crown M $\cong$ $0.05 \dfrac{w \times L^2}{8}$ .... (1.7)

Hence, the critical section in the arch girder is subjected to a compressive force H and a bending moment M. The tie beam, on the other hand, is subjected to an axial tension force, H. It should be noted the bending moment induced in the tie beam due to its own weight is negligible. The span of the tie beam under its own weight is the distance between the posts.

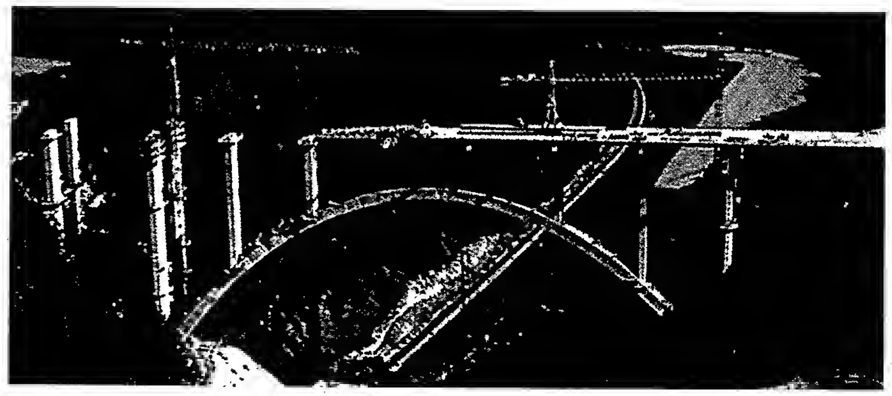


Photo 1.2 An arched bridge during construction

1.2.3 Layout of a Hall Covered by Arched Girders

In large–span big covered halls, reinforced concrete arches with ties are usually used as the main supporting elements. In order to get relatively reasonable dimensions of the arches, the spacing between arches should be in the range of 5.0 ms to 7.0 ms. Hangers are provided in order to prevent saging of the ties under its own weight.

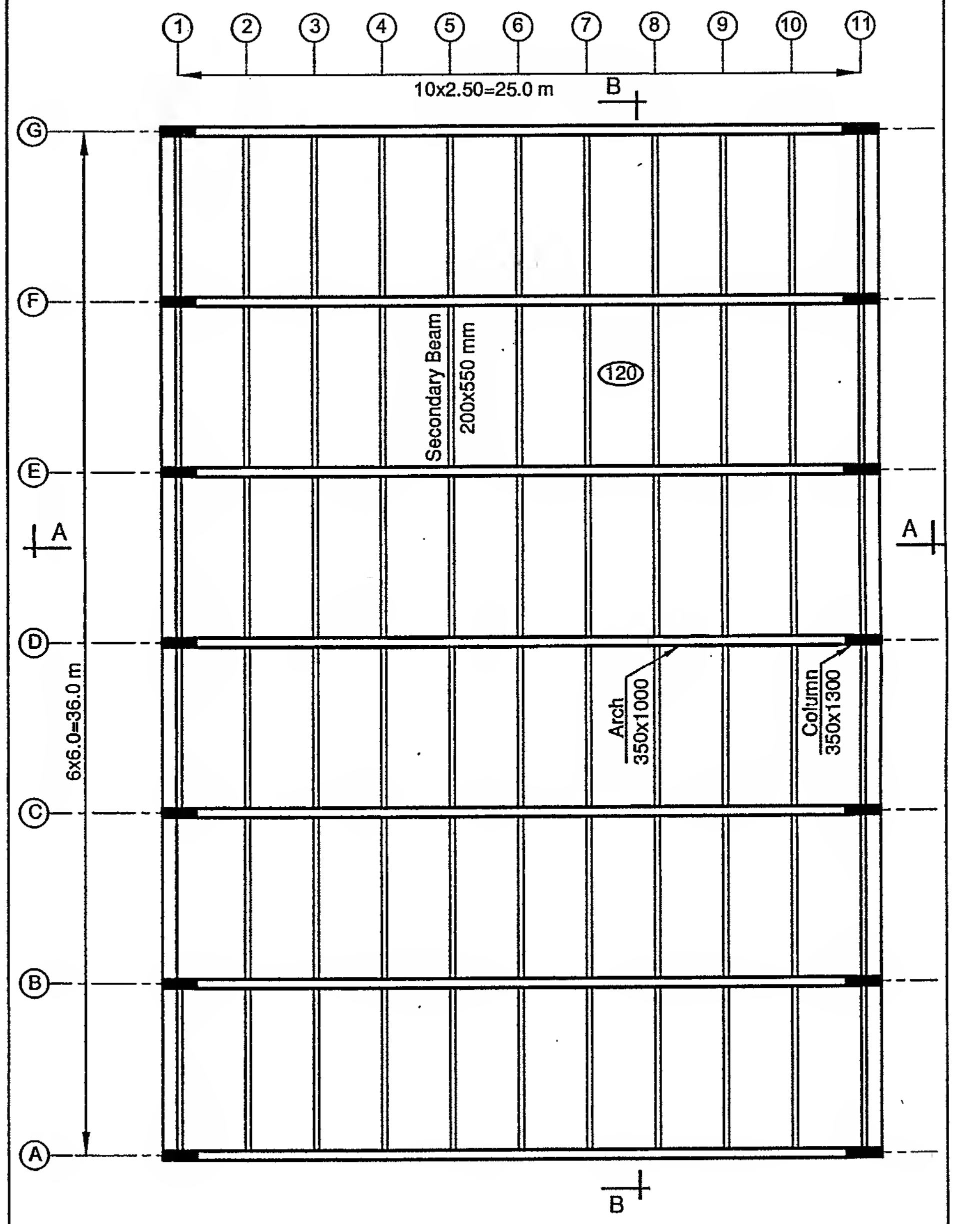
Figure 1.3 shows the layout of the supporting elements of a hall that is 25.0 ms wide, 36.0 ms long and 5.0 ms clear height. The main supporting element is chosen as an arch with a tie spanning in the short direction arranged every 6. 0 ms.

The arch rests on reinforced concrete columns. These columns should provide a reasonable bearing area to the arch. Out-of-the plane of the arch, these columns are connected together with the semelles, the wall beams and the top beams.

The horizontal roof consists of horizontal continuous beams supporting the roof slabs. These beams transmit their loads to the curved girder of the arch through a system of short members called posts. These short members serve also to reduce the span of the tie and consequently reduce the moments due to its own weight (they hang-up the tie).

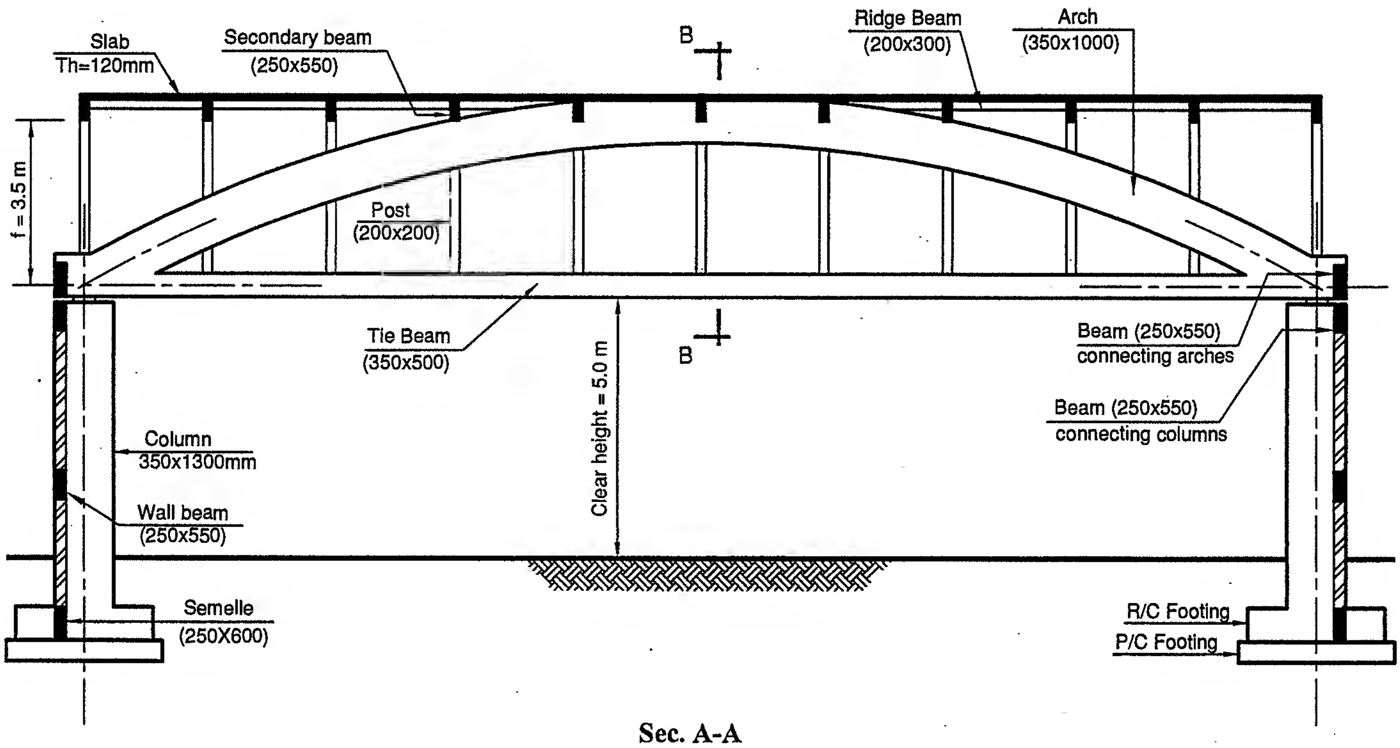
It should be mentioned that the girder of the arch is subjected to a compression force and is susceptible to buckling. The arched girders are connected in the out-of-plane direction through continuous beams located at the level of the tie as well as at the level of crown. These beams reduce the buckling length of the girder out-of-the plane of the arch.

The columns supporting the arches are connected in the out-of-plane direction using a continuous beam at the top level of the columns and a wall beam at the mid-height. In case of a weak soil, semelles are provided at the foundation level in order to connect the footings together to reduce the effect of differential settlements. Otherwise, they could be provided at the bottom end of the frame leg to support the wall above and reduce the buckling length.



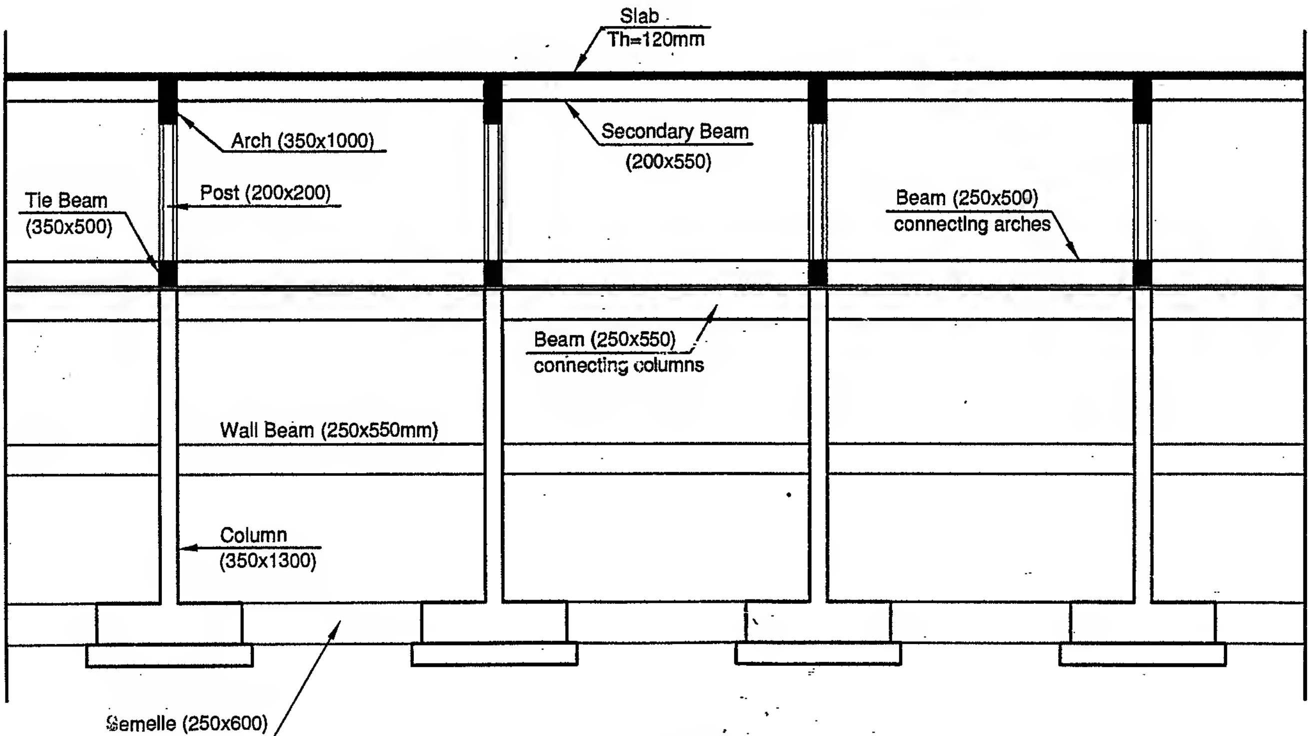
Plan

Fig. 1.3a A hall covered with arches (horizontal roof)



Sec. A-A

Fig. 1.3b A Hall covered with arches (continued)



Sec. B-B

Fig. 1.3c A hall covered with arches (continued)

The dimensions of the arch system can be estimated according to Table 1.2.

Table 1.2 Recommended dimensions for arch with a tie system

Item	Dimension
t_g	Span/25
b_g	250-400 mm
t_{tie}	$t_g/2$
Arch spacing	4→7 m
f	$Span/(6 \rightarrow 8)$
Roof Angle	24° - 32°
Column thickness	h/15 or Span/20
Secondary beam thickness	Arch spacing $/(8 \rightarrow 10)$
Post spacing	2→4 m
Post dimensions	200 × 200 mm
Ridge beam thickness	Post spacing $/(8 \rightarrow 10)$

Figure 1.4 shows an arch with a tie that is used as the main supporting element for the roof of a hall 25.0 ms wide. The roof has variable levels that allows for indirect lighting. The horizontal part of the roof consists of horizontal slab supported on continuous beams. These beams are supported by the posts. The middle part of the roof is a curved slab supported also on continuous beams that transmit the loads to the arched girder.

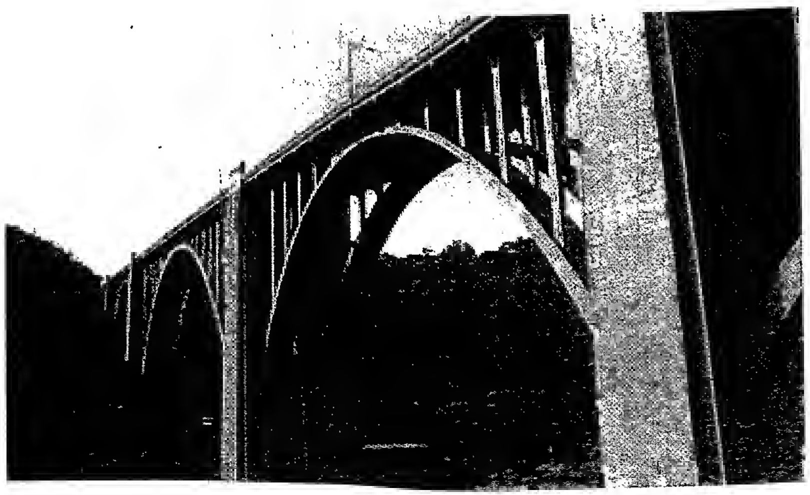


Photo 1.3 Arched bridge

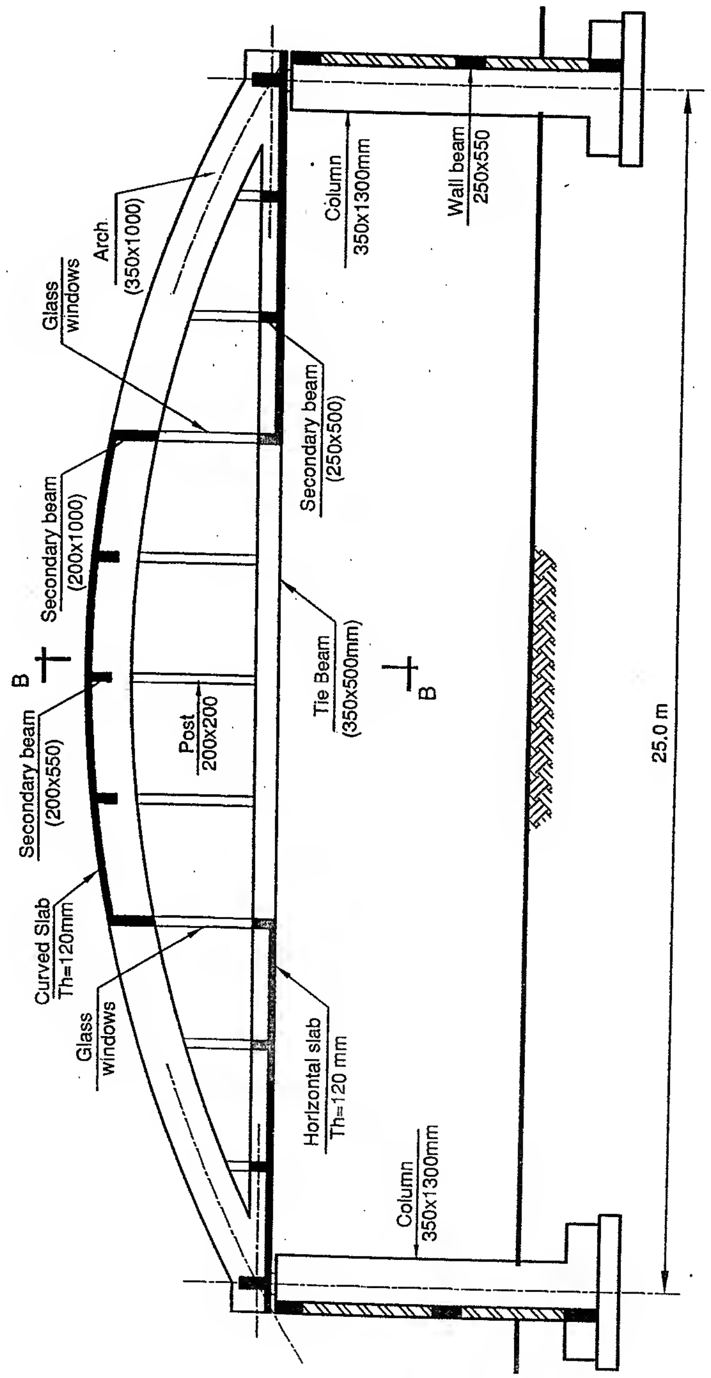


Fig. 1.4 An arched girder supporting a roof with variable levels

1.3 Saw-tooth Roof Structures

In industrial buildings, it is generally recommended not to allow any direct sunrays to fall inside the hall. In such cases, the saw-tooth roofs in which the light from the windows is directly reflected by the roof inside the hall gives a convenient solution. Windows are arranged to face the north. The choice of the appropriate system depends on the span and the direction of the north.

1.3.1 North Direction Normal to the Span

As mentioned before, windows are arranged to face the north as shown in Fig. 1.5. The main supporting element should be arranged in the short direction of the hall. If the short direction is not more than about 20.0 ms, it is recommended to use frames. If, on the other hand, the short direction is more than 20.0 ms, it might be more economical to use an arch with a tie as the main supporting element.

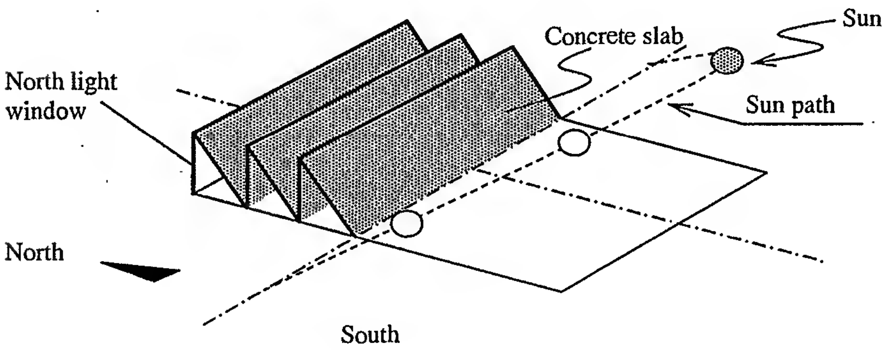


Fig. 1.5 North-light window in saw-tooth roof

1.3.1.1 Frames as the Main Supporting Elements

Figure 1.6 shows the general layout of a hall 20.0 ms wide and 33.0 ms long in which the north is normal to the short direction and is covered so that a uniform distribution of natural light is provided. Reinforced concrete frames are utilized as the main supporting elements and are arranged parallel to the short direction of the hall.

The convenient slope of the roof slab lies between 20^o and 30^o with the horizontal in order to be able to cast concrete without the use of double shuttering.

The statical system can be summarized as follows:

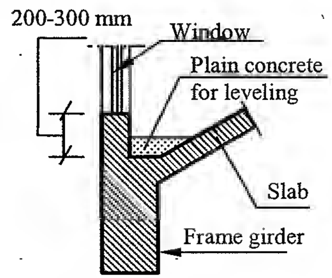
- The frames are arranged every 5.5 ms in order to obtain a reasonably economic system.
- The inclined roof consists of a system of one-way slabs that are supported on a system of inclined simply supported beams.
- The inclined simply supported beams are supported directly on the frames at one side and on the posts at the other side.
- The posts are supported directly on the frames and are connected together in the plane of the frame by the ridge beam. These posts can be assumed to resist axial forces only.

Table 1.3 Recommended dimensions for the frame system

Item	Dimension
t_g	Span/(12 $\rightarrow$ 14)
bg	300-400 mm
f	*Frame Spacing* /(2)
Frame spacing	4$\rightarrow$7 m
Roof Angle	24° - 32°
Column thickness	(0.8 $\rightarrow$ 1.0) t_g
Secondary beam thickness	Frame spacing /(8 $\rightarrow$ 10)
Post spacing	2$\rightarrow$4 m
Post dimensions	200x200 mm
Ridge beam thickness	Post spacing /(8 $\rightarrow$ 10)

One should note the difference between Sec. A-A and Sec. B-B. At Sec. B-B the inclined simple beams and the roof slabs are at the level of the frame. At Sec. B-B, these elements are at the top level of the posts. As shown in Sec. C-C, the frames are connected together in the out of plane direction at three levels, namely at the girder level, approximately at mid-height of the column (frame leg) and at the level of the foundations where semelles are provided to support the walls.

Note: As the rain water is accumulated at the lowest point of the slab, it is essential to choose the shape of cross-section of the girder of the frame in the form of a Y-shape so that there is sufficient space for the rain water and the necessary slopes for the gutter.



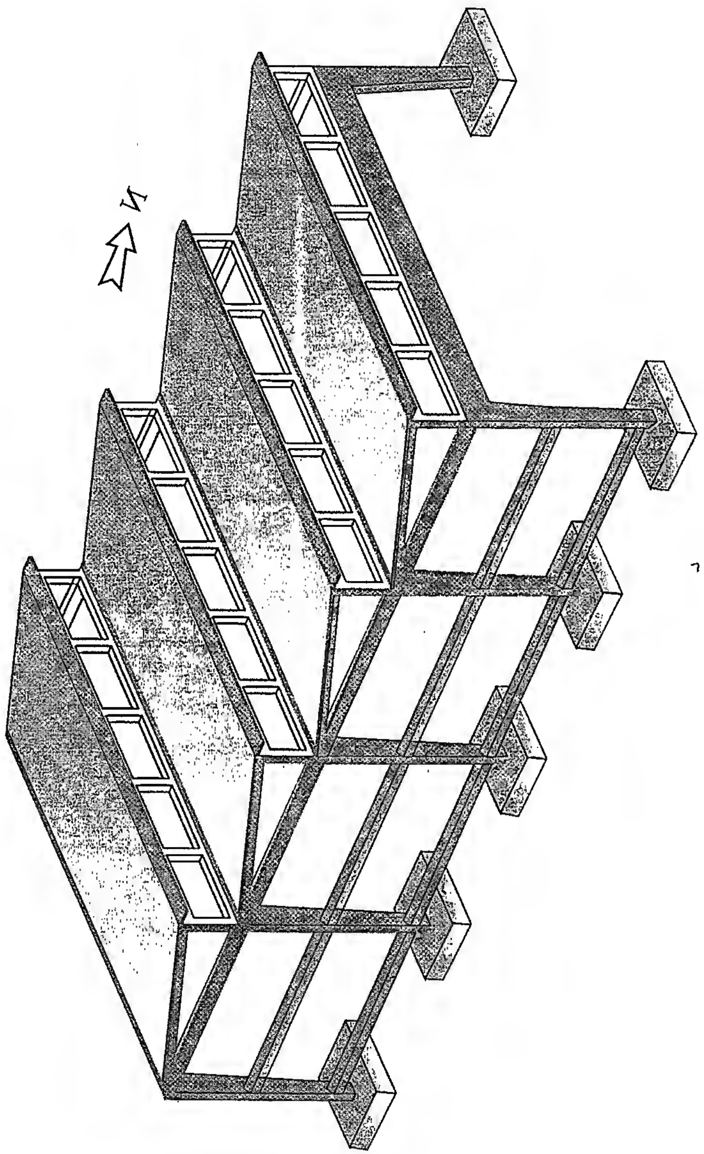
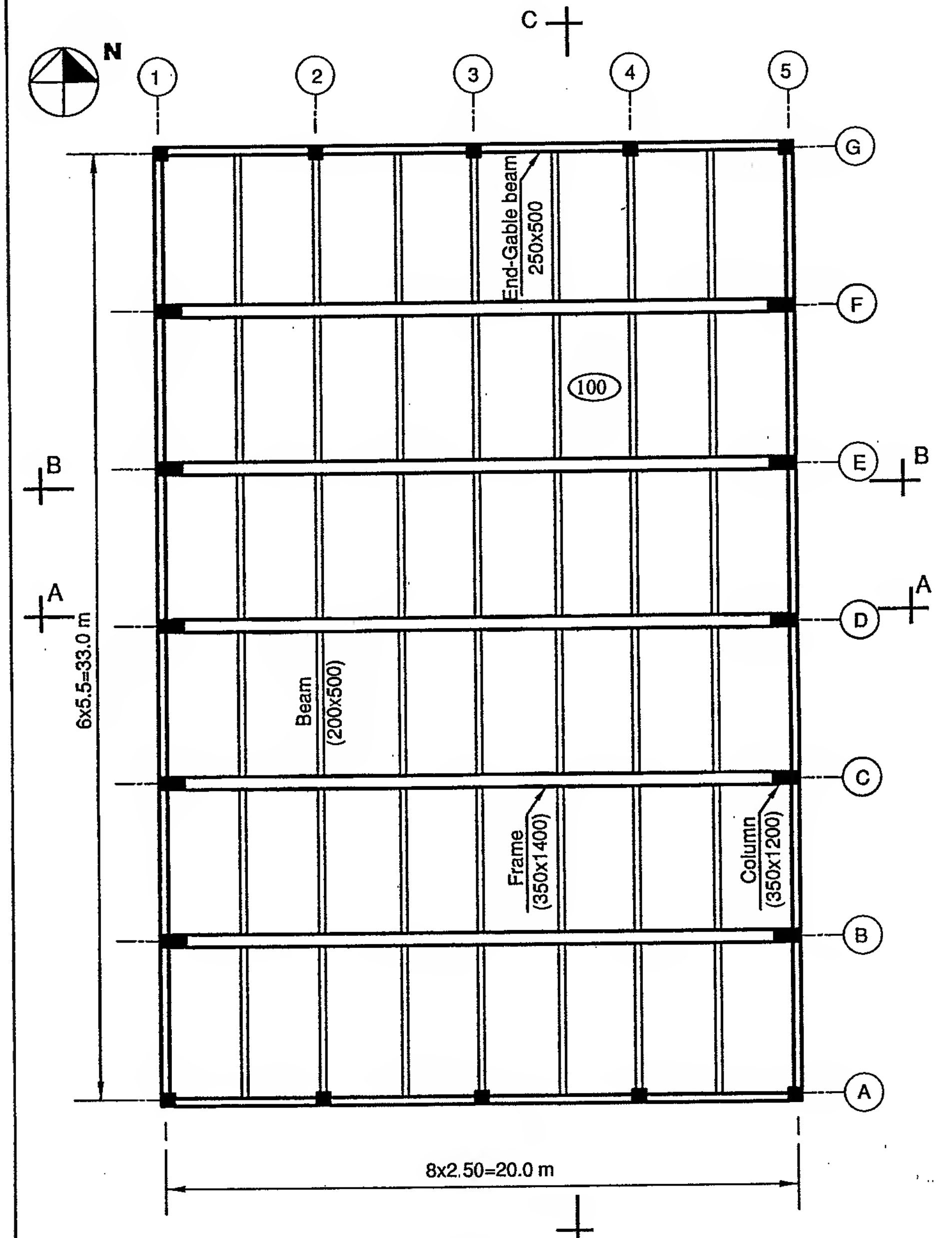
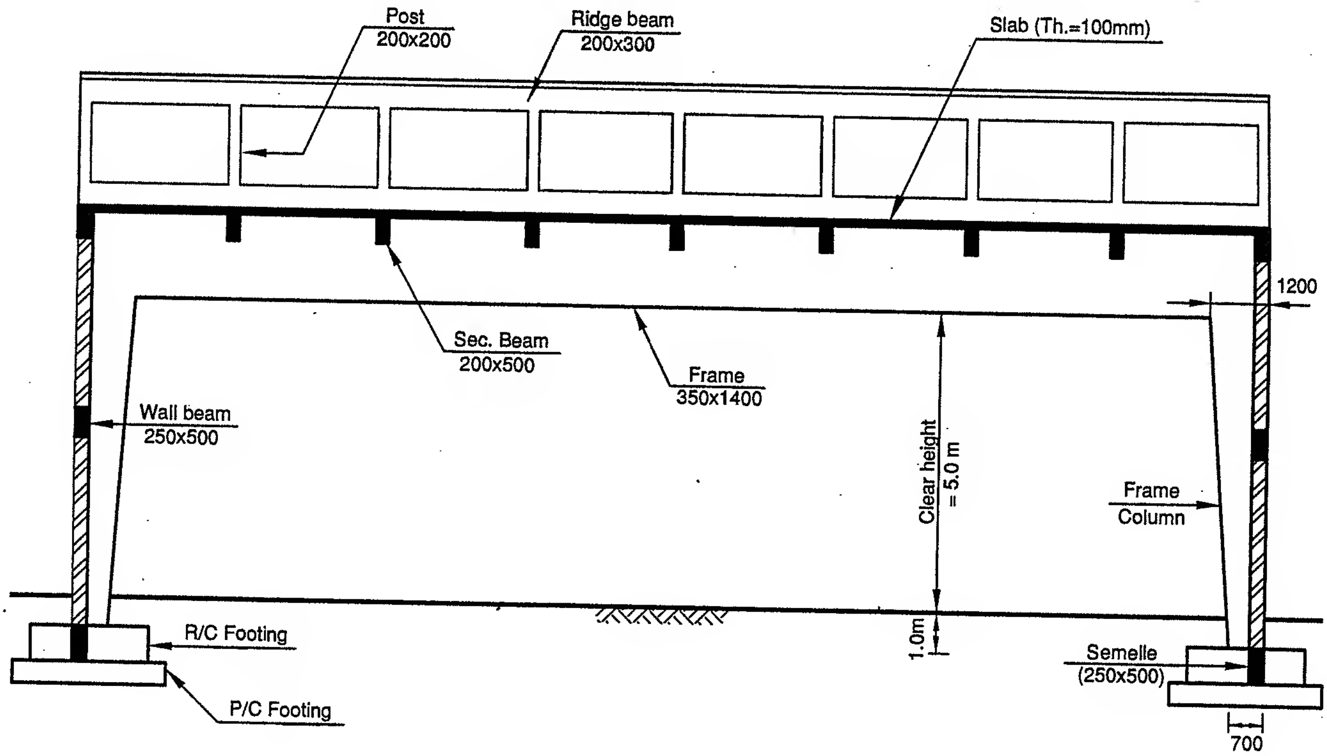


Fig. 1.6a Isometric for a frame system in which the north is perpendicular to the span



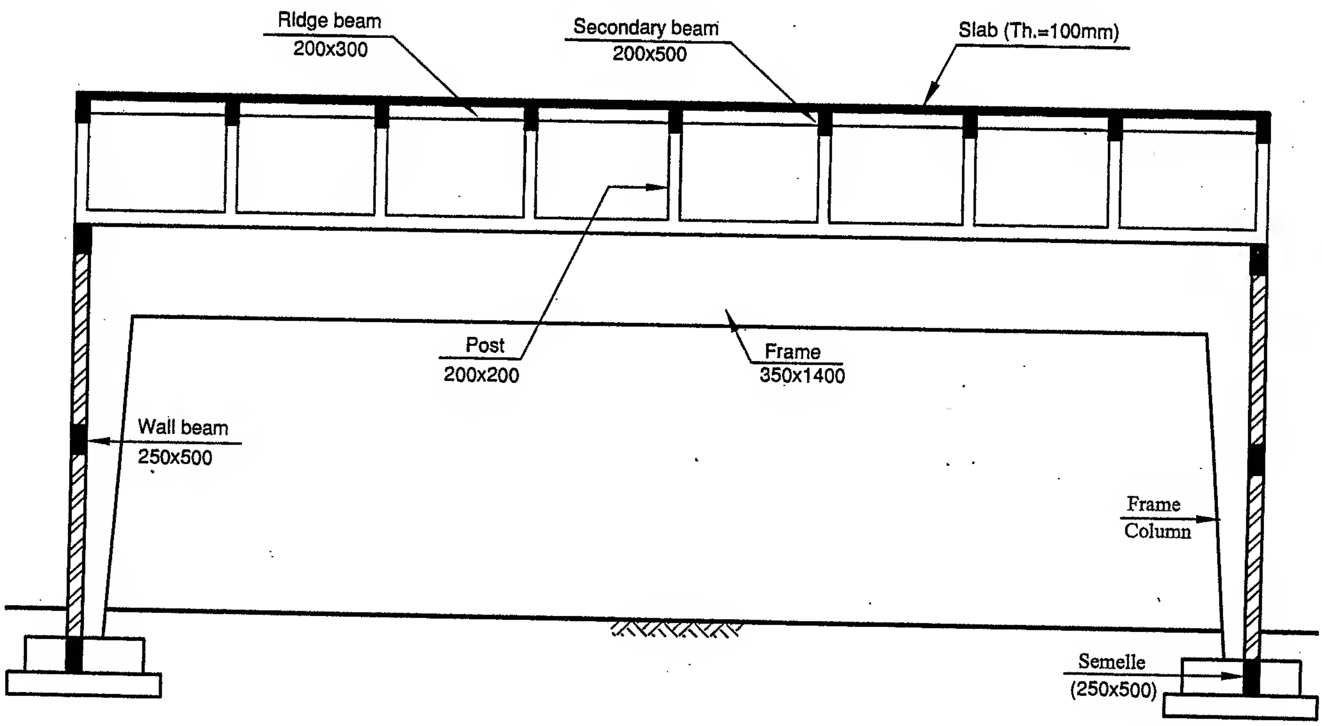
Plan

Fig. 1.6b North light saw-tooth system (Frames)
(North direction is normal to the span)



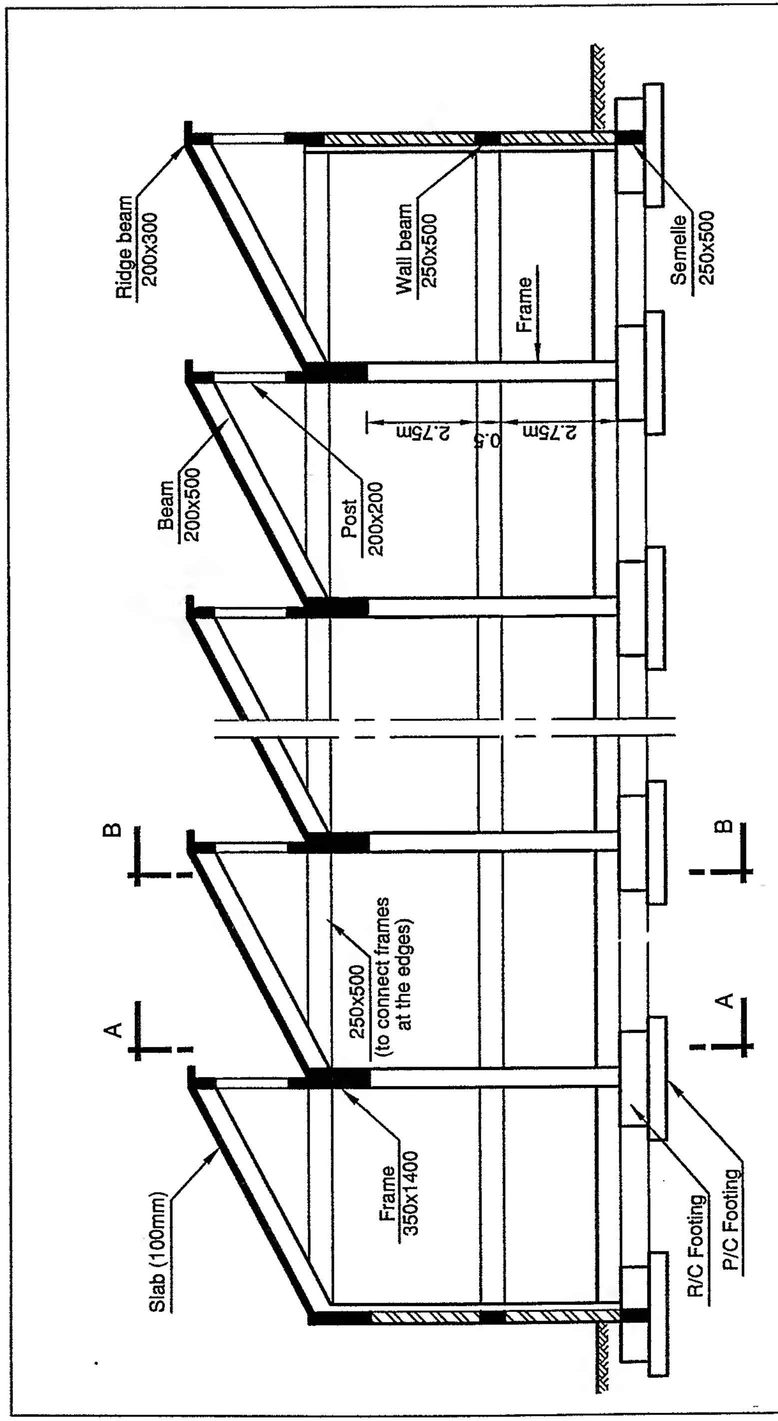
Sec. A-A

Fig. 1.6c North light saw tooth system (Frames) (cont.)



Sec. B-B

Fig. 1.6d North light saw tooth system (frames) (cont.)



Sec. C-C

Fig. 1.6e North light saw tooth system (frames) (cont.)

1.3.1.2 Arches as the Main Supporting Elements

Figure 1.7 shows the general layout of a hall 28.0 ms wide and 36.0 ms long in which the north is normal to the short direction and is covered so that a uniform distribution of natural light is provided. Reinforced concrete arches are utilized as the main supporting elements and are arranged parallel to the short direction of the hall.

The statical system can be summarized as follows:

- The arches are arranged every 6.0 ms in order to obtain a reasonably economic system.
- The inclined roof slab consists of a system of one-way slabs that are supported on a system of inclined simply supported beams.
- The inclined simply supported beams are supported on the posts acting as compression members at one side and on the posts acting as tension members at the other side.
- The posts are supported directly on the arches and are connected in the plane of the arch by the ridge beam.

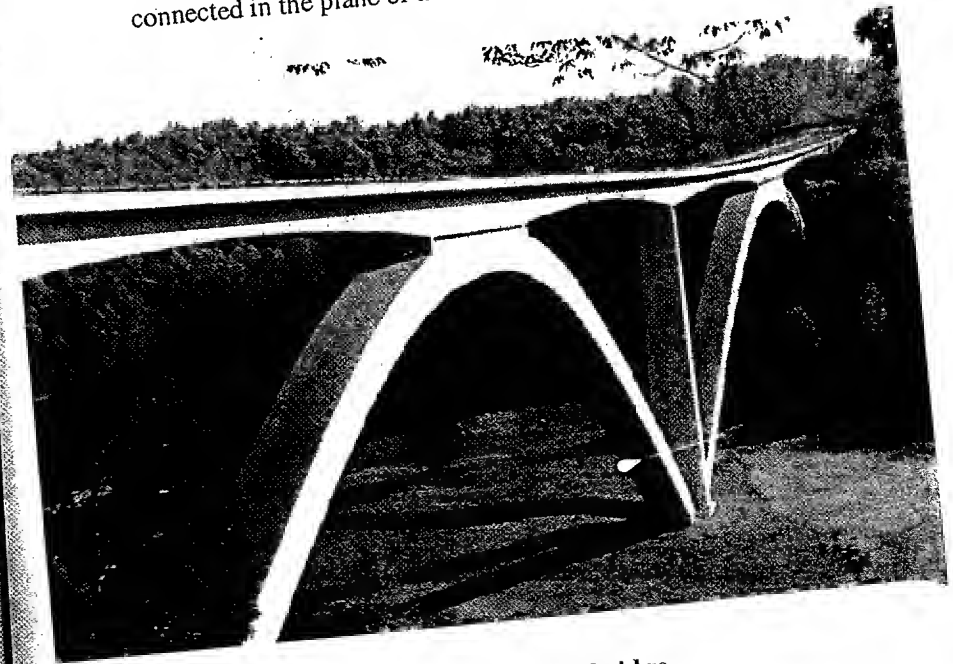


Photo 1.4 Arched bridge

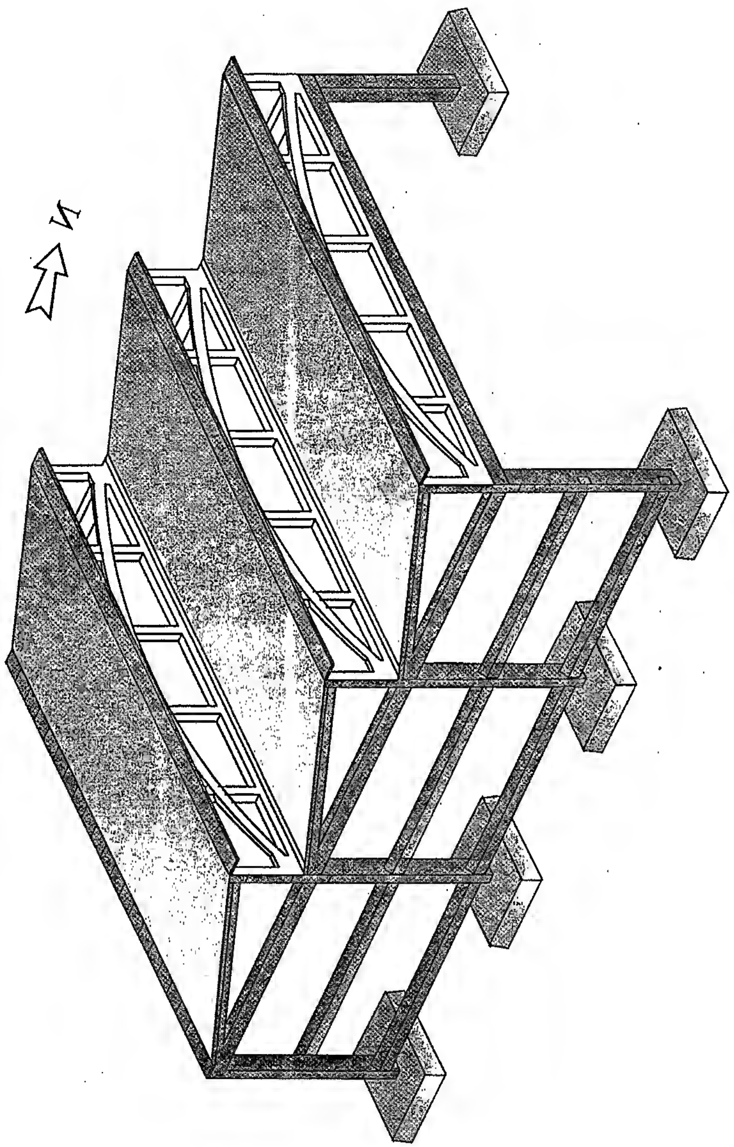
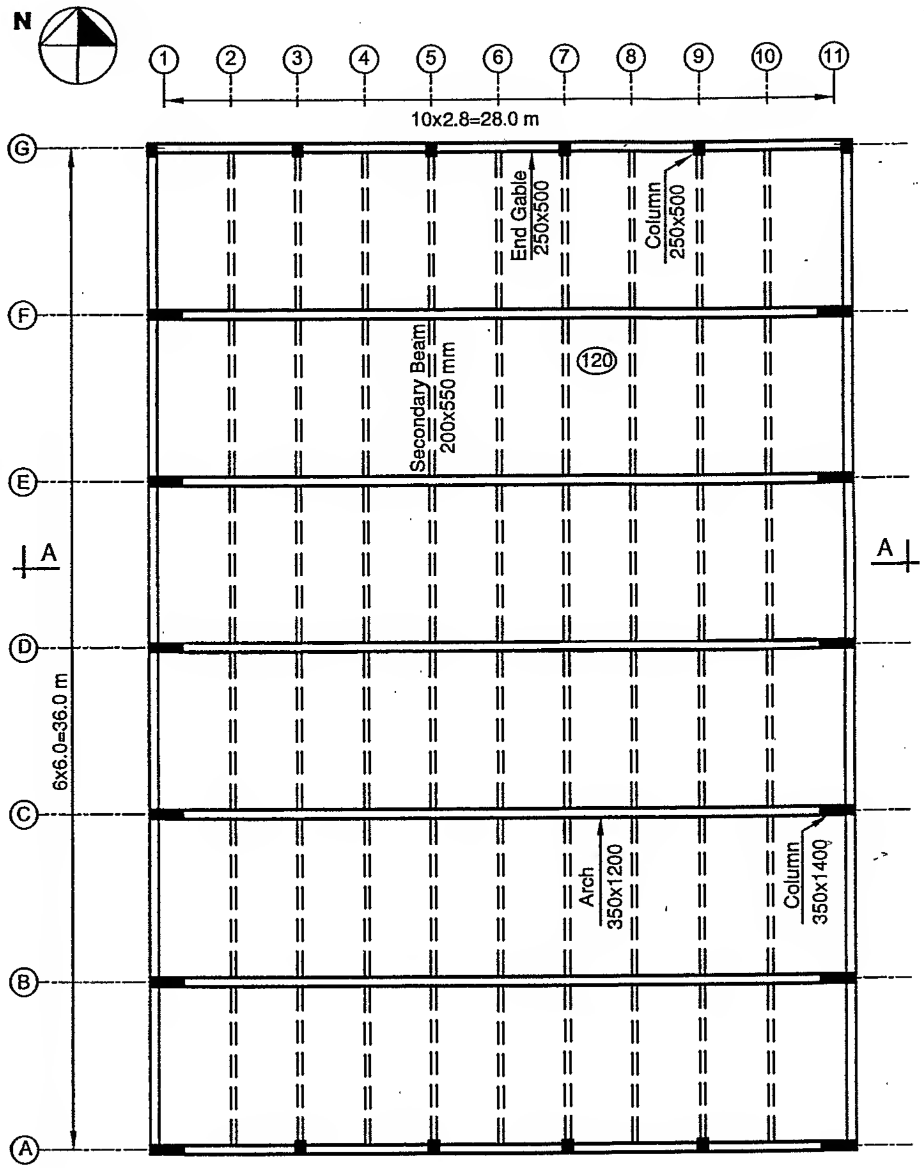
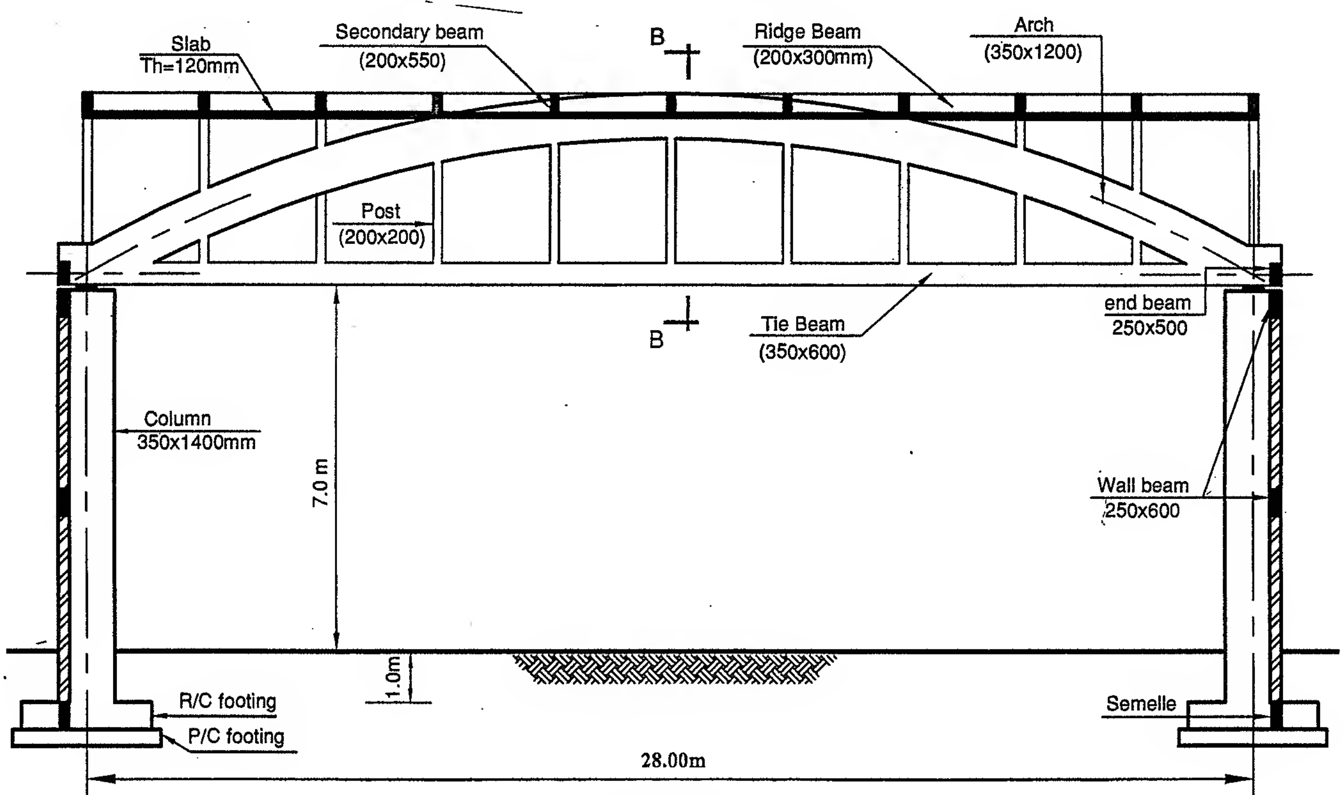


Fig. 1.7a Isometric for an arch system with north-light saw-tooth



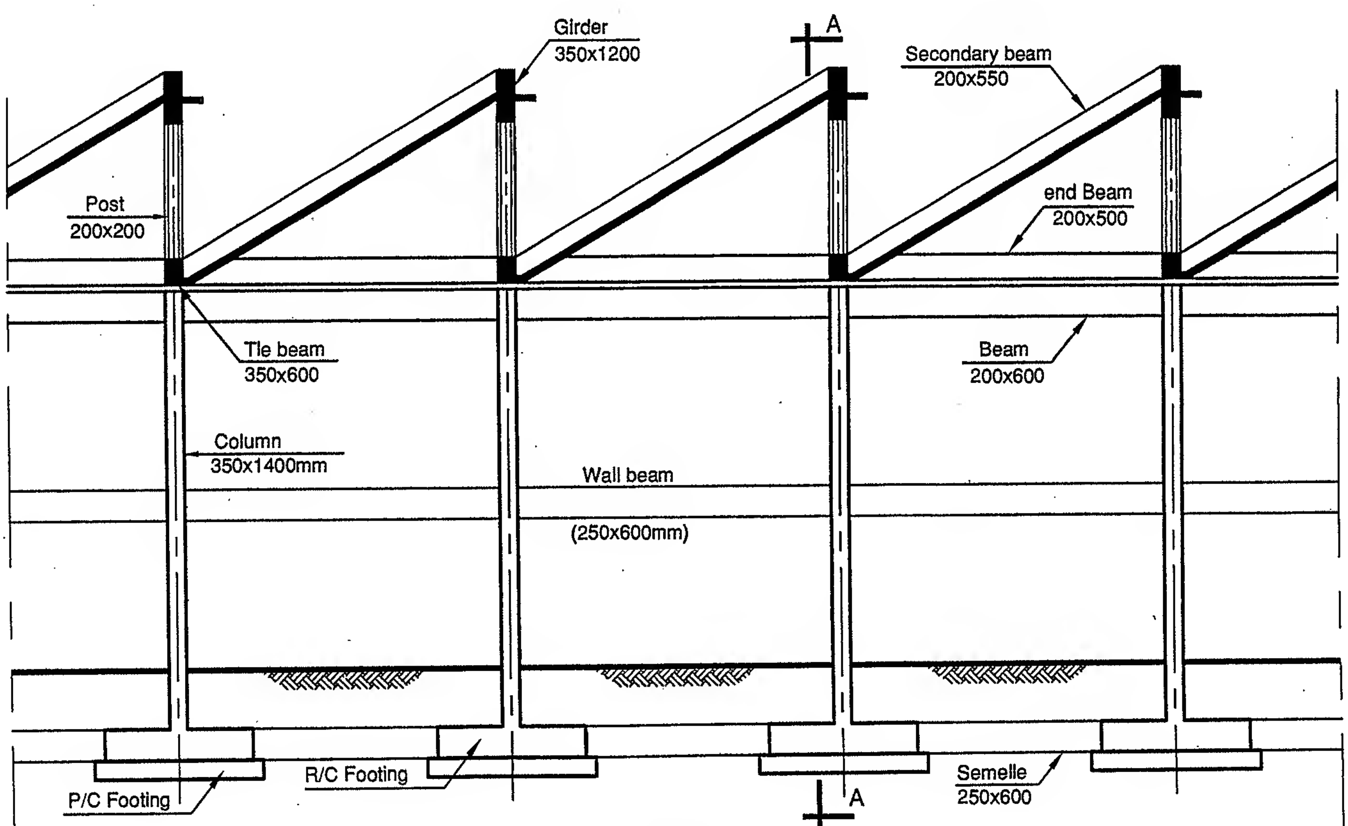
Plan

Fig. 1.7b North-light saw-tooth system (Arches)



Sec. A-A

Fig. 1.7c North-light saw-tooth system (Arches) (cont.)



Sec. B-B

Fig. 1.7d North-light saw-tooth system (Arches) (cont.)

1.3.2 North Direction is Parallel to the Span

The concept of having the main supporting elements arranged in the short direction of the hall is still valid in case the north direction is parallel to the span. Also, the windows have to be arranged to face the north direction. If the short direction is not more than about 20.0 ms, it is recommended to use frames. If, on the other hand, the short direction is more than 20.0 ms, it will be more economic to use trusses.

1.3.2.1 Frames as the Main Supporting Elements

Figure 1.8 shows the general layout of a hall 22.0 ms wide and 27.0 ms long in which the north is parallel to the short direction and is covered so that a uniform distribution of natural light is provided. Reinforced concrete frames are utilized as the main supporting elements and are arranged parallel to the short direction of the hall.

The statical system can be summarized as follows:

- The frames are arranged every 5.4 ms in order to obtain a reasonably economic system.
- A system of horizontal continuous beams (called the Y-beams) is supported on the frames.
- The inclined roof consists of a system of one-way slabs that are supported on a system of inclined simply supported beams.
- The inclined simply supported beams are supported on the posts at one side and directly on the Y-beam at the other side.
- The posts are supported directly on the Y-beams and are connected in the plane of the Y-beams by the ridge beam. These posts can be assumed to resist axial forces only.

Table 1.4 gives guide lines for choosing the dimensions of the system.

Table 1.4 Recommended dimensions for the frame system

Item	Suggested dimensions
t_g	Span/(12→14)
Frame spacing	4→7 m
Y-beam spacing	4→6 m
Roof height (f)	Y-beam spacing/2
Roof Angle	24° - 32°
Column thickness @top	(0.80 → 1.0) t_g
Column thickness @bottom	(0.4 → 0.60) t_g
Secondary beam thickness	Y-beam spacing /(8→10)
Y-beam thickness	Frame spacing/(6→8)
Post spacing	2→4 m
Post dimensions	200x200 mm
Ridge beam thickness	Post spacing /(8→10)

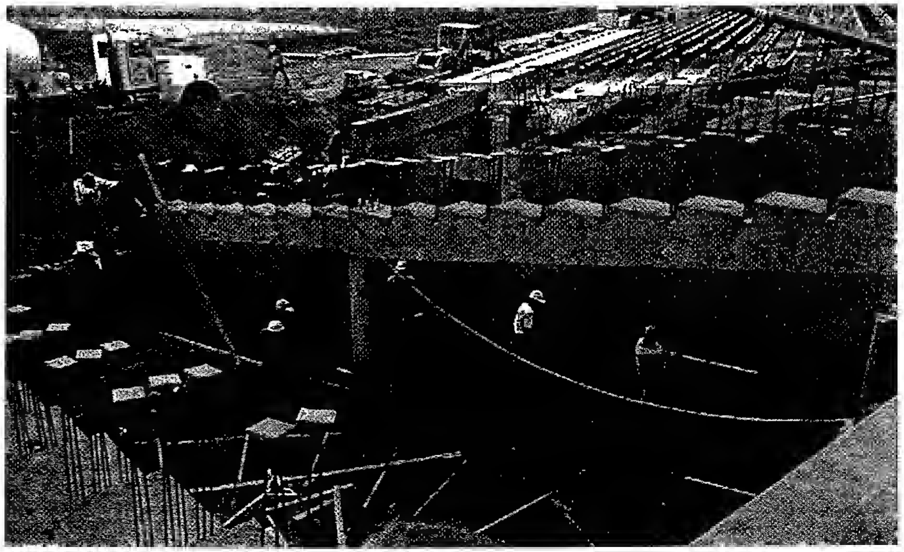


Photo 1.5 Stadium during construction

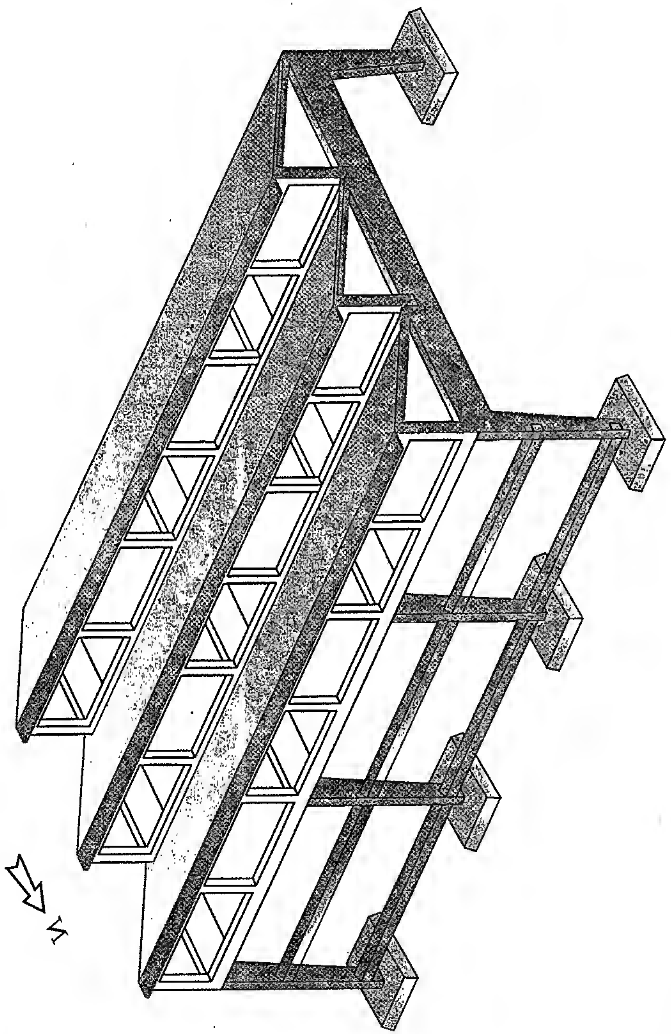
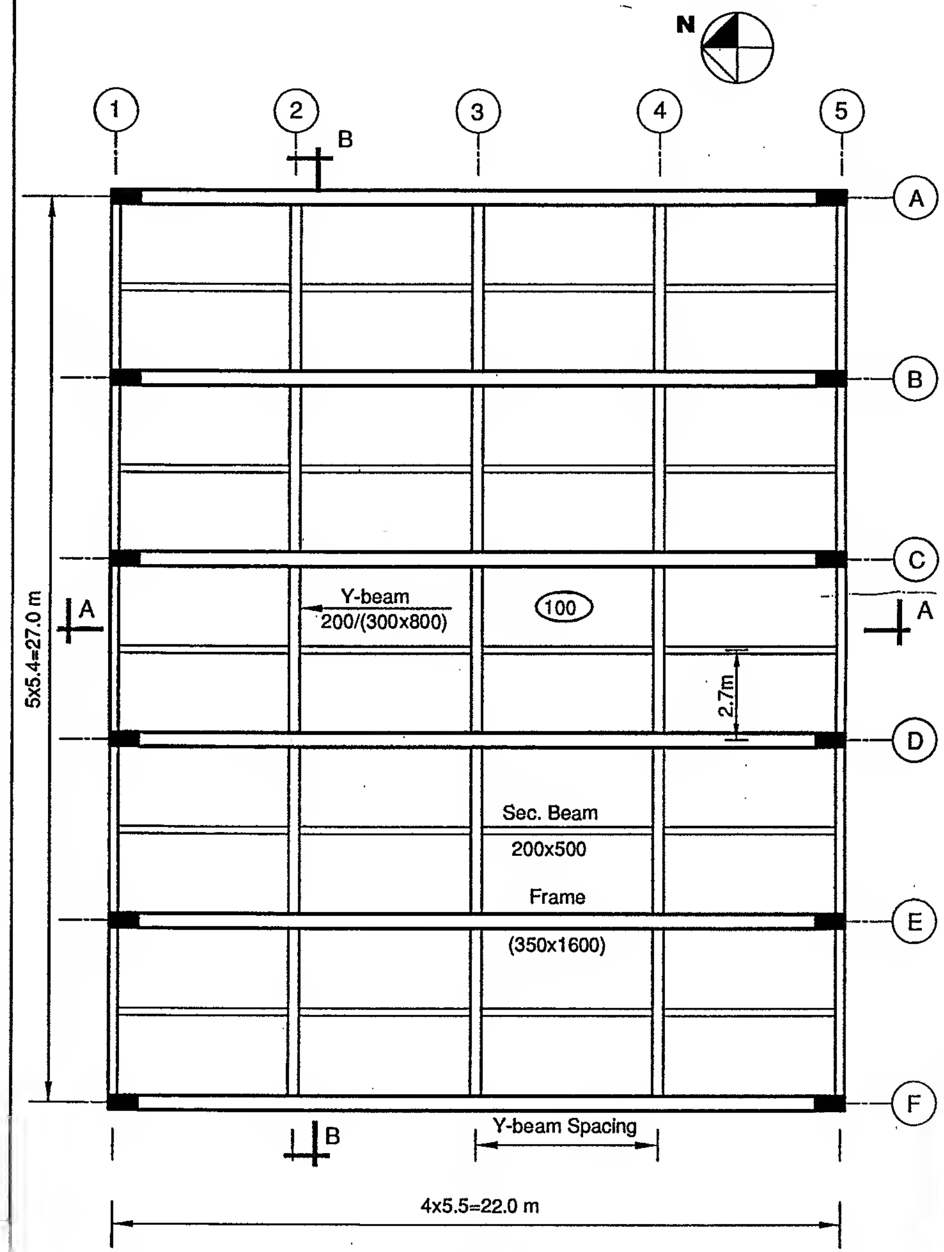


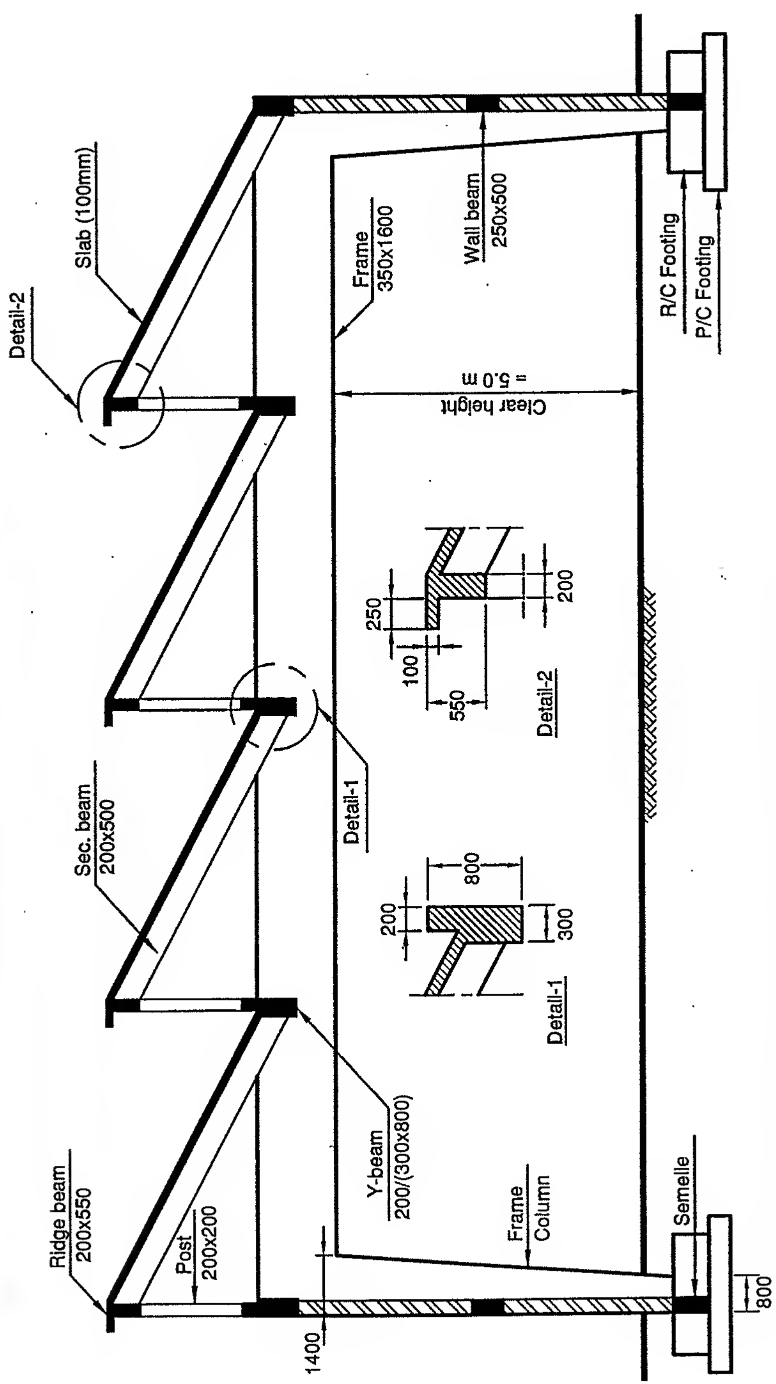
Fig. 1.8a Isometric for a frame system in which the north is parallel to the span



Plan

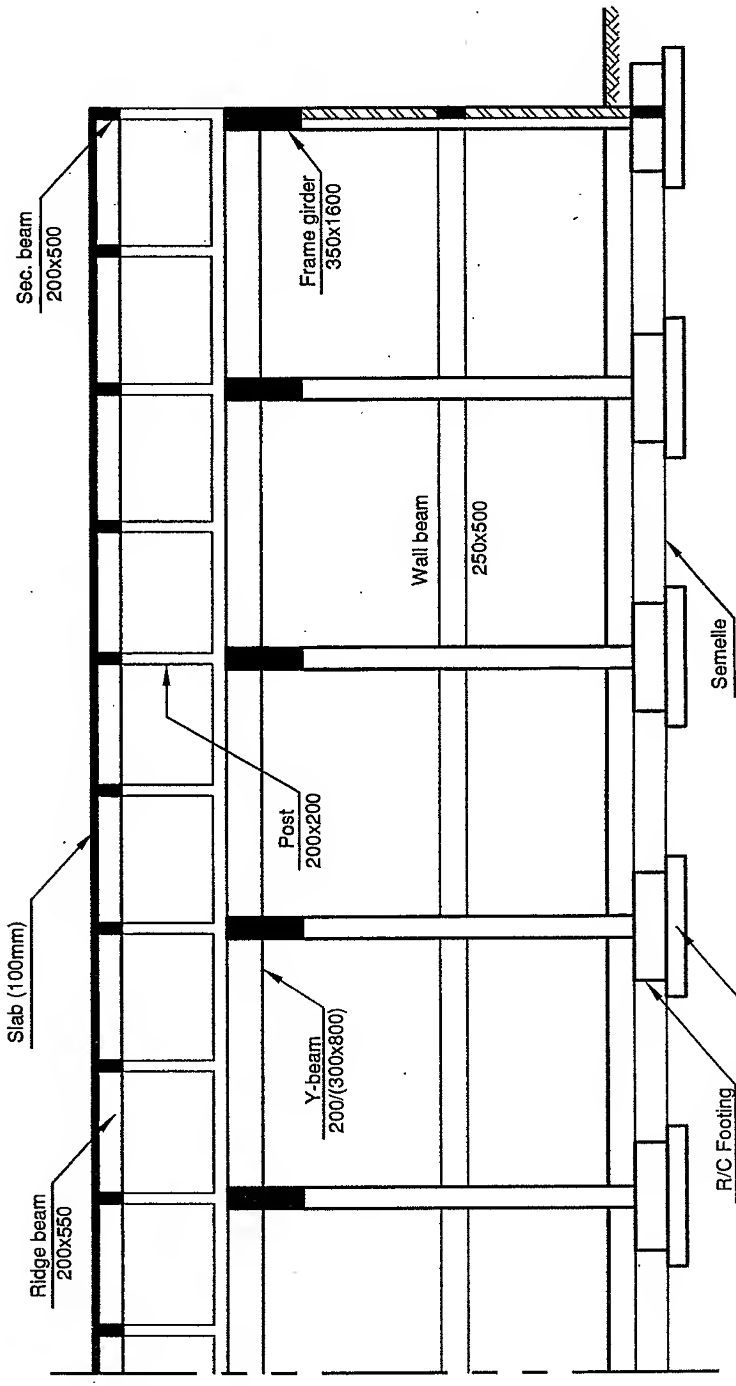
Fig. 1.8b North-light saw-tooth system (Frames)

(North direction is parallel to the span)



Section A-A

Fig. 1.8c North-light saw-tooth system (Frames) (cont.)



Section B-B

Fig. 1.8d North-light saw-tooth system (Frames) (cont.)

1.3.2.2 Trusses as the Main Supporting Elements

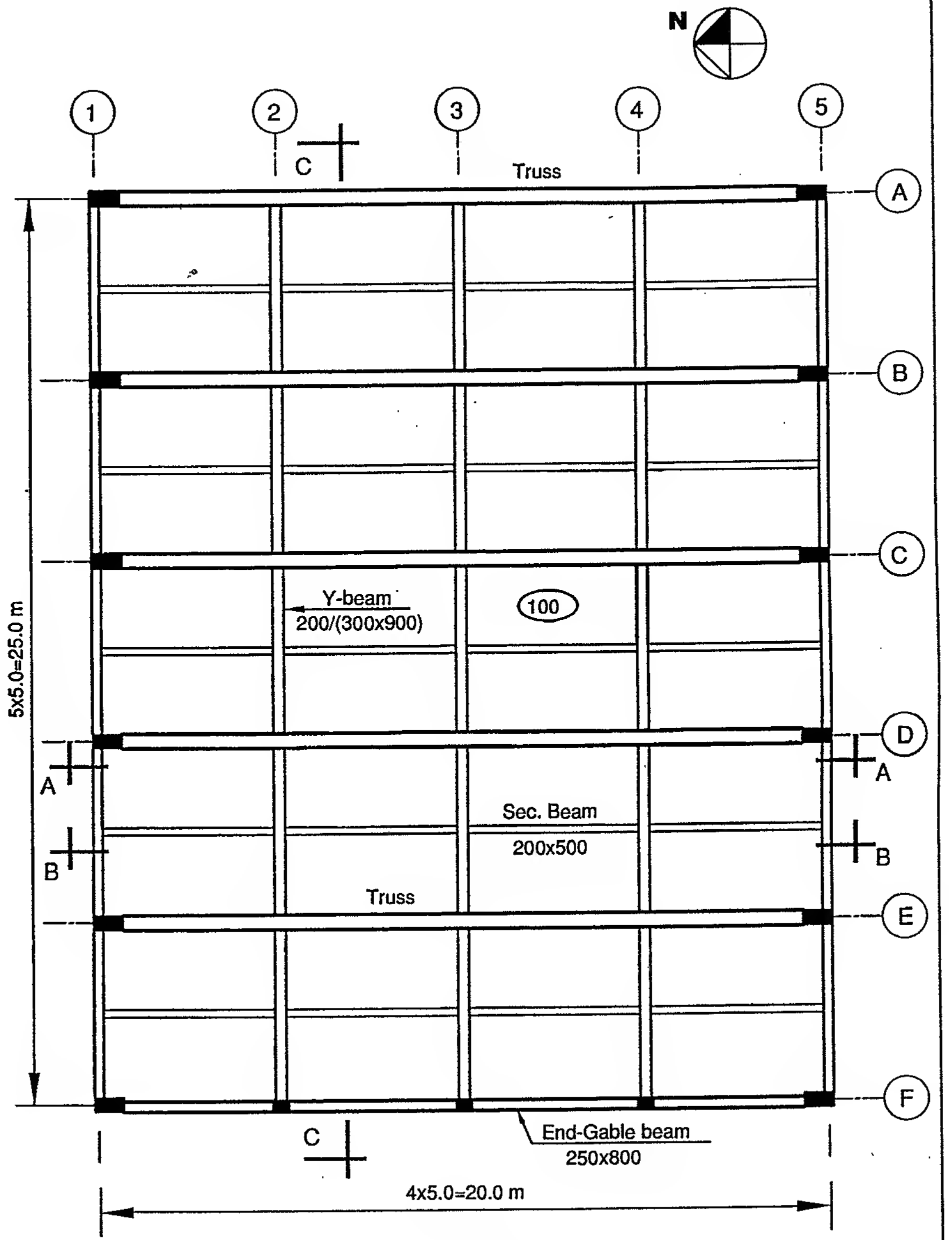
Trusses in reinforced concrete are seldom used and their shape is generally chosen similar to those constructed in steel.

One of the disadvantages of reinforced concrete trusses is that formwork of concrete and the detailing of reinforcement are complicated. However, in special cases of saw-tooth roofs in which the north is parallel to the span of the hall, the truss may give a convenient solution.

Figure 1.9 shows the general layout of a hall 20.0 ms wide and 25.0 ms long in which the north is parallel to the short direction and is covered so that a uniform distribution of natural light is provided. Reinforced concrete trusses are utilized as the main supporting elements and are arranged parallel to the short direction of the hall.

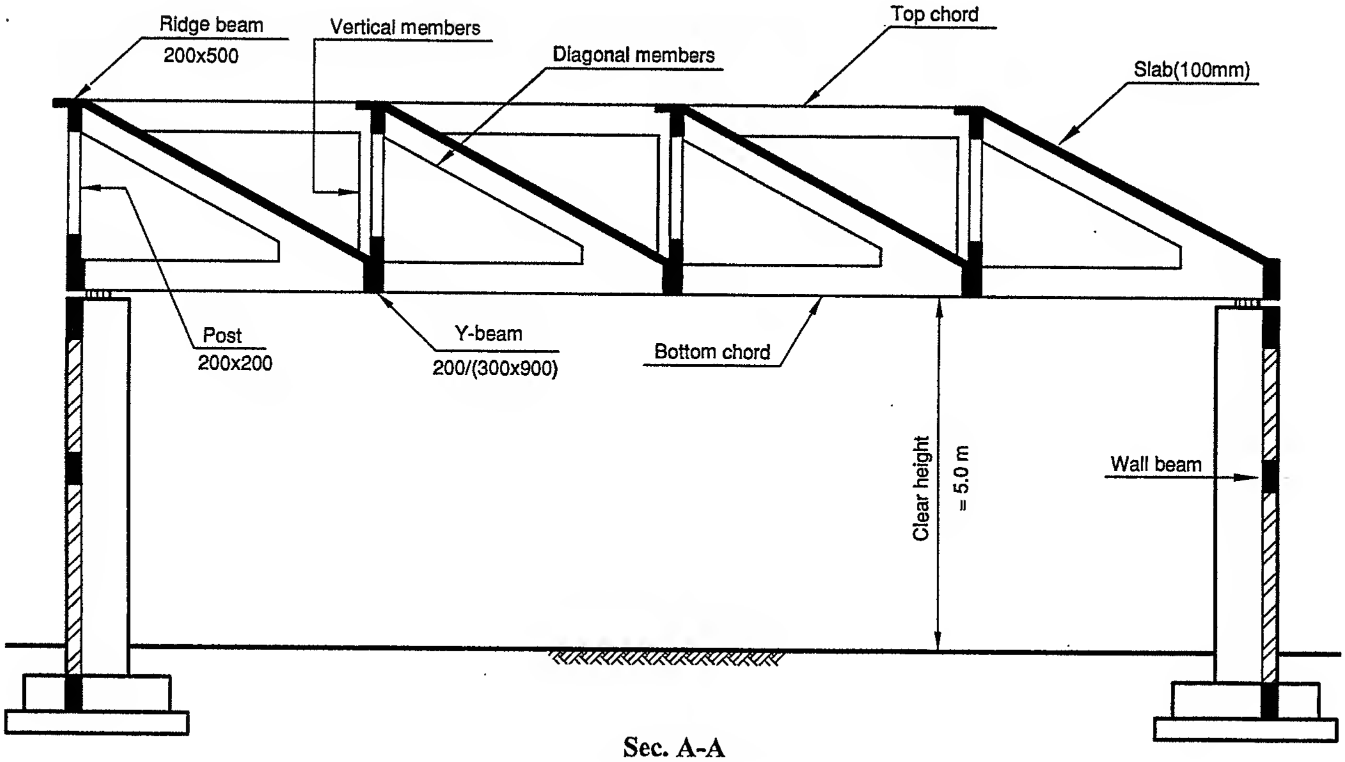
The statical system can be summarized as follows:

- The trusses are arranged every 5.0 ms in order to obtain a reasonably economic system.
- A system of horizontal continuous beams (called the Y-beams) are supported on the trusses every 5.0 ms.
- The inclined roof consists of a system of one-way slabs. At the location of the trusses, the slabs are supported on the diagonal members. Between trusses, inclined beams are provided to support the slabs. The inclined beams are simply supported at posts from one side and directly at the Y-beam from the other side.
- The posts are supported directly on the Y-beams and are connected in the plane of the Y-beams by the ridge beam. These posts can be assumed to resist axial forces only.



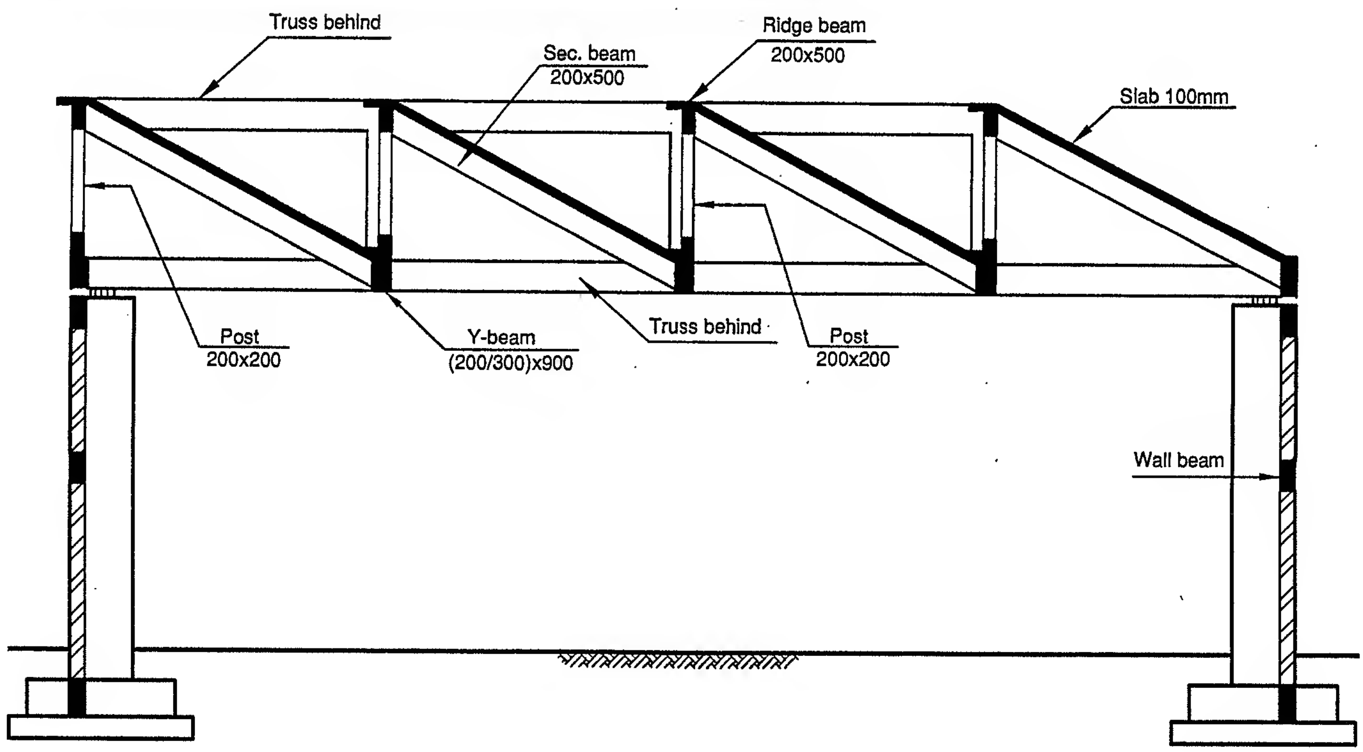
Plan

Fig. 1.10a North-light saw-tooth system (Truss)
(Noth direction is parallel to the span)



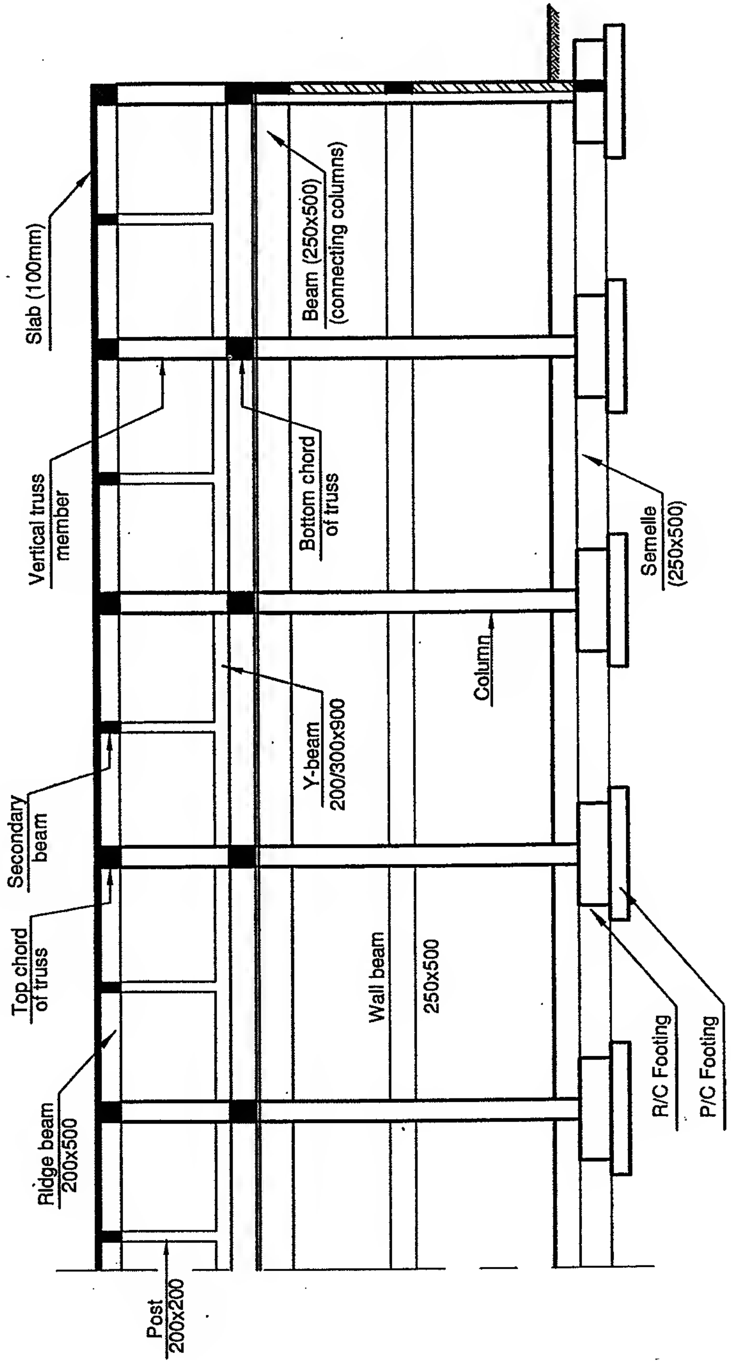
Sec. A-A

Fig. 1.10b North-light saw-tooth system (Truss) (cont.)



Sec. B-B

Fig. 1.10c North-light saw-tooth system (Truss) (cont.)



Sec. C-C

Fig. 1.10d North-light saw-tooth system (truss) (cont.)

Approximate Analysis of R/C Trusses

Unlike steel trusses, reinforced concrete trusses are subjected to direct loading from the surrounding slabs. Moreover due to the rigidity of the members connecting bending moments are induced. Truss members are mainly subjected to normal force, and therefore bent bars are not used in trusses and the reinforcement is distributed symmetrically

The internal forces in the members of a truss are:

1- The axial forces due to the concentrated loads at the joints.

2- The bending moments and shearing forces due to the direct loads on the members of the truss on which the slabs are supported, and

3- The bending moments and shearing forces due to the fact that in reinforced concrete trusses, the joints are partially rigid.

The internal forces can be obtained using the computer programs. It should be noted that the effect of the partial rigidity of the joints has to be evaluated.

The approximate dimensions of the truss can be obtained from Fig. 1.10.

$$t_1 = t_2 = t_3 = \frac{span}{36}$$

$$t_4 = (0.8 - 0.9)\, t_1$$

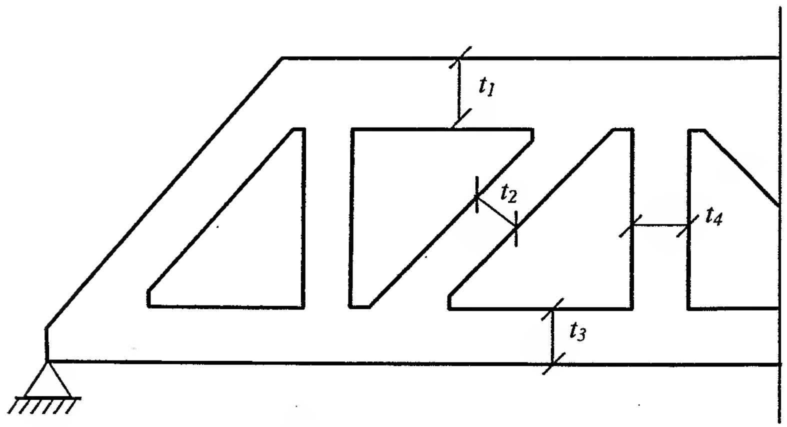


Fig. 1.10 Truss approximate dimensions

1.4 Vierendeel Girders

Vierendeel girders are similar to trusses except that they do not have diagonals as shown in Fig. 1.11. Moreover the connecting joints are rigid such that they develop moment.

Vierendeel girders are often used in transfer floors of high-rise buildings to support planted columns giving a wide space in the floor below as shown in Fig. 1.12. They also can be used in saw-tooth roofs when indirect sunlight is required as shown in Fig. 1.13a and Fig 1.13b.

A Vierendeel girder consists of a top chord, a bottom chord and vertical members. The system is externally statically determinate, while it is internally indeterminate. Internally, it is *3n* times statically indeterminate, where n is the number of panels.

The exact analysis of a Vierendeel girder is quite complicated. In the past, approximate solutions were used to calculate the force in the different members. Nowadays, computer programs are used to compute the straining actions. In such a case, the members are modeled as 2D frame elements, while the joints are modeled as rigid joints that permit moment transfer among members.

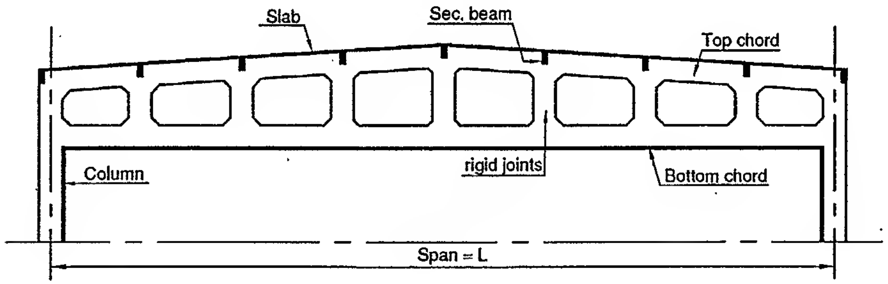
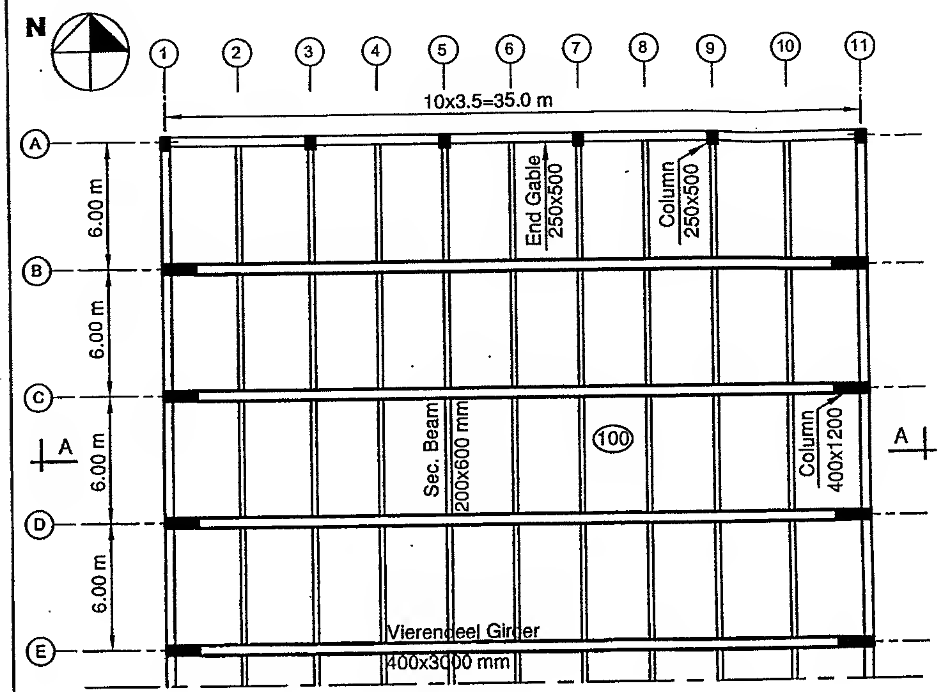
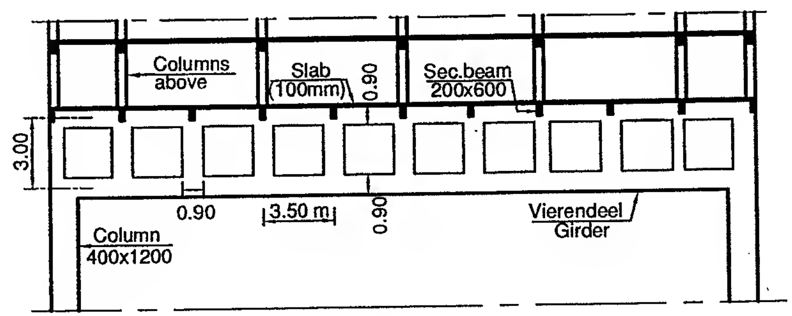


Fig. 1.11 Vierendeel Girders

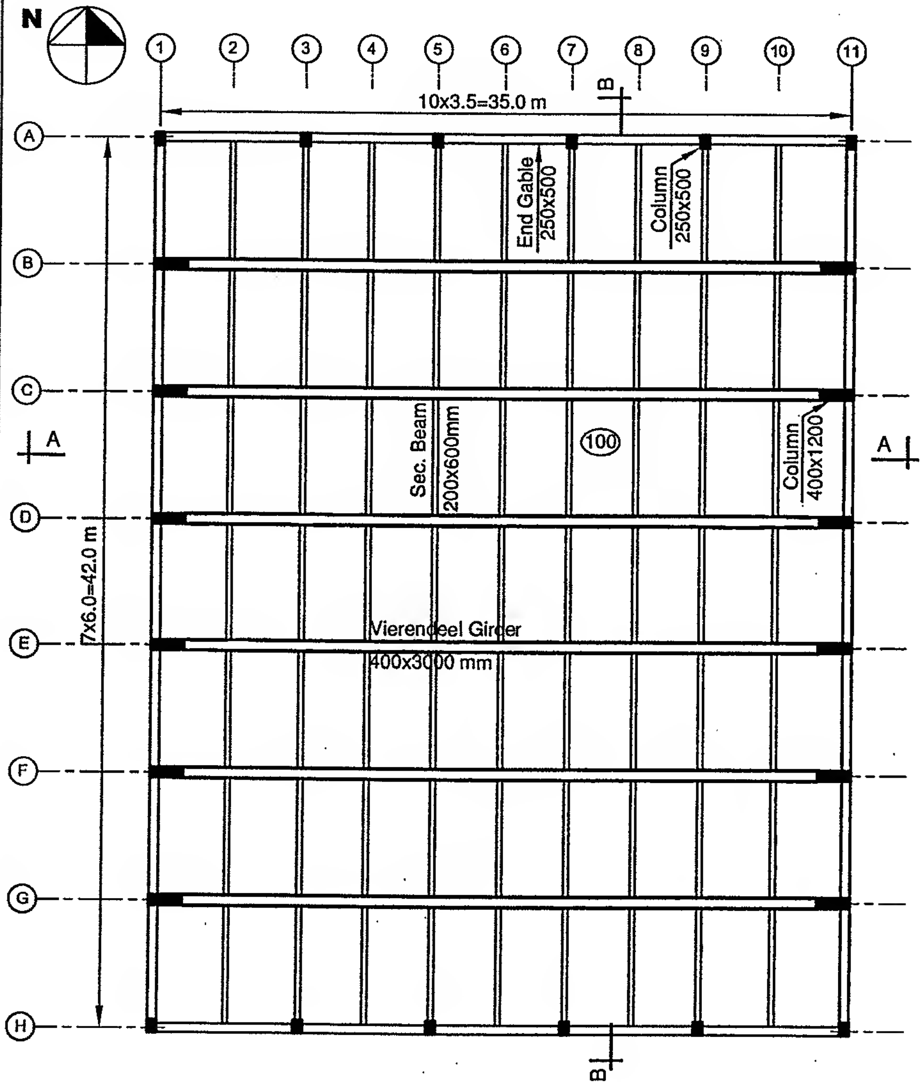


Plan



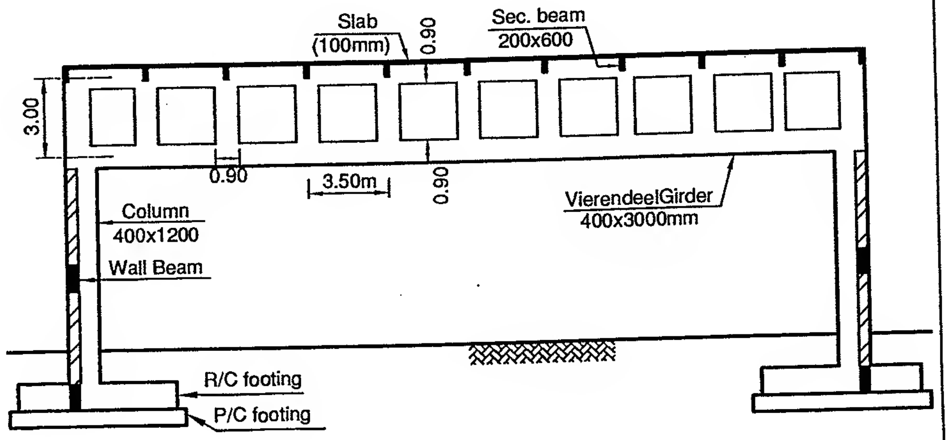
Section A-A

Fig.1.12 Vierendeel girder

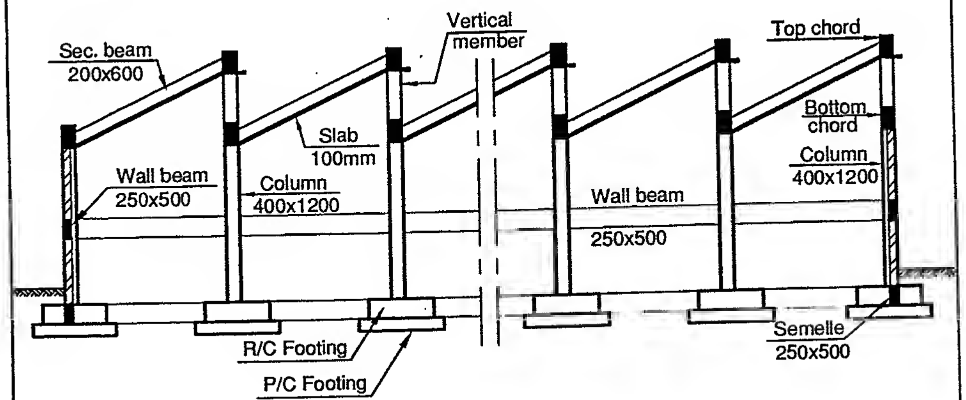


Plan

Fig.1.13a Vierendeel girder



Section A-A



Section B-B

Fig.1.13b Vierendeel girder

Summary

Figure 1.14 gives guidelines for choosing the appropriate structural system when indirect light is needed according to the span and the direction of the north.

1.5 Expansion and Settlement Joints

In the construction of reinforced concrete structures, two types of joints may be considered namely, expansion joints and settlement joints

1.5.1 Expansion Joints

Expansion joints are provided to reduce the effect of temperature. Thermal effects induce additional straining actions that lead to additional reinforcement in the structural members. For example, continuous beams running over several spans are affected by the stresses induced due to temperature. The ECP 203 specifies the use of these joints when the dimensions of the building exceed 30-35 m in hot regions and 40-45 in warm regions.

Expansion joints are achieved by a complete vertical separation in the super-structure at the location of the joint. This is usually achieved by placing foam sheets with a thickness of 20 mm. No need, however, for separating the foundations at the location of the expansion joint as shown in Fig. 1.15. Expansion joints are made in such a way to prevent water and moisture from penetrating the building by providing insulation material. The insulation material is provided at roof level as shown in Fig. 1.16a as well as at the different floors levels as shown in Fig. 1.16b.

It should be mentioned that the requirements of the ECP 203 can be waived if the designer carry out an analysis that takes into account the temperature effects.

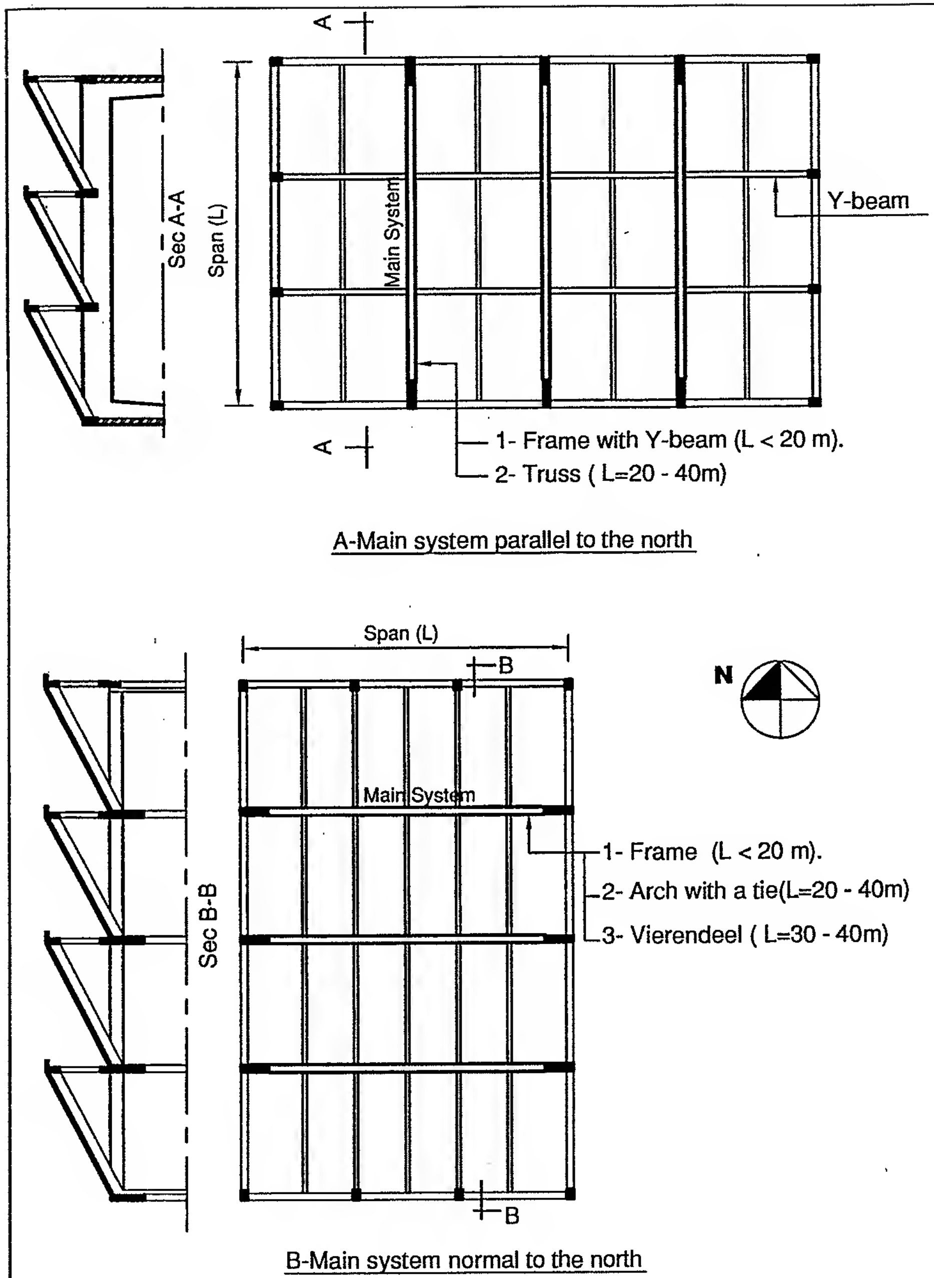


Fig. 1.14 Choosing the appropriate structural system according to the span and the direction of the north

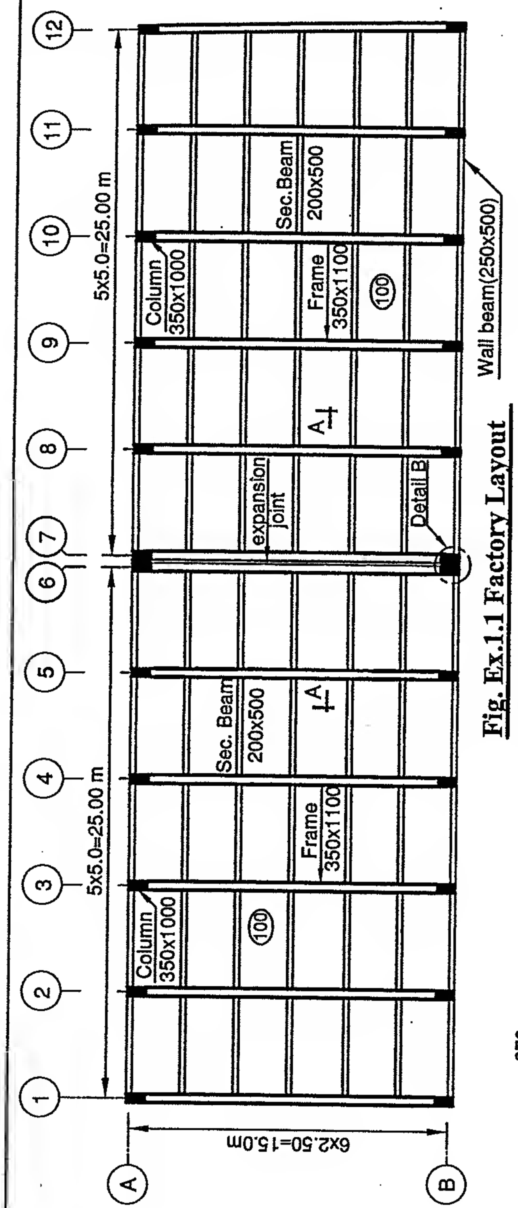
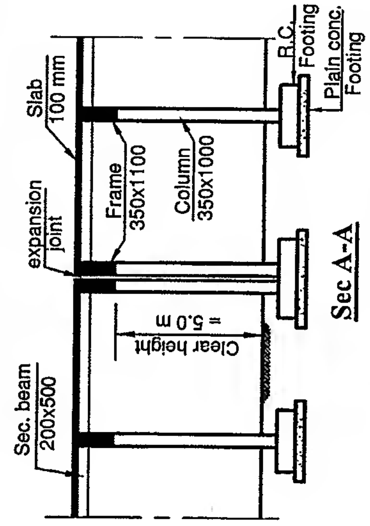
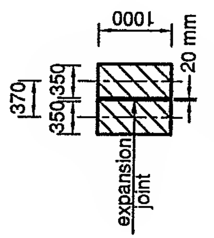


Fig. Ex.1.1 Factory Layout

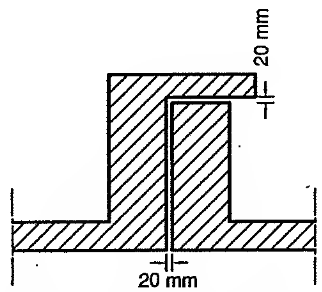


Sec A-A

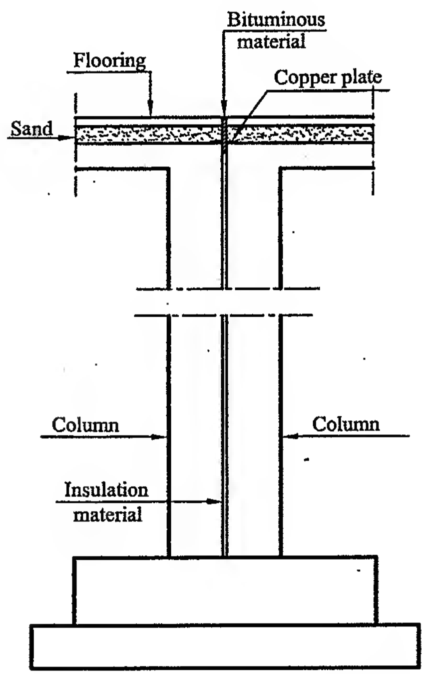


Detail B

Fig. 1.15 Expansion Joints in buildings



a-Detail of Expansion Joints at the roof



b-Two columns and the same footing

Fig. 1.16 Expansion joints details

1.5.2 Settlement Joints

This type of joints is related to height rather than length. If the loads of two adjacent columns differ significantly, the differential settlement becomes large and could affect the foundation. This is can occur in case of low rise building adjacent to high rise building as shown in Fig. 1.14. The footings in this case should be completely separated (20 mm apart) to allow the settlement of each column to take place independently.

Photo 1.6 Settlement joints is provided between low and high-rise buildings

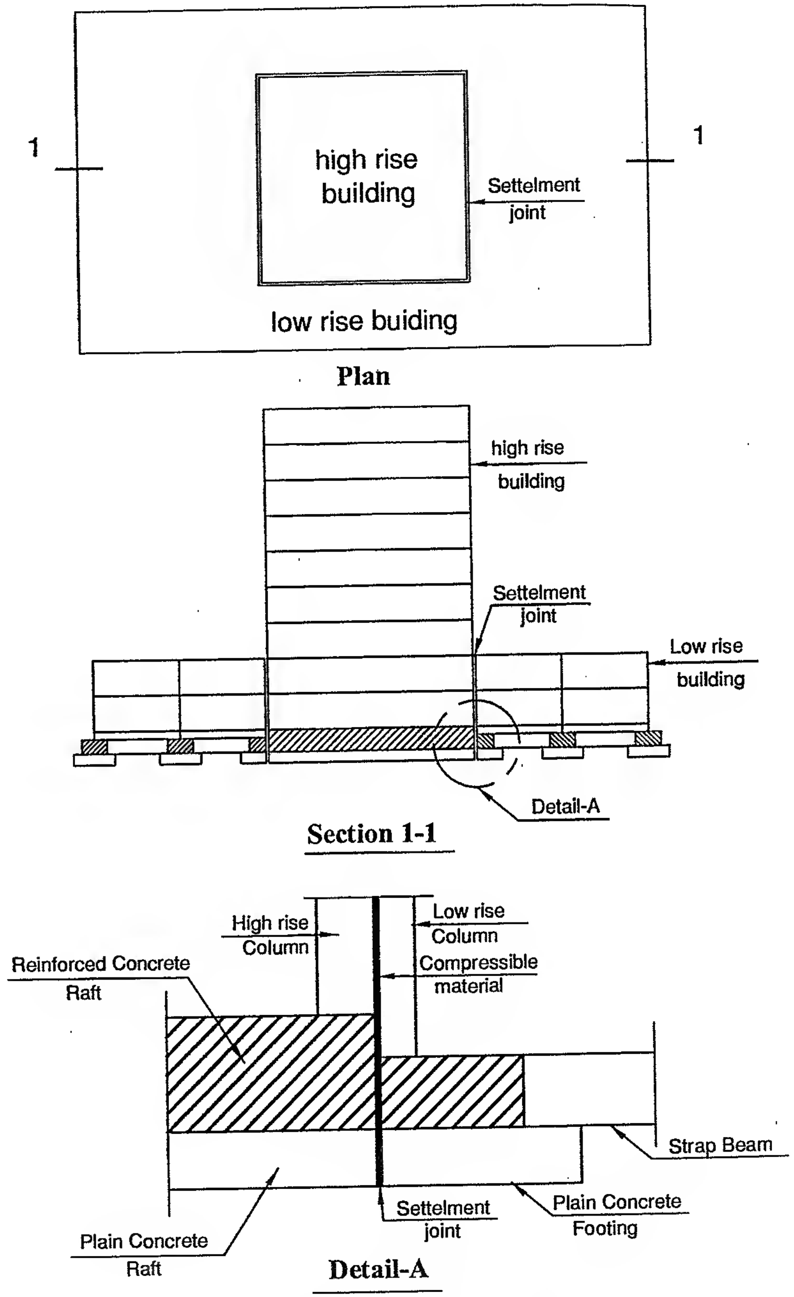


Fig. 1.17 Settelment joint details.

1.6 End gables

End gable consists of a group of columns supporting continuous beams instead of the typical frames used as the main system of the hall as shown in Fig. 1.15a. The spacing between the columns is chosen in such a way that the area of the enclosed walls should not be more than 25-30 m^2. This is to facilitate the construction of the brick wall, to reduce its buckling length, and to increase its capacity for resisting wind loads.

If a future extension of the hall is expected, another system is provided at the end as shown in Fig. 1.15b. In such a case, a frame is used at the end of the hall. End gable consists of a group of columns that are connected to the frame by dowels to reduce their buckling length and to allow for possible future demolishing the wall. The length of the dowels should not be less than the development length of the steel bars (L_d) as shown in Fig. 1.15c. Moreover, compressible material should be provided to allow for the deflection of the secondary beam

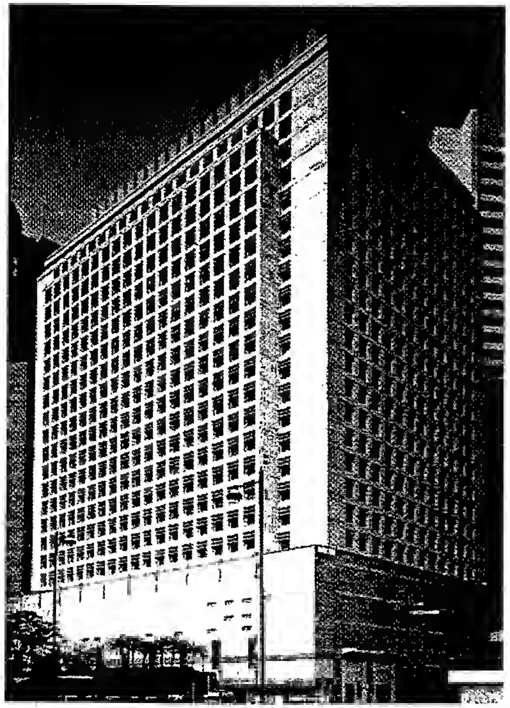


Photo 1.7 A multistory reinforced concrete building

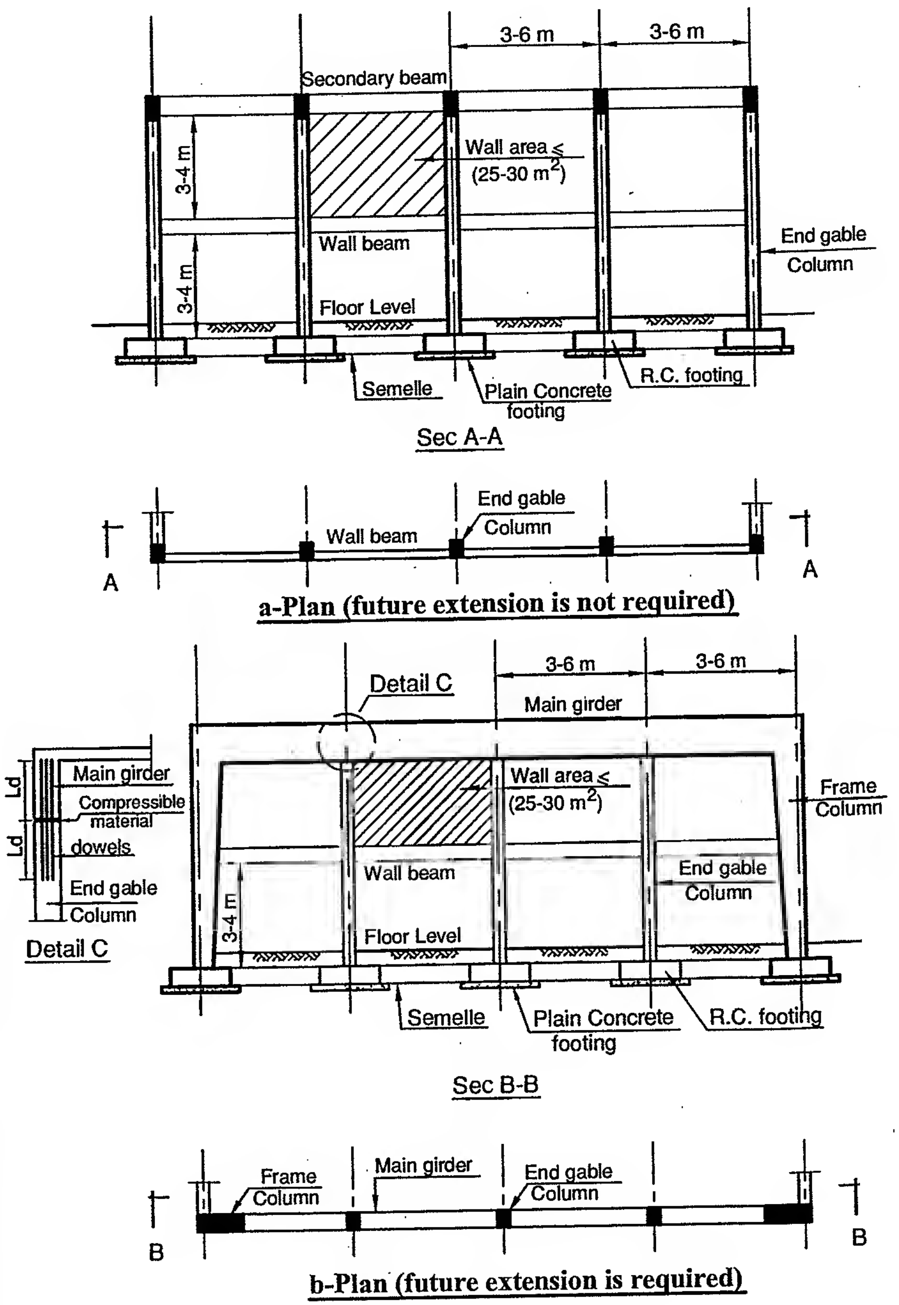
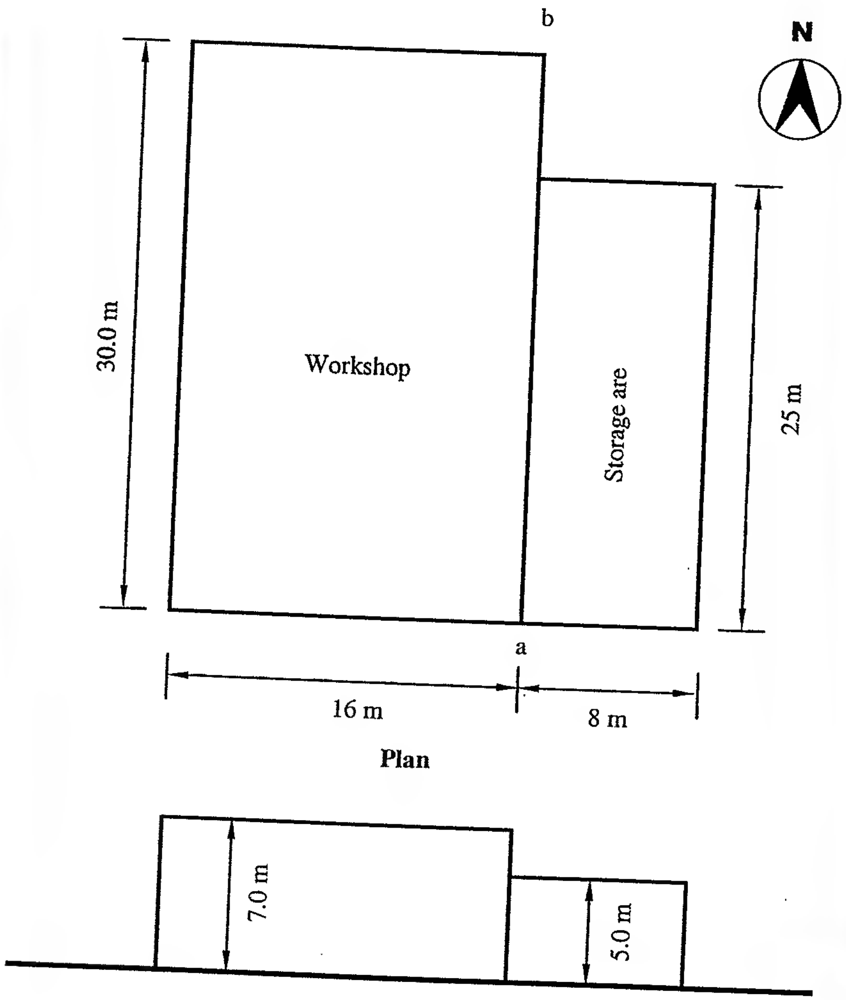


Fig. 1.18 End gable details.

Example 1.1: Structural system for workshop

The figure given below shows a workshop that consists of a large hall that spans 16.0 ms attached to storage area of 8.0 ms span. It is required to propose an appropriate structural system for such a structure knowing that an indirect lighting is required for the workshop while a horizontal roof is required for the storage area. Columns are only allowed at the outside perimeter as well as line ab.



Plan

Section

Solution

A-Wrokshop

Since the span of the hall is relatively large, and indirect lighting is required, RC frames are utilized as the main structural element. The spacing between the frames is chosen as 5.0m.

Item	Suggested dimensions	Chosen dimensions
t_g	Span/(12→14)	$=16/12 \cong 1.3m$
Frame spacing	4→7 m	5 m
Frame height (f)	Frame spacing/2	2.5 m
Column thickness	$0.80\ t_g$	$=0.80 \times 1.3 = 1.04 \cong 1.1\ m$
Secondary beam	frame spacing /(8→10)	$=5m \times 1000/10 = 500mm$
Post spacing	2→4 m	2m
Post dimensions	200×200 mm	200 × 200 mm
Ridge beam thickness	Post spacing /(8→10)	$= 2000/8 = 250\,mm$

B-Storage Area

A simple girder spanning 8.0 ms is chosen for the storage area. The spacing between these girders is taken the same as that between the frames (5.0m). Secondary beams are provided to get reasonable slab dimensions (5.0 ×4.0 ms).

$$Girder\ depth = \frac{Span}{10} = \frac{8 \times 1000}{10} = 800\ mm$$

The layout of the workshop is given in the following set of figures.

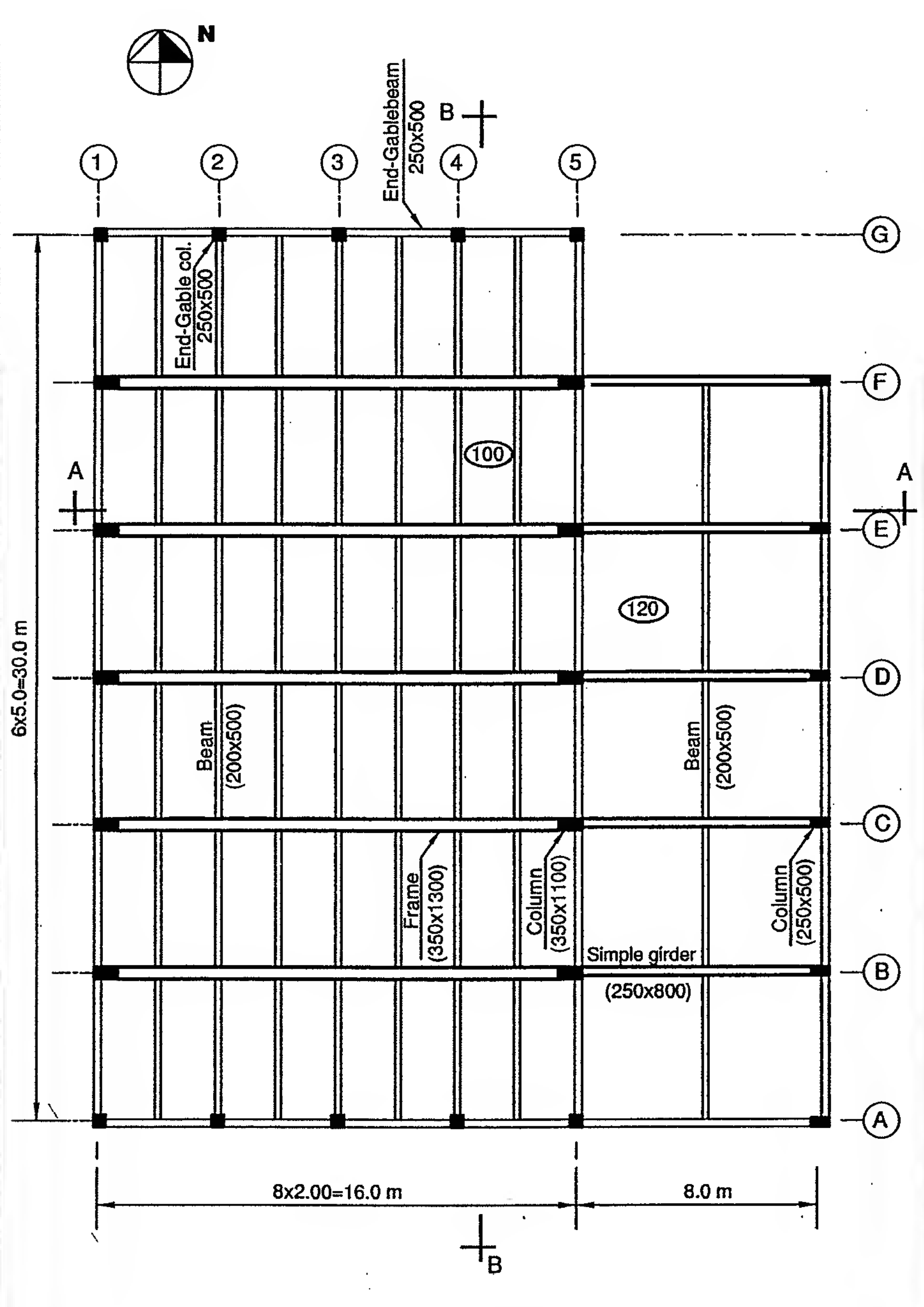


Fig. Ex.1.1a Structural system (plan)

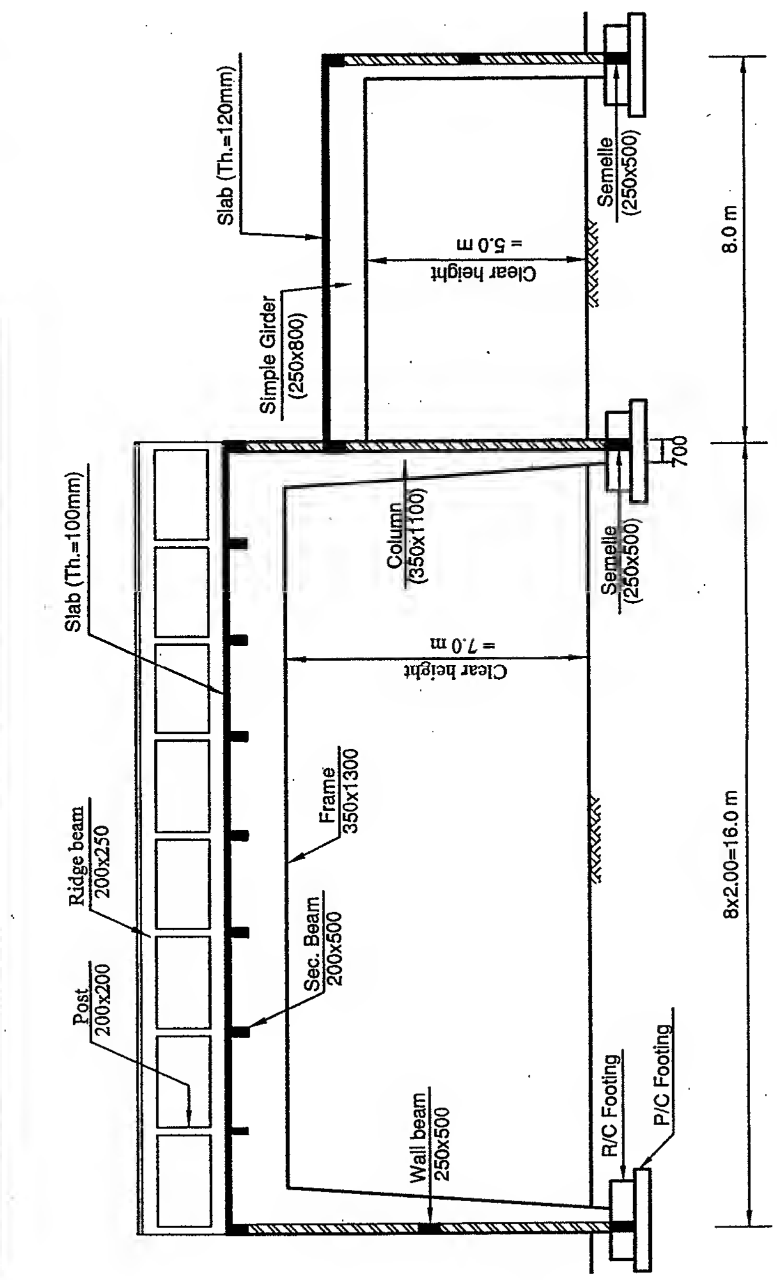


Fig. Ex.1.1b Structural system (Sec A-A)

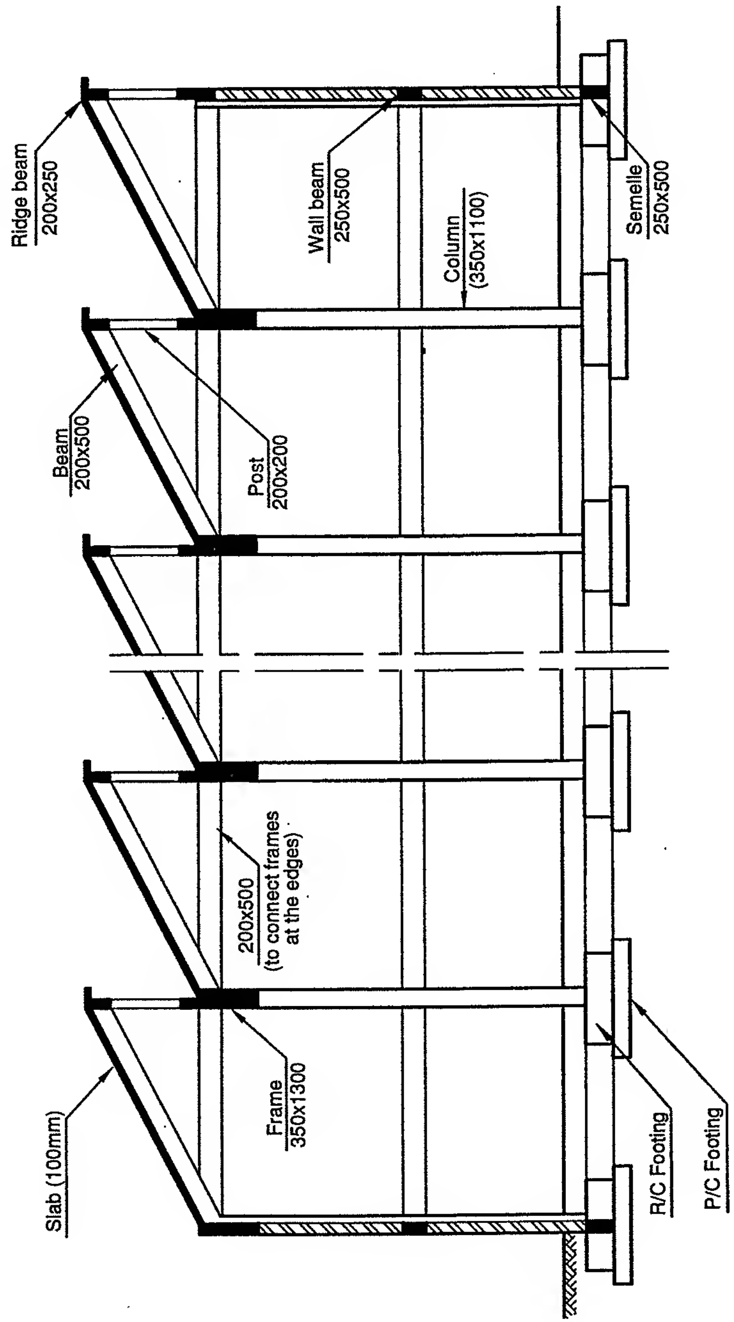
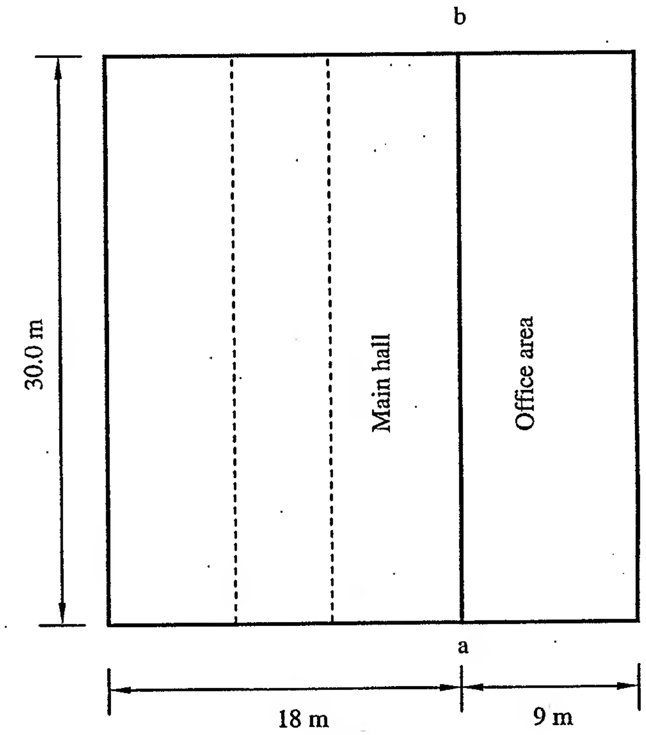


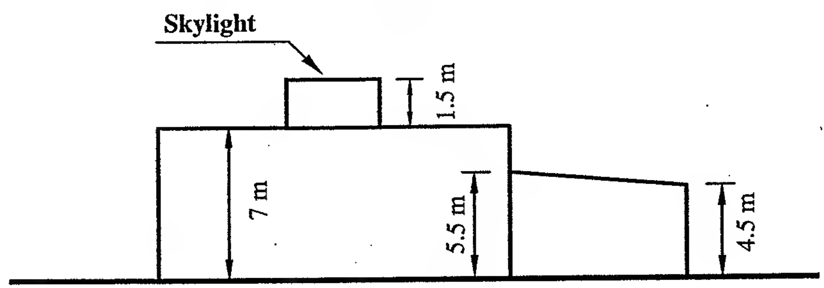
Fig. Ex.1.1c Structural system (Sec B-B)

Example 1.2: Structural system for car maintenance workshop

The figure given below shows a car maintenance workshop that consists of a main hall that spans 18.0 ms attached to an office area of 9.0 ms span. It is required to propose an appropriate structural system for such a structure. Columns are only allowed at the outside perimeter as well as on line ab.



Plan



Section elevation

Solution

A-Main Hall

Since the span of the main hall is relatively large, RC frames are used as the main structural elements. Posts supported on the girder of the frames are utilized to support the skylight roof.

The spacing between the frames is chosen as 5.0m.

Item	Suggested dimensions	Chosen dimensions
t_g	Span/(12→14)	$=18/14=1.28\ m\cong 1.3m$
Frame spacing	4→7 m	5 m
Frame height (f)	Frame spacing / 2	2.5 m
Column thickness	$0.80\ t_g$	$=0.80\times 1.3=1.04\cong 1.1\ m$
Secondary beam	Beam Span/(8→10)	$=5m\times 1000/10=500mm$
Post dimensions	200×200 mm	200×200 mm

B-Office area

An inclined simple girder spanning 9.0 ms is chosen as the main supporting element for the office area. The spacing between these girders is taken the same as that between the frames (5.0m). Secondary beams are provided to get reasonable slab dimensions (5.0 ×3.0 ms).

$$Girder\ depth=\frac{Span}{10}=\frac{9\times 1000}{10}=900\ mm$$

The layout of the car workshop is given in the following set of figures.

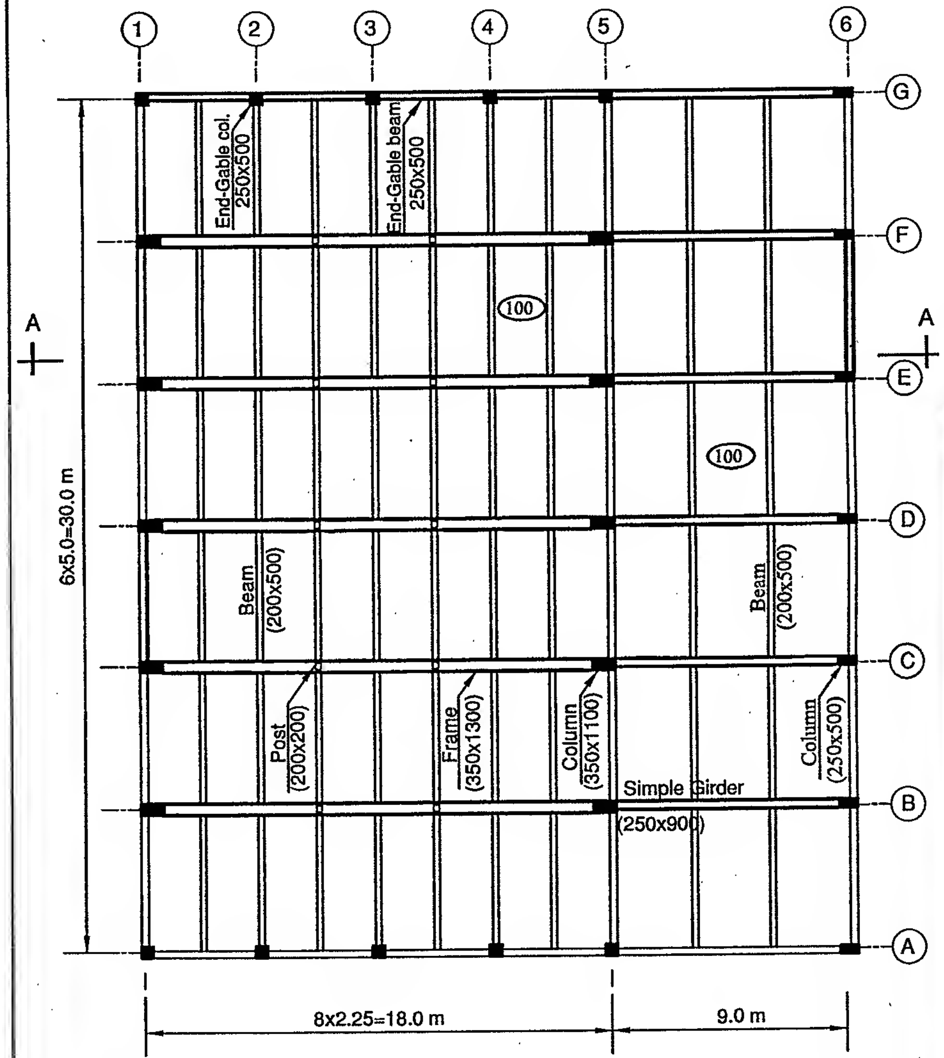


Fig. Ex.1.2a Structural system (plan)

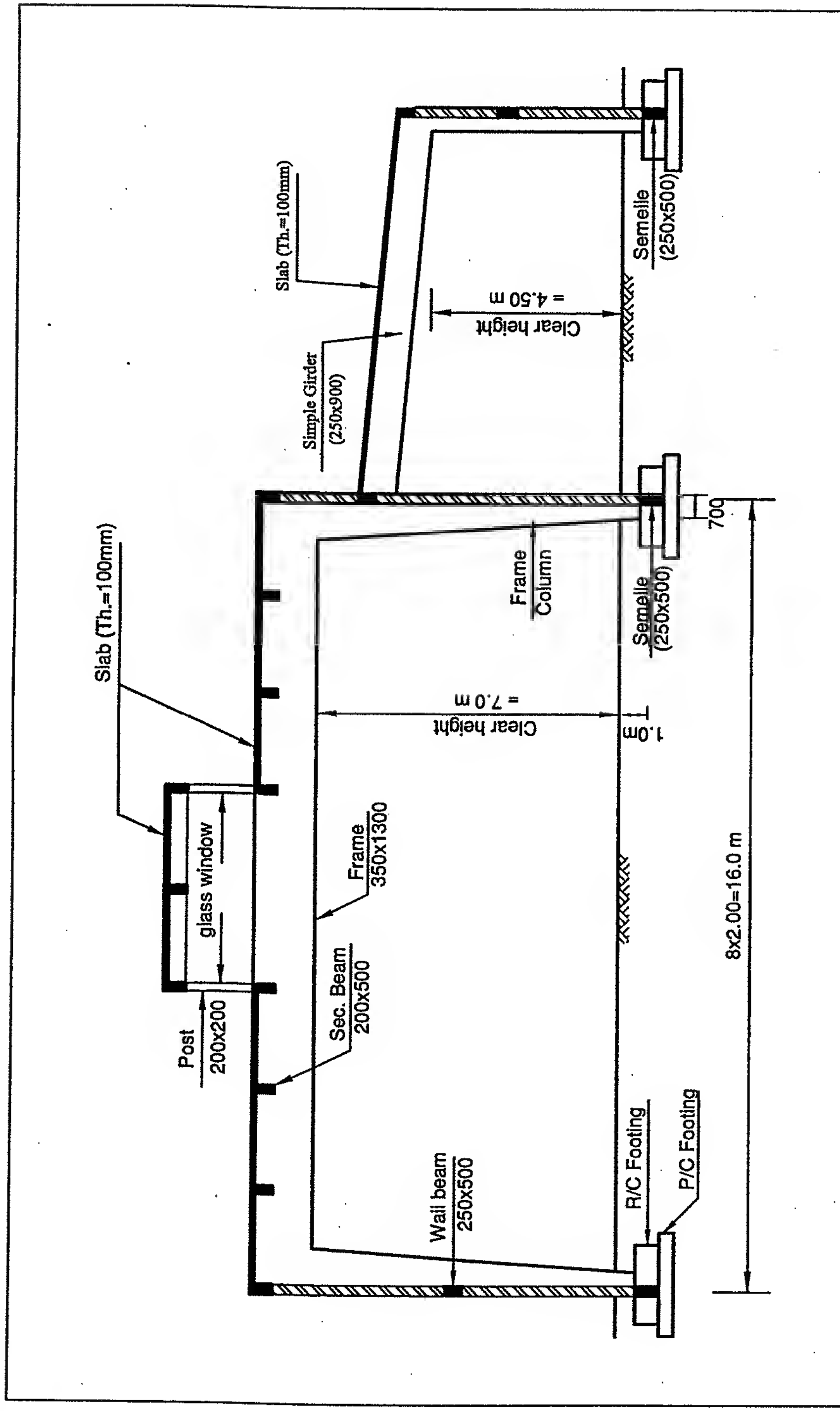
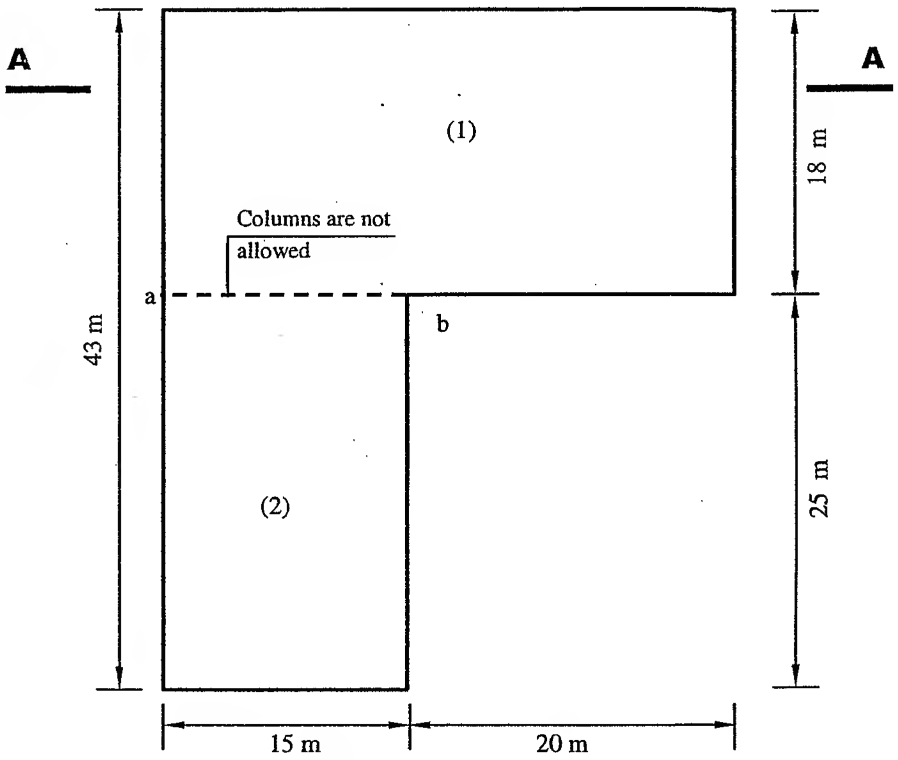


Fig. Ex.1.2b Structural system (Sec A-A)

Example 1.3: Structural system for a medical facility

The figure below shows a medical facility that consists of two large halls covered with horizontal roof. Hall (1) spans 18.0 ms while hall (2) spans 15.0 ms. Columns are only allowed on the outside perimeter. Clear height of the halls is 6.0m. It is required to suggest an appropriate structural system.



Plan

Section A-A

Solution

Hall (1)

Since the span of the hall is relatively large, and north light is not required, a frame system with secondary horizontal beams is chosen. The spacing between the frames is chosen as 5.0m, while the spacing between the secondary beams is taken as 3.0 m.

Item	Suggested dimensions	Chosen dimensions
t_g	Span/(12→14)	$=18/14=1.28\ m \cong 1.30\,m$
Frame spacing	4→7 m	5.0 m
Column thickness	0.80 t_g	$=0.80\times1.3=1.04\cong1.1m$
Secondary beam	Beam Span/(8→10)	$=5m\times1000/10=500mm$

Hall(2)

Item	Suggested dimensions	Chosen dimensions
t_g	Span/(12→14)	$=15/14=1.07\ m \cong 1.1m$
Frame spacing	4→7 m	5.0 m
Column thickness	0.80 t_g	$=0.80\times1.1=0.88\cong0.90\ m$
Secondary beam	Beam Span/(8→10)	$=5m\times1000/10=500mm$

Since columns are not allowed inside the halls, some of the frames of Hall (1) have to be supported on another frame that spans 15m. Such a frame is separated from the frame that constitutes a part of the main system of Hall (2) by an expansion joint. The expansion joint is needed since the length of the hall is more than 40.0m.

The layout of the medical facility is given in the following set of figures.

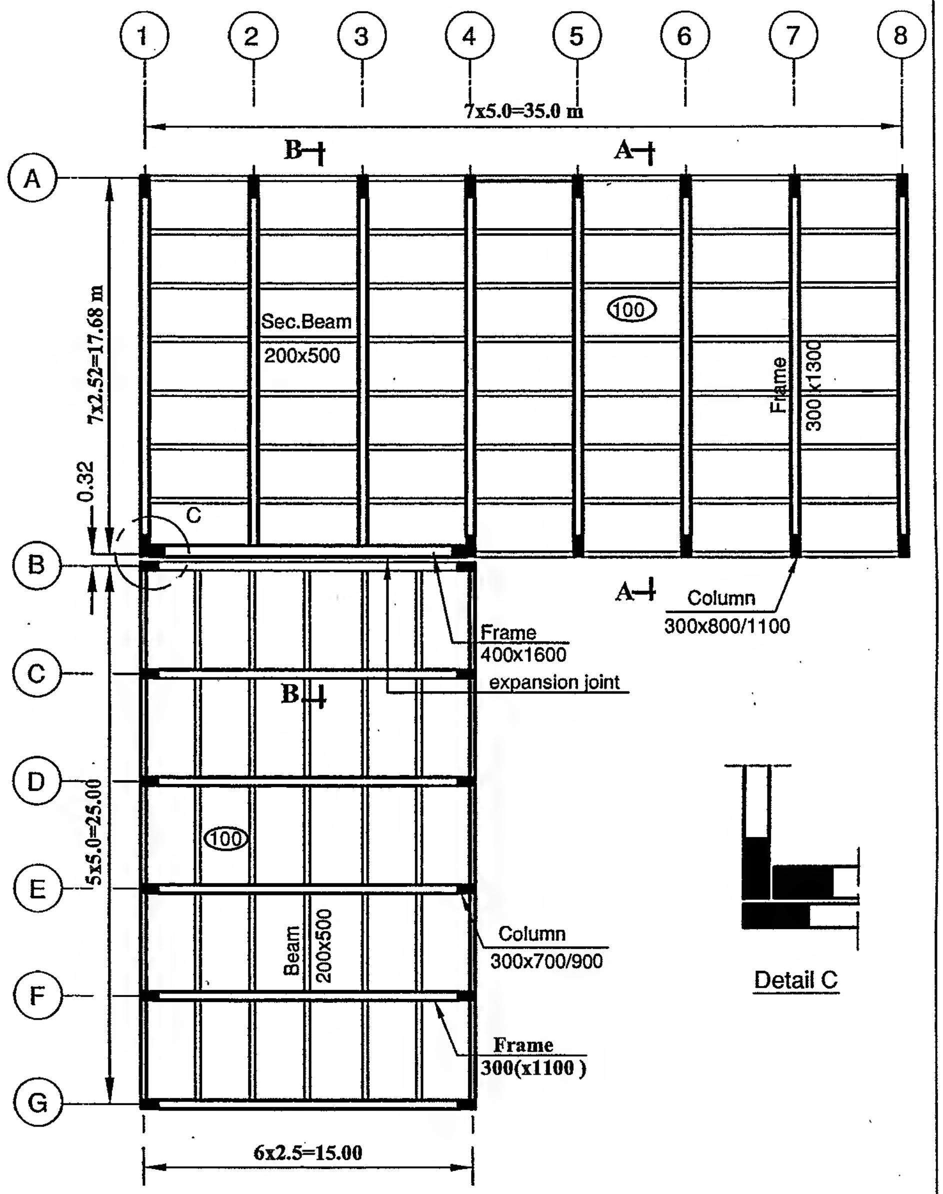


Fig. Ex.1.3a Structural system (plan)

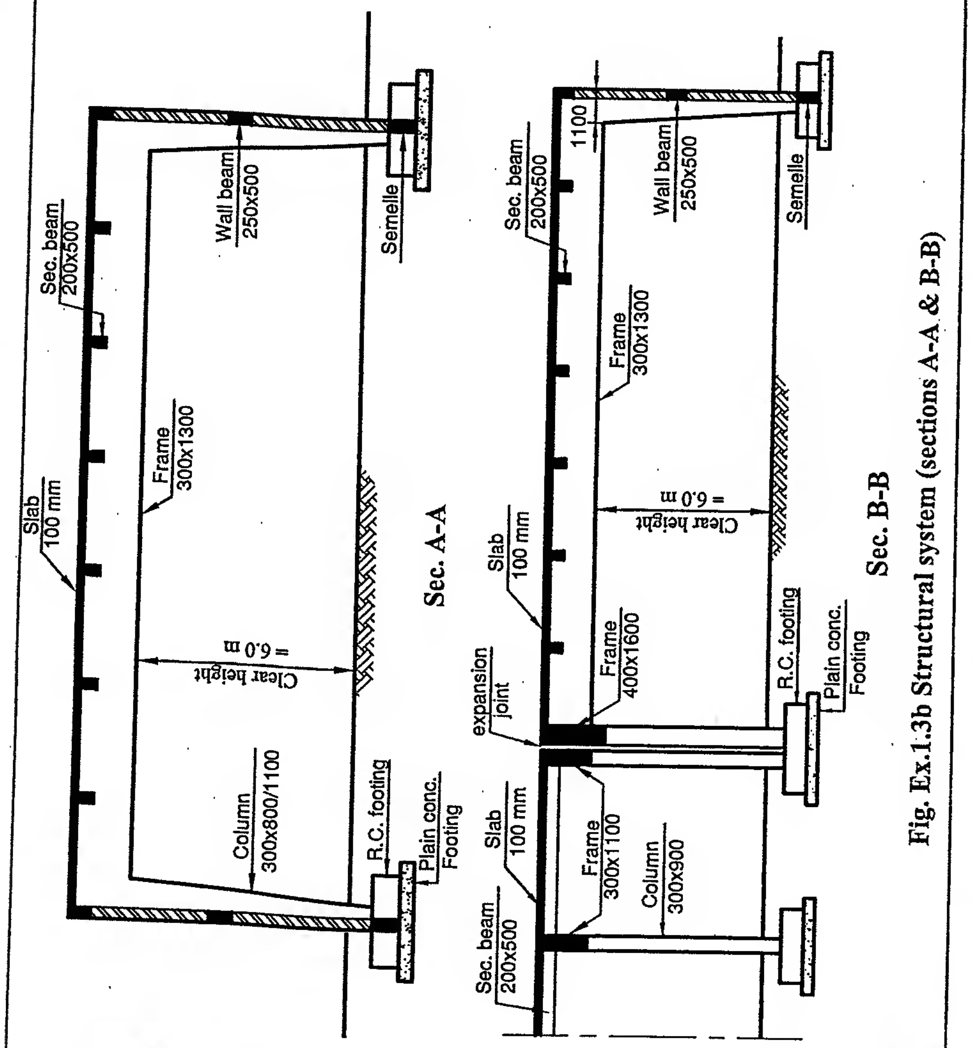
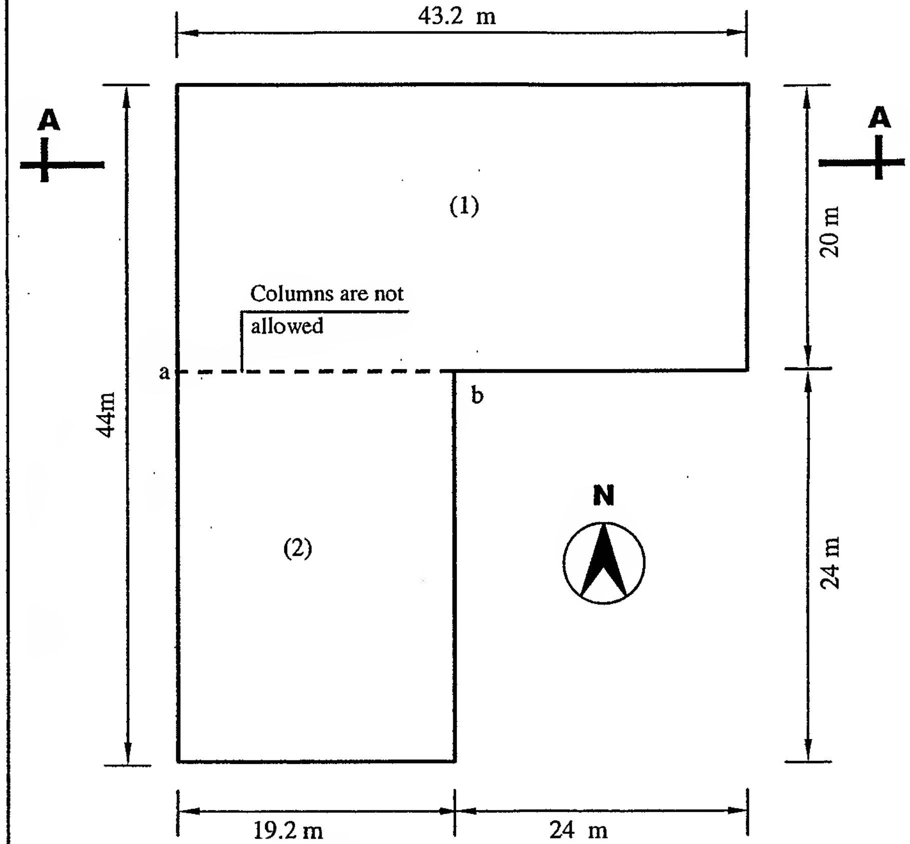


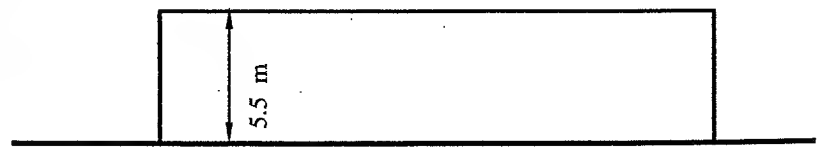
Fig. Ex.1.3b Structural system (sections A-A & B-B)

Example 1.4: Structural system for a textile factory

The figure given below shows a textile factory that consists of two large halls in which indirect lighting is needed. Hall (1) spans 20 m and Hall (2) spans 19.2m. Columns are only allowed on the outside perimeter. Clear height of the halls is 5.5m. It is required to suggest an appropriate structural system for the factory.



Plan



Section A-A

Solution

Hall (1)

Since the span of the hall is relatively large, and north light is required, a frame system with Y-beams is chosen. The spacing between the frames is chosen as 4.8m, while the spacing between the secondary beams is taken as 2.4 m to get a system of one-way slabs (2.4x5.0 ms). The spacing of the Y-beams is chosen equal to 5.0 ms.

Item	Suggested dimensions	Chosen dimensions
t_g	Span/(12→14)	$=20/14=1.43\ m \cong 1.5m$
Frame spacing	4→7 m	4.8 m
Y-beam spacing	4→6 m	5m
frame height (f)	Y-beam spacing/2	=5/2=2.5 m
Column thickness	0.80 t_g	$=0.80\times1.5=1.2$
Secondary beam	Beam span/(8→10)	$=5m\times1000/10=500mm$
Y-beam	Span/(6→8)	$=4.8/6=0.80\ m$
Post spacing	2→4 m	2.4 m
Post dimensions	200×200 mm	200×200 mm
Ridge beam thickness	Post spacing /(8→10)	$=2400/8=300mm$

Hall(2)

Since the north light is perpendicular to the span, a frame system with secondary beams is chosen. The spacing between frames is taken as 4.0m

Item	Suggested dimensions	Chosen dimensions
t_g	Span/(12→14)	$=19.2/14=1.37\ m \cong 1.4m$
Frame spacing	4→7 m	4.0 m
Frame height (f)	*Frame spacing* / 2	=4.0/2=2.0 m
Column thickness	0.80 t_g	$=0.80\times1.4=1.12\cong1.2\ m$
Secondary beam	Beam span/(8→10)	$=4m\times1000/10=400mm$
Post spacing	2→4 m	2.4 m
Post dimensions	200×200 mm	200×200 mm
Ridge beam thickness	Post spacing /(8→10)	$=2400/8=300mm$

Since columns are not allowed inside the halls, some of the frames of Hall (1) have to be supported on another frame that spans 15m. Such a frame is separated from the frame that constitutes a part of the main system of Hall (2) by an expansion joint. The expansion joint is needed since the length of the hall is more than 40.0m.

The layout of the factory is given in the following set of figures.

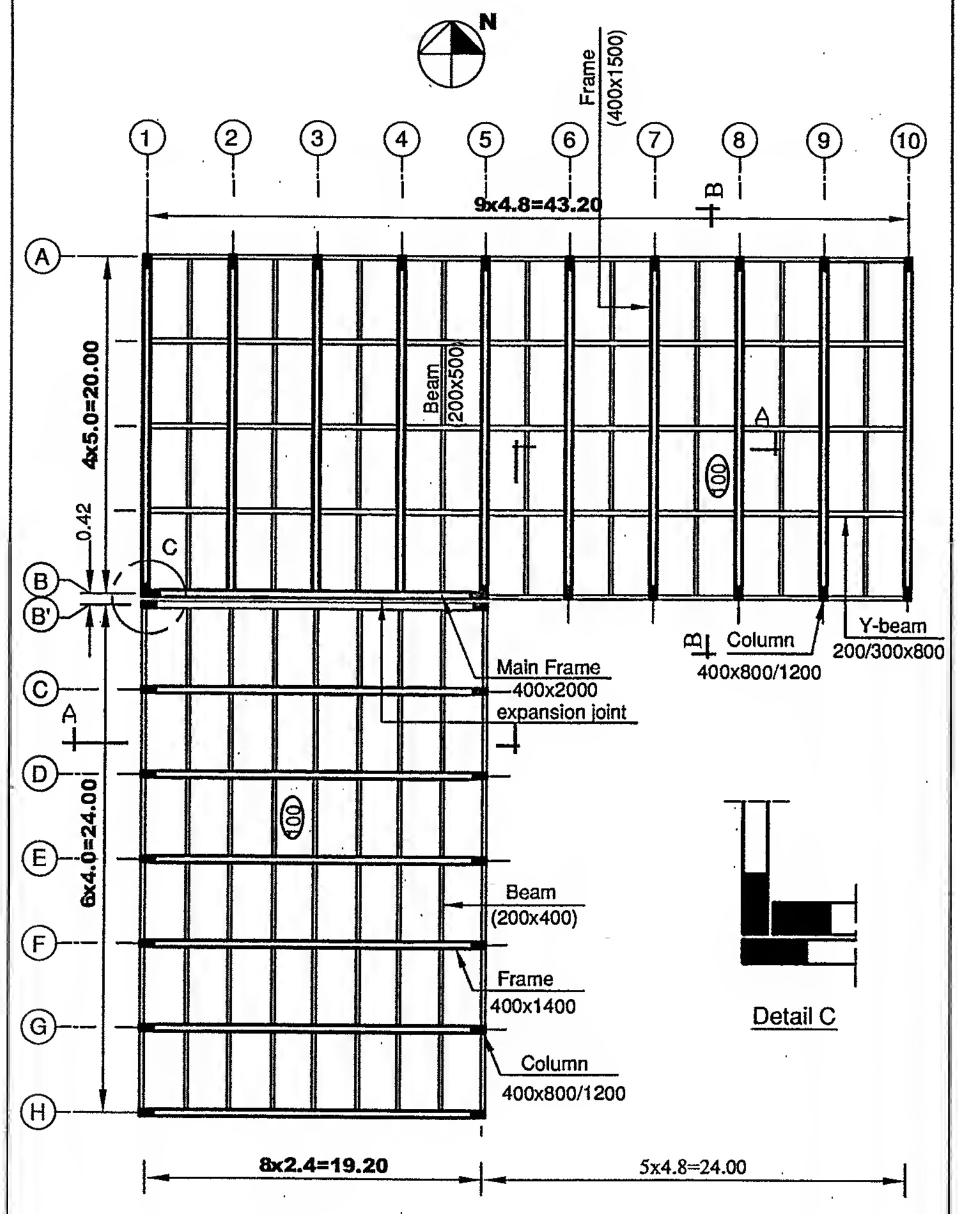
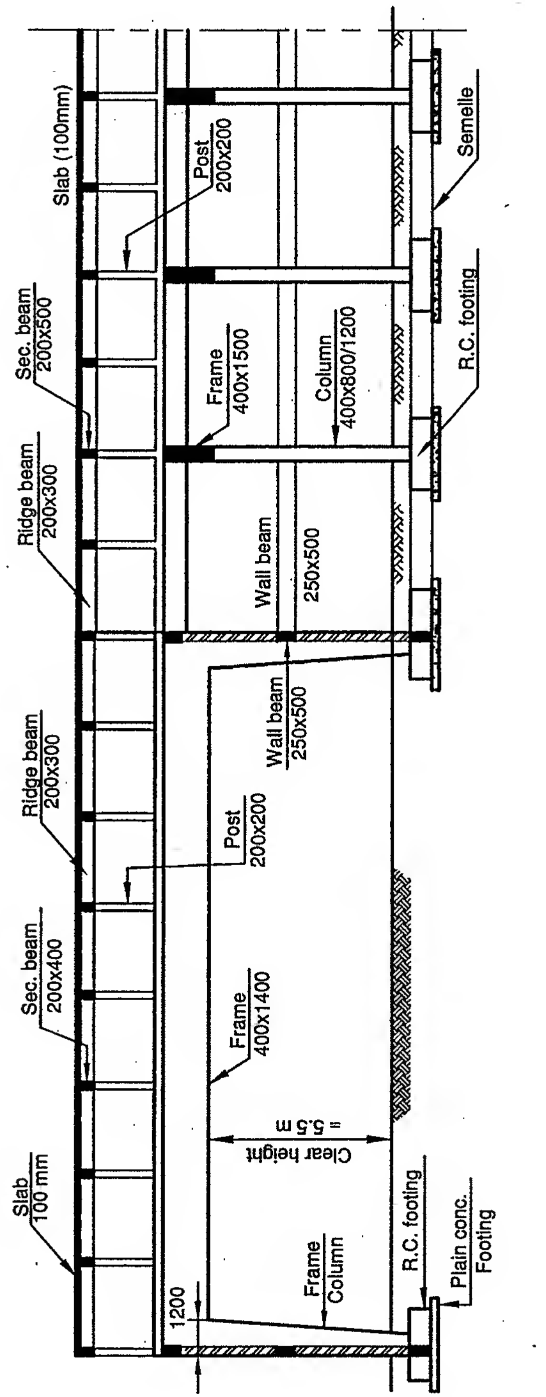
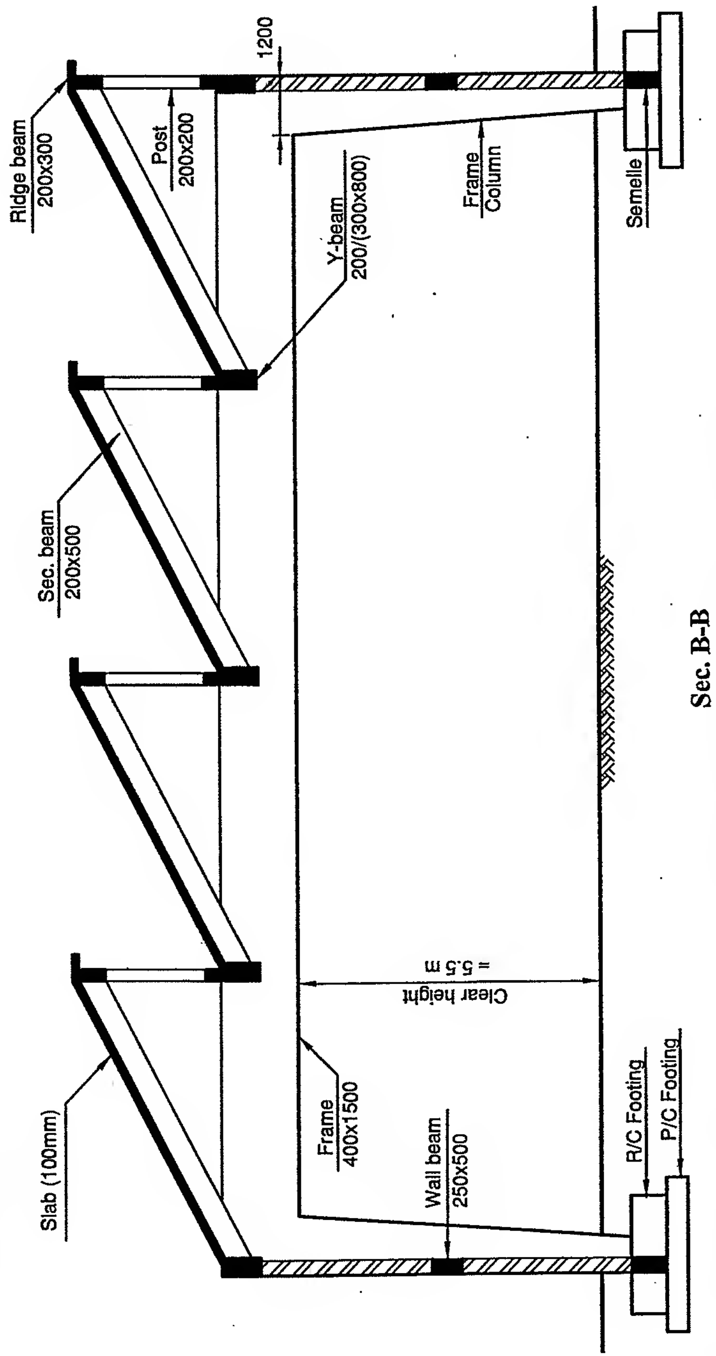


Fig. Ex.1.4a Structural system (plan)



Sec. A-A

Fig. Ex.1.4b Structural system (sections A-A)



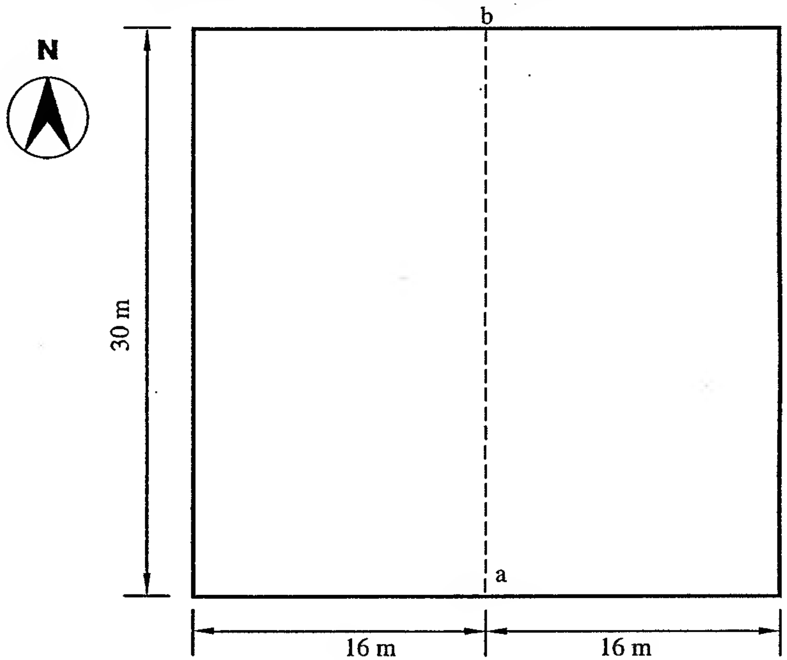
Sec. B-B

Fig. Ex.1.4c Structural system (sections B-B)

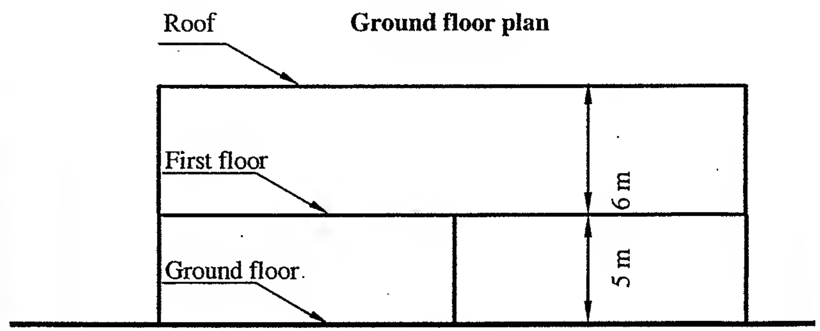
Example 1.5: Structural system of a factory

The figure given below shows a factory to be constructed in 10th of Ramadan city. The factory consists of two floors. In the ground floor level, columns are only allowed along the outside perimeter as well as on line *ab*. However, the first floor level must be free from inside columns and must have indirect lighting.

It is required to propose an appropriate structural system for the factory and to show the details of such a system in plan and sections.



Ground floor plan



Section

A: Structural System the First Floor

Since horizontal floor is required, a continuous frame system with secondary beams is chosen. The spacing between frames is taken as 6.0m

Item	Suggested dimensions	Chosen dimensions
t_g	Span/(12→14)	$=16/14=1.14\ m\cong 1.2m$
b_g	250→400 mm	0.35 mm
Frame spacing	4→7 m	6.0 m
Side Column thickness	Same as roof column	1.5 m
Middle column thickness	0.8 t_g	1.0m
Secondary beam	Beam span/(10→12)	$=6m\times1000/12\cong 550mm$

B: Roof

Since the roof has a relatively large span and north lighting is required, an arch with a tic is chosen as the main structural system.

Item	Suggested dimensions	Chosen dimensions
t_{arch}	Span/25	$=32/25=1.28\ m\cong 1.3m$
t_{tie}	$t_{arch}/2$	$=1.3/2=0.65m$
Arch spacing	4→7 m	6.0 m
Arch height (f)	$Span/(5\rightarrow 8)$	=32/8=4.0 m
Column thickness	span/20	$=32/20\cong 1.5\ m$
Secondary beam	Beam span/(8→10)	$=6m\times1000/10=600mm$
Post spacing	2→4 m	3 m
Post dimensions	200×200 mm	200×200 mm

The layout of the factory is given in the following set of figures.

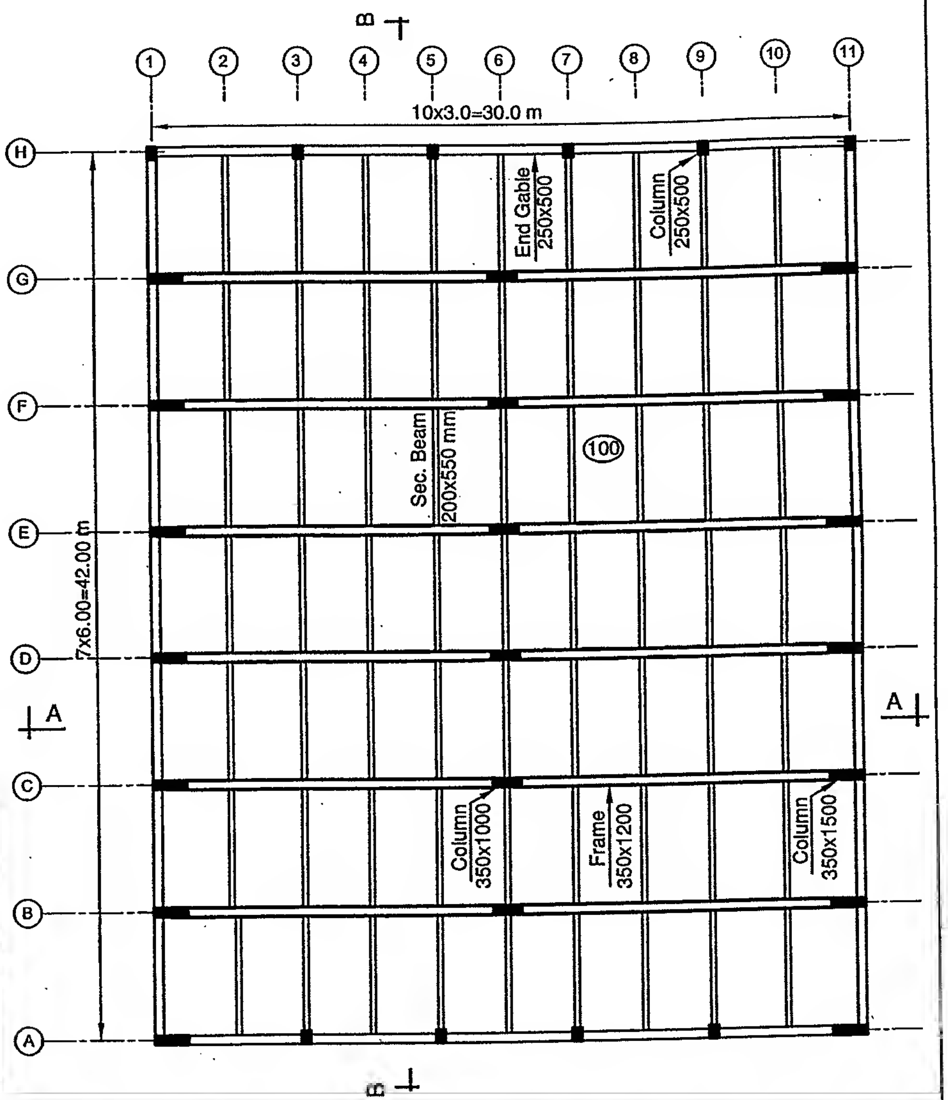


Fig. Ex.1.5a Structural system for the first floor (plan)

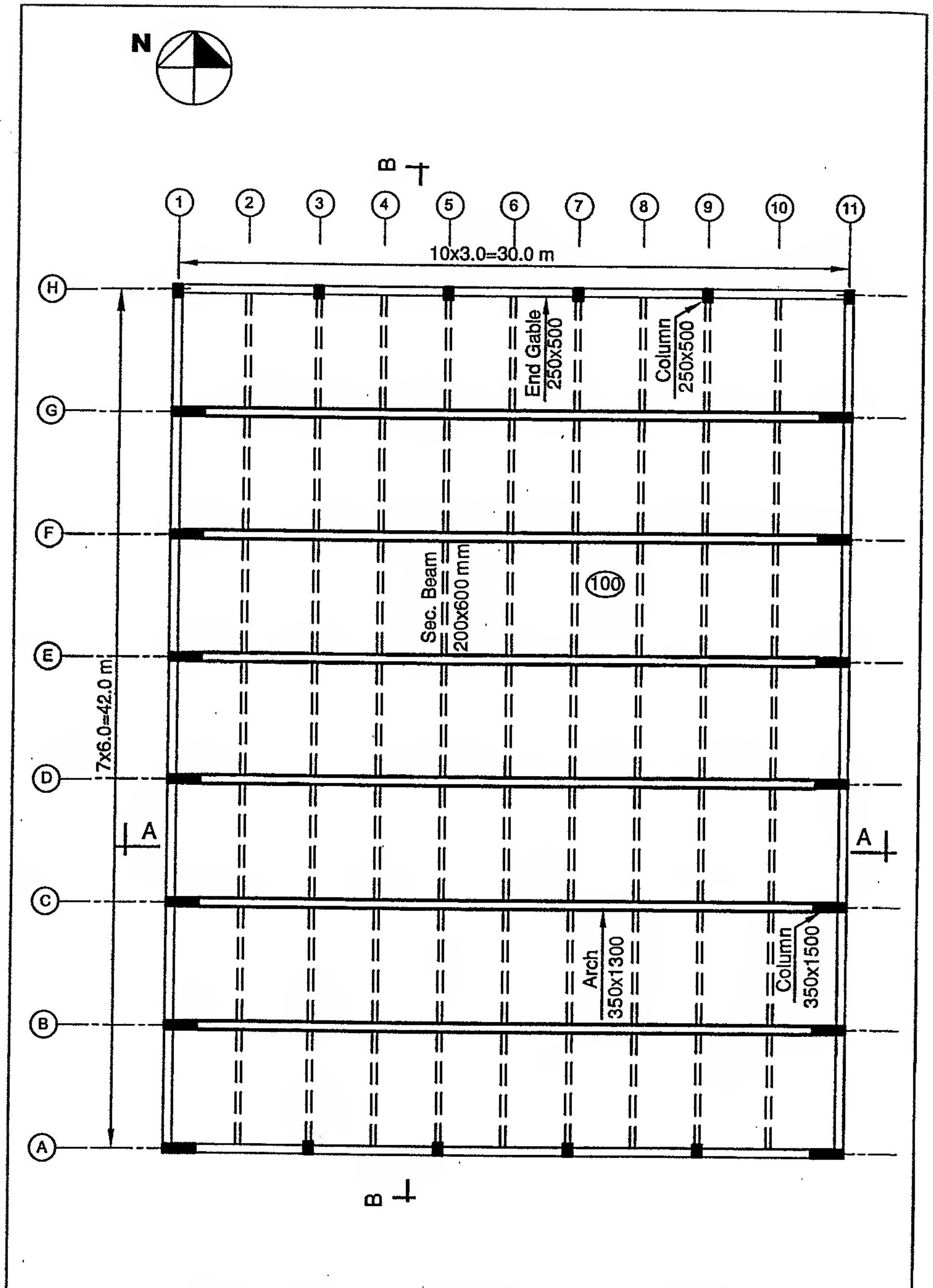


Fig. Ex.1.5b Structural system for the roof (plan)

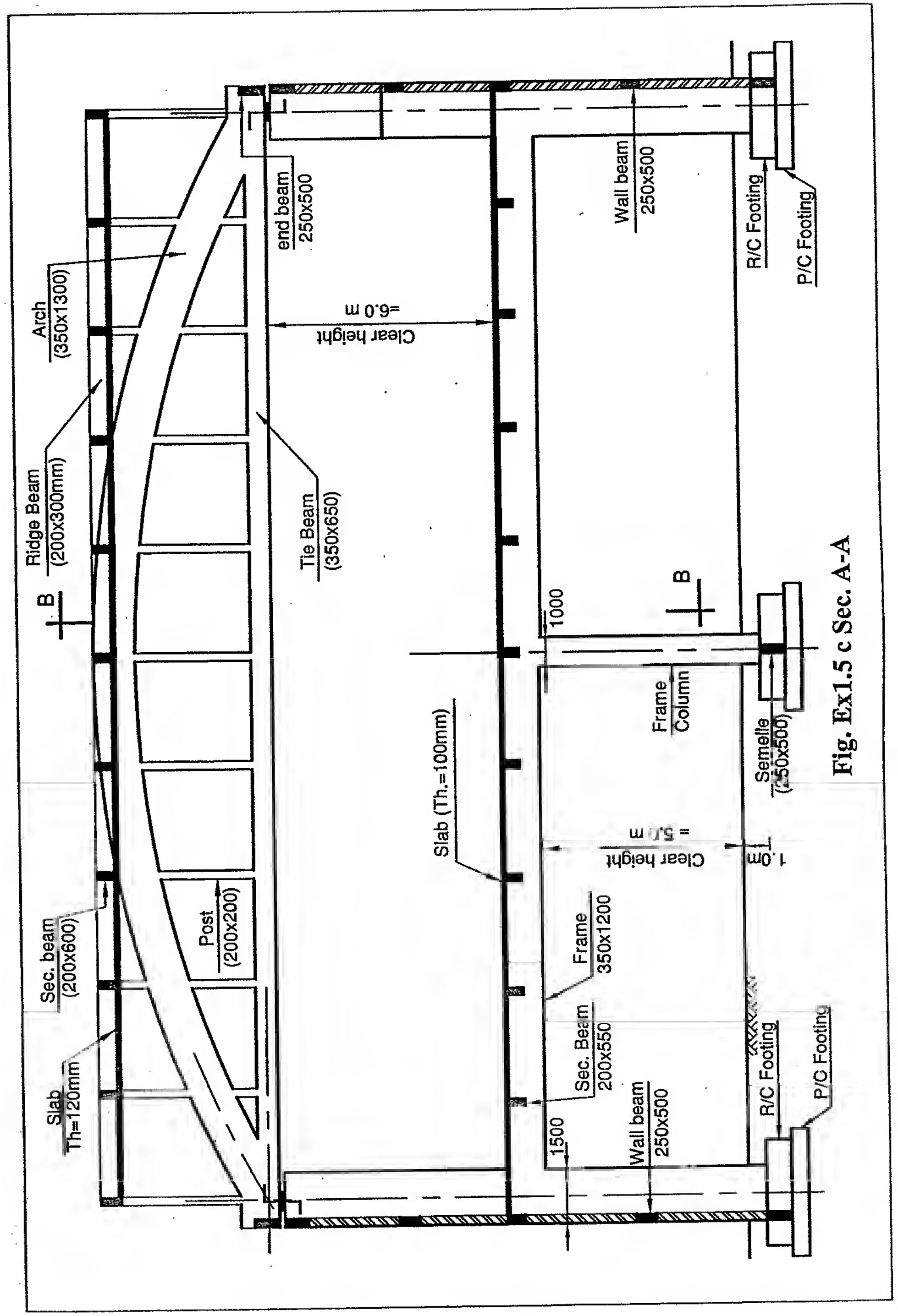


Fig. Ex1.5 c Sec. A-A

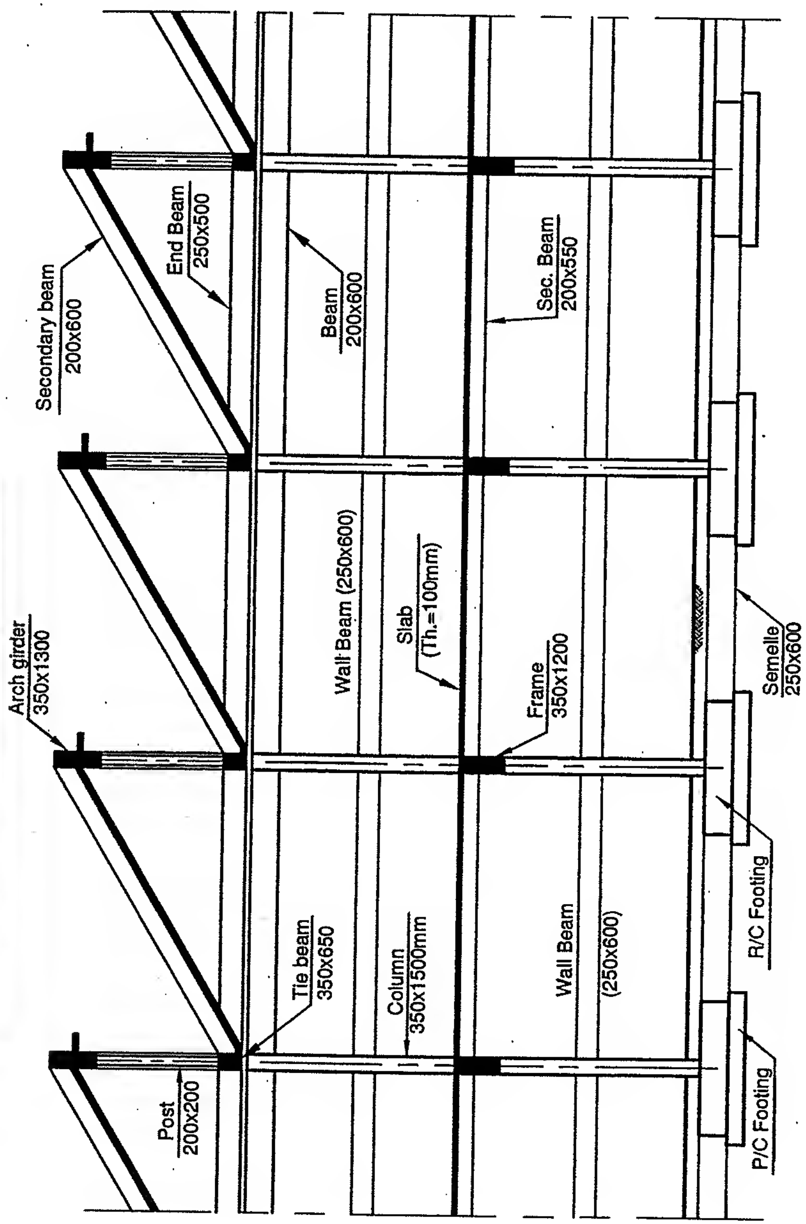
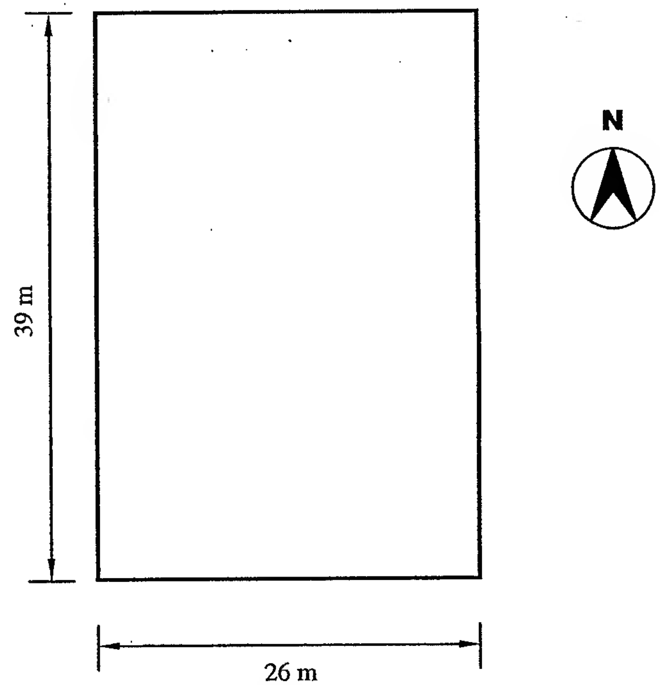


Fig. Ex1.5d Sec. B-B

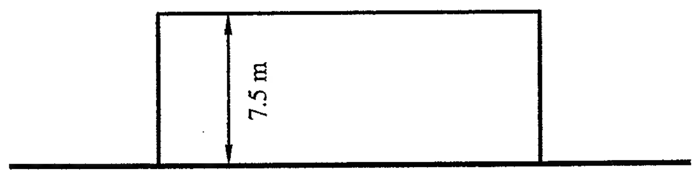
Example 1.6: Arch with a Tie

It is required to design a saw-tooth structural system for the factory shown in the figure below. The material properties are f_{cu}=30 N/mm^2 and f_y=360 N/mm^2.

Live load = 1.0 kN/m^2
Flooring load = 1.0 kN/m^2
Wind load = 0.7 kN/m^2
Clear height = 7.5 m
γ_{wall}=12 kN/m^3
Span =26.0 m



Plan



Section

Solution

The span of the factory is relatively large (>20 m) such that the choice of a frame system leads to an uneconomical solution. Since the span of the factory is normal to the north direction, an arch with a tie is chosen as the main structural system.

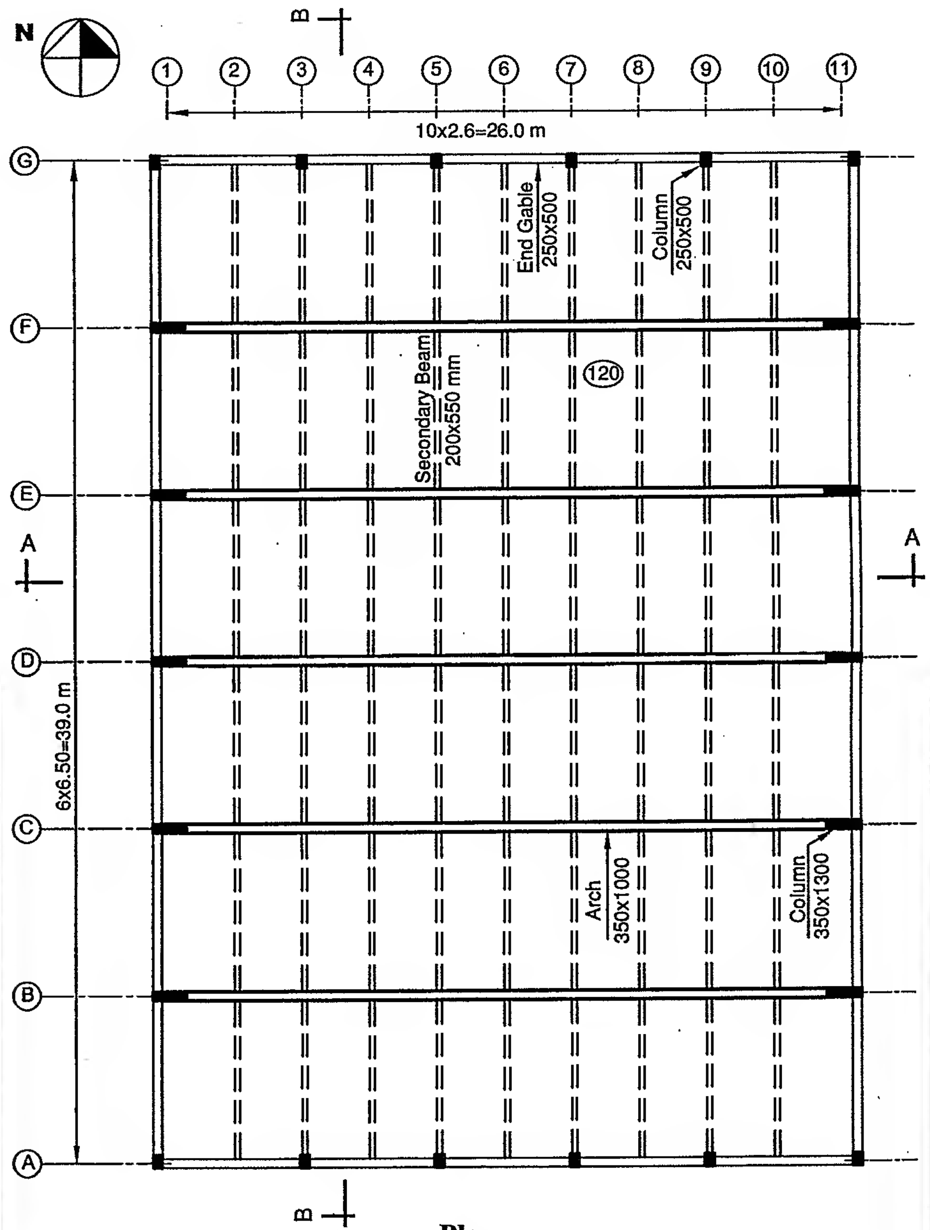
Assume the following dimensions:

Slab thickness t_s	= 120 mm		
Ridge beam	= (200 mm x 300 mm)		
Secondary beam	= (200 mm x 550 mm)		
Post dimension	= (200 mm x 200 mm)		
Span of the arch	= 26 m		
t_g = span/25	≈ 1.0 m	Girder	(350 mm x 1000 mm)
t_{tie} =0.5 tg	= 0.5 m	Tie	(350 mm x 500 mm)
Spacing between arches	= 6.5 m		
Secondary beams spacing	= 2.60 m		

The rise of the arch equals $(f) = \dfrac{span}{6-8} = \dfrac{26.0}{6-8} = (3.25-4.33) = 3.5\,m$

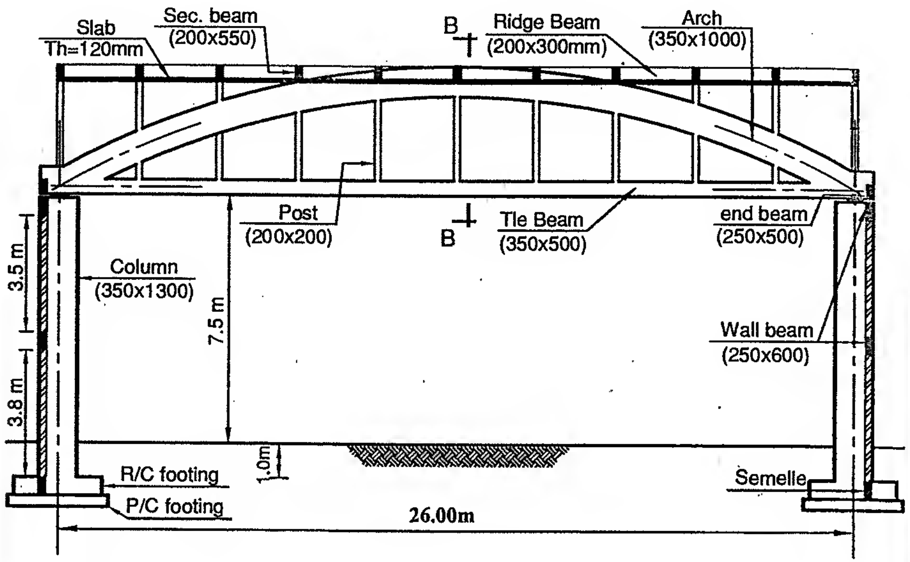
$$\text{The thickness of the column equals}\begin{cases}\dfrac{h}{15} = \dfrac{8.5}{15} = 0.56\,m \\[2ex] \dfrac{span}{20} = \dfrac{26}{20} = 1.3\ m\end{cases}$$

Choose the column cross section (350 mm x1300 mm)

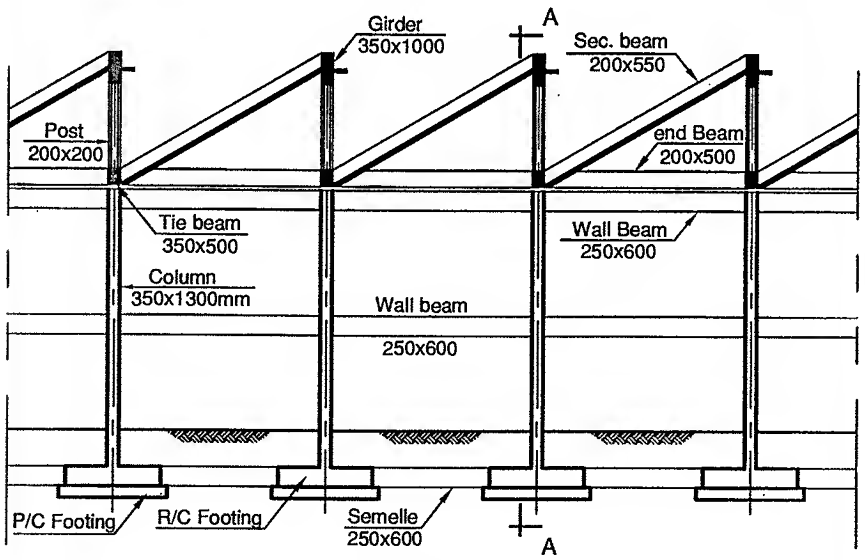


Plan

Fig. Ex.1.6a: North-light saw-tooth system (Arches)



Sec. A-A



Sec. B-B

Fig. Ex.1.6b: North-light saw-tooth system (Arches)

Step 1: Design of solid slabs

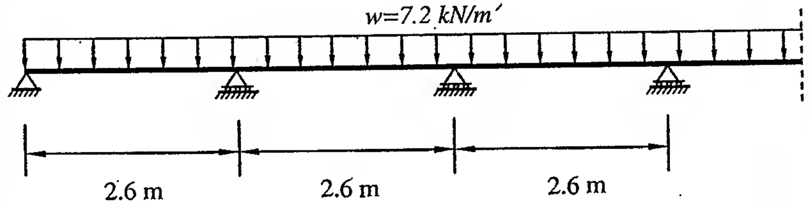
Assume that the slab thickness t_s=120 mm

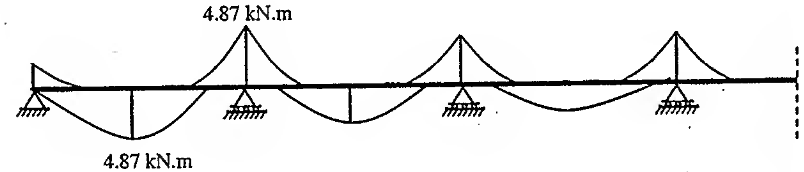
The total dead load of the slab and flooring load are equal to:

$$g_s = t_s \times \gamma_c + flooring = 0.12 \times 25 + 1.0 = 4.0\ kN/m^2$$

The slab ultimate load w_{su} equals

$$w_{su} = 1.4 \times g_s + 1.6 \times p_s = 1.4 \times 4 + 1.6 \times 1.0 = 7.2\ kN/m^2$$





The slab is a one-way slab (2.6 m × 6.5 m) and continuous in the short direction, thus the maximum moment is given by:

$$M_u = \frac{w_{su} \times L^2}{10} = \frac{7.2 \times 2.6^2}{10} = 4.87\ kN.m$$

Assuming 20 mm cover, the effective depth $d = 120 - 20 = 100\ mm$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{4.87 \times 10^6}{30 \times 1000 \times 100^2} = 0.0162$$

For small values of R, ω can be approximated by 1.2R. Thus, the reinforcement index ω=0.019

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.019 \times \frac{30}{360} \times 1000 \times 100 = 158.4 \ mm^2$$

$$A_{s,\min} = \frac{0.6}{f_y} b \ d = \frac{0.6}{360} \times 1000 \times 100 = 166 \, mm^2 < A_s .... o.k$$

Choose 5Φ8/m′ (251 mm²)

Step 2: Design of the Secondary Beam (200x550 mm)

The ultimate self-weight of the beam equals to:

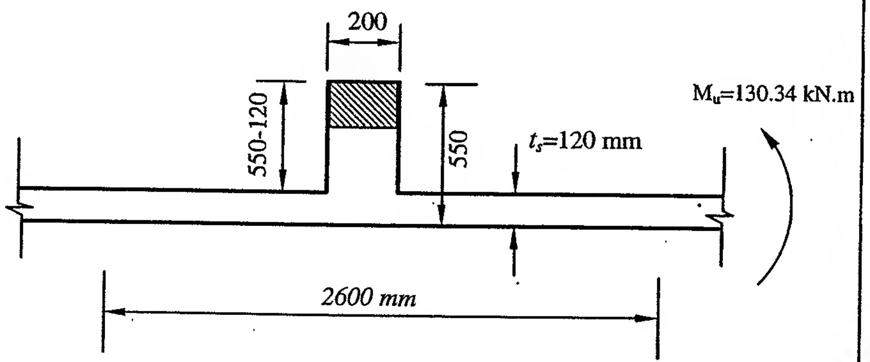
$$w_{u,o.w} = 1.4 \times \gamma_c \times b \times (t - t_s) = 1.4 \times 25 \times \frac{200}{1000} \times \frac{550-120}{1000} = 3.01 \ kN/m'$$

The spacing between the secondary beams is *2.6m,* thus the total beam load is:

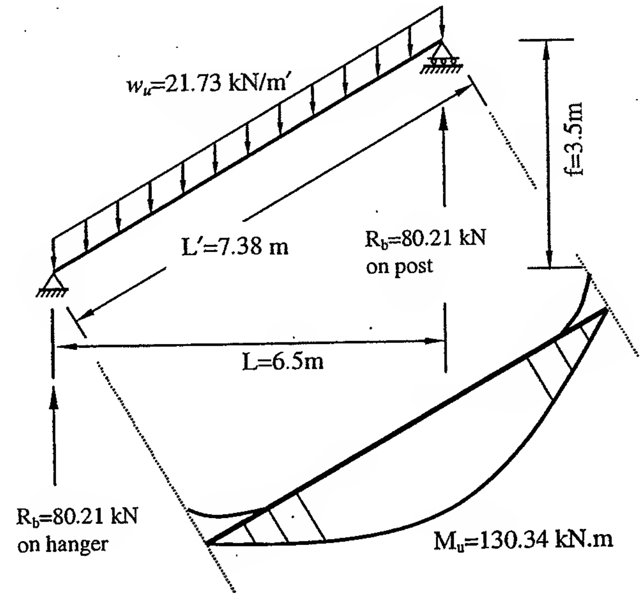
$$w_u = w_{u,o.w} + spacing \times w_{su} = 3.01 + 2.6 \times 7.2 = 21.73 \ kN/m'$$

The sloped length L′ equals to:

$$L' = \sqrt{L^2 + f^2} = \sqrt{6.5^2 + 3.5^2} = 7.38 \ m$$



Cross-section of the secondary beam (acts as a rectangular section)



The beam is simply supported on the post (compression member) on one side and on the hanger (tension member) on the other side.

$$M_u = \frac{w_u \times L \times L'}{8} = \frac{21.73 \times 6.5 \times 7.38}{8} = 130.34 \ kN.m$$

The reaction of the secondary beam is

$$R_b = \frac{w_u \times L'}{2} = \frac{21.73 \times 7.38}{2} = 80.21 \ kN$$

Since the secondary beam is an inverted beam, the section at mid-span acts as rectangular section as previously shown.

$$d = t - cover = 550 - 50 = 500 \ mm$$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{130.34 \times 10^6}{30 \times 200 \times 500^2} = 0.087$$

From the chart, the reinforcement index $\omega = 0.112$

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.112 \times \frac{30}{360} \times 200 \times 500 = 933\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{360} \times 200 \times 500 = 342\ mm^2 \\ 1.3 \times A_s = 1.3 \times 933 = 1213\ mm^2 \end{cases}$$

Choose 4Φ18 (1017 mm^2)

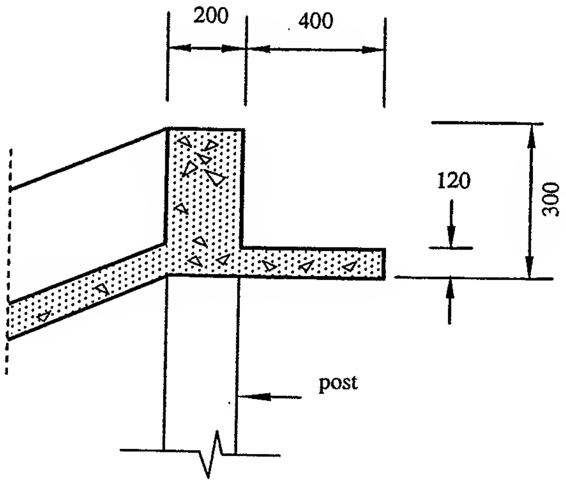
Step 3: Design of ridge beam (200 x 300 mm)

The ridge beam is a continuous beam supported on the posts.

The ultimate self-weight of the beam equals to:

$$w_{u,o.w} = 1.4 \times \gamma_c \times b \times t = 1.4 \times 25 \times 0.20 \times 0.30 = 2.1\ kN\ /\ m'$$

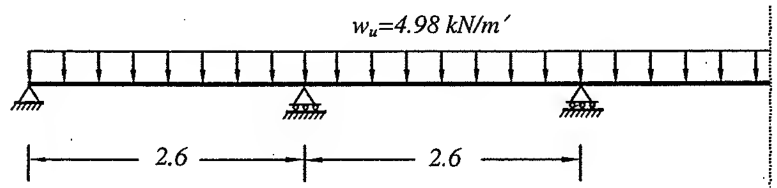
No slab load is transferred to the ridge beam. The cantilever part is considered as shown in figure below.

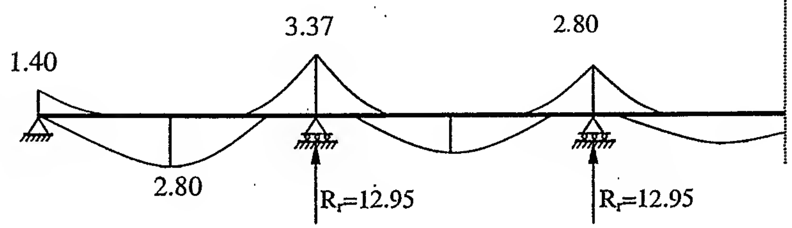


$$w_u = w_{u,o.w} + w_u \times cantilever\,length = 2.1 + 7.2 \times 0.40 = 4.98\,kN\,/\,m'$$

$$M_u = \frac{w_u \times L^2}{10} = \frac{4.98 \times 2.6^2}{10} = 3.37\ kN.m$$

$$R_r = w_u \times L = 4.98 \times 2.6 = 12.95\ kN$$





To use the R-ω curve, calculate R

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{3.37 \times 10^6}{30 \times 200 \times 250^2} = 0.009$$

Since R is below the chart ω is taken as 1.2 R = 0.0108

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.0108 \times \frac{30}{360} \times 200 \times 250 = 45\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{360} \times 200 \times 250 = 171\ mm^2 \\ 1.3A_s = 1.3 \times 45 = 58.5\ mm^2 \quad \leftarrow \end{cases}$$

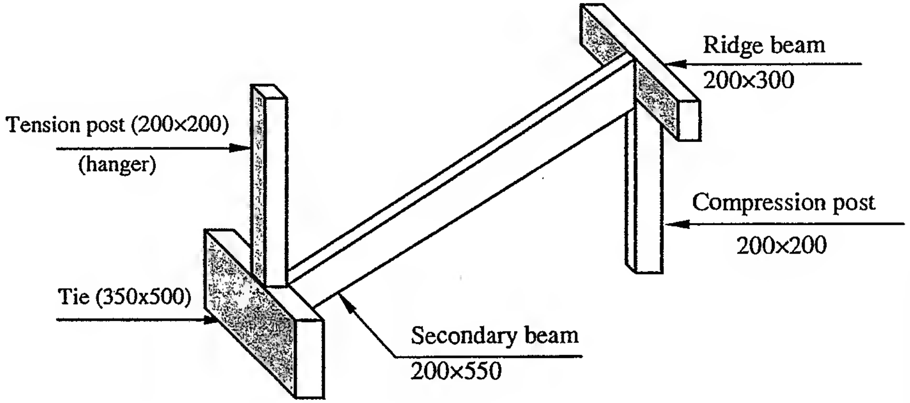
but not less than $\frac{0.15}{100} \times b \times d = \frac{0.15}{100} \times 200 \times 250 = 75\,mm^2$

Thus, $A_{s,min}$=75 mm^2

Use 2Φ12 (226.2 mm^2)

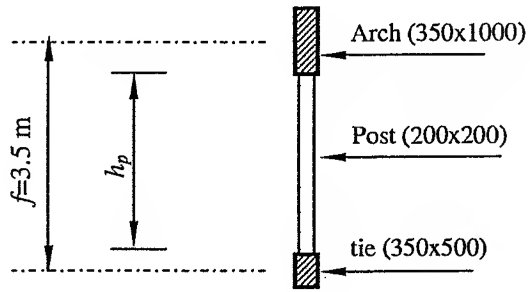
Step 4: Design of post (200x200 mm)

Posts in the arch with a tie system are subjected to tension or compression according to their locations as shown in the figure below



Step 4.1: Design of the compression post

The height of the post h_p = the height of the rise – arch thickness/2 – tie thickness /2 as shown in figure.



$$h_p = f - \frac{t_g}{2} - \frac{t_{tie}}{2} = 3.5 - \frac{1.0}{2} - \frac{0.5}{2} = 2.75\ m$$

The ultimate self-weight of the post (200 × 200 mm) equals

$$P_{o.w} = 1.4 \times \gamma_c \times b \times t \times h_p = 1.4 \times 25 \times 0.20 \times 0.20 \times 2.75 = 3.85\ kN$$

The load acting on the post results from the reactions of the ridge beam and the secondary beam

$$P_u = P_{o.w} + P_b\,(\text{secandry beam}) + P_r\,(\text{ridge beam}) = 3.85 + 80.21 + 12.95 = 97\ kN$$

Since the factory has no special system for resisting the lateral forces, it is considered unbraced. The effective length factor k can be obtained from Table 6-10 in the code. The top and the bottom part of the column are considered case 1. Thus k=1.2.

$$H_e = k \times h_p = 1.2 \times 2.75 = 3.30\ m$$

The slenderness ratio λ is given as $\lambda = \frac{H_e}{t_{post}} = \frac{3.3}{0.2} = 16.50$

Since λ greater than 10, the post is considered long and additional moment is developed.

$$\delta = \frac{\lambda^2 \times t}{2000} = \frac{16.5^2 \times 200}{2000} = 27.2\ mm \qquad (\text{note } \delta_{avg.} = \delta)$$

$$e_{\min} = \max\ of \begin{cases} 0.05 \times t = 0.05 \times 200 = 10\ mm \\ 20\ mm \end{cases} = 20\,mm < \delta$$

Thus the lateral deflection due to buckling is larger than the code minimum eccentricity. The post is subjected to axial force of P_u=97 kN and the additional moment equals

$$M_{add} = P_u \times \delta = 97 \times \frac{27.2}{1000} = 2.64\ kN.m$$

$$M_{tot} = M_u + M_{add} = 0 + 2.64 = 2.64\ kN.m$$

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{97 \times 1000}{30 \times 200 \times 200} = 0.08$$

$$\frac{M_u}{f_{cu} \times b \times t^2} = \frac{2.64 \times 10^6}{30 \times 200 \times 200^2} = 0.011$$

Assuming that the distance from the concrete to the c.g. of the reinforcement is 20 mm. Thus the factor ζ equals

$$\zeta = \frac{t - 2 \times cover}{t} = \frac{200 - 2 \times 20}{200} = 0.80$$

Using interaction diagram with uniform steel f_y=360 N/mm^2, and ζ=0.8
The point is below the chart use μ_{min}
Since the column is long the minimum reinforcement ratio μ_{min} equals

$$\mu_{min} = 0.25 + 0.052\ \lambda = 0.25 + 0.052 \times 16.5 = 1.1\%$$

$$A_{s,min} = \mu_{min} \times b \times t = \frac{1.1}{100} \times 200 \times 200 = 443\ mm^2$$

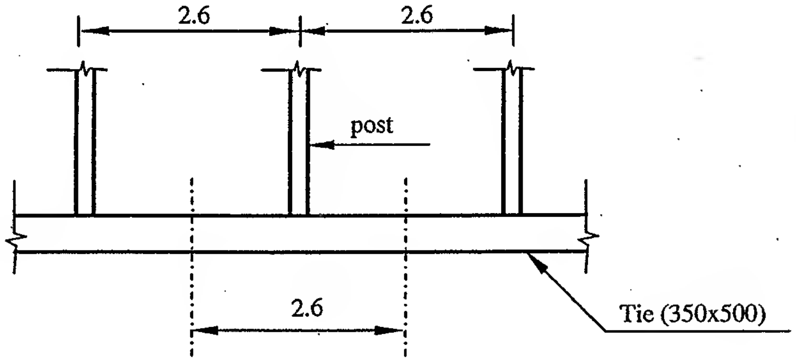
Choose (4Φ12, 452 mm^2)

Step 4.2: Design of the tension post (Hanger)

The reaction on the tension post (T)
= o.w. + Reaction from secondary beam + Tie weight

$$\text{Tie weight} = 1.4 \times \gamma_c \times b \times t \times post\ \ spacing$$

$$\text{Tie weight} = 1.4 \times 25 \times 0.35 \times 0.50 \times 2.6 = 15.925\,kN$$



$$T = 3.85 + 80.21 + 15.925 = 99.98\ kN$$

$$A_s = \frac{T}{f_y/1.15} = \frac{99.98 \times 1000}{360/1.15} = 319\ mm^2$$

Choose (4Φ12, 452 mm^2)

Step 5: Design of the arch and the tie

Step 5.1: Calculations of the loads

The total loads on the arch are the summation of the uniform and the concentrated loads.

A: Uniform loads

1. own weight of the arch

$$= 1.4 \times \gamma_c \times b \times t = 1.4 \times 25 \times 0.35 \times 1.0 = 12.25\ kN/m'$$

2. own weight of the tie

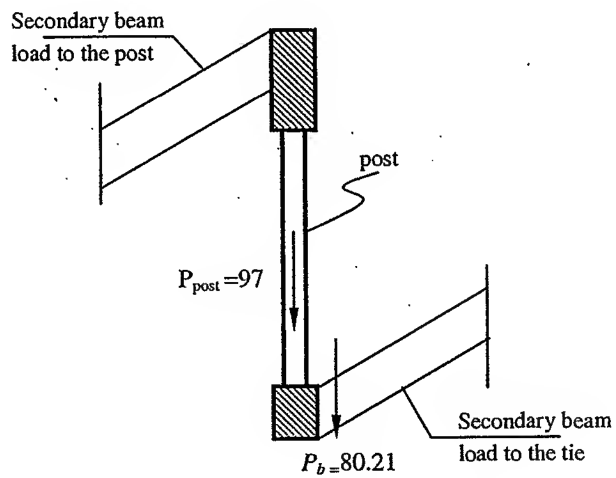
$$= 1.4 \times \gamma_c \times b \times t = 1.4 \times 25 \times 0.35 \times 0.5 = 6.125\ kN/m'$$

$$w_{o.w} = w_{arch} + w_{tie} = 12.25 + 6.125 = 18.375\ kN/m'$$

B: Concentrated loads

1. reaction from the post = 97 kN
2. reaction from the secondary beam =P_b=80.21 kN

$$P_u = P_{post} + P_b = 97 + 80.21 = 177.21\,kN$$



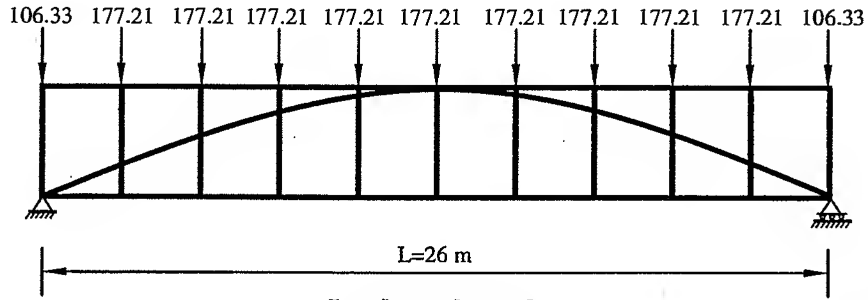
To simplify the calculations of the bending moment, the concentrated loads on the arch can be replaced by a uniform load as follows:

$$w_{eq} = \frac{\sum n \times P_u}{L} = \frac{9 \times 177.21}{26} = 61.345\,kN/m'$$

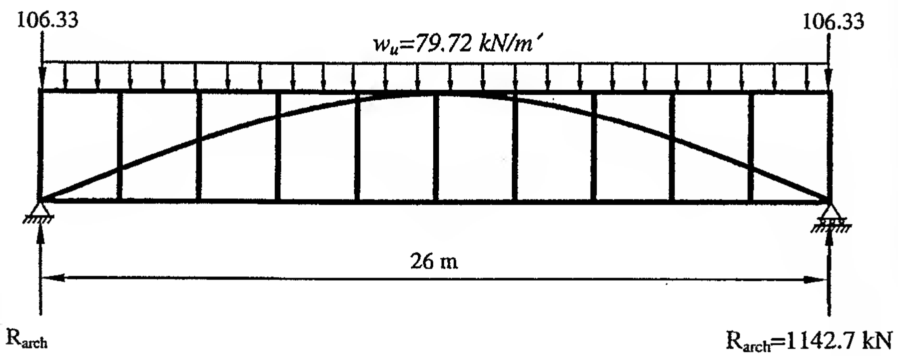
The total uniform load on the frame equals

$w_u = w_{o.w} + w_{eq} = 18.375 + 61.345 = 79.72\ \text{kN/m}'$

The concentrated loads on the sides can be estimated by 0.6 P_u=106.33 kN



Loads on the arch



Equivalent load system and reactions

The reaction from the arch to the columns equals

$$R_{arch} = 0.6 \cdot P_u + \frac{w_u \times L}{2} = 106.33 + \frac{79.72 \times 26}{2} = 1142.7\ kN$$

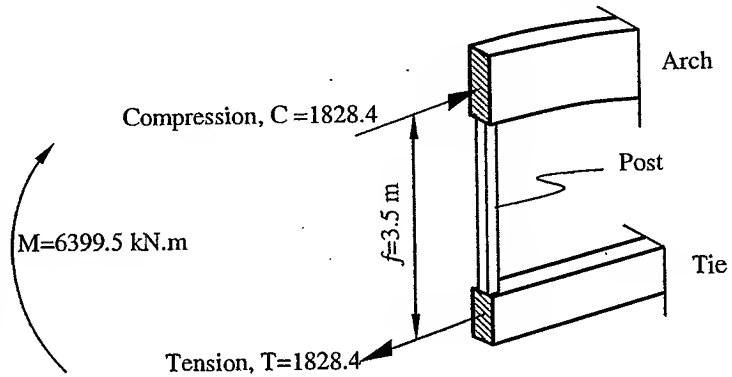
The total bending on the arch equals the simple beam bending moment $w_u L^2/8$.

$$M_{simple} = \frac{w_u \times L^2}{8} = \frac{79.71 \times 26^2}{8} = 6736\ kN.m$$

The determination of the internal force in the arch can be performed using a structural analysis program. As an approximation, the internal forces can be obtained as follows:

1. 95% of the simple bending moment is resisted by compression in the arch and tension in the tie because of the elastic deformation of the system.

$$C = T = \frac{0.95 \times M_{simple}}{f} = \frac{0.95 \times 6736}{3.5} = \frac{6399.5}{3.5} = 1828.4\ kN$$



2. 5% of the simple bending acts on the arch alone

$$M_u = 0.05 \times M_{simple} = 0.05 \times 6736 = 336.8\ kN.m$$

Step 5.2: Design of the tie (350x500)

The tie resists only tension forces=1828.4 kN

$$A_s = \frac{T}{f_y/1.15} = \frac{1828.4 \times 1000}{360/1.15} = 5840\ mm^2$$

Choose 12Φ25 (5890 mm^2) → distributed uniformly

Step 5.3: Design of main girder (350x1000)

P_u=C=1828.4 kN

M_u=336.8 kN.m

The section is subjected to a normal compression force and a bending moment. The design interaction diagram is used for the determination of the reinforcement.

$$\frac{P_u}{f_{cu}\times b\times t}=\frac{1828.4\times1000}{30\times350\times1000}=0.174$$

$$\frac{M_u}{f_{cu}\times b\times t^2}=\frac{336.8\times10^6}{30\times350\times1000^2}=0.032$$

Assuming the concrete cover is 80 mm. Thus the factor ζ equals

$$\zeta=\frac{t-2\times cover}{t}=\frac{1000-2\times80}{1000}=0.84$$

Use an interaction diagram with f_y=360 N/mm^2, and ζ=0.8 (conservative).

The intersection point is below the chart, use μ_{min} (0.006) for compression member.

$A_s=\mu\times b\times t=0.006\times350\times1000=2100\ mm^2$ for both sides

Choose (8Φ20, 2512 mm^2)

Use 4Φ20 at top of the arch and 4Φ20 at the bottom

Step 6: Design of Columns

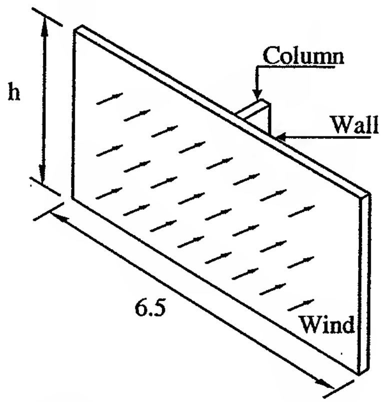
Step 6.1: Loads

The column is subjected to an axial load in addition to wind loads on the walls.

A. Wind loads

Assuming the extension of the column to the foundation is 1.0 m, the height of the column equals to:

$h=clear\ height+extention\ of\ the\ column\ to\ the\ foundation=7.5+1.0=8.5\ m$

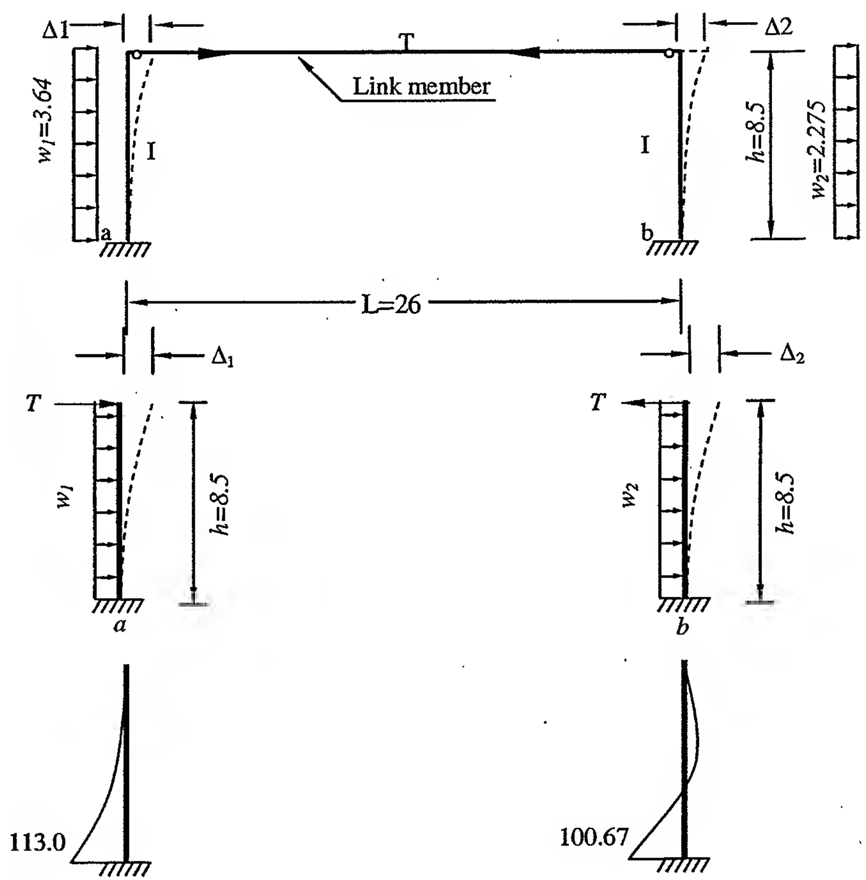


The intensity of the wind load on the walls is given as 0.7 kN/m^2. Thus, the pressure on the walls equals to:

$w_1=c_w\times q_w\times spacing=0.8\times0.7\times6.5=3.64\ kN/m'$ (windward side)

$w_2=c_l\times q_w\times spacing=0.5\times0.7\times6.5=2.275\ kN/m'$ (leeward side)

The columns are linked together with the arch-with-a-tie system. Such a system can be simulated by an *equivalent link member* subjected to either tension or compression.



The system is once statically indeterminate. The unknown is the force in the link member. Using the principle of superposition, one can obtain the deflection at the end of each column as follows:

$$\Delta = \frac{w \times h^4}{8\ EI} \pm \frac{T \times h^3}{3\ EI}$$

$$\Delta_1 = \frac{w_1 \times h^4}{8\ EI} + \frac{T \times h^3}{3\ EI} \qquad \Delta_2 = \frac{w_2 \times h^4}{8\ EI} - \frac{T \times h^3}{3\ EI}$$

Neglecting the axial deformation in the equivalent link member, the deflection of the first column Δ_1 must be equal to the lateral deflection of the second column Δ_2.

$$\Delta_1 = \Delta_2$$

$$\frac{w_1 \times h^4}{8\ EI} + \frac{T \times h^3}{3\ EI} = \frac{w_2 \times h^4}{8\ EI} - \frac{T \times h^3}{3\ EI}$$

Simplifying the terms gives the axial force in the link member (T).

$$T = \frac{3 \times h}{16}(w_2 - w_1) = \frac{3 \times 8.5}{16}(2.275 - 3.64) = -2.175\ kN \text{ (compression)}$$

$$M_1 = \frac{w_1 \times h^2}{2} + T \times h = \frac{3.64 \times 8.5^2}{2} + (-2.175) \times 8.5 = 113.0\ kN.m$$

$$M_2 = \frac{w_2 \times h^2}{2} - T \times h = \frac{2.275 \times 8.5^2}{2} - (-2.175) \times 8.5 = 100.67\ kN.m$$

B. Vertical Loads

The vertical loads on the column is the summation of the following:

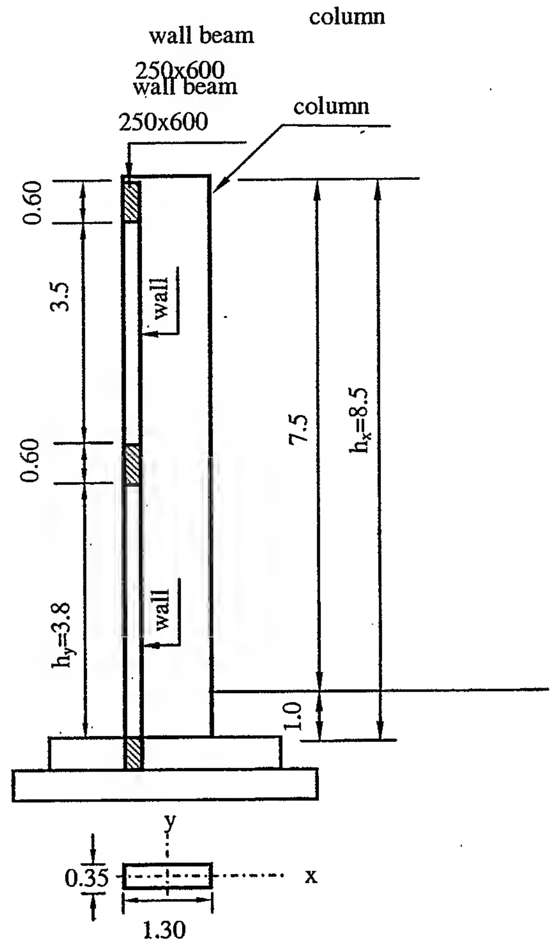
1. Self-weight $= 1.4 \times \gamma_c \times b \times t \times h = 1.4 \times 25 \times 0.35 \times 1.3 \times 8.5 = 135.36\ kN$
2. Weight of the wall beams: two wall beams are provided as shown in the following figure.

   $= 2 \times 1.4 \times \gamma_c \times b \times t \times spacing = 2 \times 1.4 \times 25 \times 0.25 \times 0.6 \times 6.5 = 68.25\ kN$
3. Wall load $= 1.4 \times \gamma_w \times b \times L \times (h - 2 \times t_{wall\ beam})$

   $= 1.4 \times 12 \times 0.25 \times 6.5 \times (8.5 - 2 \times 0.6) = 199.29\ kN$
4. Arch load $= R_{arch}$

   $$R_{arch} = 0.6 \cdot P_u + \frac{w_u \times L}{2} = 106.33 + \frac{79.72 \times 26}{2} = 1142.7\ kN$$

$$P_u = 135.36 + 68.25 + 199.29 + 1142.7 = 1545.59\ kN$$



Step 6.2: Calculation of the reinforcement

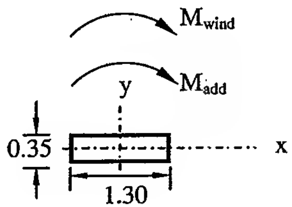
The column is considered unbraced in its plane and braced in the out-of-plane direction. The unsupported length in X-direction is 8.5 m and the unsupported length in Y-direction is 3.8 m. The calculation of the additional moment can be summarized in the following table and in the figure given below.

Item	X-Direction	Y-Direction
bracing condition	unbraced	braced
Ultimate load P_u (kN)	1545.59	1545.59
Short column if	λ<10	λ<15
H_o (m)	8.5	3.8
t (m)	1.3	0.35
k (bracing factor)	1.6 (Table 6-10)	0.90 (Table 6-9)
H_e	13.6	3.42
$\lambda = H_e / t$	10.46	9.77
Status	long(λ>10)	short (λ<15)
$\delta = \lambda^2 \times t / 2000$	0.071	0
$M_{add} = P_u.\delta$	109.95	0
M_u(wind)	113.0	0
$M_{tot} = M_u + M_{add}$	222.95	0

It is clear from the previous table that the X-direction is more critical than Y-direction. Using an interaction diagram with uniform distribution of reinforcing steel and with $f_y = 360$ N/mm^2 and $\zeta = 0.8$, one gets:

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{1545.59 \times 1000}{30 \times 350 \times 1300} = 0.113$$

$$\frac{M_{tot}}{f_{cu} \times b \times t^2} = \frac{222.95 \times 10^6}{30 \times 350 \times 1300^2} = 0.012$$



The intersection point is below the chart. Use μ_{min}.

Since the column is long, the minimum reinforcement ratio μ_{min} is given by:

$$\mu_{min} = 0.25 + 0.052\ \lambda = 0.25 + 0.052 \times 10.46 = 0.8\,\%$$

$$A_{s,min} = \mu_{min} \times b \times t = \frac{0.8}{100} \times 350 \times 1300 = 3640\ mm^2$$

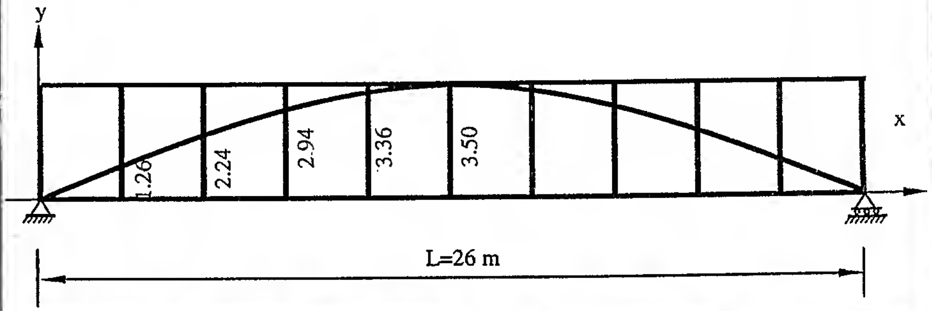
Choose (16 Φ18, 4071 mm^2) → distributed uniformly

Step 9: Geometric coordinates of the arch

The formula for the construction of the arch is given by:

$$y = \frac{4 \cdot f \cdot x \cdot (L - x)}{L^2} = \frac{4 \cdot 3.5 \cdot x \cdot (26 - x)}{26^2} = 0.0207 \cdot (26x - x^2)$$

x	2.6	5.2	7.8	10.4	13
y	1.26	2.24	2.94	3.36	3.5



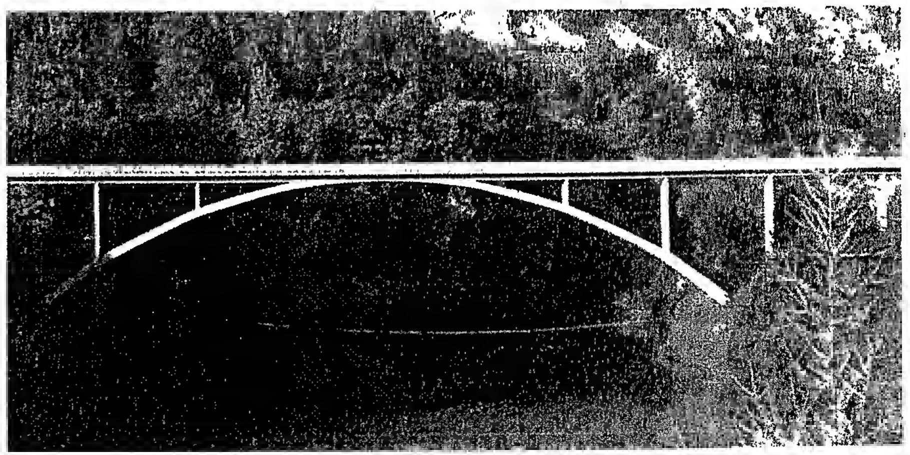
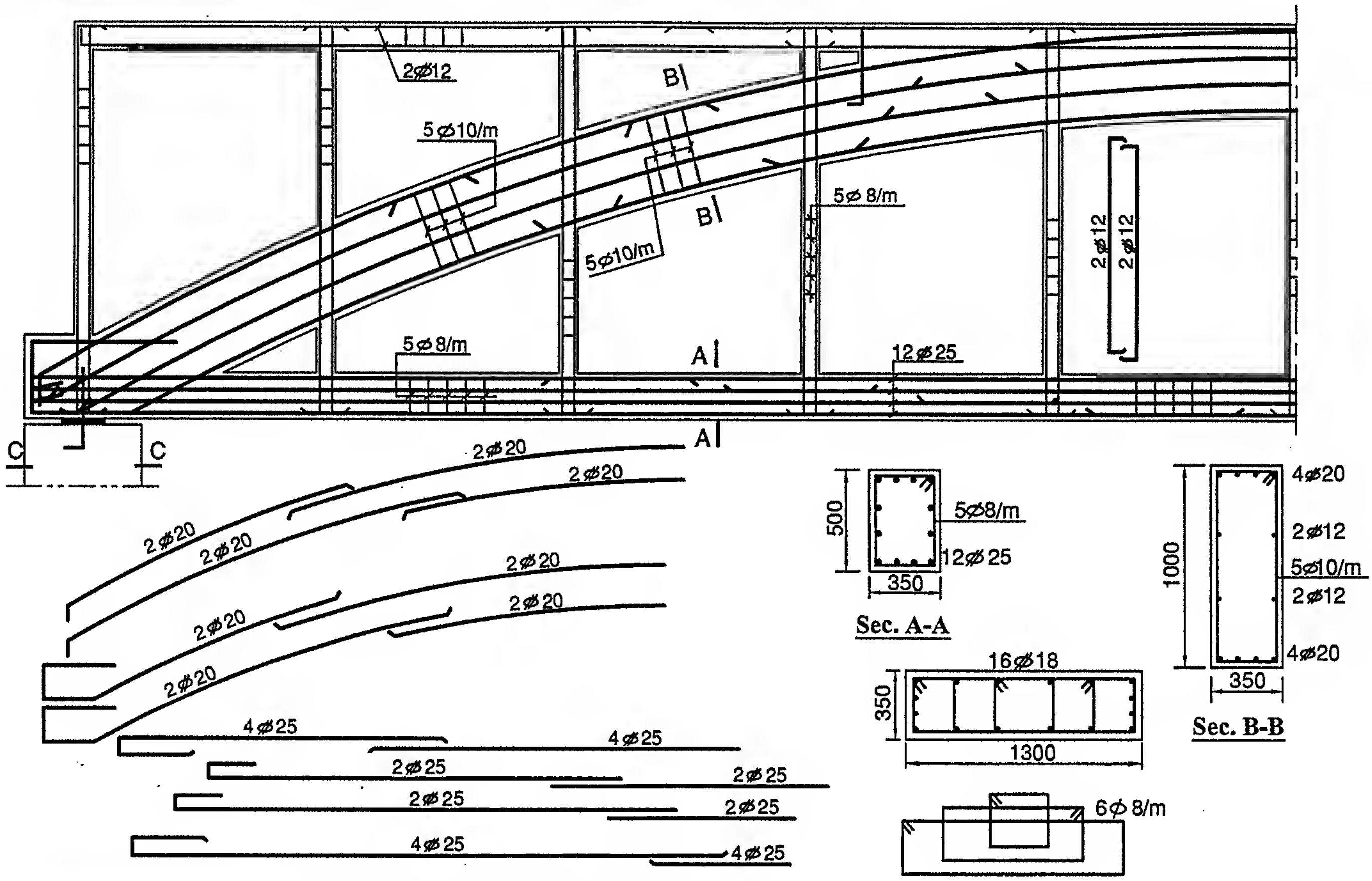
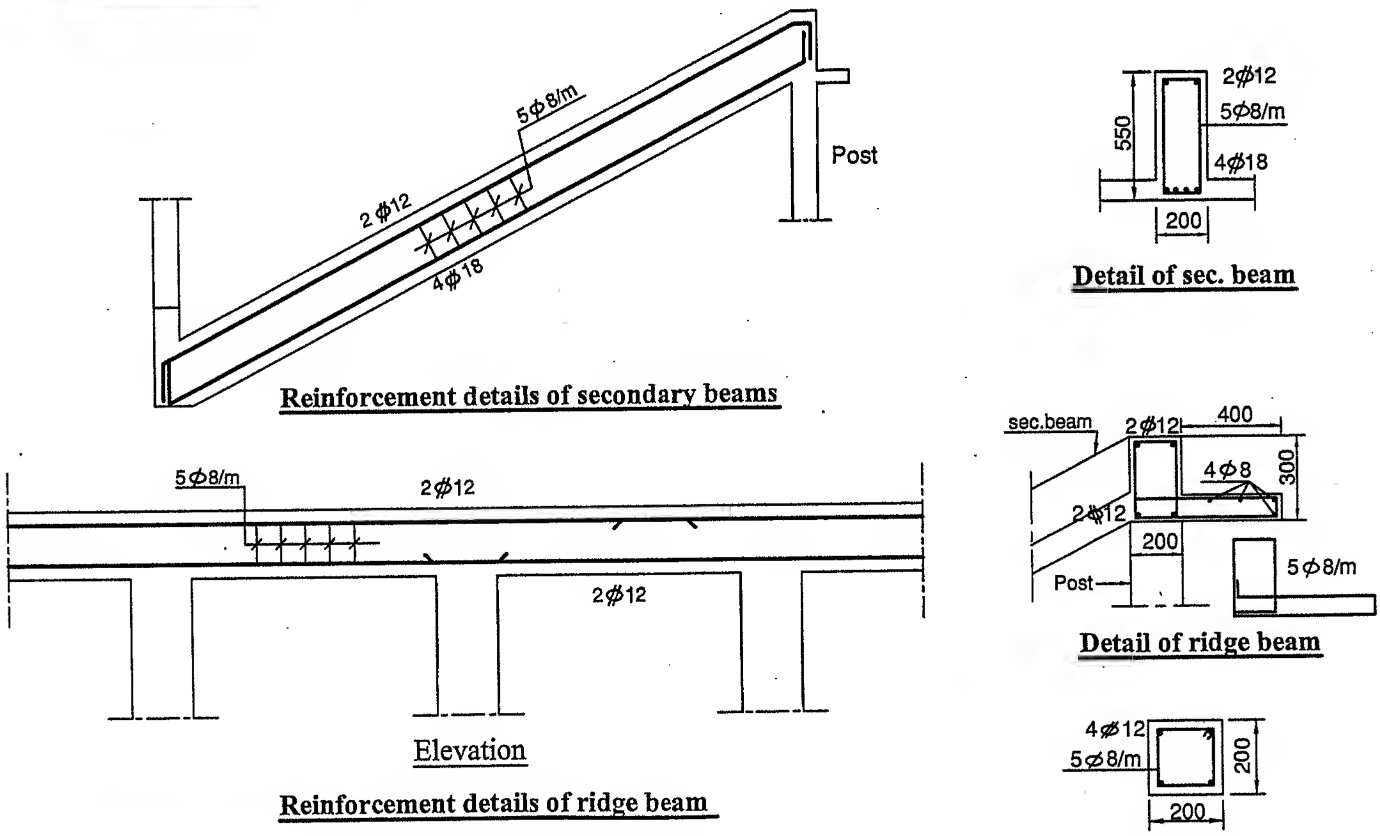


Photo 1.8 An arched bridge



Reinforcement Details of the Arch

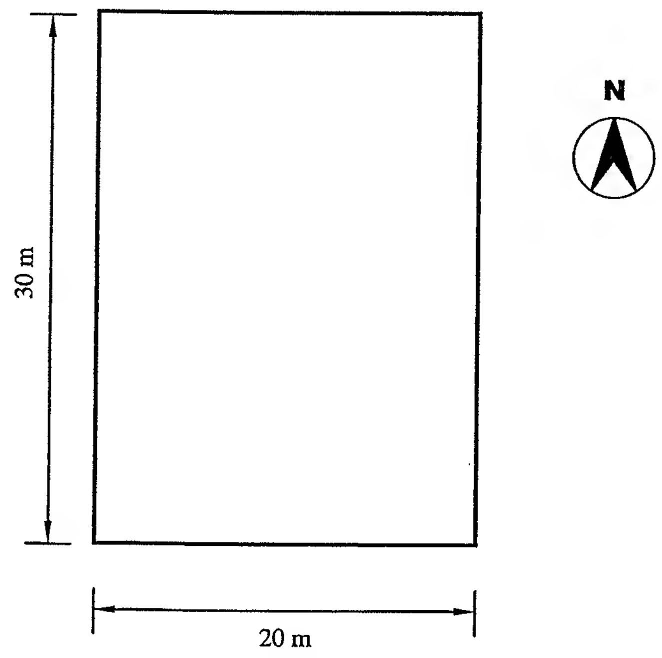


Reinforcement Details of the Arch (cont.)

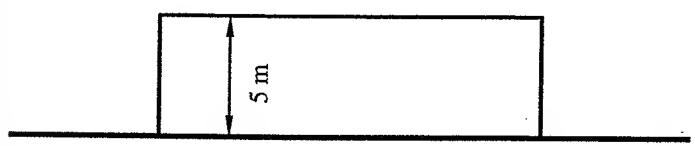
Example 1.7: Frame with the north direction is normal to span

The figure below shows the general layout of a workshop that is covered by a structural system that permits indirect lighting. It is required to carry out a complete design of the roof for such a system.

The material properties are f_{cu}=35 N/mm^2, f_y=360 N/mm^2 and f_{yst}=240 N/mm^2



Plan



Section

Solution

Step 1: Propose the concrete Dimensions

Since the north direction is perpendicular to the span and the span equals to 20.0 ms, a system of frames is chosen.
Assume the following dimensions:

t_s	= 100 mm
Ridge beam	= (200 mm x 300 mm)
Secondary beam	= (200 mm x 500 mm)
Post	= (200 mm x 200 mm)
span	= 20 m
b_g	= 0.35 m
tg = span/(12-14)	= 1.4 m
$t_{col,top}$=(0.8 t_g- t_g)	= 1.2 m
$t_{col,bot}$=(0.4 t_g- 0.6t_g)	= 0.8 m
Spacing between frames	= 5.0 m
Secondary beams spacing	=2.50 m
f= frame spacing/2	= 2.5 m

Step 2: Design of solid slabs

Assume the flooring weight is 1 kN/m^2 and that the live load is equal to 0.5 kN/m^2.

The total dead load of the slab g_s is given by:

$$g_s = t_s \times \gamma + \text{flooring} = 0.1 \times 25 + 1.0 = 3.5\ kN/m^2$$

The slab ultimate load w_u is given by:

$$w_{su} = 1.4 \times g_s + 1.6 \times p_s = 1.4 \times 3.5 + 1.6 \times .5 = 5.7\ kN/m^2$$

Taking a strip of 1.0m width $\rightarrow$ w_{su}=5.7 kN/m′

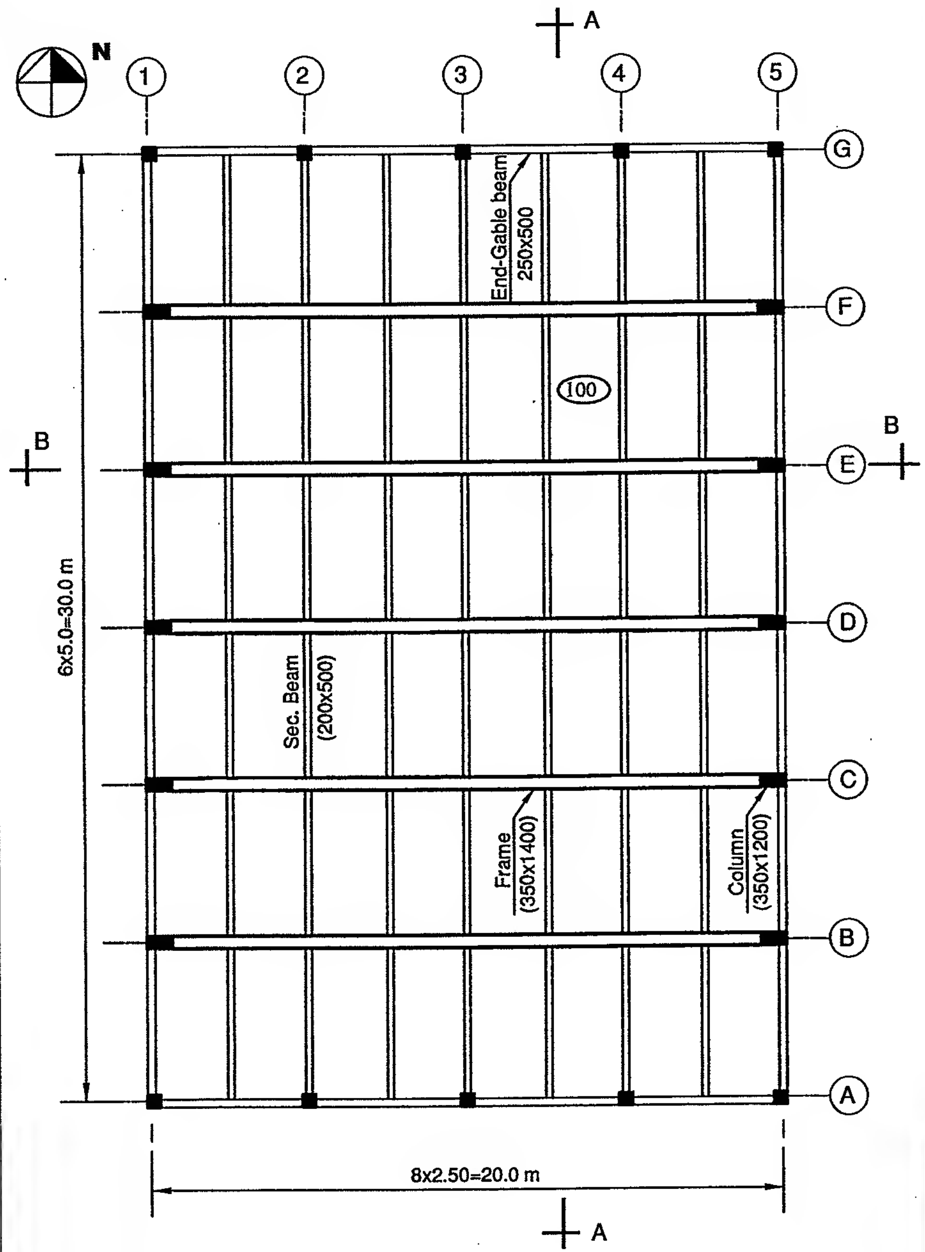
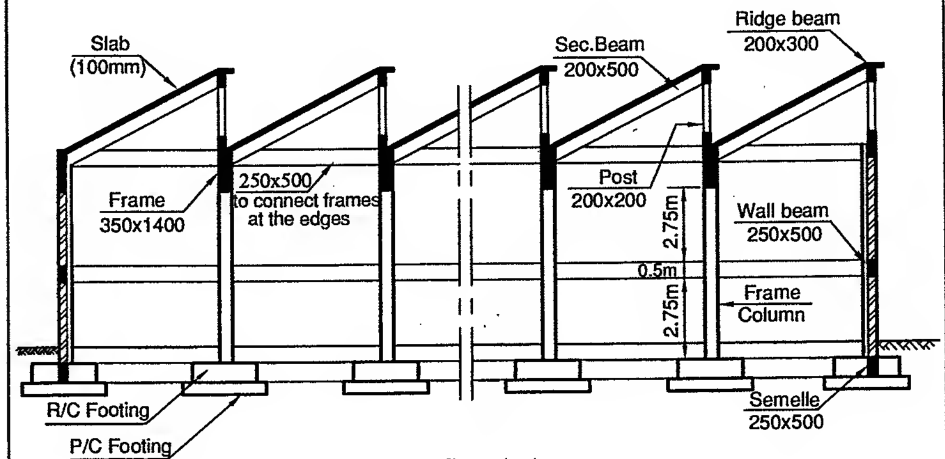
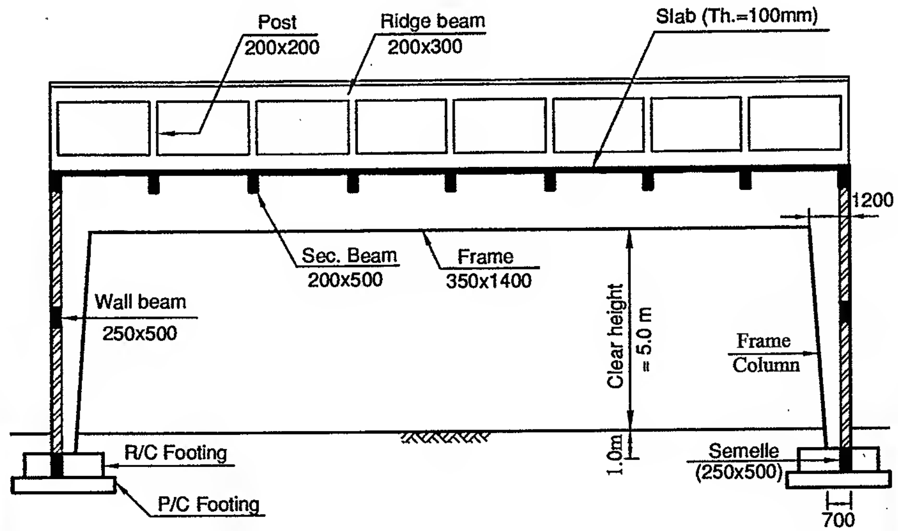


Fig. Ex 1.7a Structural System (Plan)

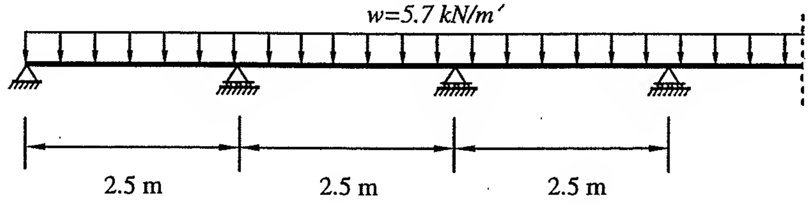


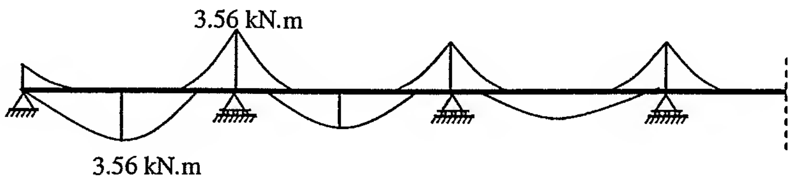
Sec. A-A



Sec. B-B

Fig. Ex 1.7b Structural System (Sections)





The roof slab is a system of one-way slabs that are continuous in the short direction, thus the maximum negative bending at the support equals to:

$$M_u = \frac{w_{su} \times L^2}{10} = \frac{5.7 \times 2.5^2}{10} = 3.56\ kN.m$$

Assuming a 20 mm cover $\rightarrow d = 100 - 20 = 80\ mm$.

Using R-ω curve, the value of R is given by:

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{3.56 \times 10^6}{35 \times 1000 \times 80^2} = 0.016$$

From the chart with R=0.016, the reinforcement index $\omega = 0.0186$

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.0186 \times \frac{35}{360} \times 1000 \times 80 = 144.8\ mm^2/m'$$

$$A_{s,\min} = \frac{0.60}{f_y} \times b \times d = \frac{0.6}{360} \times 1000 \times 80 = 133.3 mm^2/m'$$

Choose 5Φ8/m′ (250 mm^2)

Step 3: Design of the secondary beam (200 mm x 500 mm)

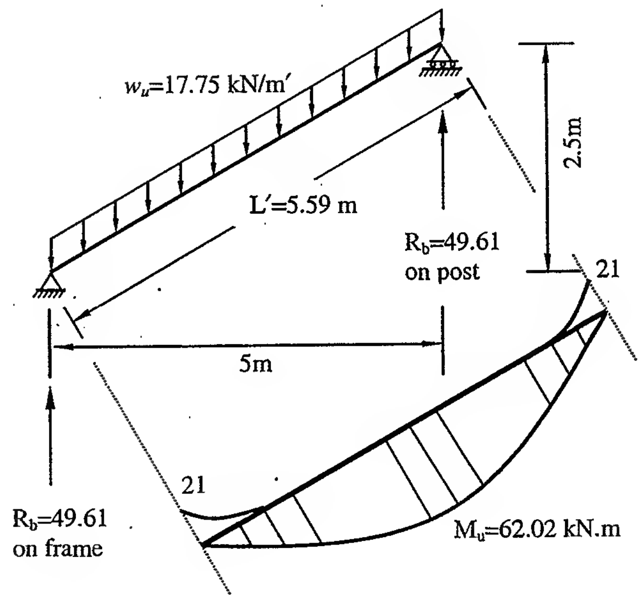
The beam cross-section is 200 mm x 500 mm (from step 1).

The ultimate self-weight of the beam equals to:

$$w_{u,ow} = 1.4 \times \gamma_c \times b \times t = 1.4 \times 25 \times 0.20 \times 0.50 = 3.5\ kN/m'$$

The spacing between the secondary beams is *2.5m*, thus the total beam load is

$$w_u = w_{u,o.w} + spacing \times w_{su} = 3.5 + 5.7 \times 2.5 = 17.75\ kN/m'$$



The inclined length L′ is equal to

$$L' = \sqrt{2.5^2 + 5^2} = 5.59\ m$$

$$M_u = \frac{w_u \times L \times L'}{8} = \frac{17.75 \times 5.0 \times 5.59}{8} = 62.02\ kN.m$$

The reaction of the secondary beam is

$$R_b = \frac{w_u \times L'}{2} = \frac{17.75 \times 5.59}{2} = 49.61\ kN$$

The section at mid-span is a T-section and the width B is given by:

$$B = the\ smaller\ of \rightarrow \begin{cases} 16t_s + b \\ \dfrac{L}{5} + b \\ CL \rightarrow CL \end{cases} = \begin{cases} 16\times100+200=1800\ mm \\ \dfrac{5590}{5}+200=1318\ mm \\ 2500\ mm \end{cases}$$

B= 1318 mm.Using C1-J curve and assume that $c < t_s$

$$C1 = d / \sqrt{\frac{M_u}{f_{cu} \times B}} = 450 / \sqrt{\frac{62.02\times10^6}{35\times1318}} = 12.27$$

The point is outside the curve, thus $c/d)_{min} = 0.125$ and $j = 0.825$

$$a = 0.8\times c = 0.8\times0.125\times450 = 45\ mm$$

Since $a < t_s$ *(100 mm)*, the assumption is valid.

$$A_s = \frac{M_u}{f_y \times J \times d} = \frac{62.02\times10^6}{360\times0.825\times450} = 464\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{35}}{360}\times200\times450 = 333\ mm^2 \\ 1.3\ A_s = 1.3\times464 = 603\ mm^2 \end{cases}$$

Choose 3Φ16 (600 mm^2)

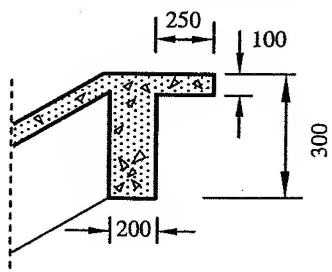
Step 4: Design of ridge beam (200 x 300 mm)

The ultimate self-weight of the beam equals

$$w_{u,o.w} = 1.4\times\gamma_c \times b \times t = 1.4\times25\times0.20\times0.30 = 2.1\ kN/m'$$

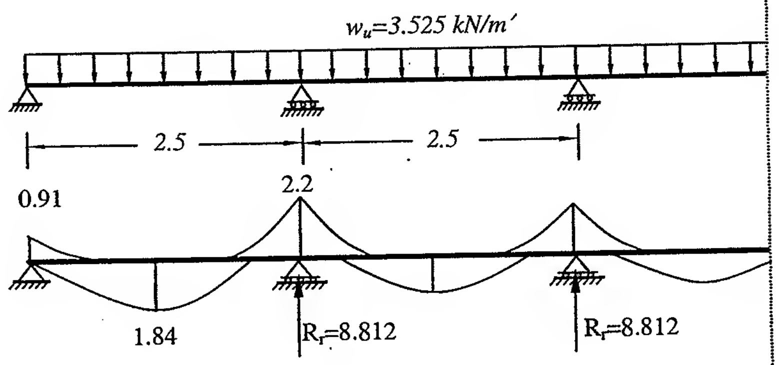
No slab load is transferred to the ridge beam. The weight of the cantilever part is calculated as shown in the following figure.

$$w_u = w_{u,o.w} + w_u \times cantilever\,length = 2.1 + 5.7\times0.25 = 3.525\ kN/m'$$



$$M_u = \frac{w_u \times L^2}{10} = \frac{3.525\times2.5^2}{10} = 2.20\ kN.m$$

$$R_r = w_u \times L = 3.525\times2.5 = 8.8125\ kN$$



To use the R-ω, calculate R

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{2.20\times10^6}{35\times200\times250^2} = 0.005$$

From the chart with R=0.005, the reinforcement index $\omega = 0.0058$.

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.0058\times\frac{35}{360}\times200\times250 = 28.3\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{35}}{360}\times200\times250 = 185\ mm^2 \\ 1.3A_s = 1.3\times28.3 = 36\ mm^2 \end{cases}$$

but not less than $\dfrac{0.15}{100}\times b\times d = \dfrac{0.15}{100}\times200\times250 = 75\,mm^2$

Thus, $A_{s,min}$=75 mm^2 > A_s, use A_{smin}

Use 2Φ12 (226.2 mm^2)

Step 5: Design of the post (200x200 mm)

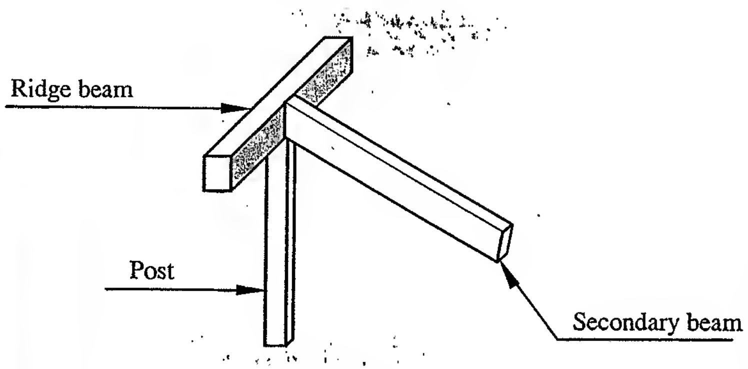
The factored self-weight of the post (200 x 200 mm) equals to:

$$P_{o.w} = 1.4 \times \gamma_c \times b \times t \times h = 1.4 \times 25 \times 0.20 \times 0.20 \times 2.5 = 3.5\ kN$$

The post supports loads from the ridge beam and from the secondary beam.

$$P_u = P_{o.w} + P_b\,(\text{secandry beam}) + P_r\,(\text{ridge beam})$$

$$P_u = 3.5 + 49.61 + 8.8125 = 61.92\ kN$$



$$h = f - t_{ridge} - \text{clearance} = 2.5 - 0.3 - 0.3 = 1.9\,m$$

The building is considered unbraced thus $H_e = 1.2 \times 1.9 = 2.28\,m$

$$\lambda = \frac{H_e}{b} = \frac{2.28}{0.2} = 11.4 > 10 \rightarrow \text{long member}$$

$$\delta = \frac{\lambda^2 \times t}{2000} = \frac{11.4^2 \times 0.2}{2000} = 0.013\ m < 0.02\,m \quad \rightarrow \quad \delta_{av} = \delta$$

$$M_{add} = P_u \times \delta = 61.92 \times 0.02 = 1.23\,kN.m$$

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{61.92 \times 1000}{35 \times 200 \times 200} = 0.044, \quad \frac{M_u}{f_{cu} \times b \times t^2} = \frac{1.23 \times 10^6}{35 \times 200 \times 200^2} = 0.004$$

The point is below the interaction diagram, use A_{smin}.

$$\mu = 0.25 + 0.052 \times \lambda = 0.84\%$$

$$A_{s,min} = 0.0084 \times A_c = 0.008 \times 200 \times 200 = 337\ mm^2$$

Use 4Φ12 (452 mm²)

Step 6: Design of the frame (350 mm x 1400 mm)

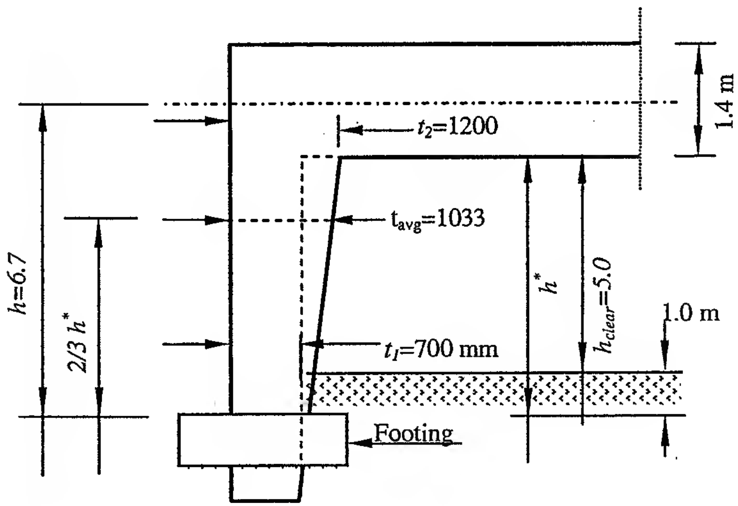
Step 6.1: Dimensioning

The thickness of the column at the top is taken as (0.8-1 t_g) and at the bottom as (0.4-0.6 t_g). The thickness of the column at the top is taken as 1200 mm and at the bottom as 700 mm. The height of the column of the frame h is measured from the top of the footing (*or semelle*) to the center-line of the frame girder.

$$h = \text{clear height} + \frac{t_g}{2} + h_f = 5 + \frac{1.4}{2} + 1.0 = 6.70\,m$$

The frame column has a variable moment of inertia. To simplify the calculations, an average column width at $2/3h^*$ is used.

$$t_{avg} = t_1 + \frac{2}{3}(t_2 - t_1) = 700 + \frac{2}{3}(1200 - 700) = 1033.33\ mm$$



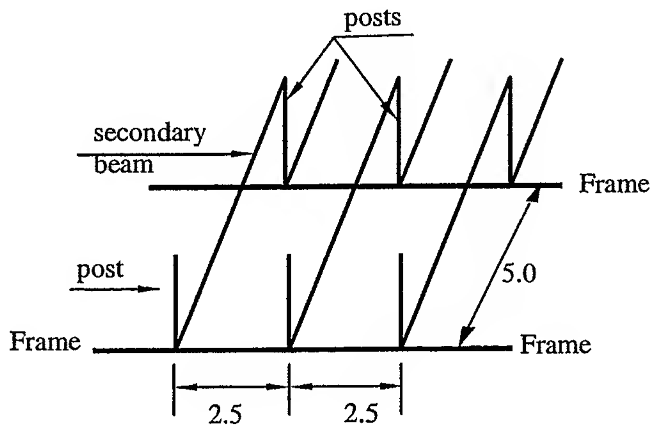
Step 6.2: Calculation of loads

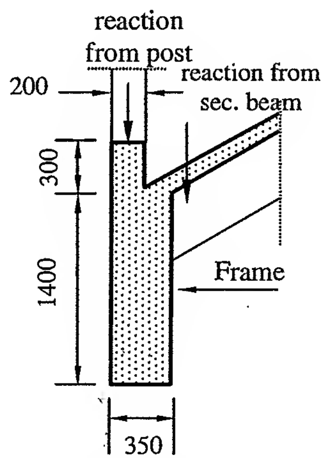
The self-weight of the frame equals

$$w_{u,ow} = 1.4 \times \gamma_c \times (b_1 \times t_1 + b_2 \times t_2) = \frac{1.4 \times 25}{1000 \times 1000}(200 \times 300 + 350 \times 1400) = 19.25\ kN/m'$$

The loads on the frame results from the reactions of the secondary beam and the post every 2.5m causing concentrated loads on these locations.

$$P_u = P_{post} + P_{sec} = 61.925 + 49.61 = 111.54\ kN$$





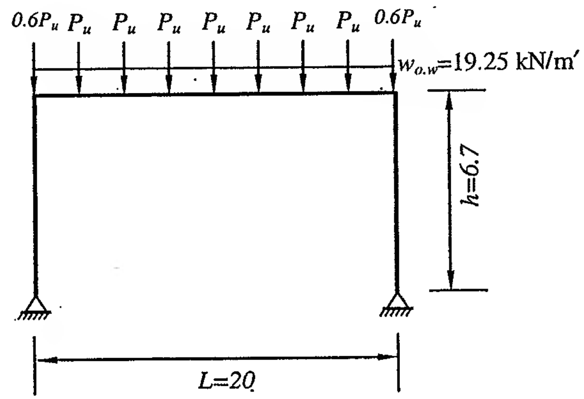
To simplify the calculations of the bending moment, the concentrated loads on the frame can be replaced into uniform load as follows:

$$w_{eq} = \frac{\sum P_u}{L} = \frac{7 \times 111.54}{20} = 39.04\ kN/m'$$

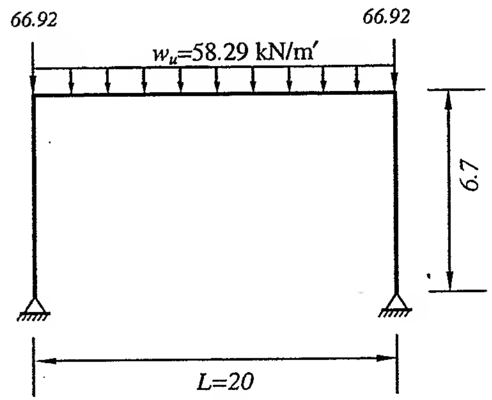
The total uniform load on the frame equals to:

$$w_u = w_{o.w} + w_{eq} = 19.25 + 39.04 = 58.29\ \text{kN/m}'$$

The concentrated loads on the sides can be estimated by 0.6 P_u=66.92 kN



Actual loading system



Equivalent loading system

Step 6.3: Calculation of straining actions

The frame is two-hinged and is once statically indeterminate. The horizontal reaction at the base for uniformly loaded frame is given by

$$H_a = H_b = \frac{w_u \times L^2}{4 \times h \times N}$$

where $K = \frac{I_b}{I_c} \times \frac{h}{L}$ and $N = 2K + 3$

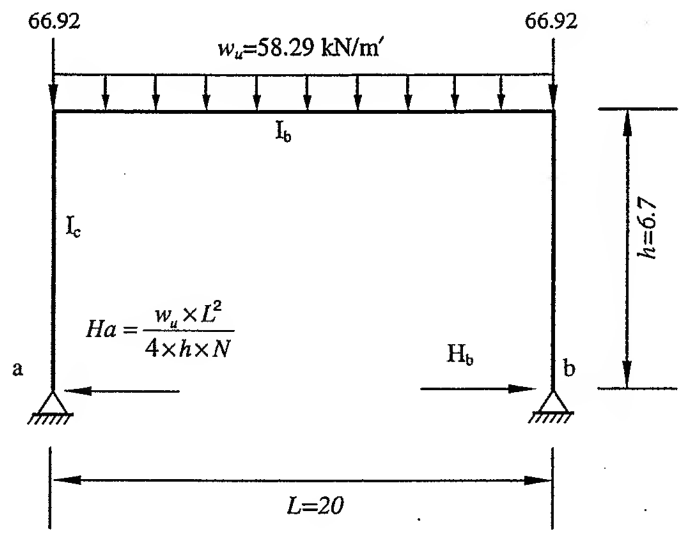
The moment of inertia for the column is calculated using t_{avg}

$$I_c = \frac{b \times t_{avg}^3}{12} = \frac{0.350 \times 1.033^3}{12} = 0.0322 m^4$$

$$I_b = \frac{b \times t^3}{12} = \frac{0.35 \times 1.4^3}{12} = 0.08 \ m^4$$

$$K = \frac{I_b}{I_c} \times \frac{h}{L} = \frac{0.08}{0.0322} \times \frac{6.7}{20} = 0.833$$

$$N = 2K + 3 = 2 \times 0.833 + 3 = 4.666$$



$$H_a = H_b = \frac{w_u \times L^2}{4 \times h \times N} = \frac{58.29 \times 20^2}{4 \times 6.7 \times 4.666} = 186.44 \ kN$$

The vertical reaction can be easily obtained as follows:

$$Y_a = \frac{w_u \times L}{2} + 0.6 \times P_u = \frac{58.29 \times 20}{2} + 66.92 = 649.81 \ kN$$

The moment at top of the column M_{col}

$= H_a \times h_{avg} = 186.44 \times 6.7 = 1249.15$ kN

The maximum moment at mid span of the frame can be obtained as follows:

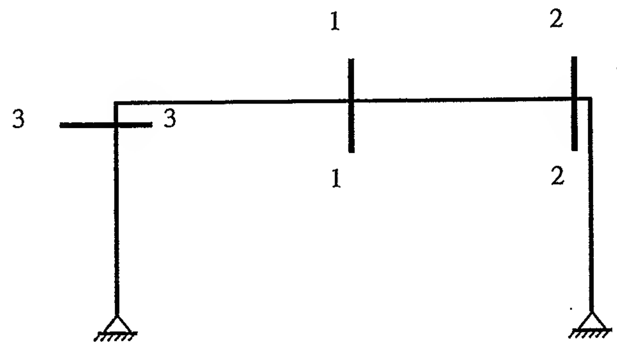
$$M_{mid} = \frac{w_u \times L^2}{8} - M_{col}$$

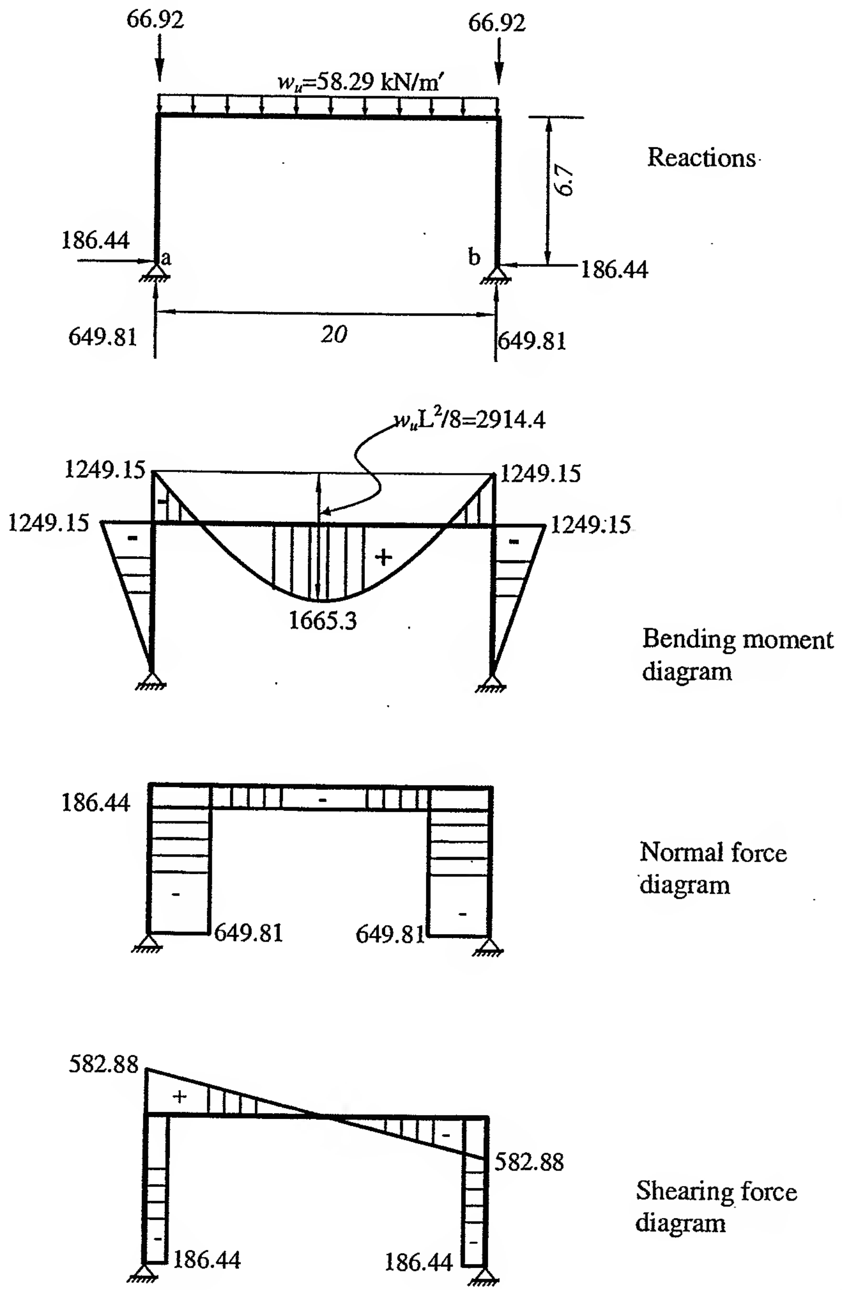
$$M_{mid} = \frac{58.29 \times 20^2}{8} - 1249.15 = 1665.3 \text{ kN.m}$$

The bending moment, shear force, and normal force diagrams for the frame are presented in the next page.

Step 6.4: Design of frame sections

The critical sections are shown in the figure below.





Straining actions for the frame

Step 6.4.1: Design of section 1 (350 mm x 1400 mm)

Section 1 is a rectangular section that is subjected to the following factored actions:

M_u=1665.3 kN.m

P_u=186.44 kN (compression)

According to the ECP 203; if ($P_u/f_{cu}\ b\ t$) is less than 0.04, the normal force can be neglected.

$$\frac{P_u}{f_{cu}\times b\times t}=\frac{186.44\times1000}{35\times350\times1400}=0.0108<0.04 \text{ ....neglect the normal force}$$

The design will be carried out as if the section is subjected to bending only. Frames are usually heavily reinforced and the reinforcing bars are arranged in two rows. Therefore, the effective depth is given by:

$$d=t-100\text{mm}=1400-100=1300\ mm$$

To use the R-ω, calculate R

$$R=\frac{M_u}{f_{cu}\times b\times d^2}=\frac{1665.3\times10^6}{35\times350\times1300^2}=0.0804$$

From the chart with R=0.0804, the reinforcement index $\omega=0.1028$

$$A_s=\omega\times\frac{f_{cu}}{f_y}\times b\times d=0.1028\times\frac{35}{360}\times350\times1300=4546\ mm^2$$

$$A_{s\min}=smaller\ \ of\begin{cases}\dfrac{0.225\sqrt{f_{cu}}}{f_y}b\ d=\dfrac{0.225\sqrt{35}}{360}\times350\times1300=1682\ mm^2\\ 1.3\ A_s=1.3\times4546=5910\ mm^2\end{cases}$$

$A_s > A_{s,min}$ →→o.k.

Use 10Φ25 (4908 mm²)

Step 6.4.2: Design of section 2 (350 mm x 1400 mm)

Section 2 is a rectangular section that is subjected to the following factored actions:

M_u= 1249.15 kN.m P_u=186.44 kN (compression)

According to the ECP 203; if ($P_u/f_{cu}\ b\ t$) is less than 0.04, the normal force can be neglected.

$$\frac{P_u}{f_{cu}\times b\times t}=\frac{186.44\times1000}{35\times350\times1400}=0.0108<0.04\ \text{........neglect normal force}$$

The design is carried out as if the section is subjected to bending only.
To use the R-ω, calculate R

$$R=\frac{M_u}{f_{cu}\times b\times d^2}=\frac{1249.15\times10^6}{35\times350\times1300^2}=0.0603$$

From the chart with R = 0.0603, the reinforcement index ω = 0.075

$$A_s=\omega\times\frac{f_{cu}}{f_y}\times b\times d=0.075\times\frac{35}{360}\times350\times1300=3310\ mm^2$$

$$A_{s\min}=smaller\ \ of\begin{cases}\frac{1.1}{f_y}b\ d=\frac{1.1}{360}\times350\times1300=1390\ mm^2\ \lrcorner<A_s\quad o.k\\ 1.3\ A_s=1.3\times3310=4303\ mm^2\end{cases}$$

Use 7Φ25 (3436 mm²)

Step 6.4.3: Design of section 3 (350 mm x 1200 mm)

Buckling in the out-of-plane direction

The frame is considered unbraced because the lack of any bracing system.

As shown in Fig. EX. 1.7b, H_o=2.75. The effective length factor k can be obtained from ECP 203 with case (1) at the top and at the bottom. Thus, k=1.2.

$$H_e=k\times H_o=1.2\times2.75=3.30\ m$$

$$\lambda=\frac{H_e}{b}=\frac{3.30}{0.35}=9.4<10\ \text{(case of unbraced columns)}$$

Thus no additional moments are induced in the out-of-plane direction.

Buckling in the in-plane direction

The frame is considered unbraced because the lack of any bracing system. The effective length factor k can be obtained from the ECP 203. The top part of the column is considered case (1) and the bottom part is considered case (3) (hinged base). Thus, k=1.6.

The height of the column is measured from the bottom of the beam to the base (h^*). However, it is customary to use the length used in the analysis h.

$$H_e=k\times h=1.6\times6.7=10.72\ m$$

The slenderness ratio λ is calculated using the average column thickness not the actual one, thus λ equals

$$\lambda=\frac{H_e}{t_{avg}}=\frac{10.72}{1.033}=10.37$$

Since λ is greater than 10, the column is considered long and additional moment is developed.

$$\delta=\frac{\lambda^2\times t_{avg}}{2000}=\frac{10.37^2\times1.033}{2000}=0.0556\ m\ \rightarrow\ (\delta_{av}=\delta)$$

$$e_{\min}=0.05\times t_{avg}=0.05\times1.033=0.052\ m<\delta$$

Thus the lateral deflection due to buckling is larger than the code minimum eccentricity. The column is subjected to axial force of P_u=649.8 kN. The additional moment equals

$$M_{add}=P_u\times\delta=649.8\times0.0556=36.13\ kN.m$$

$$M_{tot}=M_u+M_{add}=1249.15+36.13=1285.28\ kN.m$$

Due to the fact that column sections are subjected to large normal force, it is recommended to use compression steel between 40%-60% of the tension steel to ensure ductile behavior. The section is designed with the interaction diagram (α=0.6).

$$\frac{P_u}{f_{cu}\times b\times t}=\frac{649.8\times1000}{35\times350\times1200}=0.0442$$

$$\frac{M_u}{f_{cu}\times b\times t^2}=\frac{1285.28\times10^6}{35\times350\times1200^2}=0.073$$

Assuming that the distance from the concrete to the c.g. of the reinforcement is 80 mm. Thus the factor ζ equals

$$\zeta = \frac{t - 2\times cov\,er}{t} = \frac{1200 - 2\times 80}{1200} = 0.86$$

Using interaction diagram with f_y=360 N/mm², α=0.6 and ζ=0.8 (conservative)

$$\rho = 2.1$$

$$\mu = \rho \times f_{cu} \times 10^{-4} = 2.1\times 35\times 10^{-4} = 0.00735$$

$$A_s = \mu \times b \times t = 0.00735\times 350\times 1200 = 3087\ mm^2 \quad (7\Phi 25,\ 3436\ mm^2)$$

$$A_s' = \alpha \cdot A_s = 0.6\times 3087 = 1852\ mm^2 \quad (4\Phi 25,\ 1963\ mm^2)$$

$$A_{s,total} = A_s' + A_s = 3436 + 1963 = 4939\ mm^2$$

Since the column is long the minimum reinforcement ratio is given by:

$$\mu_{min} = 0.25 + 0.052\ \lambda = 0.25 + 0.052\times 10.37 = 0.789\ ,\ \text{use}\ \mu_{min}=0.008$$

$$A_{s,min} = 0.008\times b \times t = 0.008\times 350\times 1200 = 3360\ mm^2 < A_{s,tot} ....o.k$$

Step 6.4.4: Design of section 4 (350x700)

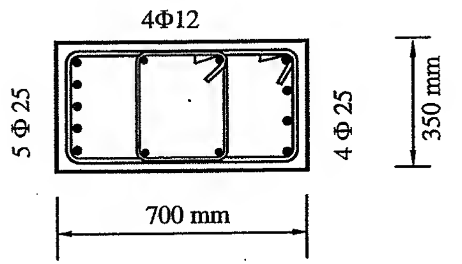
This section is subjected to a compression force (P_u=649.8 kN) and can be reinforced with the minimum area of steel.

$$A_{s,min} = 0.008\times b \times t = 0.008\times 350\times 700 = 1960\ mm^2$$

$$P_u = 0.35\times f_{cu}\times A_c + 0.67\times f_y \times A_{sc}$$

$$P_u = \frac{1}{1000}(0.35\times 35\times(350\times 700) + 0.67\times 360\times 1960) = 3474\,kN\ > (649.8).....o.k$$

Use 9Φ25, 4415 mm² >A_{smin}

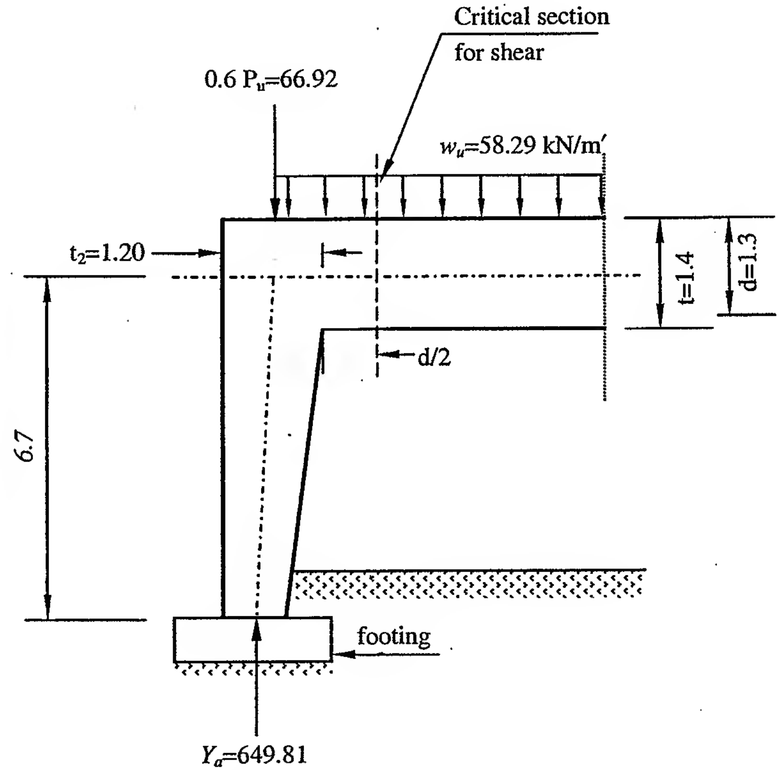


Step 7: Design for shear

The critical section for shear is at d/2 from the face of the column. Thus the design force Q_u equals to:

$$Q_u = Y_a - P_{edge} - w_u\left(\frac{t_2}{2} + \frac{d}{2}\right) = 649.81 - 66.92 - 58.29\times\left(\frac{1.2}{2} + \frac{1.3}{2}\right) = 510.02\ kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{510.02\times 1000}{350\times 1300} = 1.12\ \text{N/mm}^2$$



The presence of the compression force increases the shear capacity of the beam, however, this force is relatively small that its effect can be neglected (*conservative*)

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{35}{1.5}} = 1.15\ N/mm^2$$

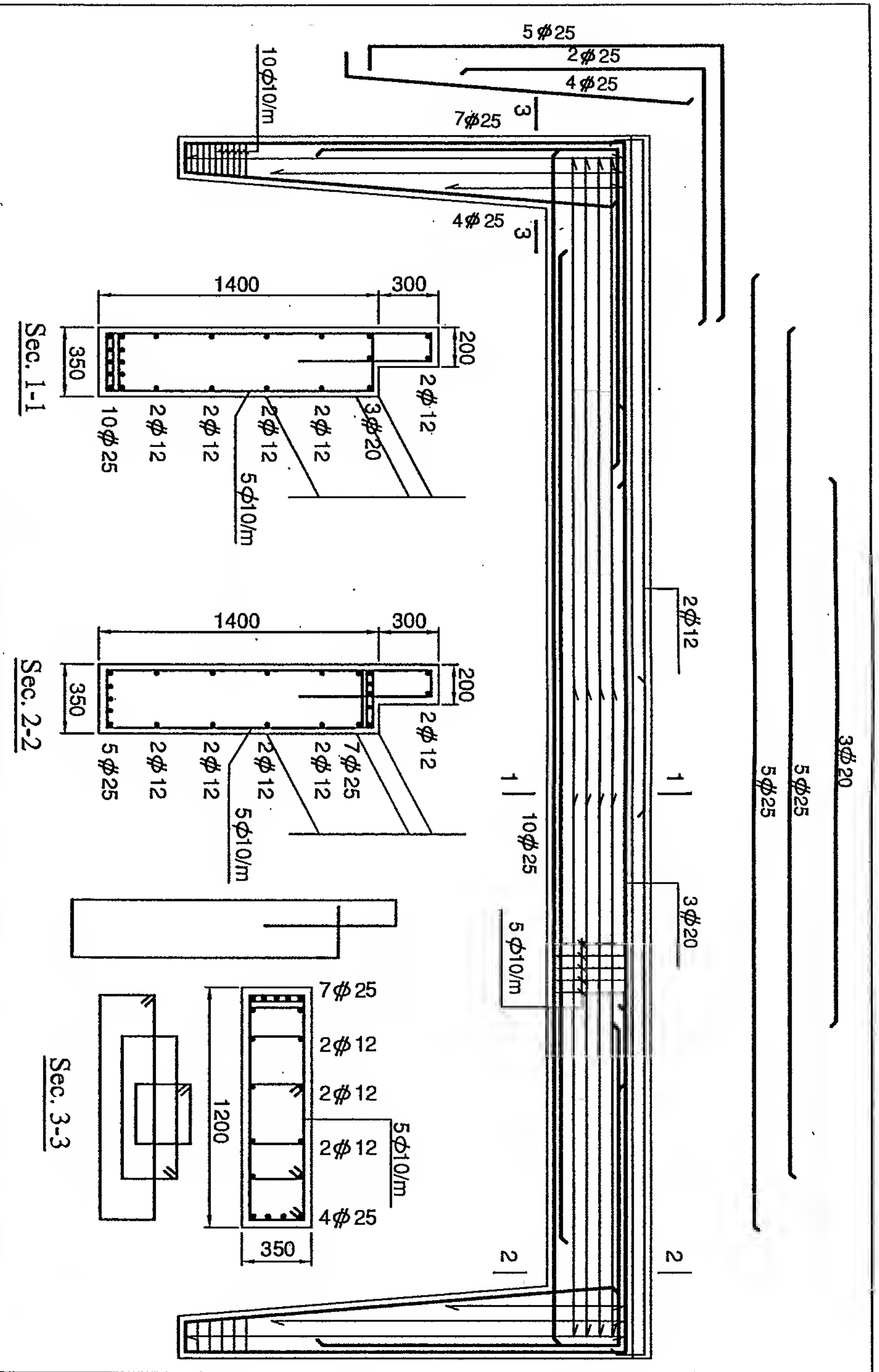
Since $q_u < q_{cu}$, provide minimum amount of stirrups.

Assume that stirrups spacing is 200 mm

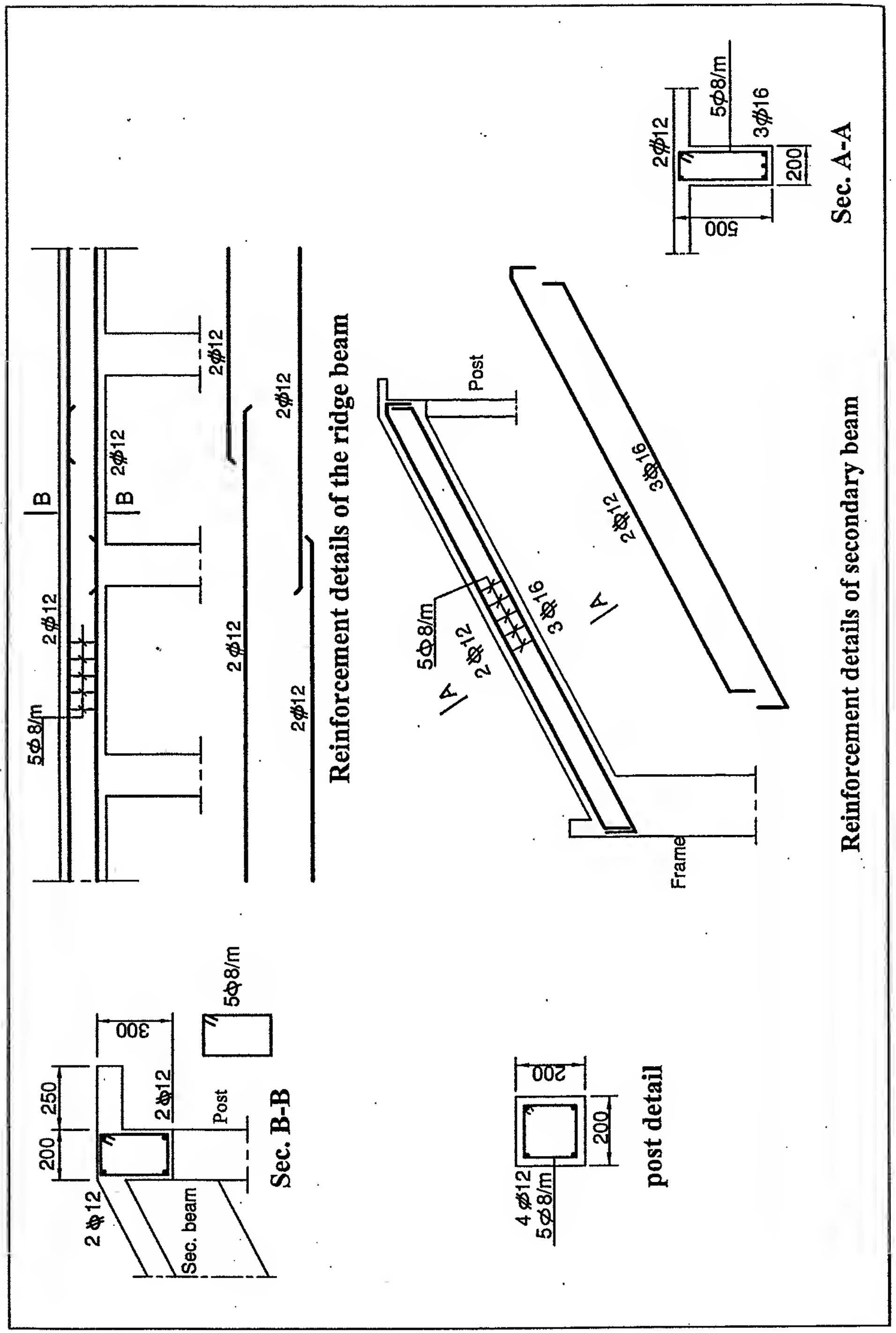
$$A_{st,\min} = \frac{0.4}{f_y} \times b \times s = \frac{0.4}{240} \times 350 \times 200 = 117\ mm^2$$

$A_{st,min}$ is the area of two branches. For one branch, A_{st}=58 mm^2 (ϕ10=78.5 mm^2)

Choose ϕ 10@ 200 mm (5ϕ10/m′)



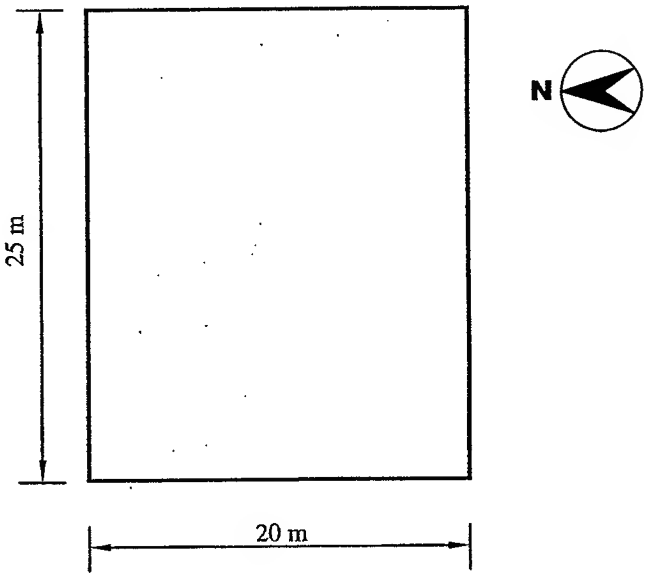
Reinforcement Details of the Frame



Example 1.8: Frame with north direction parallel to the span

The figure given below shows the general layout of a factory covered by a structural system that permits indirect lighting. The main structural system of the workshop is reinforced concrete frames. It is required to carry out a complete design of the roof of such a system together with the frame.

The material properties are f_{cu}=30 N/mm^2, f_y=400 N/mm^2 and f_{yst}=280 N/mm^2



Plan

Section

Solution

Step 1: Propose the concrete dimensions

Since the north is parallel to the span, a system of frames with Y-beams is chosen.

Assume the following dimensions

t_s	= 100 mm
Ridge beam	= (200 mm ×550 mm)
Secondary beam	= (200 mm × 500 mm)
Y-beam	= (200/300 mm × 800 mm)
Post	= (200 mm × 200 mm)
b_g	= 350 mm
Span	= 20 m
t_g = span/(12→14)	= 1.6 m
$t_{col,top}$=(0.8 t_g→t_g)	= 1.4 m
$t_{col,bot}$=(0.4 t_g→ 0.6t_g)	= 0.8 m
Spacing between frames	= 5.0 m
Spacing between Y-beams	= 5.0 m
Secondary beams spacing	=2.50 m
f = frame spacing/2	= 2.5 m

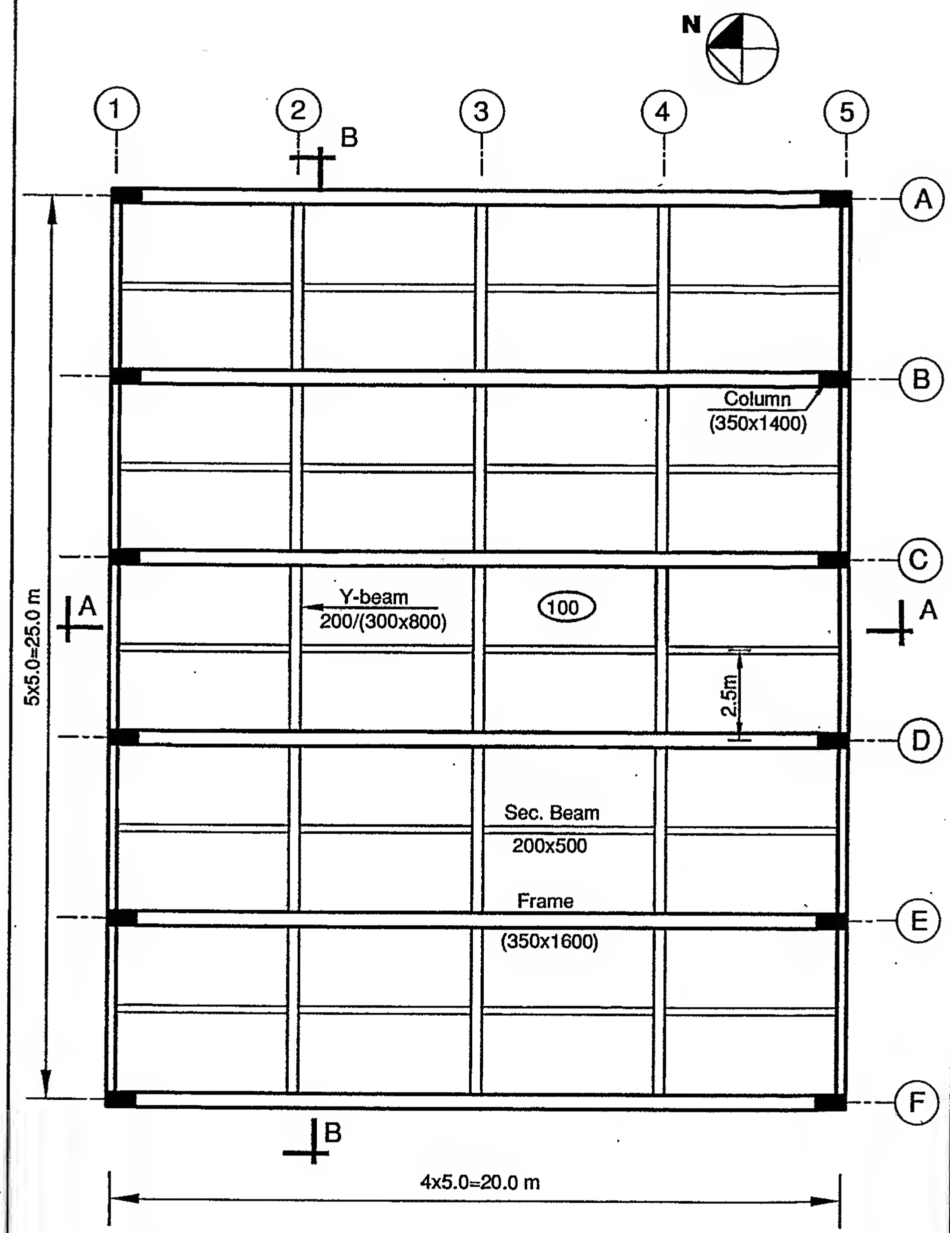
Step 2: Design of the solid slabs

Assuming that the flooring load is 1.0 kN/m^2, the total dead load of the slab g_s is given by

$$g_s = t_s \times 25 + flooring = 0.1 \times 25 + 1.0 = 3.5\ kN/m^2$$

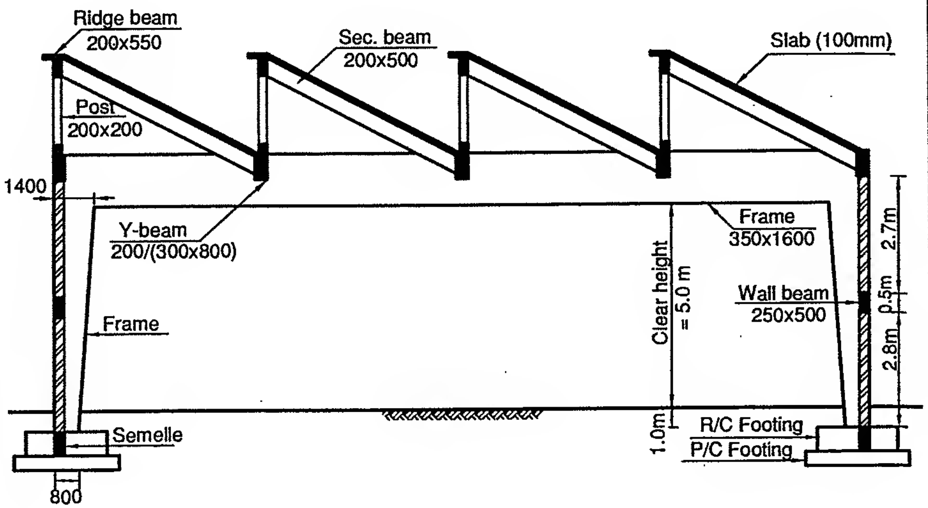
Assuming that the live load is equal to 0.5 kN/m^2, the ultimate load w_{su} is given by

$$w_{su} = 1.4 \times g_s + 1.6 \times p_s = 1.4 \times 3.5 + 1.6 \times .5 = 5.7\ kN/m^2$$

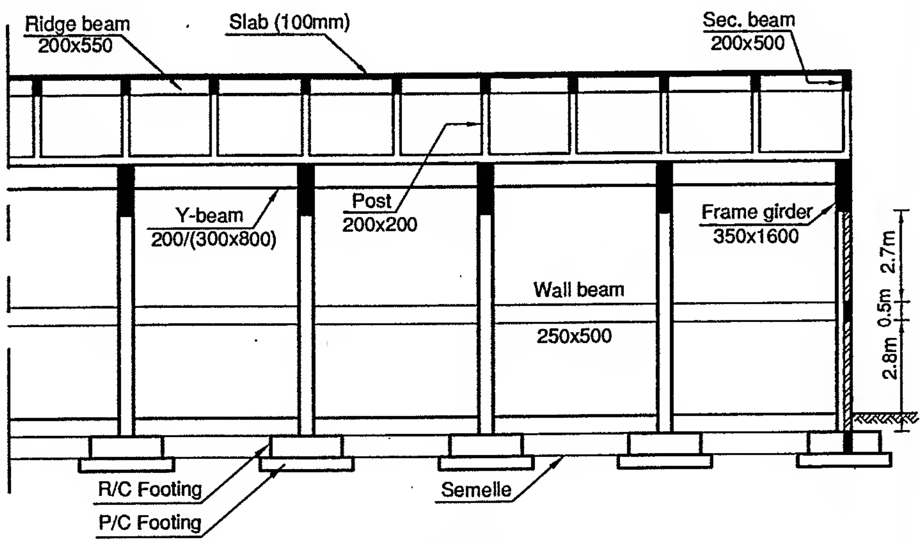


Plan

Fig. Ex 1.8a Structural system (plan)



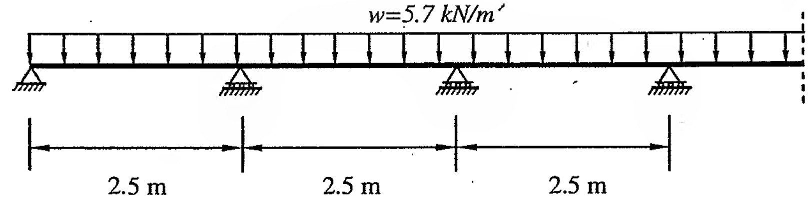
Section A-A

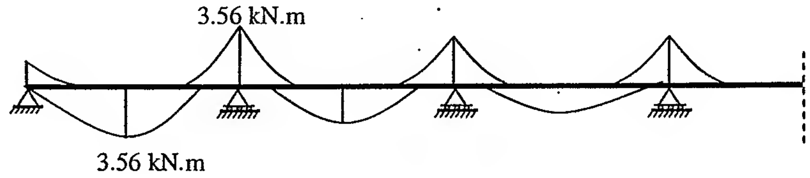


Section B-B

Fig. Ex 1.8b Structural system (sections)

The roof is a system of one-way slabs that are continuous in the short direction. Thus, the maximum moment can be obtained as shown in the following figure.





$$M_u = \frac{w_{su} \times L^2}{10} = \frac{5.7 \times 2.5^2}{10} = 3.56 \; kN.m$$

Assuming 20 mm cover, the effective depth $d = 100 - 20 = 80 \; mm$

Taking a strip of 1.0 m width and using R-ω curve, the value of R is given by:

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{3.56 \times 10^6}{30 \times 1000 \times 80^2} = 0.0185$$

From the chart with R=0.0185, the reinforcement index $\omega = 0.022$

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.022 \times \frac{30}{400} \times 1000 \times 80 = 132 \; mm^2 / m$$

$$A_{s,\min} = \frac{0.15}{100} \times 1000 \times 80 = 120 \, mm^2 / m \; < A_s \ldots \text{o.k}$$

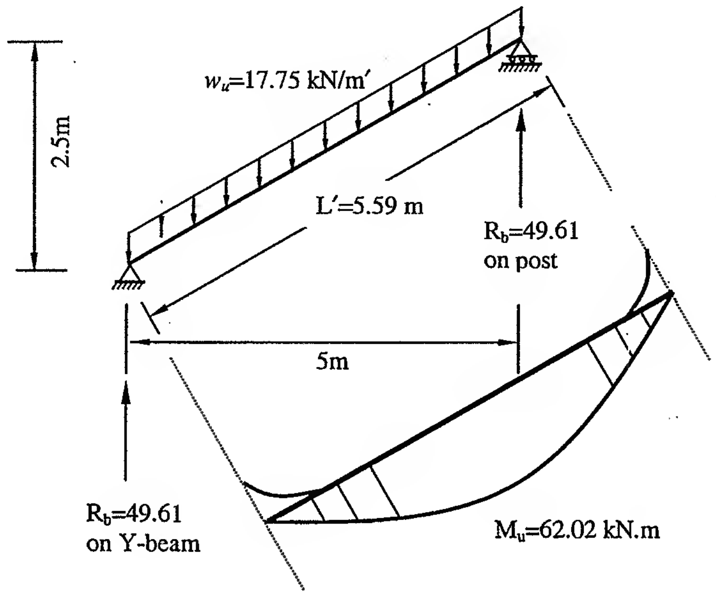
Choose 5Φ8/m′(250 mm^2)

Step 3: Design of the secondary beam (200 mm x 500 mm)

Assume that the beam -ection is 200 mm x 500 mm (from step 1).

The factored self-weight of the beam equals

$$w_{u,o.w} = 1.4 \times 25 \times b \times t = 1.4 \times 25 \times 0.20 \times 0.50 = 3.5 \; kN / m'$$



The spacing between secondary beams is *2.5m,* thus the total beam load is given by:

$$w_u = w_{u,o.w} + spacing \times w_{su} = 3.5 + 5.7 \times 2.5 = 17.75 \ kN/m'$$

The inclined length L′ is equal to

$$L' = \sqrt{2.5^2 + 5^2} = 5.59 \ m$$

$$M_u = \frac{w_u \times L \times L'}{8} = \frac{17.75 \times 5.0 \times 5.59}{8} = 62.02 \ kN.m$$

The reaction of the secondary beam is

$$R_b = \frac{w_u \times L'}{2} = \frac{17.75 \times 5.59}{2} = 49.61 \ kN$$

The section at midspan is a T-section and the effective width B is taken as:

$$B = the\ smaller\ of \rightarrow \begin{cases} 16t_s + b \\ \dfrac{L}{5} + b \\ CL \rightarrow CL \end{cases} = \begin{cases} 16 \times 100 + 200 = 1800 \ mm \\ \dfrac{5590}{5} + 200 = 1318 \ mm \\ 2500 \ mm \end{cases}$$

B=1318 mm

Using C-J curve, and assuming $a < t_s$

$$C1 = d / \sqrt{\frac{M_u}{f_{cu} \times B}} = 450 / \sqrt{\frac{62.01 \times 10^6}{30 \times 1318}} = 11.36$$

The point is located outside the curve, thus $c/d)_{min}$=0.125 and j=0.825

$$a = 0.8 \times c = 0.8 \times 0.125 \times 450 = 45 \ mm$$

Since $a < t_s$ *(100 mm)*, the assumption is valid.

$$A_s = \frac{M_u}{f_y \times J \times d} = \frac{62.01 \times 10^6}{400 \times 0.825 \times 450} = 417.6 \ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{400} \times 200 \times 450 = 277 \ mm^2 \\ 1.3A_s = 1.3 \times 417 = 542 \ mm^2 \end{cases}$$

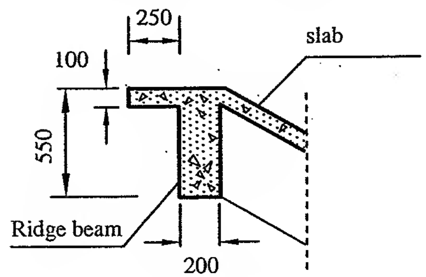
Choose 3Φ16 (600 mm^2)

Step 4: Design of the ridge beam (200 mm x 550 mm)

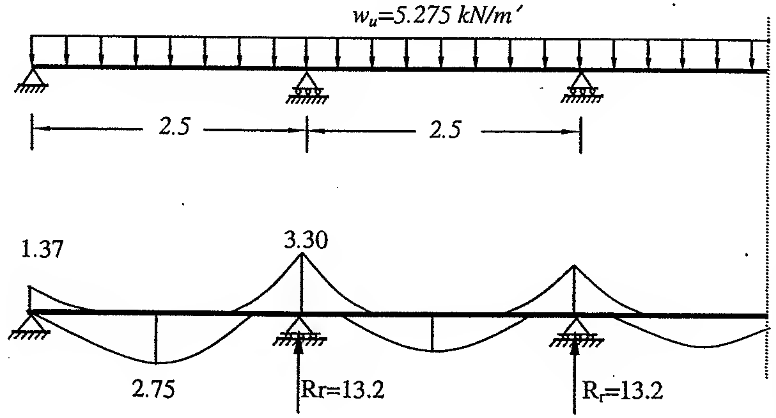
The cross-section of the ridge beam is (200x550 mm). It is a continuous beam that is supported on posts.
The ultimate self-weight of the beam equals

$$w_{u,o.w} = 1.4 \times 25 \times b \times t = 1.4 \times 25 \times 0.20 \times 0.55 = 3.85 \ kN/m'$$

No slab load is transferred to the ridge beam. The weight of the cantilever part is calculated as shown in the figure below.



$$w_u = w_{u,o.w} + w_u \times cantilever\, length = 3.85 + 5.7 \times 0.25 = 5.275\, kN/m'$$



$$M_u = \frac{w_u \times L^2}{10} = \frac{5.275 \times 2.5^2}{10} = 3.30\, kN.m$$

$$R_r = w_u \times L = 5.275 \times 2.5 = 13.1875\, kN$$

To use the R-ω, calculate R

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{3.30 \times 10^6}{30 \times 200 \times 500^2} = 0.0022$$

The point is located outside the charts. For small values of R the reinforcement index ω may be equal to 1.2R, $\omega = 0.0026$

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.0026 \times \frac{30}{400} \times 200 \times 500 = 19.5\, mm^2$$

$$A_{s\,min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\, d = \dfrac{0.225\sqrt{30}}{400} \times 200 \times 500 = 308\, mm^2 \\ 1.3 A_s = 1.3 \times 19.5 = 25.4\, mm^2 \end{cases}$$

but not less than $\frac{0.15}{100} \times b \times d = \frac{0.15}{100} \times 200 \times 500 = 150\, mm^2$

Thus, $A_{s,min}$=150 mm², Use 2Φ12 (226.2 mm²)

Step 5: Design of the post (200 mm x 200 mm)

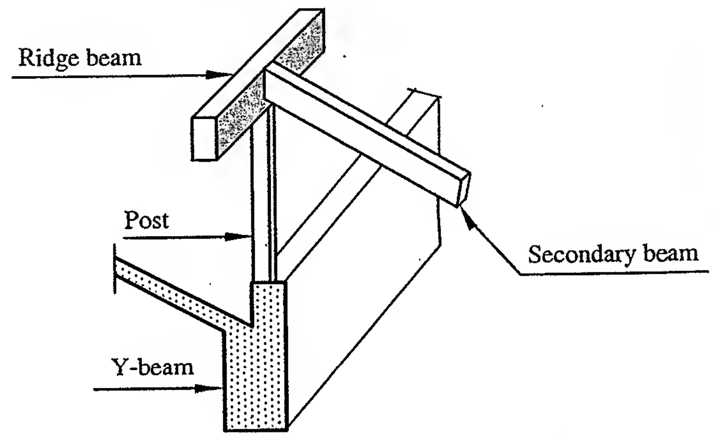
The factored self weight of the post (200 mm x 200 mm) equals to:

$$P_{o.w} = 1.4 \times 25 \times b \times t \times h = 1.4 \times 25 \times 0.2 \times 0.2 \times 2.5 = 3.5\, kN$$

The post supports loads from the ridge beam and from the secondary beam.

$$P_u = P_{o.w} + P_b(secandry beam) + P_r(ridge beam)$$

$$P_u = 3.5 + 49.61 + 13.1875 = 66.3\, kN$$



$$h = f - t_{ridge} - \text{Top part of Y-beam} = 2.5 - 0.55 - 0.3 = 1.65\, m$$

The building is considered unbraced thus $H_e = 1.2 \times 1.65 = 1.98\, m$

$$\lambda = \frac{H_e}{b} = \frac{1.98}{0.2} = 9.9 < 10 \rightarrow \text{short member}$$

The axial capacity for the post equals

$$P_u = 0.35 \times f_{cu} \times A_c + 0.67 \times f_y \times A_{sc}$$

Subsisting for P_u and computing the required area of steel

$$66.3 \times 1000 = 0.35 \times 30 \times 200 \times 200 + 0.67 \times 400 \times A_{sc}$$

A_s = negative (use $A_{s,min}$ for the post)

$$A_{s,min} = 0.008 \times A_c = 0.008 \times 200 \times 200 = 320\, mm^2$$

Use 4Φ12 (452 mm²)

Step 6: Design of the Y-beam (200/300 mm x 800 mm)

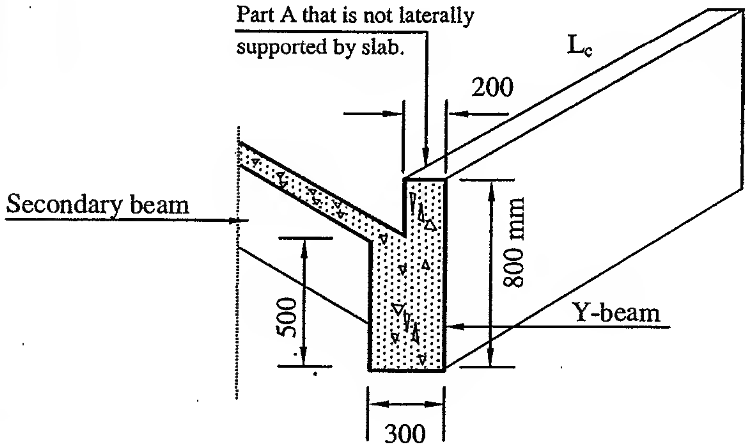
Step 6.1: Loads and straining actions

The cross-section of the Y-beam is shown below. The effective cross-section of the beam can be taken as (300 mm x 500 mm) or (200 mm x 800 mm). The last choice is more economical because it permits larger depth.

Lateral torsional buckling of the compression flange might occur because the upper part of the beam is not connected to the slab (*part A*). To avoid that, the ECP 203 requires that the unsupported length between the inflection points be less than

$$L_{c,\max} = \textit{the smaller of} \rightarrow \begin{cases} 40b = 40 \times 200 = 8000\ mm \\ \dfrac{200 \times b^2}{d} = \dfrac{200 \times 200^2}{750} = 10666\, mm \end{cases}$$

Since the span of the beam is 5000 mm, it is accepted to use b=200 mm.



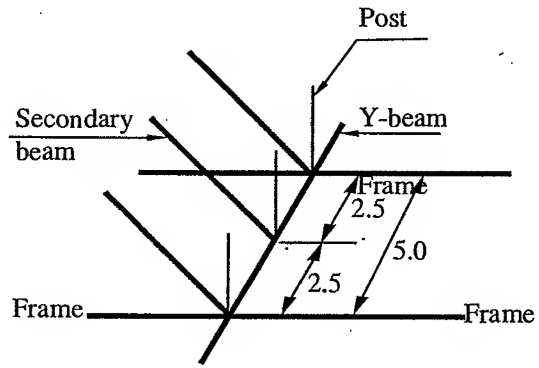
The factored self-weight equals to

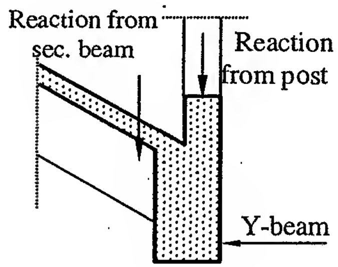
$$w_{u,o.w} = 1.4 \times \gamma_c \times (b_1 \times t_1 + b_2 \times t_2)$$

$$w_{u,o.w} = 1.4 \times 25(0.2 \times 0.80 + 0.10 \times 0.50) = 7.35\ kN/m'$$

The loads on the Y-beam result from the secondary beam and fro the post every 2.5 m, causing concentrated loads at these locations

$$P_u = 66.3 + 49.61 = 115.91\ kN$$





The Y-beam is a continuous beam having more than three equal spans. The reactions and the bending moments can be determined using a computer program or a simplified analysis. Using the simplified analysis, the bending moments can be computed as the superposition of the bending moments due to the concentrated loads and those due the uniform loads. Theses values can be obtained in text books of structural analysis.

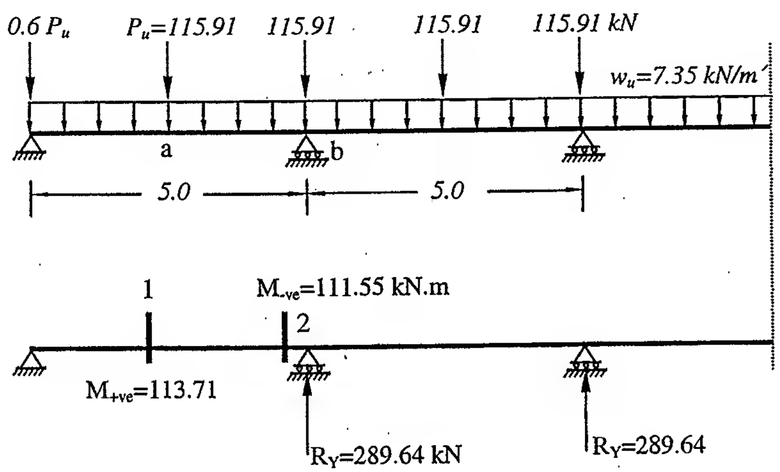
The value of the bending moment at the support due to the concentrated load is (P_u x L /6.22), while that at mid-span is (P_u x L /5.89). On the other hand the values of the bending moments due to uniform loads at the support and at mid-span are $wL^2/10$ and $wL^2/12$, respectively.

$$M_{b(-ve)} = \frac{w_u \times L^2}{10} + \frac{P_u \times L}{6.22} = \frac{7.35 \times 5^2}{10} + \frac{115.91 \times 5}{6.22} = 111.55\ kN.m$$

$$M_{a(+ve)} = \frac{w_u \times L^2}{12} + \frac{P_u \times L}{5.89} = \frac{7.35 \times 5^2}{12} + \frac{115.91 \times 5}{5.89} = 113.71\ kN.m$$

The maximum reaction at any interior support due to the concentrated loads and due to the uniform loads are equal to (2.15 P_u x L) and (1.1 w_u x L), respectively.

$$R_y = 1.1 \times w_u \times L + 2.15 \times P_u = 1.1 \times 7.35 \times 5 + 2.15 \times 115.91 = 289.64\ kN$$



Step 6.2: Calculation of the reinforcement (Sec-1)

Since the upper part of the Y-beam is not attached to slab, all sections are designed as Rectangular sections with maximum moment of 113.71 kN.m

To use the R-ω, calculate R

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{113.71 \times 10^6}{30 \times 200 \times 750^2} = 0.0337$$

From the chart with R = 0.0337, the reinforcement index $\omega = 0.0403$

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.0403 \times \frac{30}{400} \times 200 \times 750 = 454\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{400} \times 200 \times 750 = 462\ mm^2 \\ 1.3A_s = 1.3 \times 454 = 590\ mm^2 \end{cases}$$

Use 4Φ14 (615 mm^2)

Since the bending moment at section 2 is very close to that of section 1, the same reinforcement is used.

Step 6.2: Design for shear

The shear on the Y-beam can be calculated as follows:

$$Q_u = \frac{w_u\, L}{2} + \frac{P_u}{2} + \frac{M_{-ve}}{L} = \frac{7.35 \times 5}{2} + \frac{115.91}{2} + \frac{111.55}{5} = 98.65\, kN$$

The critical section is at d/2 from the frame girder

$$Q_u = 98.65 - 7.35 \times (\frac{0.35}{2} + \frac{0.75}{2}) = 94.6\ kN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{94.6 \times 1000}{200 \times 750} = 0.63\, N/mm^2$$

$$q_{cu} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

Since $q_u < q_{cu}$, provide minimum stirrups. Assume a spacing of 200 mm.

$$A_{st,min} = \frac{0.4}{f_y} b \times s = \frac{0.4}{280} \times 200 \times 200 = 57\ mm^2 \qquad \text{(for two branches)}$$

Area for one branch =28.5 mm^2 (ϕ 8mm = 50 mm^2)

Use 5ϕ8/m'

Step 7: Design of the frame (350 mm x 1600 mm)

Step 7.1: Dimensioning

From step 1, the dimensions of the frame girder are 350 mm × 1600 mm.

The thickness of the column at the top is taken as (0.8-1 t_g) and at the bottom as (0.4-0.6 t_g). Thus, the thickness of the column at the top is taken equal to 1400 mm and at the bottom is taken equal to 800 mm. The own weight of the frame equals to:

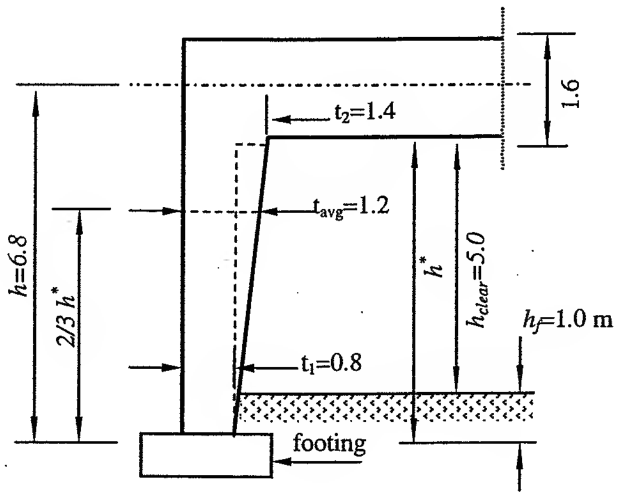
$$w_u = 1.4 \times \gamma_c \times b \times t_g = 1.4 \times 25 \times 0.350 \times 1.60 = 19.60\ kN/m'$$

The frame carries its own weight and the reaction of the Y-beam. The concentrated loads are equal to the reactions of the Y-beam (289.64 kN). At the edges the reaction can be estimated as *0.6* R_Y =173.78 kN. The height of the frame leg h is measured from the footing to the centerline of the girder.

$$h = clear\,height + \frac{t_g}{2} + h_f = 5 + \frac{1.6}{2} + 1.0 = 6.80\,m$$

The frame column has a variable moment of inertia. To simplify the calculations, an average column thickness measured at 2/3h is used.

$$t_{avg} = t_1 + \frac{2}{3}(t_2 - t_1) = 0.8 + \frac{2}{3}(1.4 - 0.8) = 1.2\ m$$



Step 7.2: Calculation of the straining actions

The frame is two-hinged and is once statically indeterminate. The horizontal reaction at the base can be estimated by:

$$H_a = H_b \rightarrow \begin{cases} \dfrac{w_u \times L^2}{4 \times h \times N} & uniform\,load \\ \dfrac{3 \times P_u}{2 \times h \times L \times N} \times (a \times b) & concentrated\,load \end{cases}$$

where

$$K = \frac{I_b}{I_c} \times \frac{h}{L}$$

$$N = 2K + 3$$

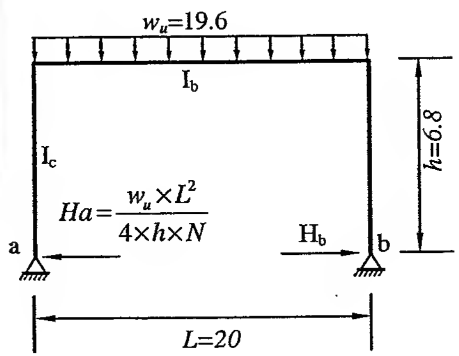
The moment of inertia for the column is calculated using t_{avg}

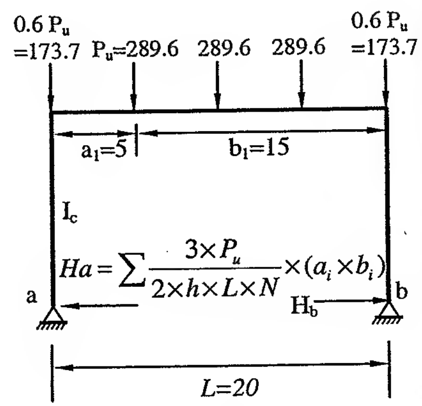
$$I_c = \frac{b \times t_{avg}^3}{12} = \frac{0.350 \times 1.2^3}{12} = 0.0504\,m^4$$

$$I_b = \frac{b \times t^3}{12} = \frac{0.35 \times 1.6^3}{12} = 0.119$$

$$K = \frac{I_b}{I_c} \times \frac{h}{L} = \frac{0.119}{0.0504} \times \frac{6.8}{20} = 0.806$$

$$N = 2K + 3 = 2 \times 0.806 + 3 = 4.61$$





Using the principle of superposition, the total horizontal reaction of the frame due to the uniform load and the three concentrated loads equals to:

$$H_a = \frac{w_u \times L^2}{4 \times h \times N} + \frac{3 \times P_u}{2 \times h \times L \times N} \times (a_1 \times b_1 + a_2 \times b_2 + a_3 \times b_3)$$

$$H_a = \frac{19.6 \times 20^2}{4 \times 6.8 \times 4.61} + \frac{3 \times 289.64}{2 \times 6.8 \times 20 \times 4.61} \times (5 \times 15 + 10 \times 10 + 15 \times 5) = 235.67\,kN$$

The vertical reaction can be obtained easily due to symmetry as follows:

$$Y_a = \frac{w_u \times L}{2} + 0.6 \times P_u + \frac{3 \times P_u}{2} = \frac{19.6 \times 20}{2} + 0.6 \times 289.64 + \frac{3 \times 289.64}{2} = 804.24\,kN$$

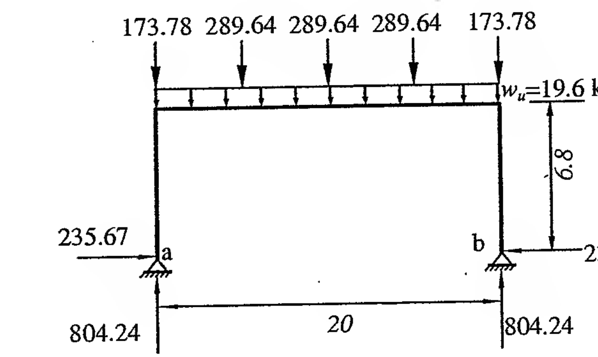
The moment at top of the column $M_{col} = H_a \times h = 1602.6$ kN.m

The maximum moment at mid-span of the girder can be calculated as the superposition of the moments due to the uniform load and those due to the concentrated loads.

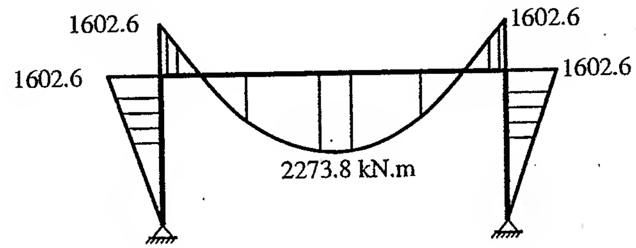
$$M_{mid} = \frac{w_u \times L^2}{8} + \frac{P_u \times L}{4} + P_u \times a - M_{col}$$

$$M_{mid} = \frac{19.6 \times 20^2}{8} + \frac{289.64 \times 20}{4} + 289.64 \times 5 - 1602.6 = 2273.81 \text{ kN.m}$$

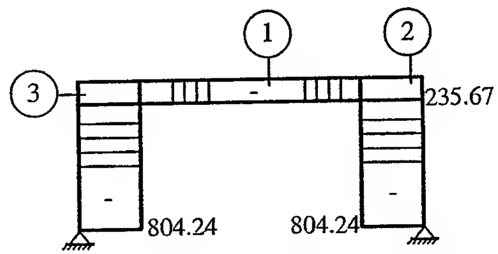
The bending moment, the shear force, and the normal force diagrams for the frame are given in figure below.



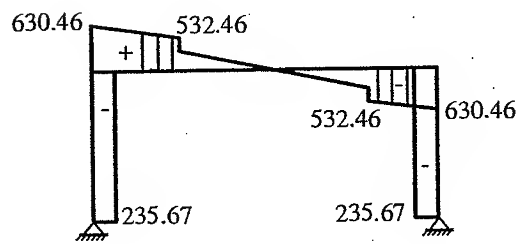
Reactions



Bending moment diagram



Normal force diagram



Shearing force diagram

Step 7.3: Design of the frame sections

Step 7.3.1: Design of section 1 (350 mm x 1600 mm)

Section 1 is a rectangular section that is subjected to

M_u=2273.81 kN.m & P_u=235.67 kN

If $(P_u/f_{cu}\, b\, t)$ is less than 0.04, the normal force can be neglected.

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{235.67 \times 1000}{30 \times 350 \times 1600} = 0.014 < 0.04 \text{ ....neglect the normal force}$$

The design will be carried out as if the section is subjected to bending only. Since frames are usually heavily reinforced, the bars are usually arranged in at least two rows. $\rightarrow$ $d = t - 100 = 1600 - 100 = 1500\ mm$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{2273.81 \times 10^6}{30 \times 350 \times 1500^2} = 0.096$$

From the chart with R = 0.096, the reinforcement index ω = 0.126

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.126 \times \frac{30}{400} \times 350 \times 1500 = 4961\ mm^2$$

$$A_{s\,\min} = \textit{the smaller of} \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{400} \times 350 \times 1500 = 1617\ mm^2 \\ 1.3 A_s = 1.3 \times 4961 = 6449\ mm^2 \end{cases}$$

Use 9Φ28 (5541 mm^2)

The stirrup hangers are taken as 15% of A_s, which gives 831 mm^2 (3Φ20). The shrinkage bars should not be less than 8% from A_s with a maximum distance between bars of 300 mm. This gives 443 mm^2 (8Φ12)

Step 7.3.2: Design of section 2 (350 mm x 1600 mm)

Section 2 is a rectangular that is subjected to

M_u= 1602.57 kN.m & P_u=235.67 kN

If $(P_u/f_{cu}\, b\, t)$ is less than 0.04, the normal force can be neglected.

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{235.67 \times 1000}{30 \times 350 \times 1600} = 0.014 < 0.04 \text{ ....neglect the normal force}$$

The design will be carried out as if the section is subjected to bending only.

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{1602.57 \times 10^6}{30 \times 350 \times 1500^2} = 0.067$$

From the chart with R=0.067, the reinforcement index ω=0.085

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.085 \times \frac{30}{400} \times 350 \times 1500 = 3346\ mm^2$$

$$A_{s\,\min} = \textit{the smaller of} \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{400} \times 350 \times 1500 = 1617\ mm^2 \\ 1.3 A_s = 1.3 \times 3346 = 4350\ mm^2 \end{cases}$$

Use 8Φ25 (3926 mm^2), *see reinforcement details*

Step 7.3.3: Design of section 3 (350 mm x 1400 mm)

Buckling in the out-of-plane direction

The frame is considered unbraced in the out-of-plane direction because of the lack of any bracing system.

From Fig. Ex. 1.8b, it can be determined H_o=2.8. The effective length factor k is obtained with case (1) at top and bottom. Thus, k=1.2.

$$H_e = k \times H_o = 1.2 \times 2.8 = 3.36\ m$$

$$\lambda = \frac{H_e}{b} = \frac{3.36}{0.35} = 9.6 < 10 \text{ (case of unbraced columns)}$$

Thus, no additional moments are induced in the out-of-plane direction.

Buckling in the in-plane direction

The frame is considered unbraced because the lack of any bracing system. The top part of the column is considered case (1) and the bottom part is considered case (3) (hinged base). Thus k=1.6.

The height of the column is measured from the bottom of the beam to the base (h^*). However, it is customary to use the length used in the analysis h.

$H_e = k \times h = 1.6 \times 6.8 = 10.88\ m$

The slenderness ratio λ is calculated using an average column thickness not the actual one, thus λ equals

$$\lambda = \frac{H_e}{t_{avg}} = \frac{10.88}{1.2} = 9.0667$$

Since λ is less than 10, the column is considered short and no additional moment is developed.

M=1602.57 kN.m & P_u=804.24 kN

Due to the fact that column sections are subjected to large normal force, it is recommended to use compression steel between 40%-60% of the tension steel to ensure ductile behavior. Use the interaction diagram (α=0.6).

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{804.24 \times 1000}{30 \times 350 \times 1400} = 0.0547$$

$$\frac{M_u}{f_{cu} \times b \times t^2} = \frac{1602.57 \times 10^6}{30 \times 350 \times 1400^2} = 0.077$$

Assuming that the distance from the concrete to the c.g. of the reinforcement is 80 mm. Thus the factor ζ equals

$$\zeta = \frac{t - 2 \times cover}{t} = \frac{1400 - 2 \times 80}{1400} = 0.89$$

Using a interaction diagram with f_y= 400 N/mm², α=0.6, and ζ=0.9 → $\rho = 1.9$

$$\mu = \rho \times f_{cu} \times 10^{-4} = 1.9 \times 30 \times 10^{-4} = 0.0057$$

$A_s = \mu \times b \times t = 0.0057 \times 350 \times 1400 = 2793\ mm^2$ (8Φ25, 3926 mm²)

$A_s' = \alpha \cdot A_s = 0.6 \times 2793 = 1676\ mm^2$ (4Φ25, 1963 mm²)

$$A_{s,total} = A_s' + A_s = 3926 + 1963 = 5889\ mm^2$$

Since the column is short, the minimum reinforcement ratio is 0.008.

$$A_{s,min} = 0.008 \times b \times t = 0.008 \times 350 \times 1400 = 3920\ mm^2 < A_{s,tot} ....o.k$$

Step 7.3.4: Design of section 4 (350 mm x 800 mm)

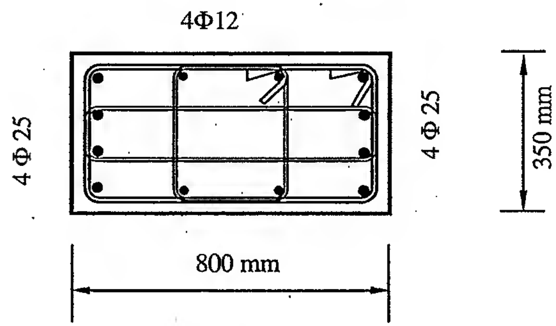
This section is subjected to a pure compression force (P_u=804.24 kN) and can be reinforced with the minimum area of steel.

$$A_{s,min} = 0.008 \times b \times t = 0.008 \times 350 \times 800 = 2240\ mm^2$$

$$P_u = 0.35 \times f_{cu} \times A_c + 0.67 \times f_y \times A_{sc}$$

$$P_u = \frac{1}{1000}(0.35 \times 30 \times (350 \times 800) + 0.67 \times 400 \times 2240) = 3540\,kN > (804.24)....o.k$$

From the frame reinforcement details A_s=8Φ25, 3927 mm² > $A_{s,min}$



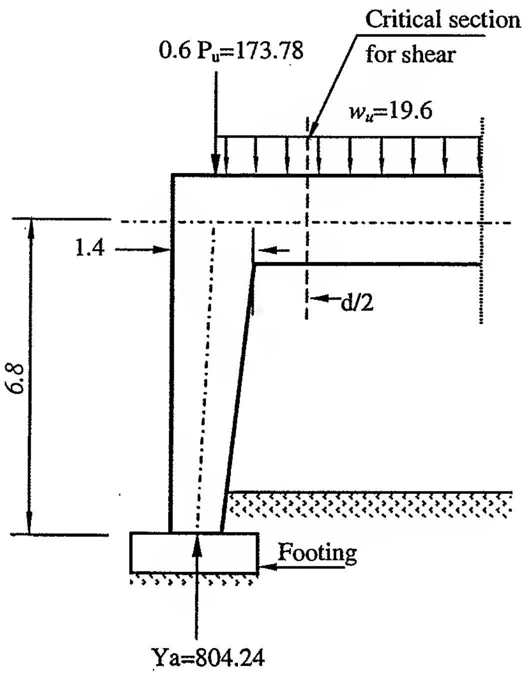
Step 8: Design for shear

The critical section for shear is at d/2 from the face of the column. Thus the design force Q_u equals to:

$$Q_u = Y_a - 0.6 P_u - w_u \left(\frac{t_2}{2} + \frac{d}{2}\right) = 804.24 - 173.78 - 19.6\left(\frac{1.4}{2} + \frac{1.5}{2}\right) = 602.04\ kN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{602.04 \times 1000}{350 \times 1500} = 1.147\ \text{N/mm}^2$$

The presence of the compression force increases the shear capacity of the girder. however, this force is relatively small and can be neglected (*conservative*).



$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$ (Neglect the effect of P_u)

Since $q_u > q_{cu}$, shear reinforcement is required.

$$q_{su} = q_u - \frac{q_{cu}}{2} = 1.147 - \frac{1.07}{2} = 0.61\ N/mm^2$$

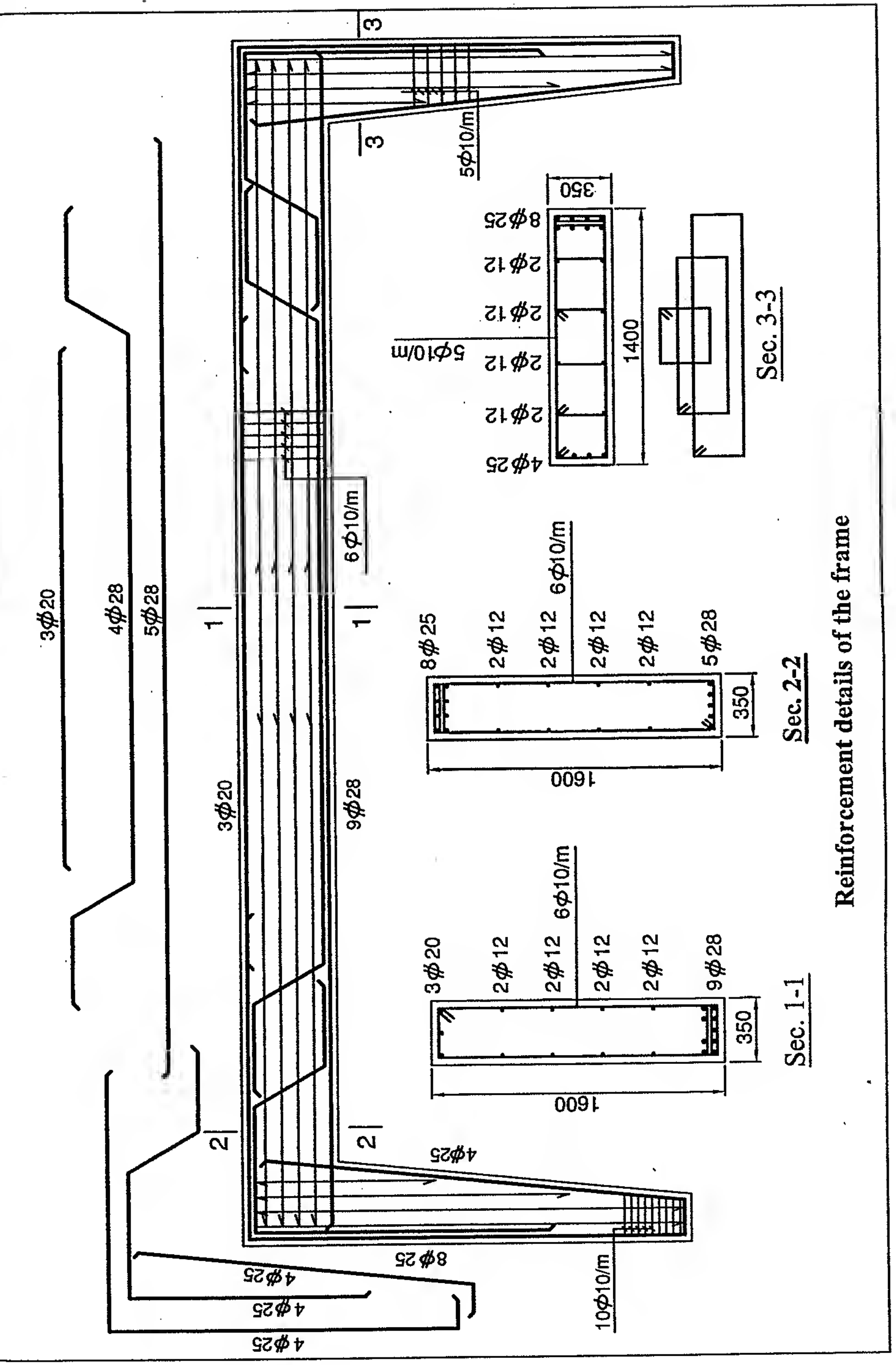
Try ϕ 10/m′ (A_{st}=2×78.5=157 mm^2) (10 mm diameter is chosen because of the heavy reinforcement of the frame)

$$q_{su} = \frac{A_{st} \times f_y / 1.15}{b \times s}$$

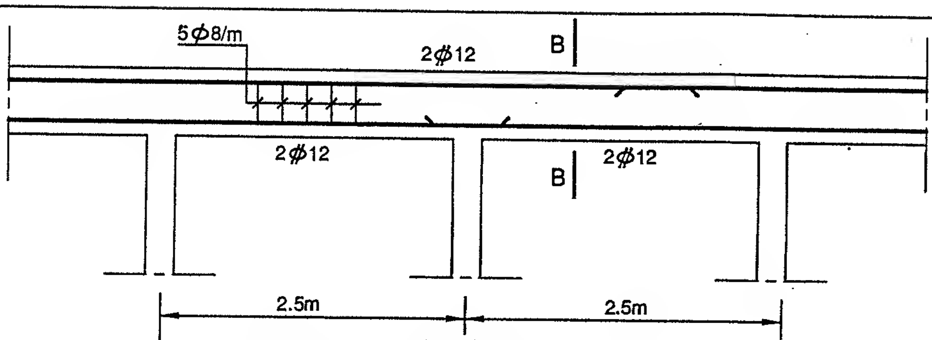
$$0.61 = \frac{157 \times 280 / 1.15}{350 \times s}$$

s = 179 mm , Use 6ϕ10/m (s =166 mm)

$$A_{st,min} = \frac{0.4}{f_y} \times b \times s = \frac{0.4}{280} \times 350 \times 166 = 83 mm^2 < A_{st}(157\ mm^2).....o.k$$

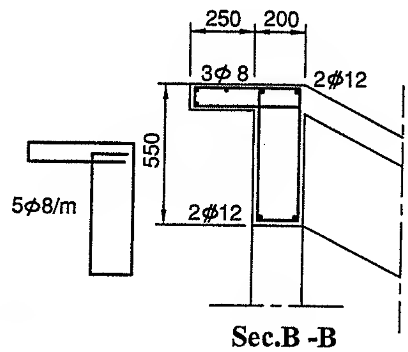


Reinforcement details of the frame

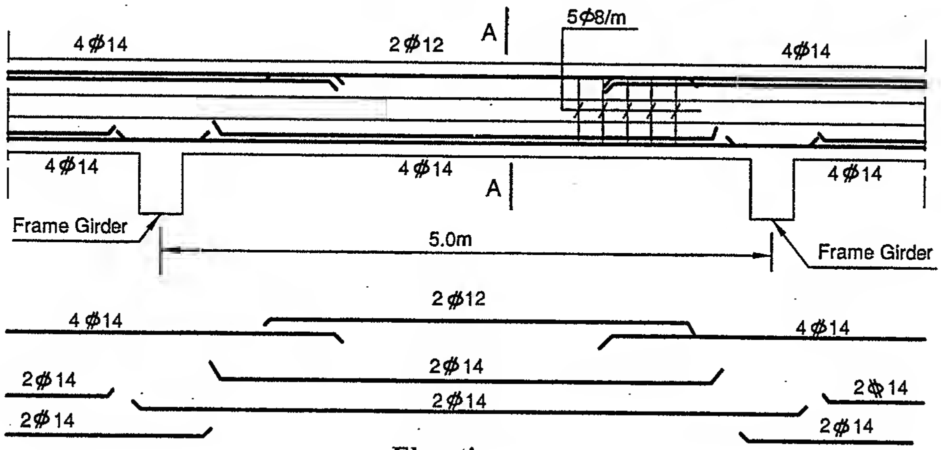


Elevation

Reinforcement details of ridge beam

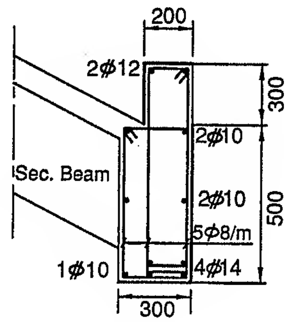


Sec.B -B

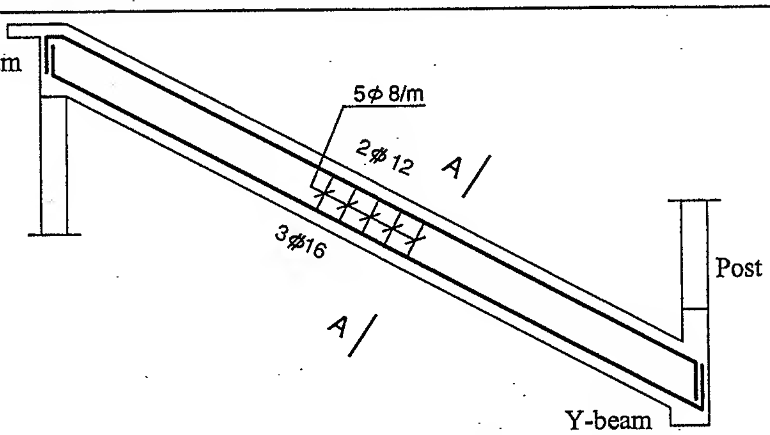


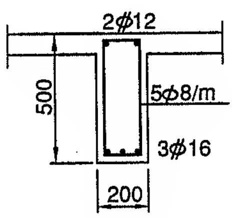
Elevation

Reinforcement details of Y-beams



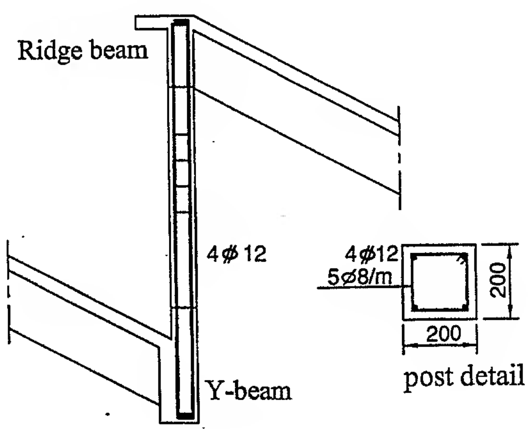
Sec. A-A





Sec. A-A

Reinforcement details of secondary beams



post detail

1.7 Arched Slab Systems

1.7.1 Introduction

Arched slabs are commonly used to cover relatively large spans. The spans can range from 12.0 ms to 25.0 ms. The forces developed in the arched slabs are mainly compression combined with small amount of bending moments. Therefore, such a structural system is a very efficient reinforced concrete structure. The advantages of using arched slabs can be summarized as follows:

- Permits covering large areas free of columns.
- Leads to shortening the construction period.
- Allows economical use of the construction materials because concrete mainly subject to compression.

1.7.2 Structural system of the Arched Slab

The arched slab is usually analyzed as a three-hinged arch with a tie. The intermediate hinge is assumed to form at the crown of the slab. This is usually achieved by reducing the thickness of the slab at the point of the crown. The thickness of the slab is then increased at the supports to resist the increased compression forces and concentration of stresses at the intersection with the connecting beams. The tie is provided to carry all the thrust forces (outward forces) resulting from the arch action. The slab is basically a one-way slab resting on the vertical beams as shown in Fig. 1.19.

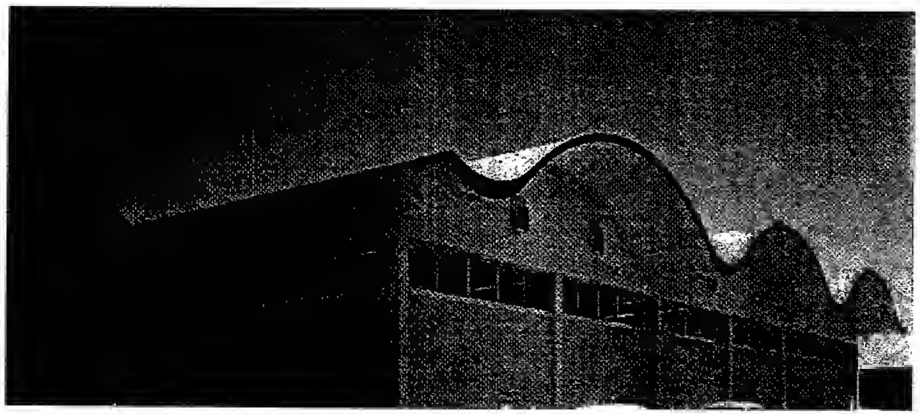


Photo 1.9 An arched slab system

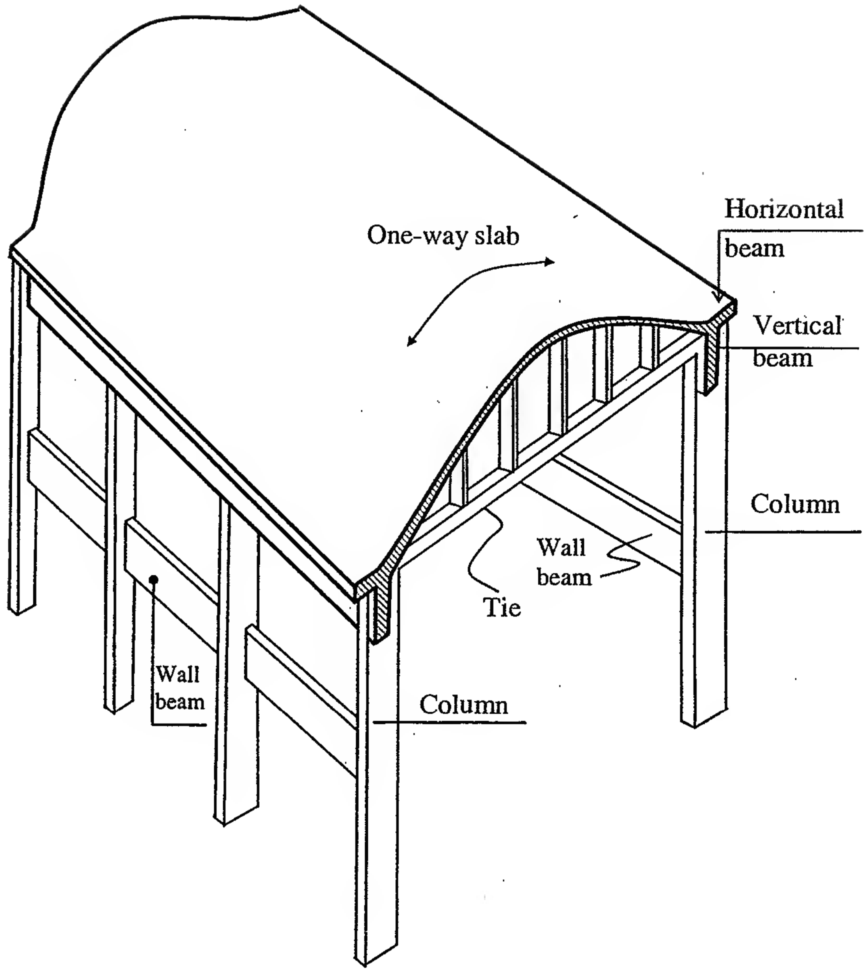
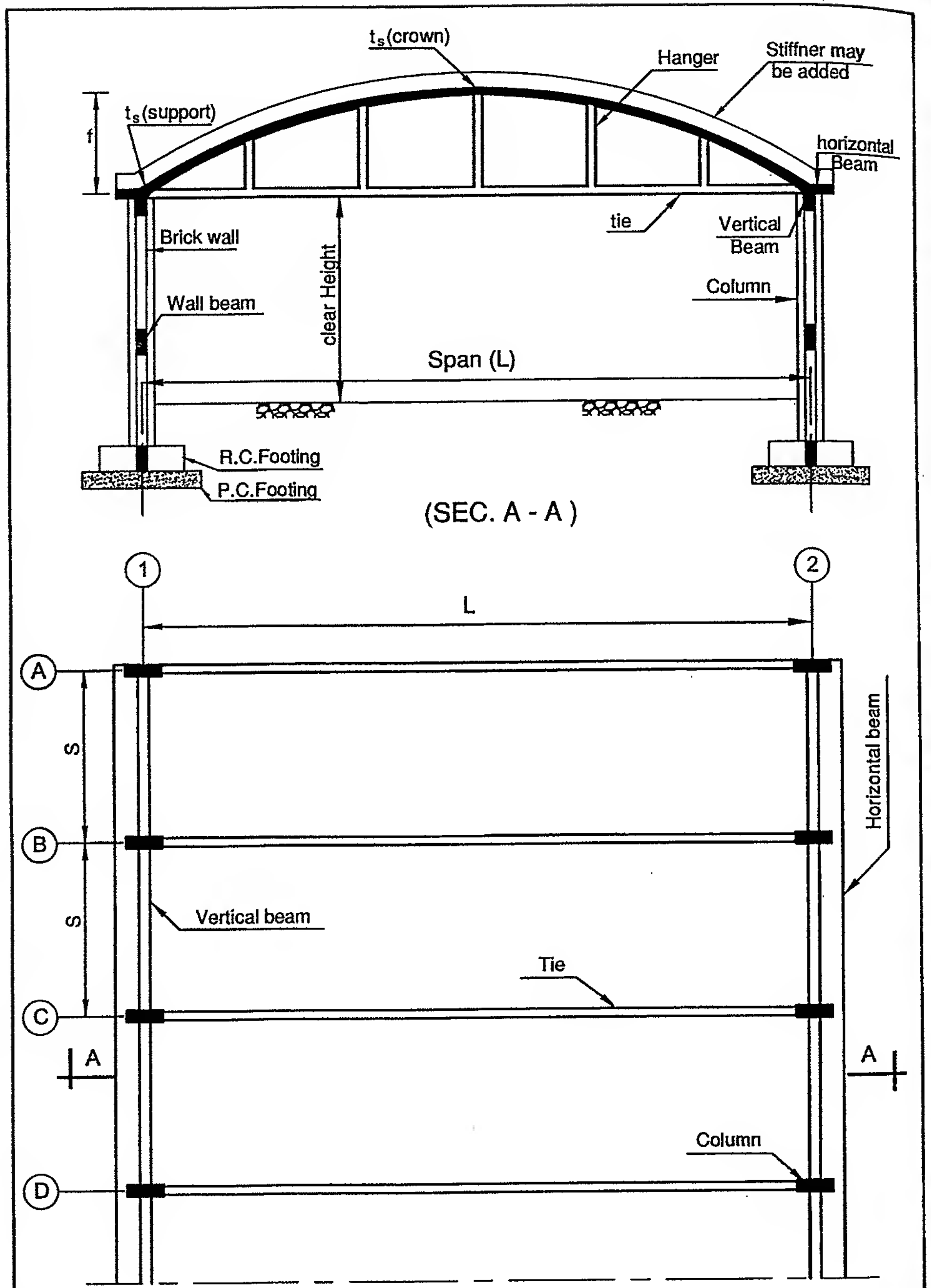


Fig. 1.19a Arched slab with a tie

The arched slab system is composed of the following elements:

- Curved slab
- Horizontal ties
- Vertical beams
- Horizontal beams
- Columns
- Wall beams



Plan

Fig. 1.19b Arched slab with a tie

The arched slab acts as a one-way curved slab as shown in Fig. 1.20. The rise of the arch f is determined as a ratio of the span L. The recommended rise/span ratio is about 1/4 to 1/8. The main reinforcement is provided in the span direction and the secondary reinforcement is provide in the longitudinal direction. The amount of secondary reinforcement is usually 20%-25% of the cross sectional area of the main steel.

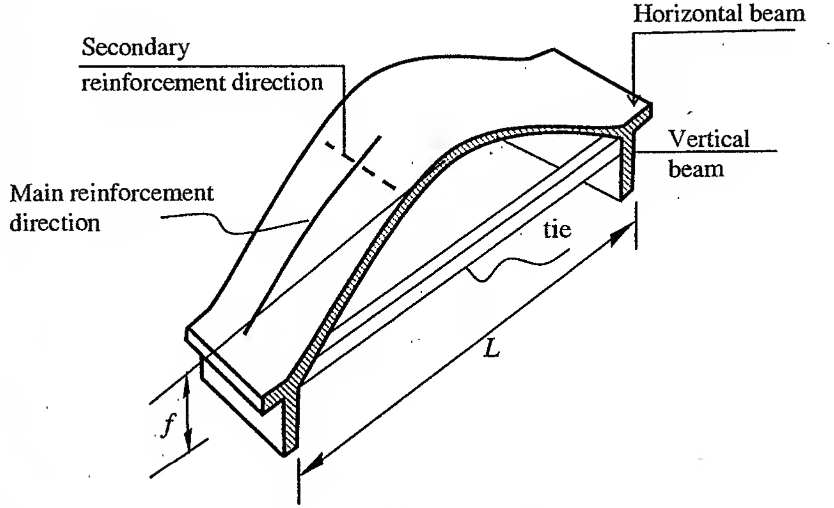


Fig. 1.20 A segment of the arched slab

The reaction of the arched slab at the support is inclined as shown in Fig. 1.21. This inclined reaction can be analyzed in the vertical and horizontal directions. It is customary to provide vertical and horizontal beams to resist these forces.

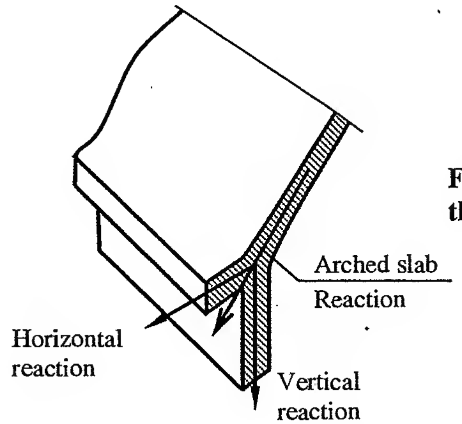


Fig. 1.21 Supporting beams of the arched slab.

The vertical beam provides a support to the vertical component of the reaction as shown in Fig. 1.22. It is analyzed as continuous beam supported on columns. On the other hand, the horizontal beam provides a support to the horizontal component of the reaction. It is analyzed as continuous beam supported on the ties. If the tie is not provided, the horizontal beam will be directly supported on columns. In such a case, the columns will be subjected to large concentrated forces at the top resulting in large bending moments.

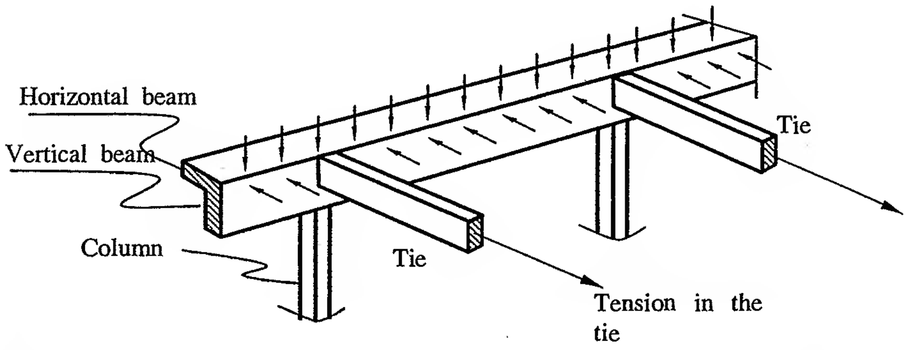


Fig. 1.22 Vertical and horizontal beams of arched-slab system

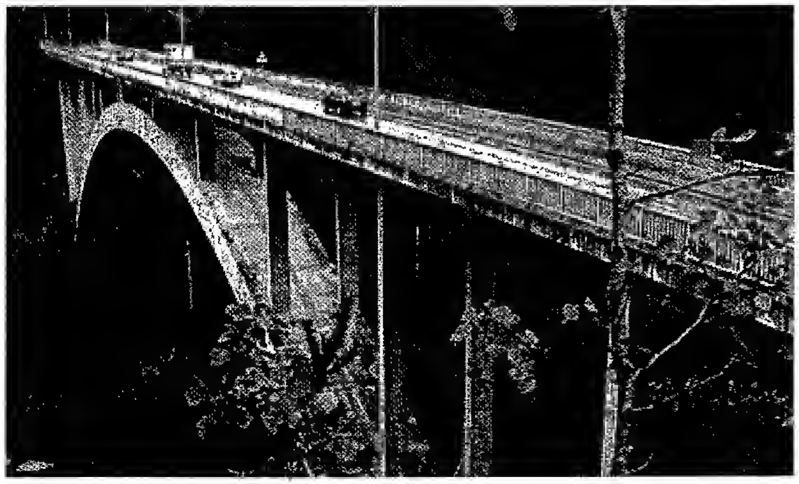


Photo 1.10 Reinforced concrete arch that support compression posts

1.7.3 Structural Analysis of Arched Slabs

Two types of arched slabs are commonly used; a) parabolic arched slab and b) circular arched slab. Parabolic arched slabs are more efficient systems because the centerline of the arch coincides with the line of pressure, resulting in zero bending moment.

Two sections are usually considered when designing the arched slab, namely; the section at the quarter point and the section at the support. The section at the quarter point is subjected to both compression and bending moment, while the section at the support is subjected to normal force (compression) only.

1.7.3.1 Circular Arched Slabs

Referring to Fig. 1.23, the equation of the axis of the circular arch is given by:

$$r^2 = x^2 + y^2 \quad \text{.........} \quad (1.8)$$

$$y = \sqrt{r^2 - x^2} \quad \text{.........} \quad (1.9)$$

where

r is the radius of the arch, and x and y are coordinates of any point on the arch.

The radius of the arch may obtained using the rise of the arch f and the span L by observing the triangle *mno* as follows

$$r^2 = (L/2)^2 + (r - f)^2 \quad \text{.........} \quad (1.10)$$

$$r = \frac{(L/2)^2 + f^2}{2f} \quad \text{.........} \quad (1.11)$$

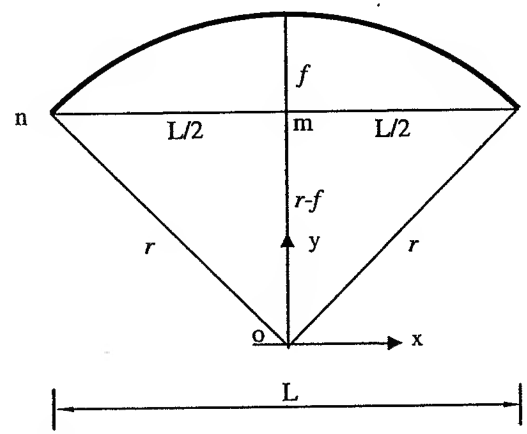


Fig. 1.23 Geometry of a circular arched slab

Section at the quarter point

The quarter point is subjected to a bending moment and a compression force. The quarter point of the arch is obtained by bisecting the angle θ to point O_1 as shown in Fig. 1.24. It is a well known geometric fact that the tangent slope (α) at point O_1 must equal θ/2, thus

$$\alpha = \theta / 2 \quad \text{......................................} \quad (1.12)$$

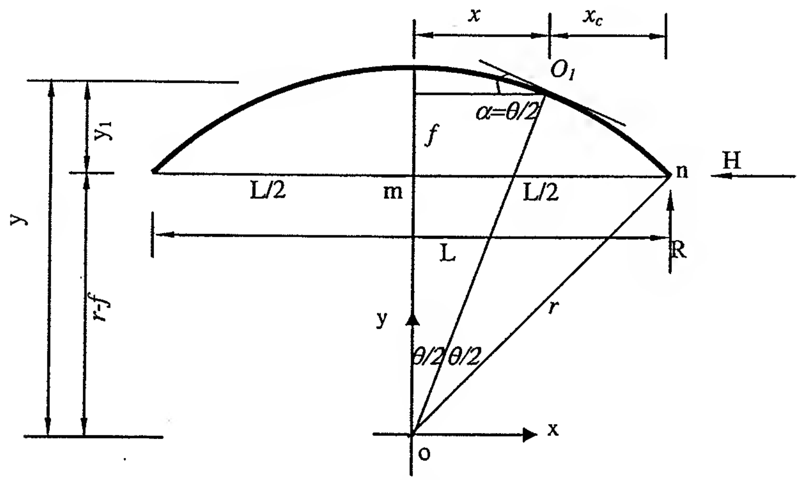


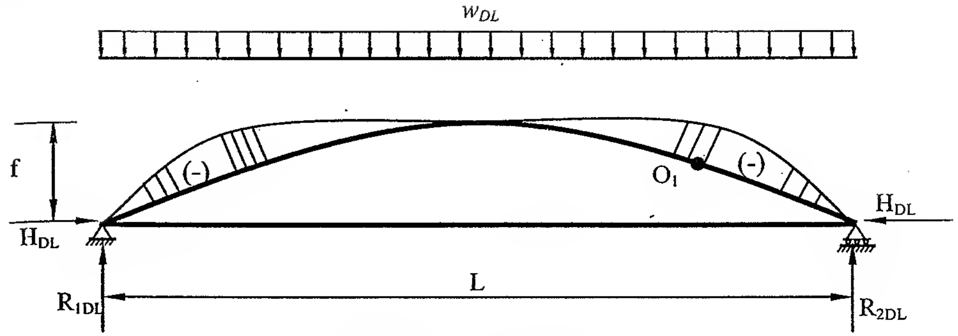
Fig. 1.24 Analysis of a circular arch at the quarter point O_1

The critical load combination is the dead load covering the whole span and the live load covering half of the span. Figure 1.25 shows the bending moment for different load cases.

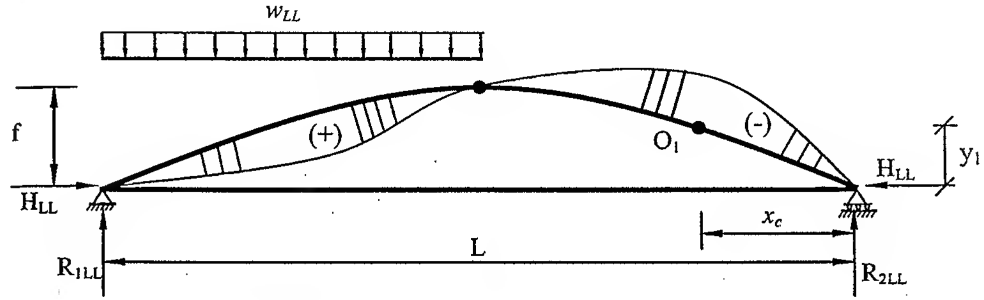
The values of both vertical and horizontal reactions due to dead and live loads are given in Table 1.5.

Table 1.5 Values of the reactions in circular arched slabs

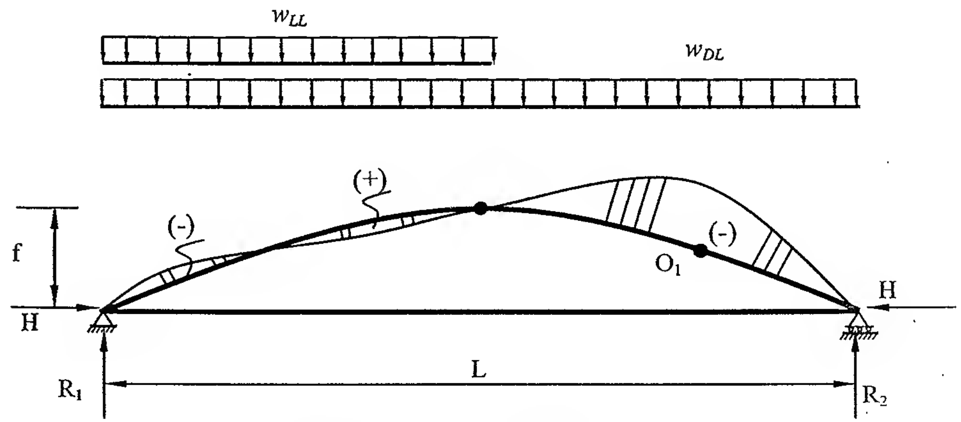
Item	Dead load covering the whole span	Live load covering half of the span
R_1	$w_{DL} \times L/2$	$3w_{LL} \times L/8$
R_2	$w_{DL} \times L/2$	$w_{LL} \times L/8$
H	$w_{DL} \times L^2/(8f)$	$w_{LL} \times L^2/(16f)$



a: Moments due to dead load



b: Moments due to live load covering half of the slab



c: Moments due to combineddead and live loads

Fig. 1.25 Bending moments in circular arched slabs

For flat arches the normal force at the quarter point can be approximated by:

$$P_u = \frac{H}{\cos \alpha} \quad \text{......................................} \quad (1.13)$$

However, the exact value of the compressive P_u and shearing force Q_u can be obtained from simple structural analysis at point O_1 as follows:

$$P_u = H \cos \alpha + Q \sin \alpha \quad \text{..............................} \quad (1.14)$$

$$Q_u = Q \cos \alpha - H \sin \alpha \quad \text{..............................} \quad (1.15)$$

Where H and Q are the horizontal and vertical forces, respectively, at the quarter point as shown in Fig. 1.26.

Referring to Fig. 1.24 and from triangle *omn*, one can get the following relation:

$$\theta = \sin^{-1} \frac{L/2}{r} \quad \text{......................................} \quad (1.16)$$

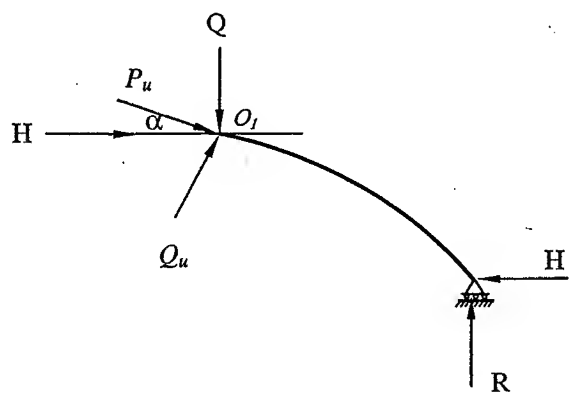


Fig. 1.26 Force analysis at the quarter point

Table 1.6 lists the values of the exact normal force P_u at the quarter point using Eq. 1.14. The bending moment at the quarter point of a circular arch varies according to ratio of the rise (f) to the span (L). The bending moments can be obtained using the conventional structural analysis of the arch, or using Table 1.6. The values in Table 1.6 were obtained using the following steps (refer also to Fig. 1.24).

$$x = r \sin\left(\frac{\theta}{2}\right) \quad \text{......................................} \quad (1.17)$$

$$y = \sqrt{r^2 - x^2} \quad \text{......................................} \quad (1.18)$$

$$y_1 = y - (r - f) \quad \text{......................................} \quad (1.19)$$

$$x_c = L/2 - x \quad \text{......................................} \quad (1.20)$$

Carrying out the structural analysis, the moment at the quarter point O_1 may be obtained. For example, the negative moment at the quarter point due to live loads on half of the span equals to:

$$M_{(-ve)LL} = R_{2LL} \cdot x_c - H_{LL}\ y_1 \quad \text{..........................} \quad (1.21)$$

Where

$R_{2LL} = w_{LL} \times L/8$

$H_{LL} = w_{LL} \times L^2/(16f)$

Table 1.6 Values of the bending moments and the normal forces at the quarter point of a circular arch.

f/L	$M_{(-ve)DL}$ k_1	$M_{(-ve)LL}$ k_2	$M_{(+ve)LL}$ k_3	P_u *(exact)* k_4
0.10	-0.00094	-0.01609	0.01515	1.0205
0.15	-0.00214	-0.01666	0.01453	1.0474
0.20	-0.00384	-0.01745	0.01361	1.0872
0.25	-0.00607	-0.01844	0.01237	1.1416
0.30	-0.00885	-0.01962	0.01077	1.2123
0.35	-0.01221	-0.02097	0.00876	1.3007
0.40	-0.01615	-0.02247	0.00632	1.4081
0.45	-0.02071	-0.02412	0.00341	1.5355
0.50	-0.02589	-0.02589	0.00000	1.6834

The values of the different forces can be obtained using the following set of equations:

$$M_{(-ve)DL} = k_1 \times w_{DL} \times L^2 \quad \text{.................................} (1.22)$$

$$M_{(-ve)LL} = k_2 \times w_{LL} \times L^2 \quad \text{.................................} (1.23)$$

$$M_{(+ve)LL} = k_3 \times w_{LL} \times L^2 \quad \text{.................................} (1.24)$$

$$P_u = k_4 \times H \quad \text{...} (1.25)$$

Section at the Support

The section at the support is subjected to a normal force only. To obtain the maximum forces (p_{umax}), the whole span should be covered with both dead and live loads as shown in Fig 1.27.

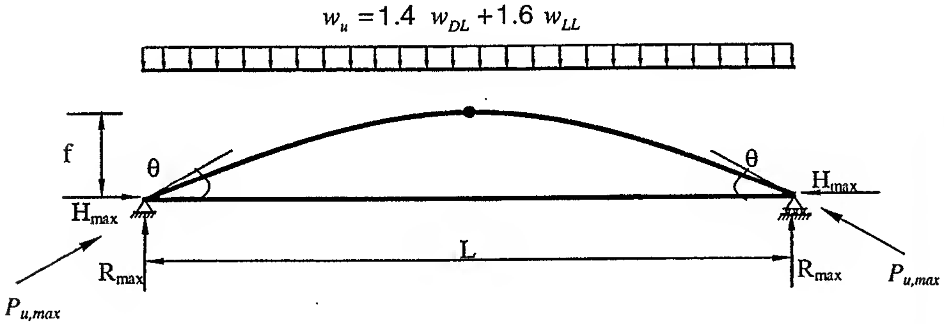


Fig. 1.27 Analysis of a section at the support

$$H_{max} = \frac{w_u\ L^2}{8f} \quad \text{..} (1.26)$$

$$R_{max} = \frac{w_u\ L}{2} \quad \text{..} (1.27)$$

$$P_{u,max} = H_{max} \cos\theta + R_{max} \sin\theta \quad \text{...........................} (1.28)$$

Noting that the angle of the tangent at the support equals θ, one gets:

$$\theta = \sin^{-1}\left(\frac{L/2}{r}\right) \quad \text{..} (1.29)$$

1.7.3.2 Parabolic Arched Slabs

The equation of the axis of the parabolic arch according to Fig. (1.28) is given by:

$$y = \frac{4f\ x}{L^2} \times (L - x) \quad \text{..} (1.30)$$

where

f = the rise of the arch
L = the span of the arch

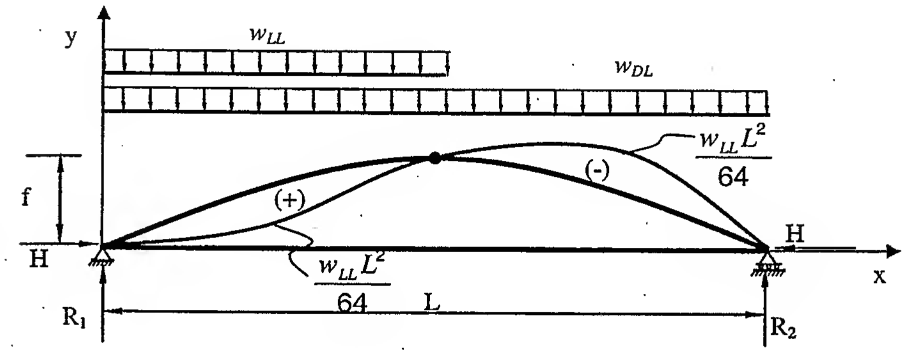


Fig. 1.28 Bending moments in parabolic arched slabs

Section at the quarter point

The height of the quarter point for a parabolic arch equals ¾ *f*. The maximum compression force due to a uniform dead load covering the whole span and a live load covering half of the span can be approximately given by:

$$P_u = H\sqrt{1 + \left(\frac{2f}{L}\right)^2} \quad \text{.................................} (1.31)$$

The exact value of the compression force can be obtained from simple structural analysis as shown in Fig. 1.29 and is given by:

$$P_u = H\ \cos\alpha + Q \sin\alpha \quad \text{.................................} (1.32)$$

Where H and Q are the horizontal and vertical forces at the quarter point.

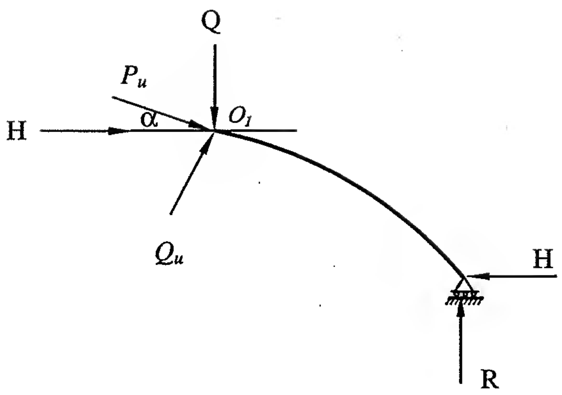


Fig. 1.29 Force analysis at the quarter point

The bending moment at the quarter point due to a uniform load covering the whole span equals to zero. Therefore, the maximum bending moment at the quarter point is obtained by placing the live load on half of the span. The value of the bending moment equals to:

$$M_u = \frac{1.6\, w_{LL} \times L^2}{64} \quad \text{...} (1.33)$$

The values of both the vertical and the horizontal reactions due to dead and live loads are given in Table 1.7 (Refer to Fig. 1.28).

Table 1.7 values of the reactions in parabolic arches

Item	Dead load covering the span	Live load covering half the span
R_1	$w_{DL} \times L/2$	$3\, w_{LL} \times L/8$
R_2	$w_{DL} \times L/2$	$w_{LL} \times L/8$
H	$w_{DL} \times L^2/(8f)$	$w_{LL} \times L^2/(16f)$
Moment (M)	0	$\pm w_{LL} \times L^2/64$

Section at the Support

The section at the support is subjected to a normal force only. To obtain the maximum forces (P_{umax}), the whole span should be covered with both dead and live loads as shown in Fig. 1.30.

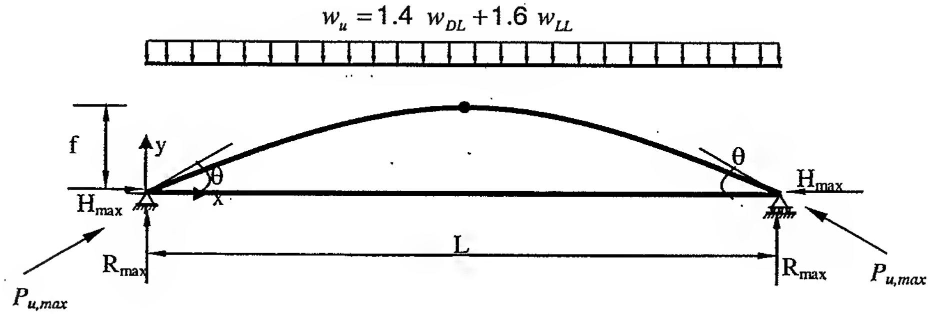


Fig. 1.30 Analysis of a section at the support

$$H_{max} = \frac{w_u\ L^2}{8f} \quad \text{...} (1.34)$$

$$R_{max} = \frac{w_u\ L}{2} \quad \text{...} (1.35)$$

$$P_{u,max} = H_{max} \cos\theta + R_{max} \sin\theta \quad \text{......................} (1.36)$$

where θ is the tangent angle at the support and can be obtained by differentiating Eq. 1.30, and substituting with x=0.

$$y' = \tan\theta = \frac{4f}{L} \quad \text{...} (1.37)$$

Alternatively, P_{umax} can be obtained by

$$P_{u,max} = \sqrt{H_{max}^2 + R_{max}^2} \quad \text{...} (1.38)$$

Example 1.9: Design of a Circular arched slab

A machinery room spans 20 ms is shown in Fig. EX 1.9. It is required to carry out a complete design of the roof that is covered by a circular arched slab. The clear height of the room is 6.4 ms and the height of the crown is 4.0 ms. The material properties are: f_{cu}=30 N/mm^2, f_y =400 N/mm^2, and f_{yst} =240 N/mm^2. The weight of plastering and finishing materials may be assumed 0.60 kN/m^2. The live load may be assumed 0.90 kN/m^2.

Solution

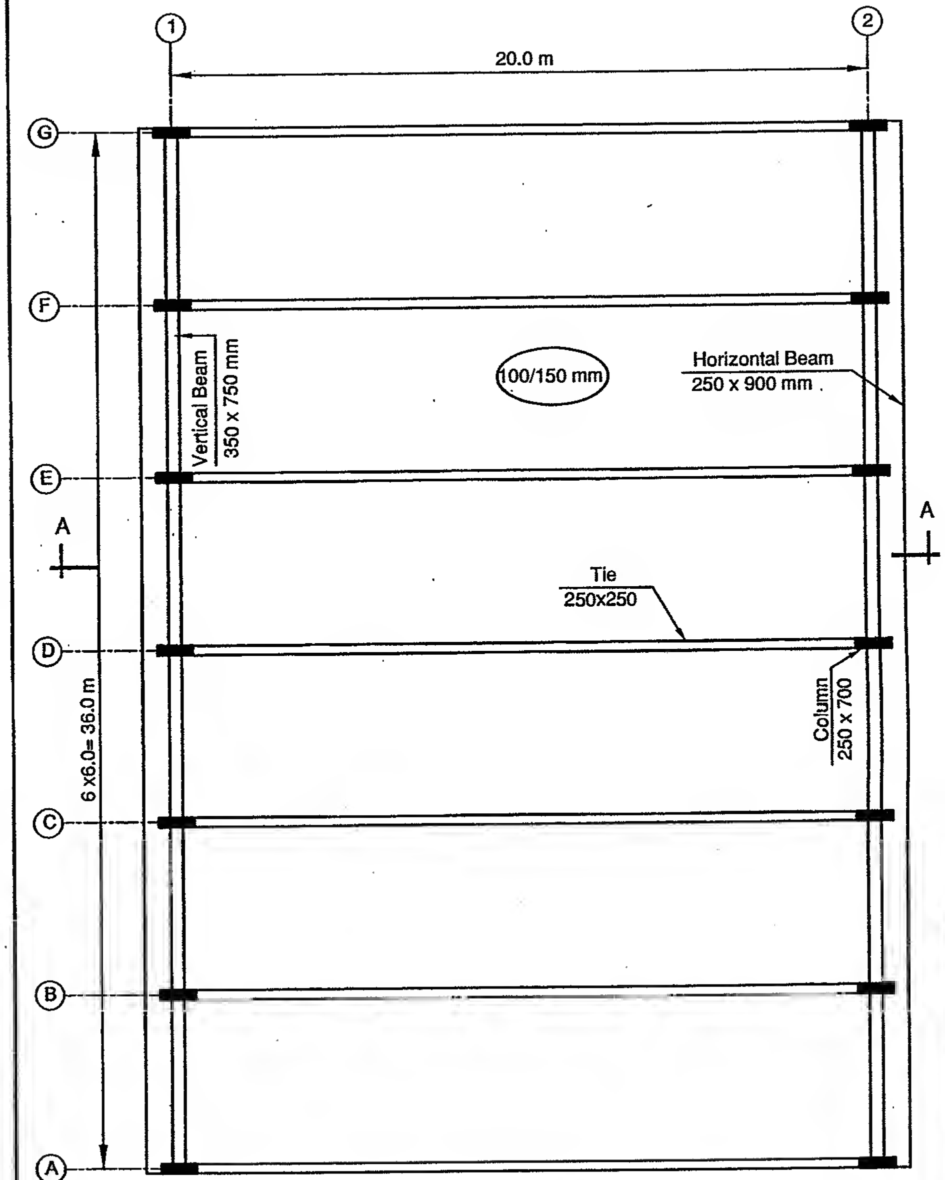
Step 1: Propose the concrete dimensions

The arched circular slab is the chosen as the main system with the following dimensions:

Arched slab

t_s (midspan)	= 100 mm
t_s (quarter point)	= 125 mm
t_s (edge)	= 150 mm
Vertical beam	= (350 mm x 750 mm)
Horizontal beam	= (250 mm x 900 mm)
Tie	= (250 mm x 250 mm)
Hanger	= (250 mm x 250 mm)
Columns	= (250 mm x 700 m)

The spacing between the ties = 6.0 m



Plan

Fig. EX. 1.9a A Hall covered with an arched slab

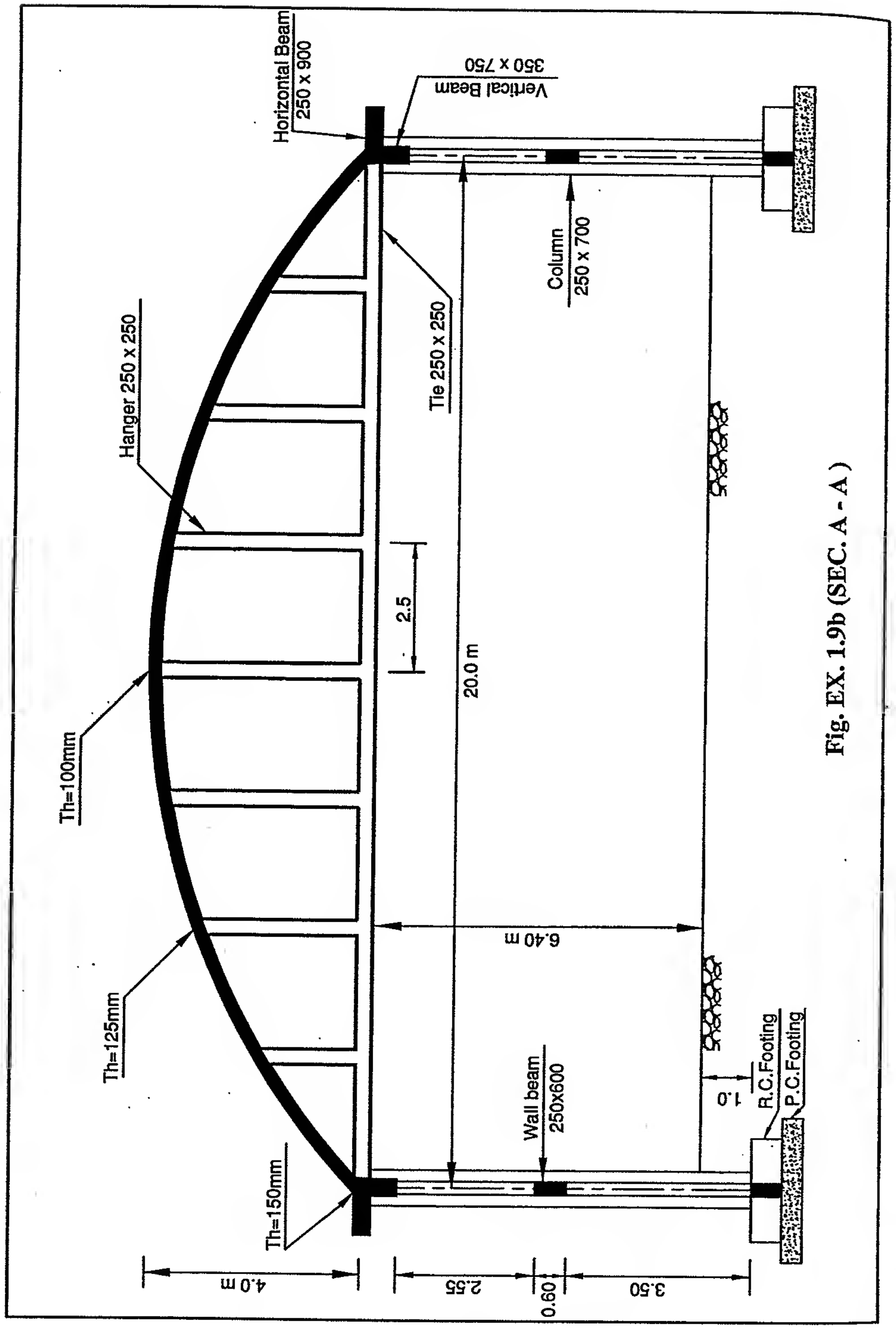


Fig. EX. 1.9b (SEC. A - A)

Step 2: Calculations of loads

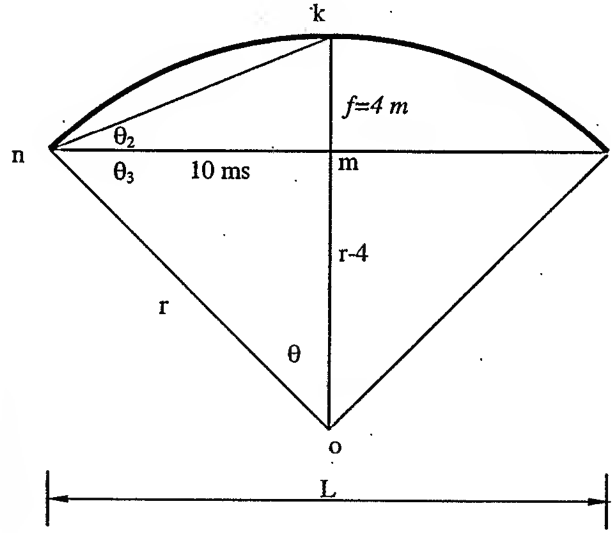
In order to calculate the forces acting of the arched slab system it is necessary to determine the radius of the arc. Referring to Eq. 1.11 and triangle *mno* in the figure below, one can get:

$$r = \frac{(L/2)^2 + f^2}{2f} = \frac{10^2 + 4^2}{2 \times 4} = 14.5\ m$$

From triangle *mno* the angle θ equals

$$\theta = \sin^{-1}\frac{L/2}{r} = \sin^{-1}\frac{10}{14.50} = 43.6^o$$

The length of the arc $L' = r\ \theta\ = 14.5 \times \frac{43.6}{180} \times 2\pi = 22.07\ m$



The self weight of the arched slab may be calculated using the thickness at the quarter point (125 mm).

$$ow. = \gamma_c \times t_{avg} = 25 \times 0.125 = 3.125\ kN/m^2$$

The factored dead load is given by:

$$w_{UDL} = 1.4\ (o.w. + plaster\ weight) = 1.4 \times (3.125 + 0.6) = 5.21\ kN/m^2$$

The dead load calculated for the horizontal projection is given by:

$$w_{UDL} = w_{UDL} \times \frac{L'}{L} = 5.21 \times \frac{22.07}{20} = 5.75\ \ kN/m^2 \qquad (H.P.)$$

Noting that the live loads on curved surfaces are always given on the horizontal projection, the slab factored live load (w_{ULL}) is given by:

$$w_{ULL} = 1.6 \times w_{LL} = 1.6 \times 0.9 = 1.44\ kN/m^2$$

The total factored load $w_u = w_{UDL} + w_{ULL} = 5.75 + 1.44 \cong 7.2\ kN/m^2$

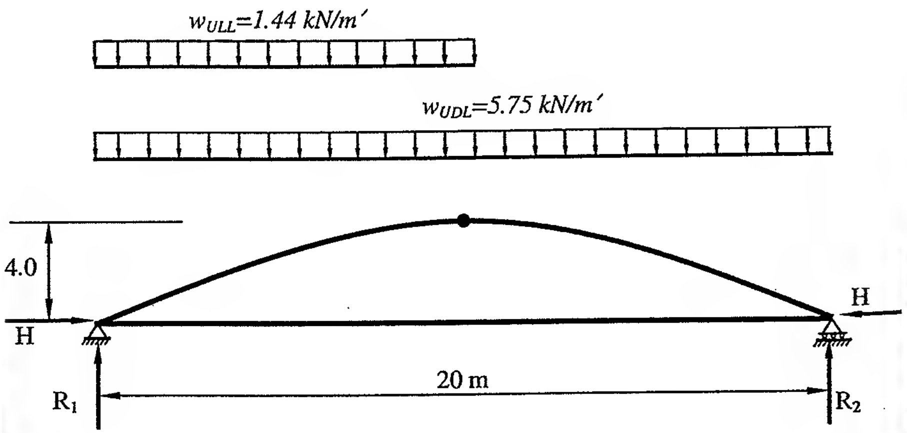
Step 3: Design the arched slab critical sections

There are two critical sections; the first section is at the quarter point and the second one is at the support. The point at mid-span is assumed to act as a hinge due to its reduced thickness. Taking 1m width of the slab, the acting loads are shown in the following figure.

Step 3.1: Section at the quarter point (t_s=125 mm)

Step 3.1.1: Straining actions

To obtain the maximum moment at the quarter point, only half of the arch is covered by the live load.



Equivalent loading system and reactions

$$R_2 \times 20 = 5.75 \times 20 \times \frac{20}{2} + 1.44 \times 10 \times 5 \qquad R_2 = 61.1\ kN$$

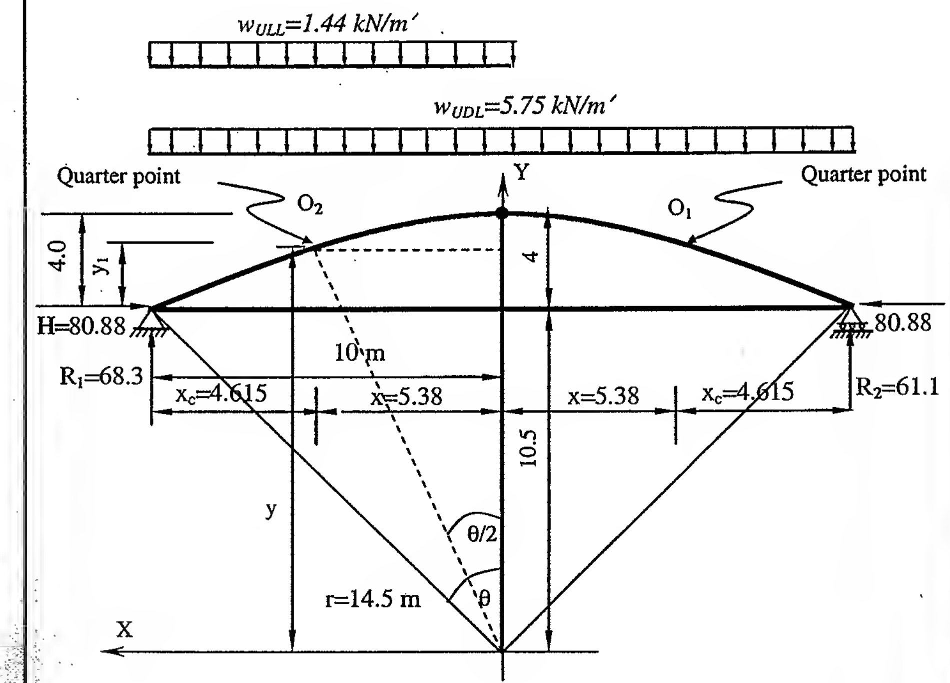
$$R_1 \times 20 = 5.75 \times 20 \times \frac{20}{2} + 1.44 \times 10 \times 15 \qquad R_1 = 68.3\ kN$$

To obtain the horizontal thrust H, the moment is taken at the middle hinge as follows:

$$H = \frac{(w_{UDL} + \frac{w_{ULL}}{2}) \times L^2}{8\ f} = \frac{(5.75 + \frac{1.44}{2}) \times 20^2}{8 \times 4} = 80.88\ kN$$

OR

$$H \times 4 = R_1 \times 10 - w_u \times 10 \times 5 = 68.3 \times 10 - 7.19 \times 10 \times 5 \ \rightarrow\ \text{H=80.88 kN}$$



The coordinates of the quarter point can be obtained as follows:

$$\theta = \sin^{-1}\frac{L/2}{r} = \sin^{-1}\frac{10}{14.50} = 43.6^o$$

The horizontal distance from the center of the arch is given by:

$$x = r\sin\left(\frac{\theta}{2}\right) = 14.5\times\sin\left(\frac{43.6}{2}\right) = 5.385\ m$$

$$x_c = L/2 - x = 10 - 5.385 = 4.615\ \ m$$

The height of the arc (y_1) at the quarter point equals to:

$$y = \sqrt{r^2 - x^2} = \sqrt{14.5^2 - 5.38^2} = 13.46\ m$$

$$y_1 = y - (r - f) = 13.64 - (10.5) = 2.96\ m$$

The moment at O_1 equals to:

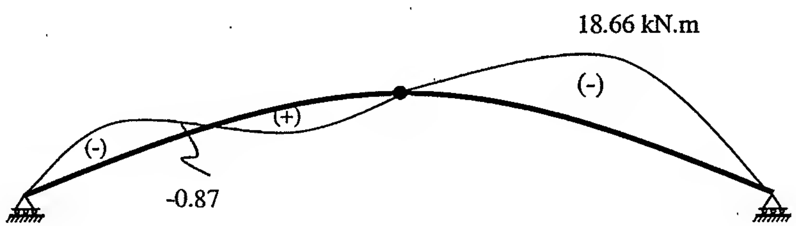
$$M_u = R_2\ x_c - (w_{UDL})\times x^2_c/2 - H\times y_1$$

$$M_u = 61.1\times 4.615 - (5.75)\times 4.615^2/2 - 80.88\times 2.96 = -18.66\,kN.m$$

The maximum moment at O_2 equals to:

$$M_u = R_1\ x_c - (w_{UDL} + w_{ULL})\ x_c^2/2 - H\times y_1$$

$$M_u = 68.3\times 4.615 - 7.2\times 4.615^2/2 - 80.88\times 2.96 = -0.87\ kN.m$$



Bending moment diagram

It is clear that the maximum moment occurs at point O_1. The corresponding normal and shear forces may be obtained using the following equations:

$$P_u = H\ \cos\alpha + Q\sin\alpha$$

$$Q' = Q\cos\alpha - H\ \sin\ \alpha$$

Where H and Q are the horizontal and vertical forces, respectively, at that section.

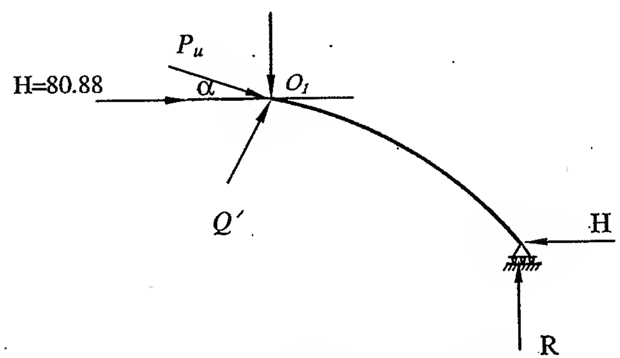
H= 80.88 kN, and

$$Q = 61.1 - 5.75\times 4.615 = 34.6\ kN$$

$$\alpha = \frac{\theta}{2} = \frac{43.6}{2} = 21.8^o$$

$$P_u = 80.88\ \cos\ 21.8 + 34.6\sin 21.8 = 87.94\,kN$$

$$Q' = 34.6\cos 21.8 - 80.88\ \sin\ 21.8 = 2.08\ kN$$



Force analysis at the quarter point

Alternatively, the bending moment and the normal force may be obtained using Table 1.6 as follows:

$$\frac{f}{L} = \frac{4}{20} = 0.2$$

From the table with f/L=0.2, one can determine that,

k_1=-0.00384, k_2=-0.01745, k_3=0.01361, k_4=1.0872

$$M_{(-ve)DL} = k_1\times w_{UDL}\times L^2 = -0.00384\times 5.75\times 20^2 = -8.84\ kN.m$$

$$M_{(-ve)LL} = k_2\times w_{ULL}\times L^2 = -0.01745\times 1.44\times 20^2 = -10.05\,kN.m$$

$$M_{(+ve)LL} = k_3\times w_{ULL}\times L^2 = 0.0136\times 1.44\times 20^2 = +7.83\ kN.m$$

$$M_{(-ve)} = -8.84 + (-10.05) = -18.8\ kN.m$$

$$M_{(+ve)} = -8.84 + (7.83) = -1.01\ kN.m$$

$$P_u = k_4\times H = 1.0872\times 80.93 = 87.99\,kN$$

Step 3.1.2: Calculate the reinforcement

The section at the quarter point is subjected to combined compression force and bending moment. The thickness of the arch at this location is 125 mm.

$$\frac{P_u}{f_{cu}\ b\ t}=\frac{87.99\times10^3}{30\times1000\ \times125}=0.023<0.04$$

Thus the normal force can be neglected, and designed for moment only.

$$d=t_s-cover=125-20=105\ mm$$

$$R_1=\frac{M_u}{f_{cu}\ b\ d^2}=\frac{18.8\times10^6}{30\times1000\ \times105^2}=0.057 \qquad \rightarrow \qquad \omega=0.07$$

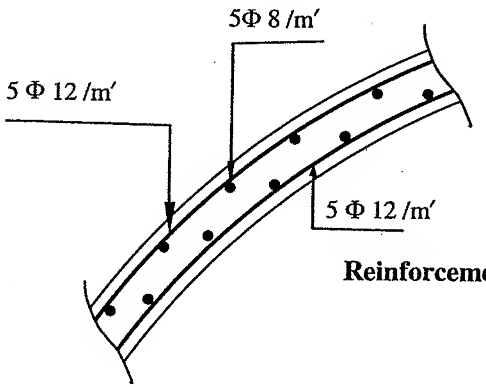
$$A_s=\omega\frac{f_{cu}}{f_y}b\times d=0.07\ \frac{30}{400}\times1000\times105=551\ mm^2$$

$$A_{s\ min}=\frac{0.6}{f_y}b\ d=\frac{0.6}{400}1000\times105=157.5\ mm^2$$

Choose 5 Φ 12/m′ (A_s = 565 mm^2) (Top reinforcement)

Thus, the main top reinforcement is taken 5 Φ 12/m′. As some parts of the slab are subjected to positive bending moment the main bottom reinforcement is also taken 5 Φ 12/m′.

The secondary reinforcement is chosen as at least 0.2 A_s. Choose 5 Φ 8 /m′. The reinforcement is arranged staggered for easy pouring of the concrete.

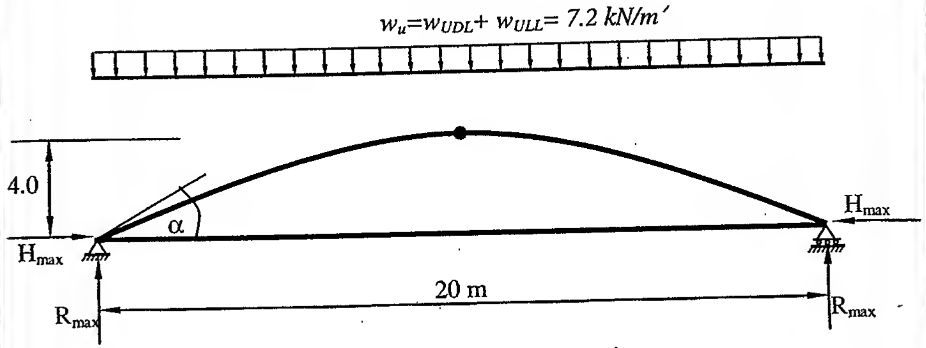


Reinforcement details for the slab

Step 3.2: Section at the support (t=150 mm)

Step 3.2.1: Straining actions

To obtain the maximum reaction at the support, the full arch is covered by the live and dead loads as shown in figure.



Equivalent load system and reactions

$$R_{max}=7.2\times\frac{20}{2}=72\,kN$$

$$H_{max}\times4=72\times10-7.2\times10\times5$$

H_{max} =90 kN

$\alpha=\theta=43.6^o$ → (refer to step 3.1.1)

The corresponding normal force and shear at this section may obtained by:

$$P_u=H\ \cos\alpha+Q\sin\alpha$$

$$Q'=Q\cos\alpha-H\ \sin\ \alpha$$

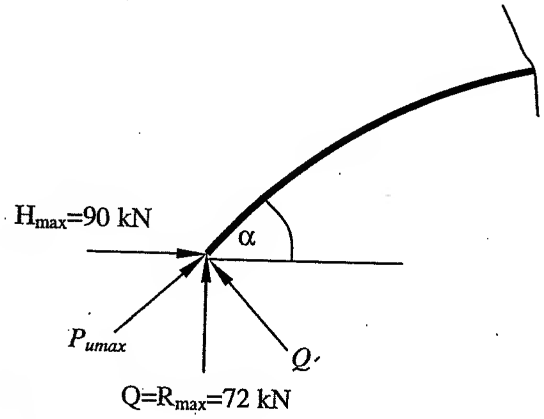
Where H and Q are the horizontal and vertical forces at the support.

H= 90 kN.

$$Q=R_{max}=72\ kN$$

$$P_{u,max}=90\ \cos\ 43.6+72\sin43.6=114.8\ kN$$

$$Q'=72\cos43.6-90\ \sin\ 43.6=-9.9\ kN$$



Step 3.2.2: Calculate the reinforcement

The section is subjected to pure compression (P_u=114.6 kN) and (M_u=0).

Assuming the total minimum area of steel of 0.6%

$$A_s = \frac{0.6}{100} \times 1000 \times 150 = 900\ mm^2 \quad \text{(top and bottom)}$$

$$A_{s/each\ face} = 450\ mm^2$$

Using the same reinforcement determined from the section at the quarter span

$A_{s,top}$=$A_{s,bot}$= 5 Φ 12/m′=565 m^2 > (450 mm^2)

$$A_{total} = 2 \times 565 = 1130\ mm^2$$

$$P_u = 0.35 f_{cu}\ A_c + 0.67 \times f_y \times A_s$$

$$P_u = 0.35 \times 30 \times 150 \times 1000 + 0.67 \times 400 \times 1130 = 1877\ kN$$

Since the applied compression force is less than the section capacity, the section is considered adequate.

Step 3.2.3: Design for shear

According to the ECP 203, the slab shear strength is calculated using the following relation:

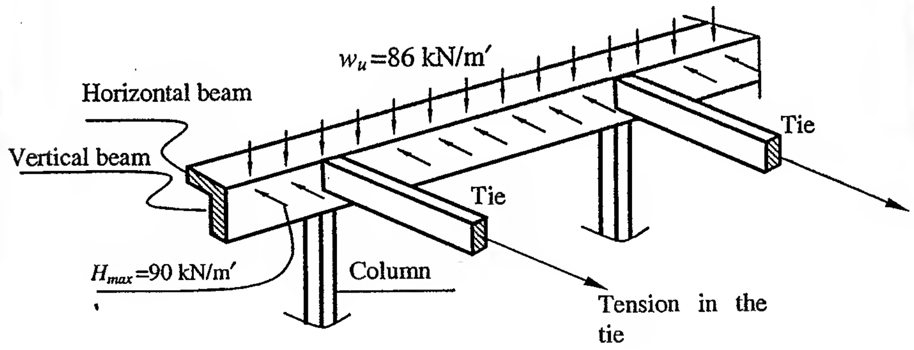
$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{1.5}} = 0.16\sqrt{\frac{30}{1.5}} = 0.715\ N/mm^2$$

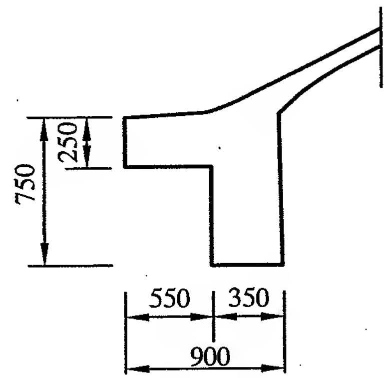
$$q_u = \frac{Q'}{b \times d} = \frac{9.9 \times 1000}{1000 \times 130} = 0.07\ N/mm^2\ (very\ safe)$$

Step 4: Design of the vertical beam (350 mm x 750 mm)

The vertical beam is analyzed as a continuous beam supported on columns.

Step 4.1: Calculations of the straining actions





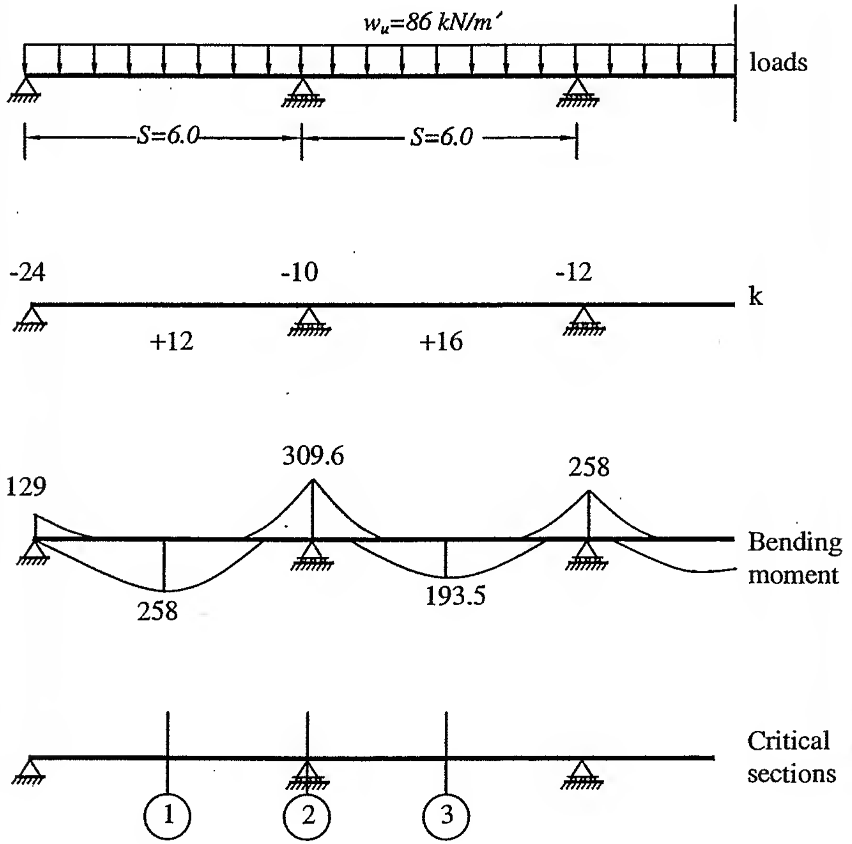
The factored weight of the vertical and horizontal beams equals:

$$ow. = 1.4 \times 25 \times (0.35 \times 0.75 + 0.25 \times 0.55)/10^6 = 14\ kN/m'$$

The total factored load on the vertical beam equals:

$$w_u = R_{max} + ow = 72 + 14 = 86\ kN/m'$$

In which R_{max} is the vertical reaction obtained from the analysis of a strip of 1.0 m width of the arched slab.



Step 4.2: flexural design

Sec. 1:

$$M_u = \frac{w\ S^2}{12} = 258 \text{ kN.m}$$

$$R_1 = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{258 \times 10^6}{30 \times 350\ \times 700^2} = 0.05 \qquad \rightarrow \qquad \omega = 0.0613$$

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.0613 \frac{30}{400} \times 350 \times 700 = 1126\ mm^2$$

Choose 6 Φ 16 (A_s= 1206 mm^2). The secondary reinforcement is chosen as at least 0.1-0.2 A_s. Choose 2 Φ 12 .

Sec. 2

$$M_u = \frac{w\ S^2}{10} = 309.6 \text{ kN.m}$$

$$R_1 = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{309.6 \times 10^6}{30 \times 350\ \times 700^2} = 0.06 \qquad \rightarrow \qquad \omega = 0.0746$$

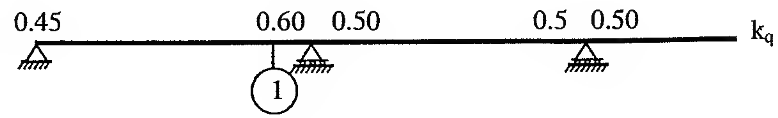
$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.0746 \frac{30}{400} \times 350 \times 700 = 1370\ mm^2$$

Choose 5 Φ 20 (A_s= 1570 mm^2)

The secondary reinforcement is chosen as 0.1-0.2 A_s. Choose 2 Φ 16 .

Step 4.3: Design for Shear

The critical section for shear is at d/2 from the face of the middle support. The width of the column is 250 mm. The critical section is at section (1) as shown in figure with code coefficient of k_q=0.6



$$Q_u = k_q\, w_u\, L - w_u \left(\frac{c}{2}+\frac{d}{2}\right) = 0.6\times 86\times 6.0 - \frac{86}{1000}\times\left(\frac{250}{2}+\frac{700}{2}\right) = 268.75\ kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{268.7\times 1000}{350\times 700} = 1.096\ N/mm^2$$

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

Since $q_u > q_{cu}$, shear reinforcement is needed.

$$q_{su} = q_s - \frac{q_{cu}}{2}$$

$$q_{su} = 1.096 - \frac{1.07}{2} = 0.56\ N/mm^2$$

$$A_{st} = \frac{q_{su}\times b\times s}{f_y/\gamma_s}$$

Assuming a spacing of 100 mm, the shear reinforcement area is given by:

$$A_{st} = \frac{0.56\times 350\times 100}{240/1.15} = 93.9\ mm^2$$

Thus, the area of one branch $= \frac{A_{st}}{2} = \frac{93.9}{2} = 46.95\ mm^2$

Use ϕ 8=50 mm^2 (A_{st}=100 mm^2) $\rightarrow$ Use $\phi 8 @ 100mm$

$$A_{st,min} = \frac{0.4}{f_y} b\times s = \frac{0.4}{240}\times 350\times 100 = 58.3\ mm^2 < A_{st}\ .....ok$$

Step 5: Design the horizontal beam (250 mm x 900 mm)

The horizontal beam is analyzed as a continuous beam supported on the ties. It carries a uniformly distributed load equals to the horizontal thrust. This uniform load equals the horizontal reaction of a 1.0 m strip of the arched slab.

Step 5.1: flexural design

Sec. 1

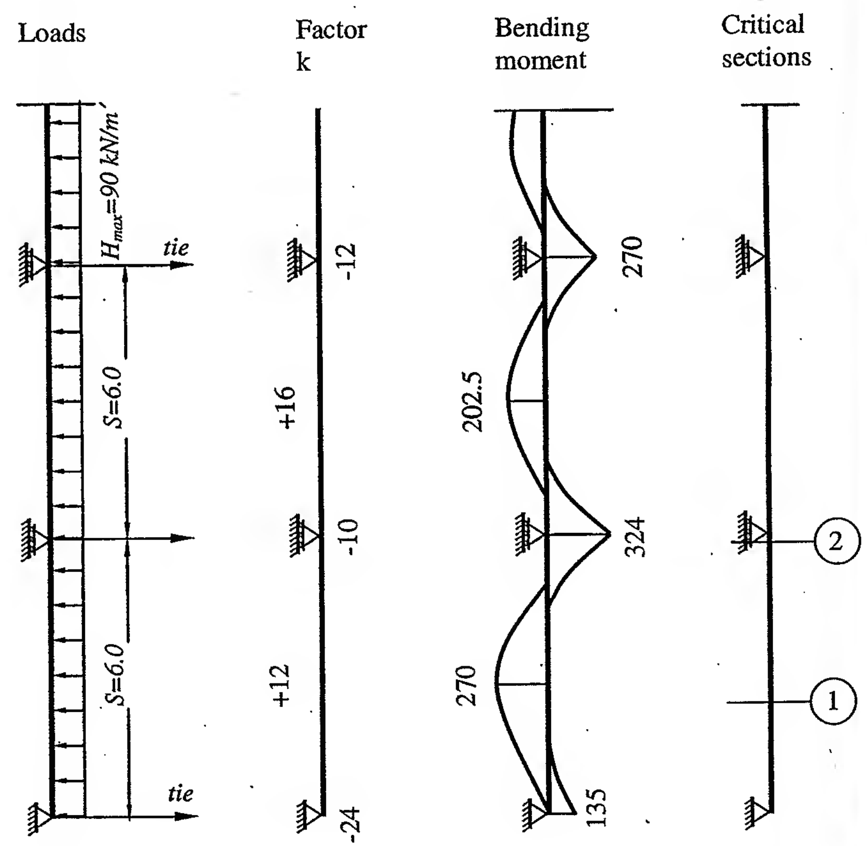
$$M_u = \frac{w\ S^2}{12} = \frac{90\times 6^2}{12} = 270\ \text{kN.m}$$

$$R_1 = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{270\times 10^6}{30\times 250\ \times 850^2} = 0.05 \qquad \rightarrow \omega = 0.061$$

$$A_s = \omega\frac{f_{cu}}{f_y} b\times d = 0.061\frac{30}{400}\times 250\times 850 = 972\ mm^2$$

Choose 5 Φ 16 (A_s= 1005 mm^2)

The secondary reinforcement is chosen as 0.1-0.2 A_s. Choose 2 Φ 12.



Sec. 2:

$$M_u = \frac{w\ S^2}{10} = \frac{90\times 6^2}{10} = 324\ \text{kN.m}$$

$$R_1 = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{324\times 10^6}{30\times 250\ \times 850^2} = 0.06 \qquad \rightarrow \qquad \omega = 0.074$$

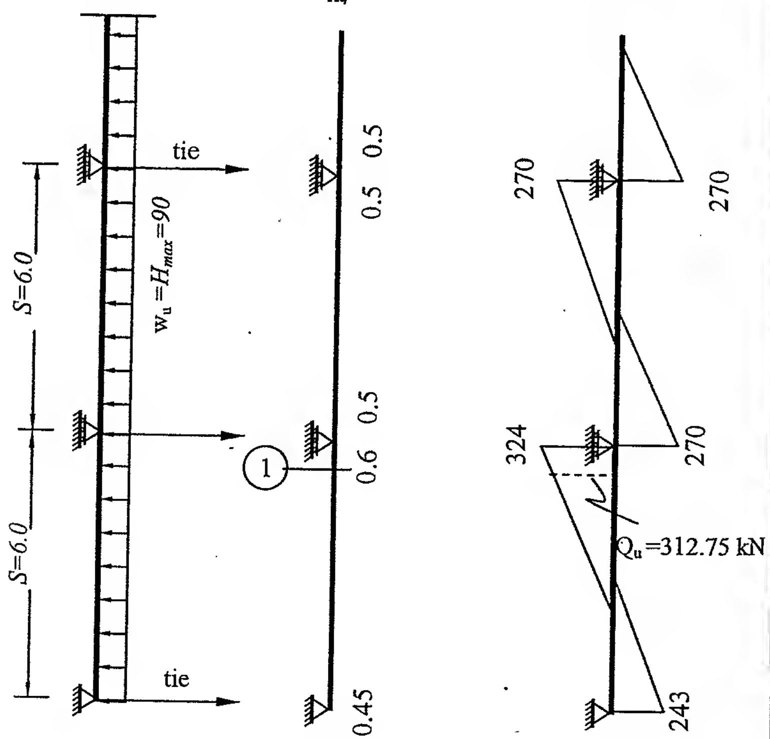
$$A_s = \omega \frac{f_{cu}}{f_y} b\times d = 0.074\frac{30}{400}\times 250\times 850 = 1180\ mm^2$$

Choose 5 Φ 18 (A_s = 1272 mm^2)

Step 5.2: Design for shear

The critical section for shear is at the face of the middle support because the support is in tension (the tie). The critical section is at section (1) as shown in figure with code coefficient of k_q=0.6.

$$Q_u = k_q\ w_u\ L - w_u\left(\frac{c}{2}\right)$$



$$Q_u = k_q\ w_u\ L - w_u\left(\frac{c}{2}\right) = 0.6\times 90\times 6.0 - 90\times\left(\frac{0.25}{2}\right) = 312.75\ kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{312.75\times 1000}{250\times 850} = 1.47\ N/mm^2$$

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

Since $q_u > q_{cu}$, shear reinforcement is needed.

$$q_{su} = 1.47 - \frac{1.07}{2} = 0.935\ N/mm^2$$

Assuming a spacing of 125 mm, the shear reinforcement area is given by:

$$A_{st} = \frac{q_{su}\times b\times s}{f_y/\gamma_s} = \frac{0.935\times 250\times 125}{240/1.15} = 140\ mm^2$$

Thus, the area of one branch $= \frac{A_{st}}{2} = \frac{140}{2} = 70\ mm^2$

Use ϕ 10=78 mm^2 (A_{st}=157 mm^2) Use $\phi 10@125mm \rightarrow 8\phi 10/m'$

$$A_{st,min} = \frac{0.4}{f_y} b\times s = \frac{0.4}{240}\times 250\times 125 = 39\ mm^2 < A_{st}\ .....\text{ok}$$

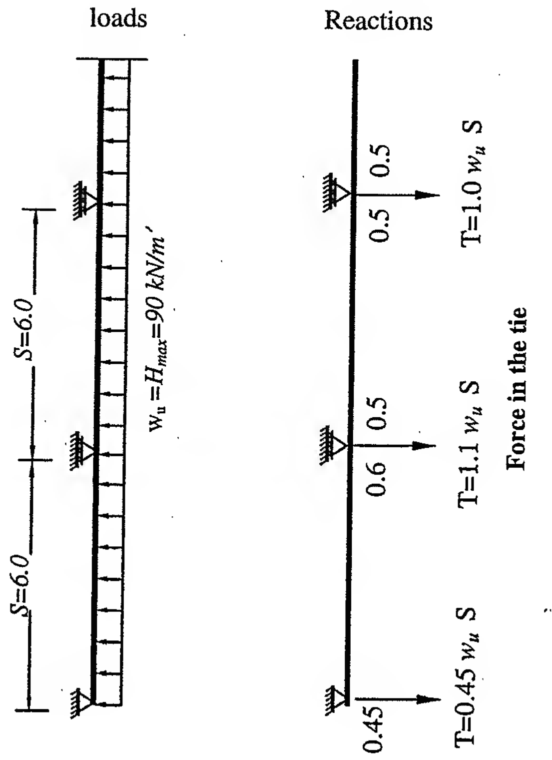
Step 6: Design of the tension tie (250 mm x 250 mm)

The tie is the main supporting element for the horizontal beam. The ECP 203 states the reaction for a continuous beam equals (1.1 w_u S) as shown in the figure below.

$$T = 1.1\times H_{max}\times S = 1.1\times 90\times 6 = 594\ kN$$

$$A_s = \frac{T}{f_y/1.15} = \frac{594\times 1000}{400/1.15} = 1707\ mm^2$$ Choose 8 Φ 18 (A_s = 2035 mm^2)

The self-weight of the tie results in a small value of the bending moment that can be neglected due to fact that under its self weight the tie acts a continuous beam supported by the hangers.



Step 7: Design of the hanger (250 mm x 250 mm)

The weight of the hanger equals to:

$$ow_H = \gamma_c \times b \times t \times h_h = 25 \times 0.25 \times 0.25 \times 4.0 = 6.25\ kN$$

The weight of the tie equals to:

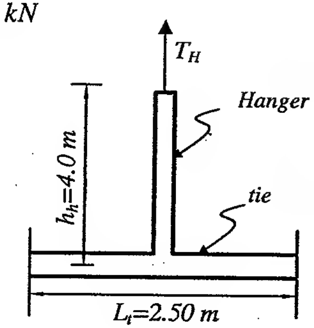
$$ow_T = \gamma_c \times b \times t \times L_t = 25 \times 0.25 \times 0.25 \times 2.5 = 3.9\ kN$$

The ultimate total weight (tension T_H)

$$T_H = 1.4 \times (6.25 + 3.9) = 14.2\ kN$$

$$A_s = \frac{T}{f_y / 1.15} = \frac{14.2 \times 1000}{400 / 1.15} = 40.8\ mm^2$$

Choose 4 Φ 10/m



Step 8: Design of the column (250 mm x 700 mm)

Step 8.1: Calculate applied loads

The columns are subjected to axial compression forces and bending moments resulting from the wind loads. The critical load combination is given by:

$$U = 0.8\ (1.4 \times D + 1.6 \times L + 1.6 W\ )$$

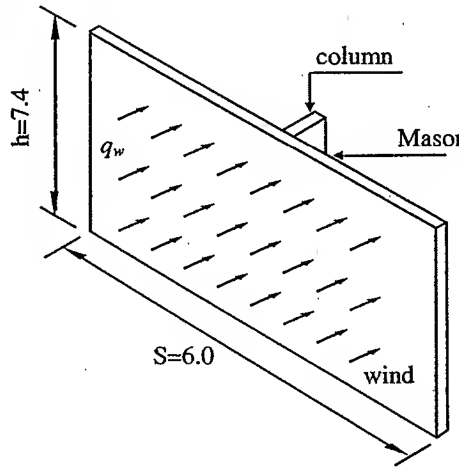
Effect of Wind loads

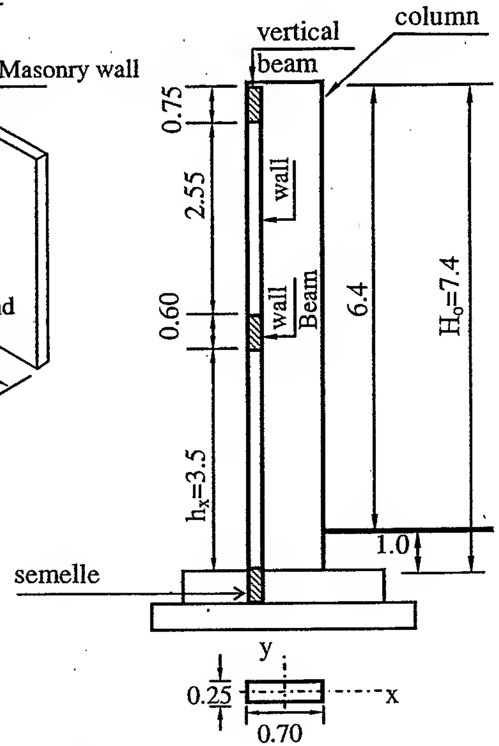
From the figure, the height of the column H_o s given by:

$$H_o = 6.4 + 1.0 = 7.4\ m$$

The wind pressure is assumed as 0.7 kN/m^2. The pressure on the walls equals

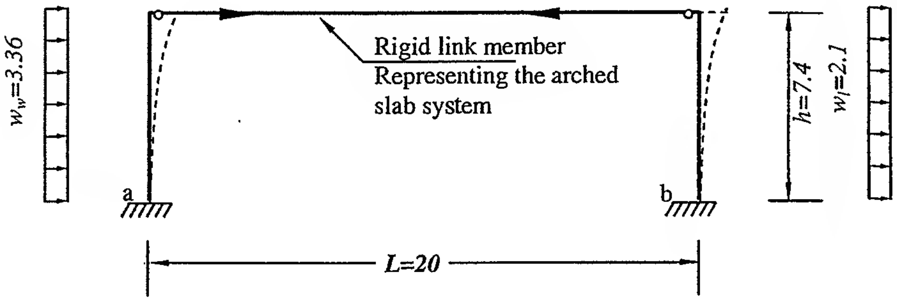
$$w_w = c_w \times q_w \times spacing = 0.8 \times 0.7 \times 6.0 = 3.36\ \ kN / m' \qquad \text{(wind-ward side)}$$

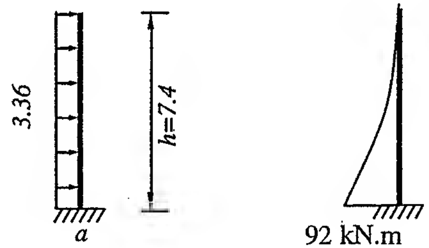




In order to determine the bending moments in the columns due to wind load, an exact analysis could be carried out. In such an analysis, the arched slab is assumed to act as a rigid link member connecting the columns as shown in the figure. The shown system is analyzed under the effect of the wind load and the bending moments in the columns are obtained.

As a conservative approximation, the bending moment in the column could be obtained by considering the case of a cantilever column subjected to uniform wind load.





The moment of a cantilever member subjected to uniform load is given by:

$$M_{wind} = \frac{w_w \times h^2}{2} = \frac{3.36 \times 7.4^2}{2} = 92\, kN.m$$

The ultimate load case is given by:

$$M_u = 0.8 \ \ (1.4 \times M_{DL} + 1.6 \times M_{LL} + 1.6 \ M_{wind})$$

Since M_{DL} and M_{LL} are equal to zero

$$M_u = 1.28 M_{wind} = 117.8 \ kN.m$$

Effect of vertical loads

The vertical loads on the column are the summation of the following

1. Self-weight $= 1.4 \times \gamma_c \times b \times t \times h = 1.4 \times 25 \times 0.25 \times 0.7 \times 7.4 = 45.3 \ kN$
2. Weight of wall beam

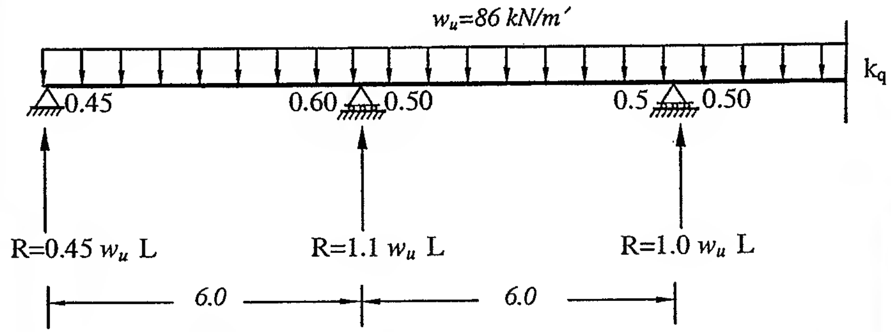
   $= 1.4 \times \gamma_c \times b \times t \times spacing \ = 1.4 \times 25 \times 0.25 \times 0.6 \times 6.0 = 31.5 \ kN$
3. Wall load $= 1.4 \times \gamma_w \times b \times S \times (h - t_{wall\ beam} - t_{vertical\ beam})$

   Assuming γ_w=18 kN/m^3 (neglecting the difference between the semelle and wall)

   $= 1.4 \times 18 \times 0.25 \times 6.0 \times (7.4 - 0.6 - 0.75) = 228.7 \ kN$
4. Arch load = Reaction from the vertical beam

   $R_{arch} = 1.1 \times w_u \,(vertical\ beam) \times S \ = 1.1 \times 86 \times 6 = 567.6 \ kN$

   $P_u = 45.3 + 31.5 + 228.7 + 567.6 = 873.1 \ kN$



Statical system and loads on the vertical beam

However the ultimate vertical load should by reduced as stated by the code as follows:

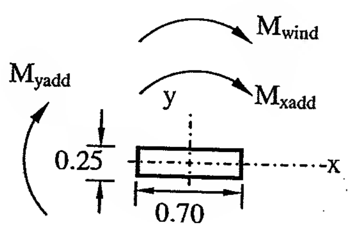
$$P_u = 0.8 \ \ (1.4 \times P_{DL} + 1.6 \times P_{LL} + 1.6 \ P_{wind})$$

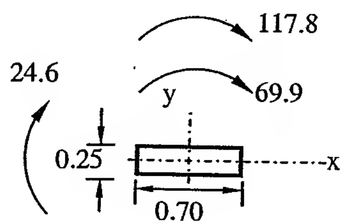
$$P_u = 0.8 \times 873.1 = 698.4 \ kN$$

Step 8.2: Calculation of the reinforcement

The column is considered unbraced in both directions because there is no lateral resisting system. The unsupported length in the X-direction is 7.4 ms and the unsupported length in the Y-direction is 3.5 ms. The calculations of the additional moments can be summarized in the following table.

Item	**X-direction**	**Y-direction**
bracing condition	unbraced	unbraced
Ultimate load P_u(kN)	698.4	698.4
Short column if	λ<10	λ<10
H_o (m)	7.4	3.5
t (m)	0.7	0.25
bracing condition at top	case 3	case 1
bracing condition at bottom	case 1	case 1
k (bracing factor)	1.6 (table 6-10)	1.2 (table 6-10)
$\lambda = k \times H_o / t$	16.91	16.8
Status	long(λ>10)	long (λ>10)
$\delta = \lambda^2 \times t / 2000$	0.1	0.035
$M_{add} = P_u . \delta$	69.9	24.6
M_u(wind)	117.8	0
$M_{total} = M_u + M_{add}$	187.7	24.6





It is clear from the previous table that the column is subjected to biaxial bending.

Since $\dfrac{M_x(187.7)}{a'(0.65)} > \dfrac{M_y(24.6)}{b'(0.20)}$, X-direction is more critical than Y-direction

The load level R_b equals to:

$$R_b = \frac{P_u}{f_{cu} \times b \times t} = \frac{698.4 \times 1000}{30 \times 250 \times 700} = 0.133$$

From ECP 203 and using R_b = 0.133, the β factor = 0.80

$$M'_x = M_x + \beta \left(\frac{a'}{b'}\right) M_y$$

$$M'_x = 187.7 + 0.8 \left(\frac{650}{200}\right) 24.6 = 251.7 \ kN.m$$

Using interaction diagram with **uniform** steel (f_y=400N/mm^2, ζ=0.8), calculate

$$\frac{M'_x}{f_{cu} \times b \times t^2} = \frac{251.7 \times 10^6}{30 \times 250 \times 700^2} = 0.068$$

from the chart ρ=3

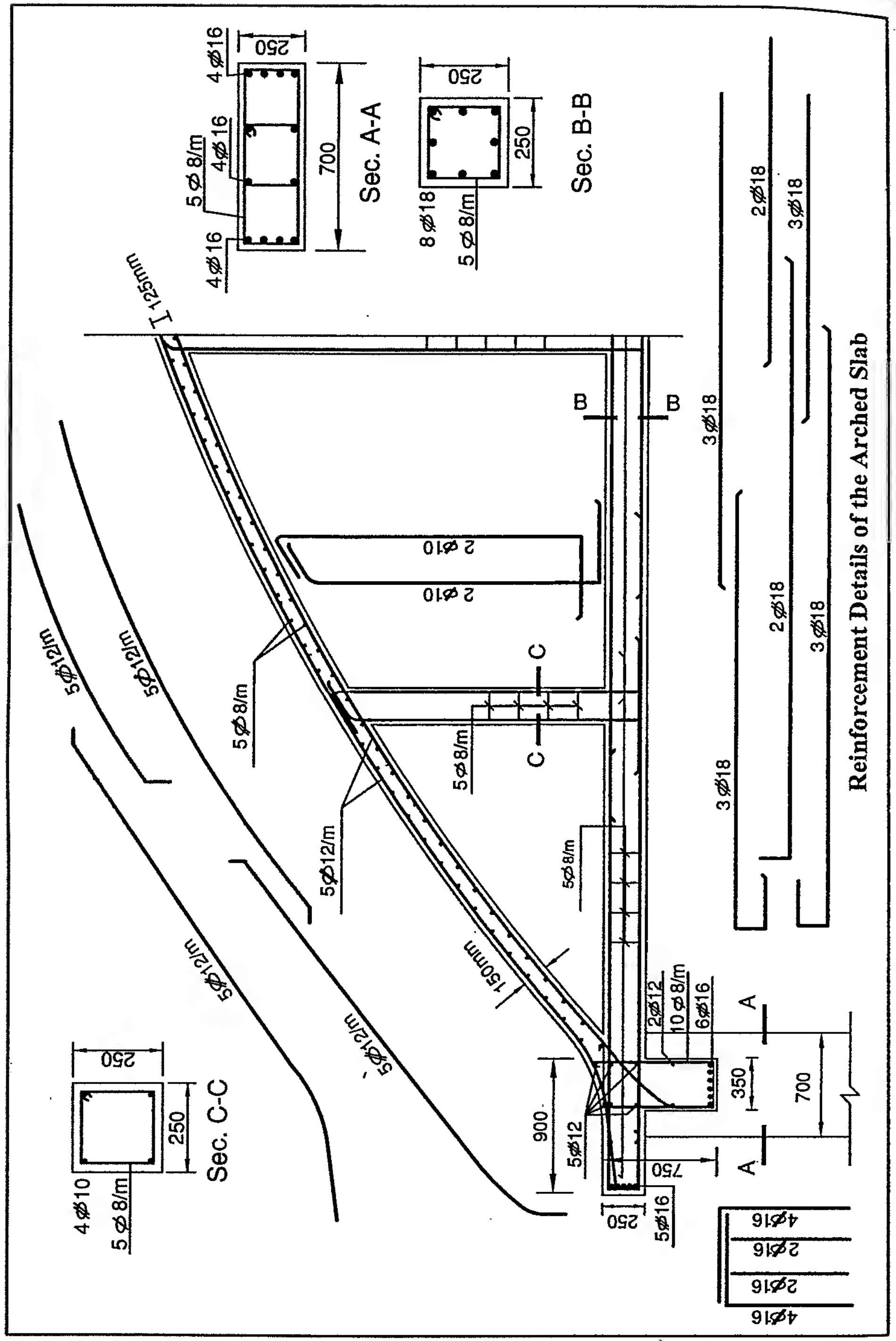
$$\mu = \rho \times f_{cu} \times 10^{-4} = 3 \times 30 \times 10^{-4} = 0.009 = 0.9\%$$

However, since the column is long the minimum reinforcement ratio μ_{min} is

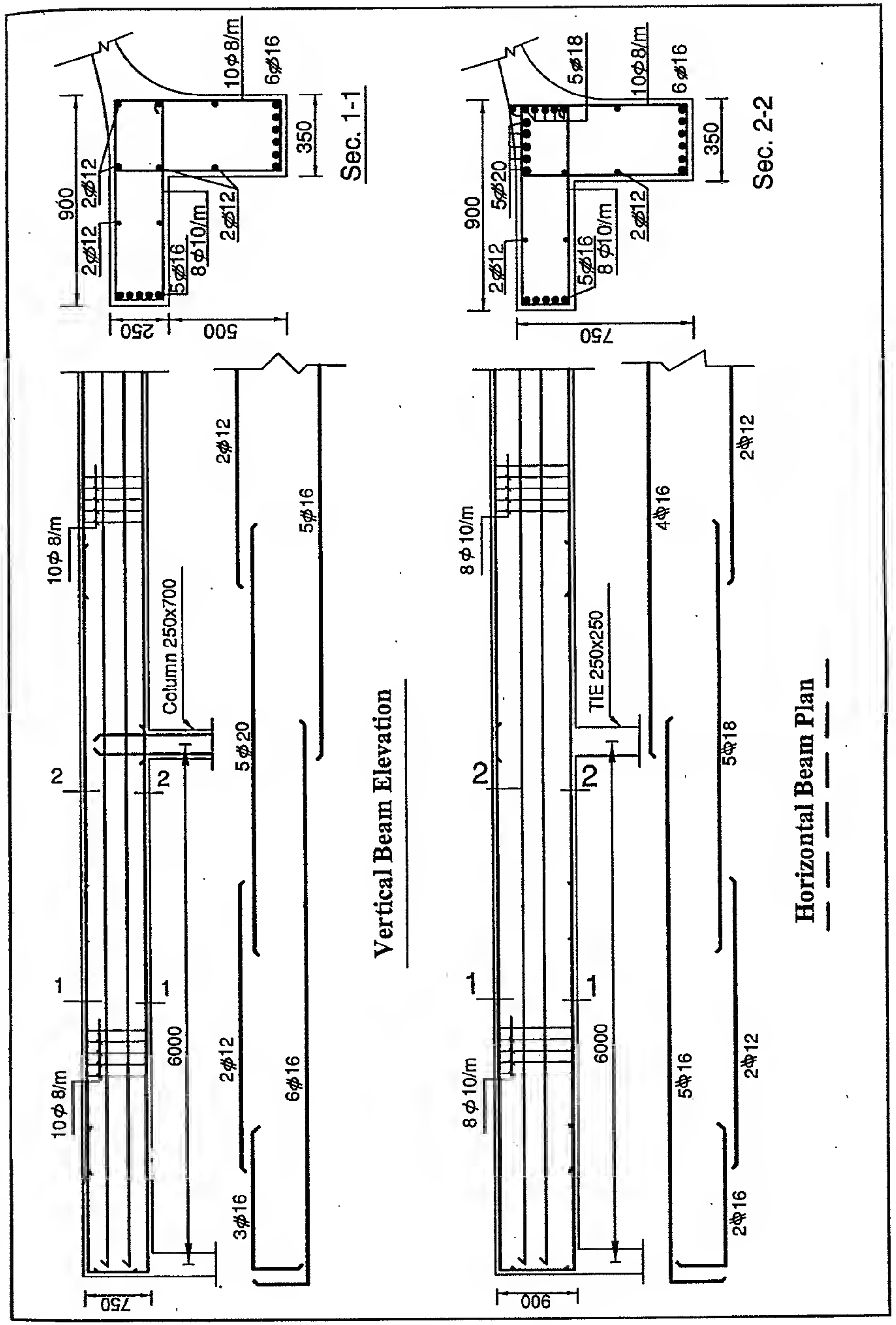
$$\mu_{min} = 0.25 + 0.052\ \lambda = 0.25 + 0.052 \times 16.91 = 1.13\%$$

$$A_{s,min} = \mu_{min} \times b \times t = \frac{1.13}{100} \times 250 \times 700 = 1976\ mm^2$$

Choose (12 Φ16, 2412 mm^2) distributed uniformly.



Reinforcement Details of the Arched Slab



Vertical Beam Elevation

Horizontal Beam Plan

Example 1.10 Parabolic arched slab

A car showroom is to be constructed on an area of (16 ms x 35 ms) as shown in Fig. EX 1.10. A parabolic arched slab was chosen as the main supporting system. Design and give details for the system knowing that the material properties are f_{cu}=25 N/mm^2, and f_y=360 N/mm^2, and f_{yst}=280 N/mm^2. Neglect the effect of wind on the design of the columns. The building may be assumed as unbraced in the in-plane direction and braced in the out-of-plane direction.

Data

D.L. =1 N/m^2 (not including own weight)
L.L. = 0.5 N/m^2
Clear height= 5.0 m

Solution

Step 1: Propose the concrete dimensions

The parabolic arched slab is the chosen main system with the following dimensions:

t_s (mid-span) = 100 mm
t_s (quarter point) = 120 mm
t_s (edge) = 140 mm

Vertical beam = (250 mm x 600 mm)
Horizontal beam = (200 mm x 700 mm)
Tie = (200 mm x 200 mm)
Hanger = (200 mm x 200 mm)

$$f = \frac{Span}{5 \rightarrow 8} = \frac{16}{5 \rightarrow 8} \approx 2.4\ ms$$

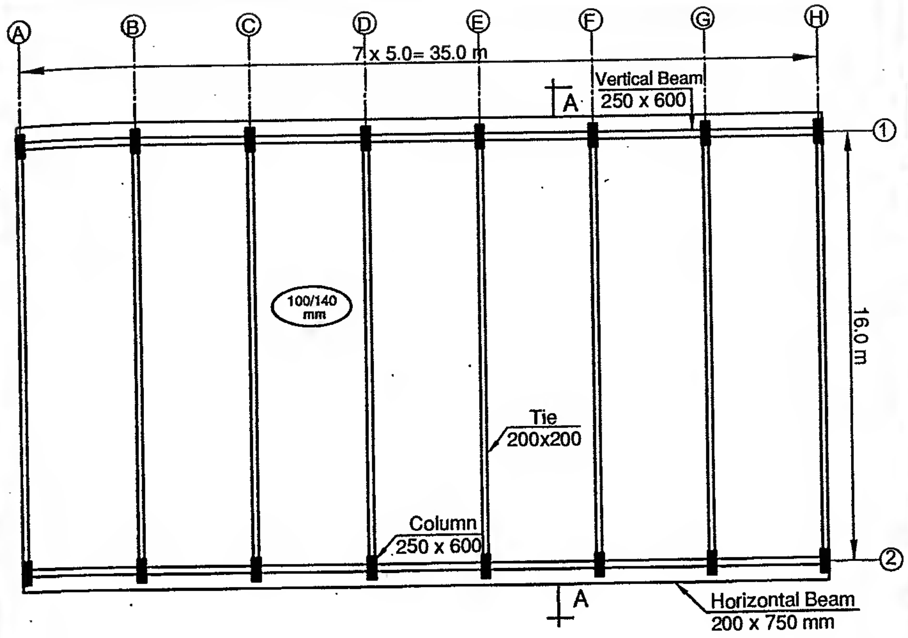
The spacing between the arches = 5.0 m
Column = 250 mm x 600 mm

The complete layout is shown in Fig. EX 1.10.

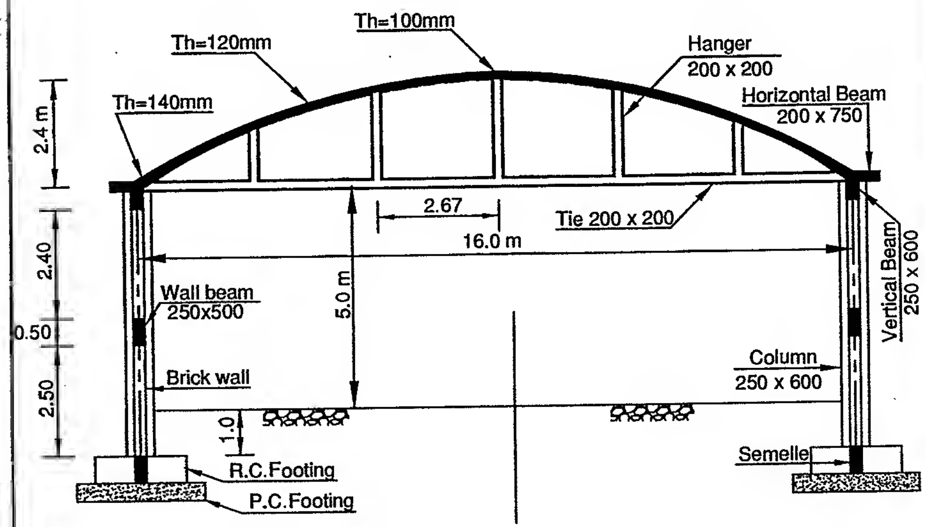
Step 2: Calculations of acting loads

In order to calculate the weight of the arched slab, the length of the parabola need to be computed. For simplicity the length of the parabola is taken as 1.1 the horizontal distance between the supports (span).

The length of the arc $L' \cong 1.1\,L = 1.1 \times 16 = 17.6\ m$



Plan



(Sec. A - A)

Fig. EX 1.10 Parabolic arched slab

The self-weight of the arched slab may be calculated using the thickness at the quarter point (120 mm).

$$o.w. = \gamma_c \times t_{avg} = 25\times0.120 = 3.0\ kN/m^2$$

The total factored dead load including plaster weight w_{UDL} is given by:

$$w_{UDL} = 1.4\ (o.w. + plaster\ weight) = 1.4\times(3.0+1.0) = 5.60\ kN/m^2$$

The value of he horizontal projection (H.P.) of this load is given by:

$$w_{UDL} = 1.4\, w_{DL} \times \frac{L'}{L} = 5.60\times\frac{17.6}{16} = 6.16\ kN/m^2 \qquad (H.P.)$$

Noting that the live loads on inclined surfaces are always taken on the horizontal projection, the slab factored live load w_{ULL} is given by:

$$w_{ULL} = 1.6\times w_{LL} = 1.6\times0.5 = 0.80\ kN/m^2$$

The total factored load $w_u = w_{UDL} + w_{ULL} = 6.16+0.80 = 6.96\ kN/m^2$ (H.P.)

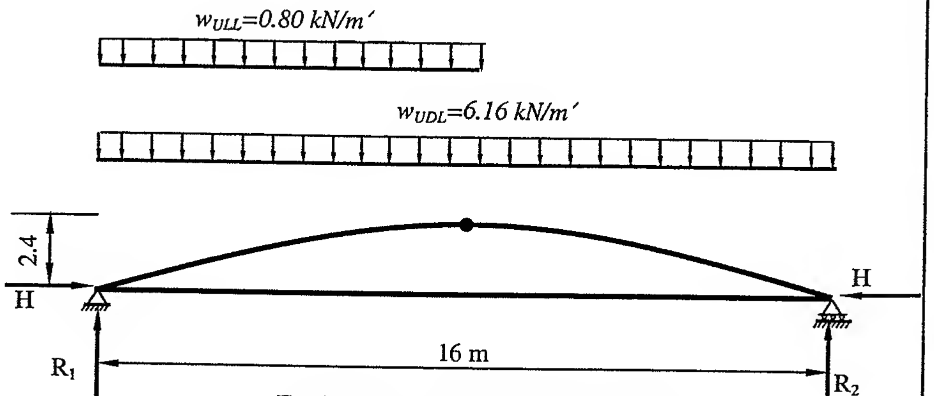
Step 3: Design the arched slab critical sections

Taking 1m width of the slab, the acting loads are shown in the following figure.

Step 3.1: Section at the quarter points (t=120 mm)

Step 3.1.1: Straining actions at the quarter points

To obtain the maximum moment at the quarter point, only half of the arch is to be covered by the L.L. as shown in figure below.



Equivalent load system and reactions

$$R_2\times16 = 6.16\times16\times\frac{16}{2} + 0.80\times8\times4 \quad\rightarrow\quad R_2 = 50.88\ kN$$

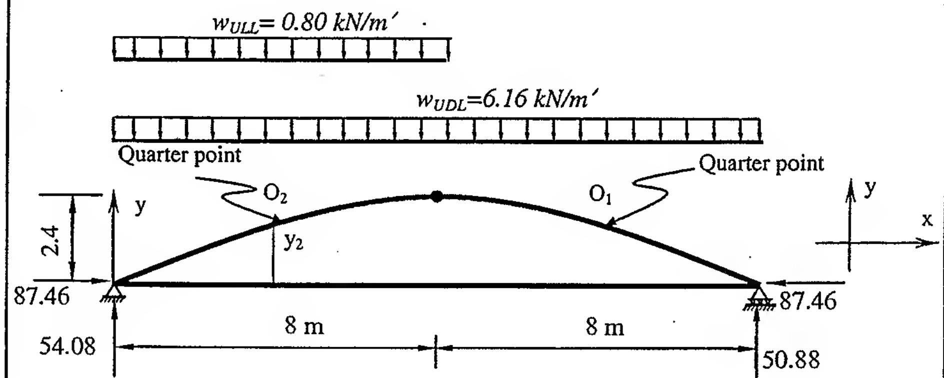
$$R_1\times16 = 6.16\times16\times\frac{16}{2} + 0.80\times16\times12 \quad\rightarrow\quad R_1 = 54.08\ kN$$

To obtain the horizontal thrust H, the moment is taken at the middle hinge as follows:

$$H = \frac{(w_{UDL} + \frac{w_{ULL}}{2})\times L^2}{8\,f} = \frac{(6.16+\frac{0.8}{2})\times16^2}{8\times2.4} = 87.46\ kN$$

The same result can be obtained by taking moment of forces at the crown.

$$H\times2.4 = R_1\times8 - w_u\times8\times4 = 54.08\times8 - 6.96\times8\times4 \quad\rightarrow\quad \text{H=87.46 kN}$$



The height of the arched slab at the quarter point may be obtained using the properties of the parabola.

$$y_2 = \frac{3}{4}f = \frac{3}{4}\times2.4 = 1.8\ m$$

It can also be obtained by substitution in the equation of the parabola with x=4.

$$y = \frac{4\cdot f\cdot x\cdot(L-x)}{L^2} = \frac{4\times2.4\times4\times(16-4)}{16^2} = 1.8\ m$$

The maximum moment at O1:

$$M_u = R_2\times4 - (1.4w_{DL})\times4\times2 - H\times y_2$$

$$M_u = 50.88\times4 - (6.16)\times4\times2.0 - 87.46\times1.8 = -3.2\ kN.m$$

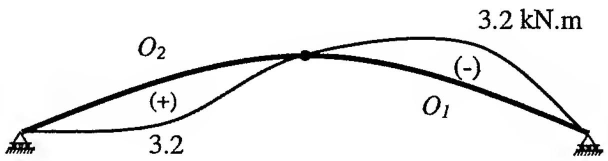
Also, it can be obtained directly from Table 1.7.

$$M_u = \pm \frac{w_{ULL} \times L^2}{64} = \pm \frac{0.8 \times 16^2}{64} = \pm 3.2\, kN.m$$

The maximum positive moment at O2 equals

$$M_u = R_1 \times 4 - (w_{UDL} + w_{ULL}) \times 4 \times 2.0 - H \times y_2$$

$$M_u = 54.08 \times 4 - 6.96 \times 4 \times 2.0 - 87.46 \times 1.8 = 3.2\ kN.m$$



Bending moment diagram

At point O1, the corresponding normal and shear forces can be obtained as:

$$P_u = H \cos \alpha + Q \sin \alpha$$

$$Q' = Q \cos \alpha - H \sin \alpha$$

H and Q are the horizontal and vertical forces at that section.

H= 87.46 kN.

$$Q = 50.88 - 6.16 \times 4 = 26.24\ kN$$

To obtain the tangent angle at the quarter point, the equation of the parabola is differentiated as follows:

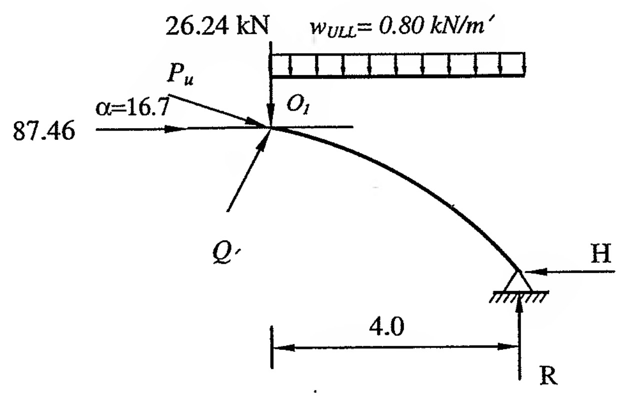
$$y = \frac{4 \cdot f \cdot x \cdot (L - x)}{L^2} = \frac{4 \times 2.4\ x\ (16 - x)}{16^2} = 0.0375\ \ (16\,x - x^2)$$

$$y' = \tan \alpha = 0.0375\ \ (16 - 2x)$$

Substituting with x=4 $\qquad \tan \alpha = 0.30 \qquad \alpha = 16.7$

$$P_u = 87.46 \cos 16.7 + 26.24 \sin 16.7 = 91.31\ kN$$

$$Q' = 26.24 \cos 16.7 - 87.46 \sin 16.7 = 0$$



The section is subjected to compression and bending moment. The thickness of the arch at this location is 120 mm.

$$\frac{P_u}{f_{cu}\, b\, t} = \frac{91.13 \times 10^3}{25 \times 1000\ \times 120} = 0.0304 < 0.04$$

Thus the normal force can be neglected, design for moment only.

$$d = t_s - cover = 120 - 20 = 100\ mm$$

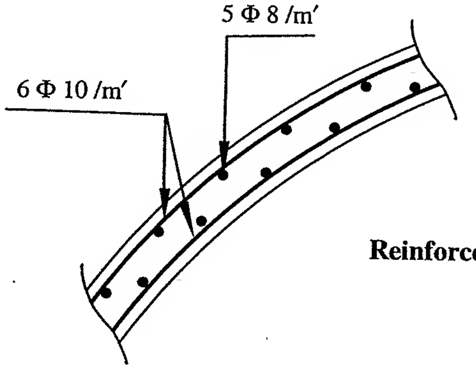
$$R_1 = \frac{M_u}{f_{cu}\, b \cdot d^2} = \frac{3.2 \times 10^6}{25 \times 1000\ \times 100^2} = 0.0128\ \rightarrow \omega = 0.015$$

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.015 \frac{25}{360} \times 1000 \times 100 = 103\ mm^2$$

$$A_{s\,min} = \frac{0.6}{f_y} b\ d = \frac{0.6}{360} 1000 \times 100 = 166\ mm^2$$

Choose 6 Φ 10/m′ (A_s = 471 mm^2)

Due to the fact that half of the arched slab is subjected to negative moment and the other half is subjected to positive moment, the main reinforcement (6 Φ 10/m′) is provided at the top and bottom. The secondary reinforcement is chosen as at least 0.2 A_s. Choose 5 Φ 8 /m′. The reinforcement is arranged staggered to avoid congestion of reinforcement.

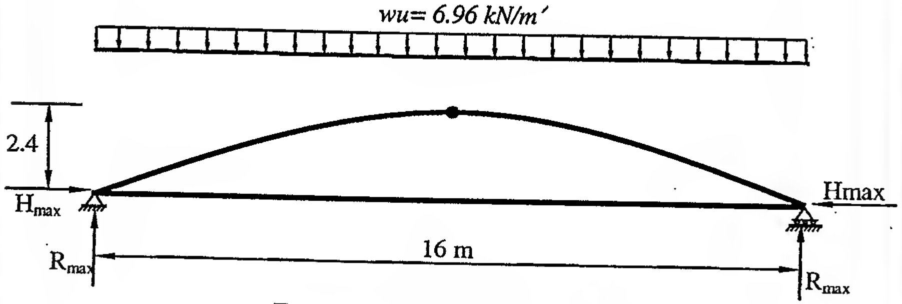


Reinforcement details for the slab

Step 3.2: Section at the support (t=140 mm)

Step 3.2.1: Straining actions

To obtain the maximum reactions at the support, the whole arch is covered by the both the dead and the live loads as shown in figure.



Equivalent load system and reactions

$$R_{max} = 6.96 \times \frac{16}{2} = 55.68\,kN$$

$$H_{max} \times 2.4 = 55.68 \times 8 - 6.96 \times 8 \times 4 \quad \rightarrow \quad H_{max} = 92.8 \text{ kN}$$

To obtain the tangent angle at the support, the equation of the parabola is differentiated as follows:

$$y = 0.0375 \ \ (16\,x - x^2)$$

$$y' = \tan\alpha = 0.0375 \ \ (16 - 2x)$$

Substituting with x=0 $\quad \tan\alpha = 0.60 \quad \alpha = 30.96$

The corresponding normal force and shear at this section can be obtained as:

$$P_u = H\cos\alpha + Q\sin\alpha$$

$$Q' = Q\cos\alpha - H\sin\alpha$$

H and Q are the horizontal and vertical forces at the support.

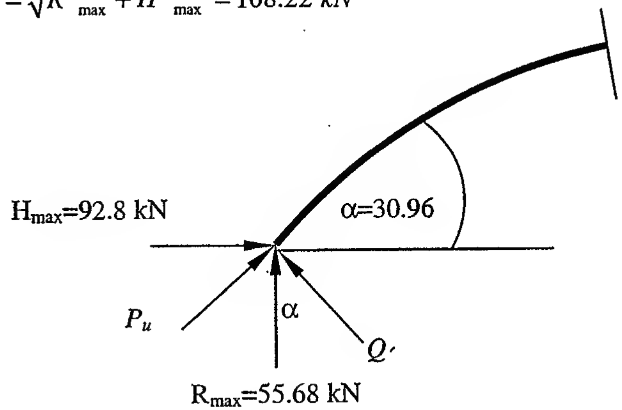
H= 92.8 kN.

$$Q = R_{max} = 55.68\ kN$$

$$P_u = 92.8\cos 30.96 + 55.68\sin 30.96 = 108.22\ kN$$

$$Q' = 55.68\cos 30.96 - 92.8\sin 30.96 = 0\ kN$$

Also $P_u = \sqrt{R^2_{max} + H^2_{max}} = 108.22\ kN$



Step 3.2.2: Design the reinforcement

The section is subjected to pure compression (P_u=108.22 kN) and (M_u=0).

Assume that the total minimum area of steel of equals to 0.6%.

$$A_s = \frac{0.6}{100} \times 1000 \times 140 = 840\ mm^2 \quad \text{(top and bottom)}$$

$$A_{s\,/each\,face} = 420\ mm^2$$

Using the same reinforcement determined from the section at the quarter span

$A_{s,top}$=$A_{s,bot}$= 6 Φ 10/m′=471 m^2 > (420 mm^2).

$A_{total} = 2 \times 471 = 942\ mm^2$

$P_u = 0.35\ f_{cu}\ A_c + 0.67 \times f_y \times A_s$

$P_u = (0.35 \times 25 \times 140 \times 1000 + 0.67 \times 360 \times 942)/1000 = 1452\ kN$

Since the applied compression force is less than the section capacity, the section is considered adequate.

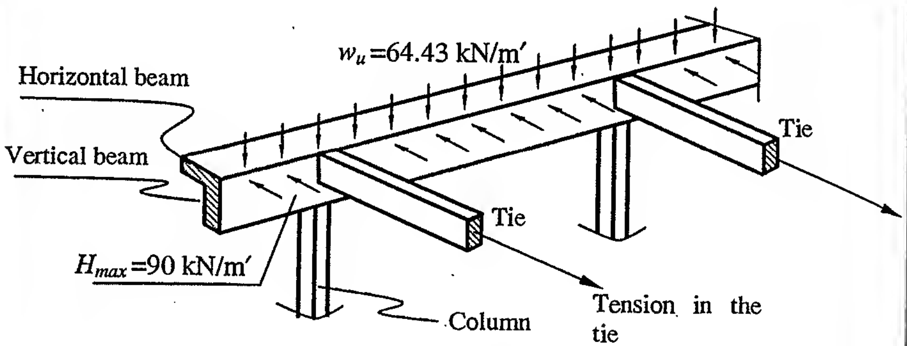
Step 3.2.3: Design for shear

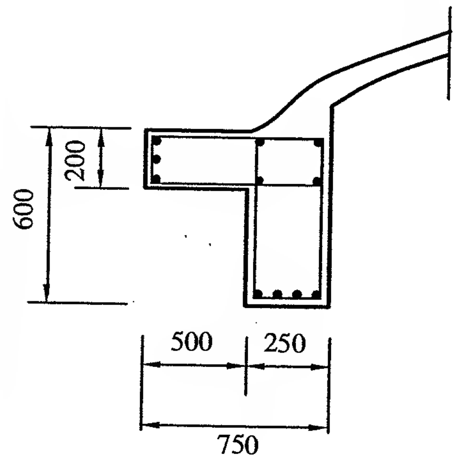
The applied shear at this section (Q′) equals to zero.

Step 4: Design the vertical beam (250 mm x 600 mm)

The vertical beam is analyzed as continuous beam supported on columns

Step 4.1: Calculate the straining actions



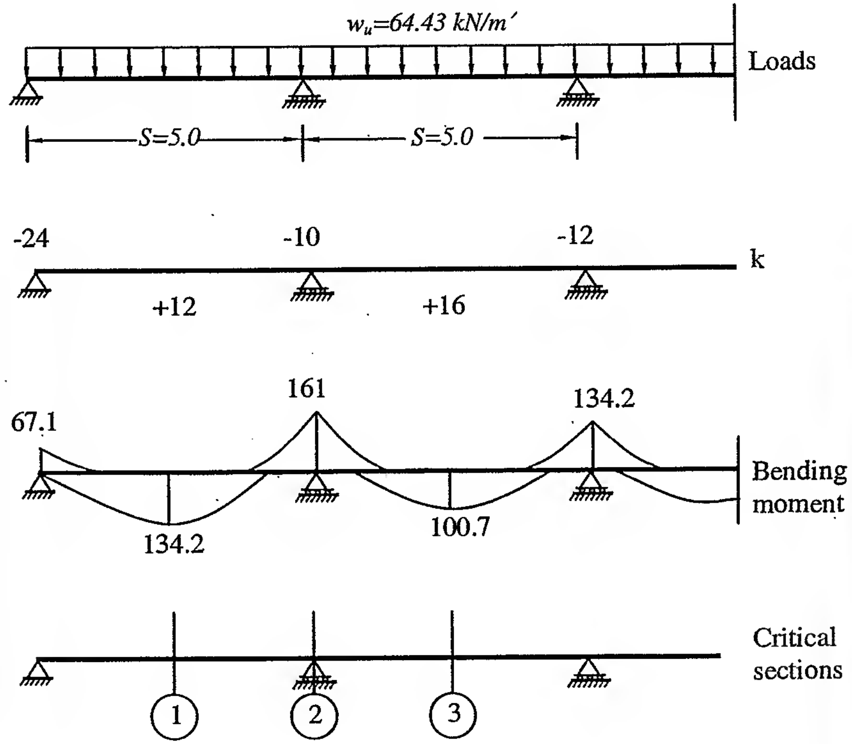


The factored weight of the vertical and horizontal beam equals to:

$ow. = 1.4 \times 25 \times (0.25 \times 0.6 + 0.20 \times 0.5) = 8.75\ kN/m'$

The total factored load on the beam equals to:

$w_u = R_{max} + ow = 55.68 + 8.75 = 64.43\ kN/m'$



Step 4.2: Flexural design

Sec. 1

$$M_u = \frac{w\ S^2}{12} = \frac{64.43 \times 5^2}{12} = 134.23\ \text{kN.m}$$

$$R_1 = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{134.23 \times 10^6}{25 \times 250 \times 550^2} = 0.071 \qquad \rightarrow \qquad \omega = 0.089$$

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.089 \frac{25}{360} \times 250 \times 550 = 850 \; mm^2$$

Choose 5 Φ 16 (A_s = 1005 mm^2)

The secondary reinforcement is chosen as 0.1-0.2 A_s. Choose 2 Φ 12 .

Sec. 2

$$M_u = \frac{w \; S^2}{10} = \frac{64.43 \times 5^2}{10} = 161.07 \text{ kN.m}$$

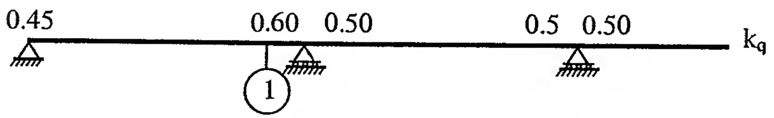
$$R_1 = \frac{M_u}{f_{cu} \; b \; d^2} = \frac{161.07 \times 10^6}{25 \times 250 \times 550^2} = 0.085 \qquad \rightarrow \quad \omega = 0.11$$

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.11 \frac{25}{360} \times 250 \times 550 = 1050 \; mm^2$$

Choose 4 Φ 20 (A_s = 1256 mm^2)

Step 4.3: Design for Shear

The critical section for shear is at d/2 from the face of the middle support. The width of the column is 250 mm. The critical section is at section (1) as shown in figure with code coefficient of k_q=0.6.



$$Q_u = k_q \; w_u \; L - w_u \left(\frac{c}{2} + \frac{d}{2} \right)$$

$$Q_u = 0.6 \times 64.43 \times 5.0 - 64.43 \times \left(\frac{0.25}{2} + \frac{0.55}{2} \right) = 167.52 \; kN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{167.52 \times 1000}{250 \times 550} = 1.22 \; N / mm^2$$

$$q_{cu} = 0.24 \sqrt{\frac{f_{cu}}{1.5}} = 0.24 \sqrt{\frac{25}{1.5}} = 0.98 \; N / mm^2$$

Since $q_u > q_{cu}$, shear reinforcement is needed.

$$q_{su} = 1.22 - \frac{0.98}{2} = 0.73 \; N / mm^2$$

$$A_{st} = \frac{q_{su} \times b \times s}{f_y / \gamma_s}$$

Using ϕ 8 and for two branches $A_{st} = 2 \times 50 = 100 \; mm^2$

The spacing of the reinforcement area is given by

$$100 = \frac{0.73 \times 250 \times s}{280 / 1.15} \qquad \rightarrow \quad s = 133 \text{ m} \qquad \rightarrow \quad \text{Use spacing of 125 mm}$$

Use $\phi 8 @ 125 mm$ or 8 ϕ 8/m′

$$A_{st,\min} = \frac{0.4}{f_y} b \times s = \frac{0.4}{280} \times 250 \times 125 = 44.6 \; mm^2 \;\; < A_{st} \; .....\text{ok}$$

Step 5: Design the horizontal beam (200 mm x 750 mm)

The horizontal beam is analyzed as a continuous beam supported on the ties. It carries a uniformly distributed load equals to the horizontal thrust. This uniform load equals the horizontal reaction of 1.0 m strip of the arched slab.

Step 5.1: Flexural design

Sec. 1

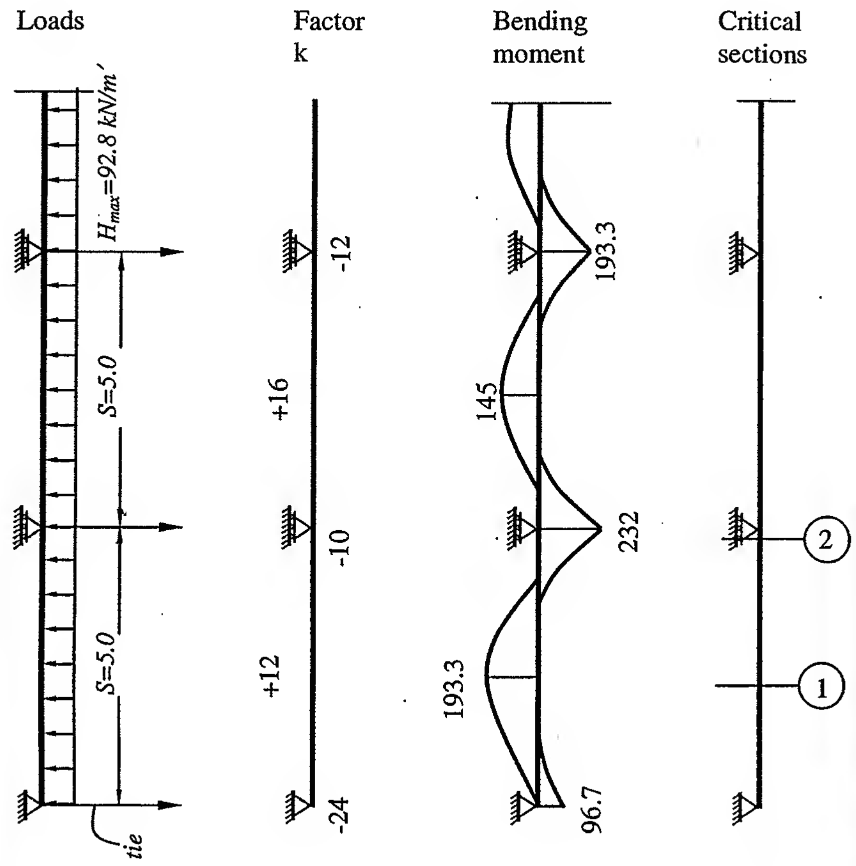
$$M_u = \frac{w \; S^2}{12} = \frac{92.8 \times 5^2}{12} = 193.3 \text{ kN.m}$$

$$R_1 = \frac{M_u}{f_{cu} \; b \; d^2} = \frac{193.3 \times 10^6}{25 \times 200 \times 700^2} = 0.079 \quad \rightarrow \quad \omega = 0.101$$

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.101 \frac{25}{360} \times 200 \times 700 = 978 \; mm^2$$

Choose 4 Φ 18 (A_s = 1017 mm^2)

The secondary reinforcement is chosen as at least 0.1- 0.2 A_s. Choose 2 Φ 12.



Sec. 2

$$M_u = \frac{w\ S^2}{10} = \frac{92.8 \times 5^2}{10} = 232 \text{ kN.m}$$

$$R_1 = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{232 \times 10^6}{25 \times 200\ \times 700^2} = 0.0946 \rightarrow \qquad \omega = 0.124$$

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.124 \frac{25}{360} \times 200 \times 700 = 1203\ mm^2$$

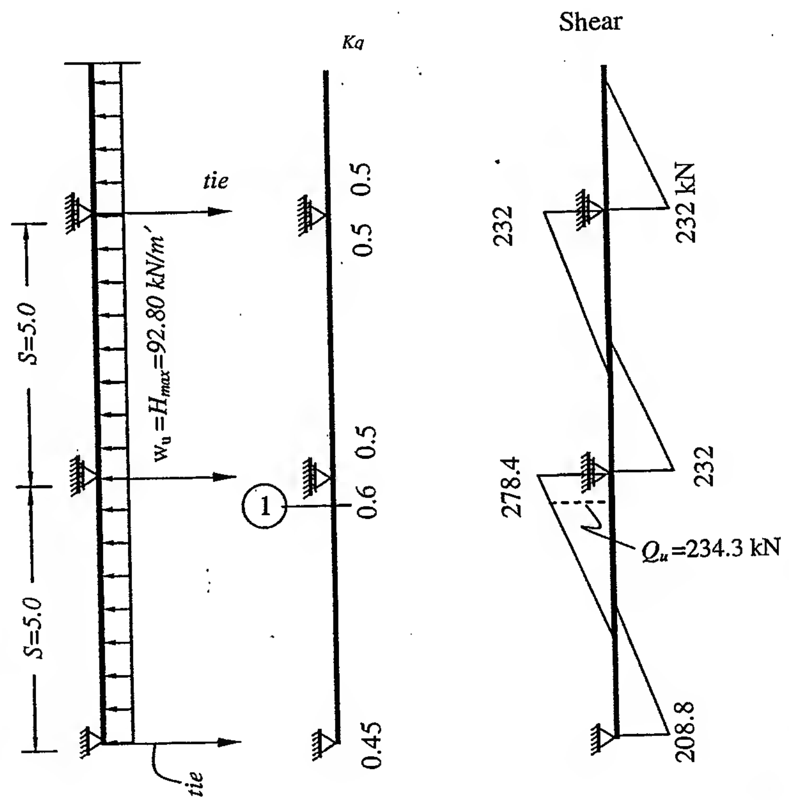
Choose 4 Φ 20 (A_s = 1256 mm²)

Step 5.3: Design for Shear

The critical section for shear is at the face of the column because the tie is in tension. The width of the column is 250 mm. The critical section is at section (1) as shown in figure with code coefficient of k_q=0.6.

$$Q_u = k_q\ w_u\ L - w_u \left(\frac{c}{2}\right)$$

$$Q_u = 0.6 \times 92.8 \times 5.0 - 92.8 \times \left(\frac{0.250}{2}\right) = 266.8\ kN$$



$$q_u = \frac{Q_u}{b \times d} = \frac{266.8 \times 1000}{200 \times 700} = 1.9\ N/mm^2$$

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{25}{1.5}} = 0.98\ N/mm^2$$

$$q_{u,max} = 0.70\sqrt{\frac{f_{cu}}{1.5}} = 0.70\sqrt{\frac{25}{1.5}} = 2.85\ N/mm^2$$

Since $q_u < q_{umax}$, then section dimensions are adequate. However, since $q_u > q_{cu}$, shear reinforcement is needed.

$$q_{su} = 1.9 - \frac{0.98}{2} = 1.4\ N/mm^2$$

$$A_{st} = \frac{q_{su} \times b \times s}{f_y / \gamma_s}$$

Assuming a spacing of 125 mm, the shear reinforcement area is given by:

$$A_{st} = \frac{1.4 \times 200 \times 125}{280/1.15} = 145\ mm^2$$

Thus, the area of one branch $= \frac{A_{st}}{2} = \frac{145}{2} = 72\ mm^2$

Use ϕ 10=78.5 mm^2 (A_{st}=157 mm^2)

Use $\phi 10 @ 125mm \rightarrow 8\phi 10/m'$

$$A_{st,min} = \frac{0.4}{f_y} b \times s = \frac{0.4}{280} \times 200 \times 100 = 28 mm^2 < A_{st} \text{ .....ok}$$

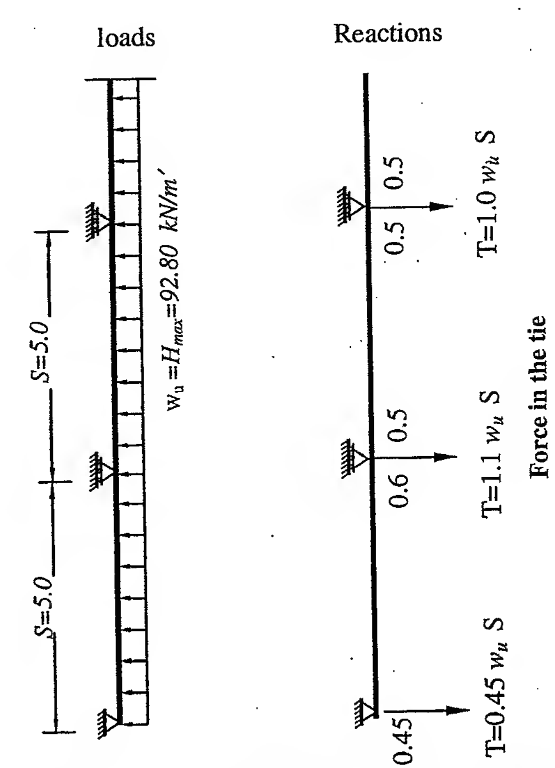
Step 6: Design of the tension tie (200 mm x 200 mm)

The tie is the main supporting element for the horizontal beam. The reaction is transferred to the tie. For continuous beams with equal loads and equal spans, the ECP 203 gives the reaction at intermediate supports as (1.1 w_u S), as shown in the figure below.

$$T = 1.1 \times H_{max} \times S = 1.1 \times 92.8 \times 5 = 510.4\ kN$$

$$A_s = \frac{T}{f_y/1.15} = \frac{501.4 \times 1000}{360/1.15} = 1630\ mm^2$$

Choose (6 Φ 18+2 Φ 16) (A_s= 1929 mm^2)



Step 7: Design of the hanger (200 mm x 200 mm)

The weight of the hanger equals to:

$$ow_H = 25 \times 0.2 \times 0.2 \times 2.4 = 2.4\ kN$$

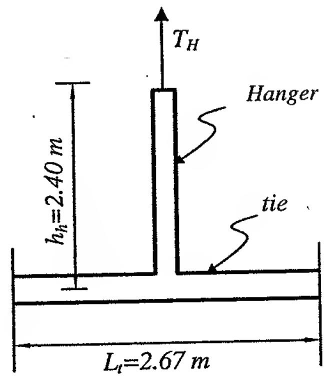
The weight of the tie equals to:

$$ow_T = 25 \times 0.2 \times 0.2 \times 2.667 = 2.667\ kN$$

The total factored weight (tension T_H)

$$T_H = 1.4 \times (2.4 + 2.667) = 7.1\ kN$$

$$A_s = \frac{T}{f_y/1.15} = \frac{7.1 \times 1000}{360/1.15} = 22.6\ mm^2$$



Choose 4 Φ 10/m' (A_s= 314 mm^2)

Step 8: Design of the column (250 mm x 600 mm)

Step 8.1: Calculate applied loads

The column is subjected to an axial load in addition to bending moments resulting from the slenderness effect. Since wind load is neglected the following load case is considered

$$U = 1.4 \times D + 1.6 \times L$$

The vertical load on the column is the summation of the following loads:

1. Self-weight $= 1.4 \times \gamma_c \times b \times t \times h = 1.4 \times 25 \times 0.25 \times 0.6 \times 6.0 = 31.5\ kN$

2. weight of the wall beam +semelle (250x500)

   $= 2 \times 1.4 \times \gamma_c \times b \times t \times spacing = 2 \times 1.4 \times 25 \times 0.25 \times 0.5 \times 5.0 = 43.75\ kN$

3. Wall load $= 1.4 \times \gamma_w \times b \times S \times (h - 2 \times t_{wall\ beam} - t_{vertical\ beam})$

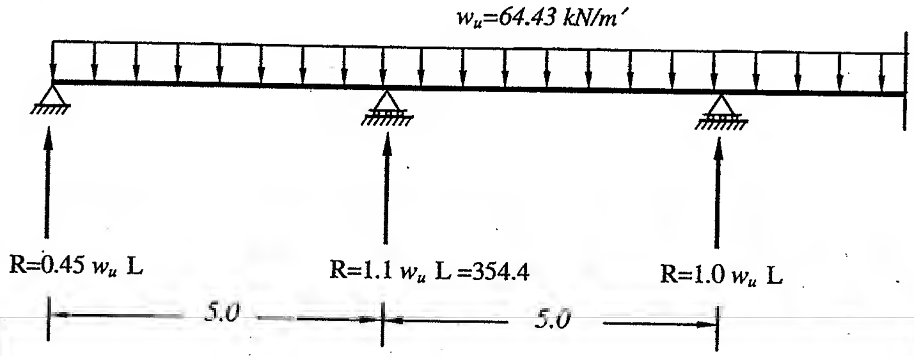
   Assuming γ_w=18 kN/m^3

   $= 1.4 \times 18 \times 0.25 \times 5.0 \times (6.0 - 2 \times 0.5 - 0.60) = 138.6\ kN$

4. Arched slab load = Reaction from the vertical beam

   $R_{arch} = 1.1 \times w_u (vertical\ beam) \times S = 1.1 \times 64.43 \times 5 = 354.4\ kN$

   $P_u = 31.5 + 43.75 + 138.6 + 354.4 = 568.3\ kN$



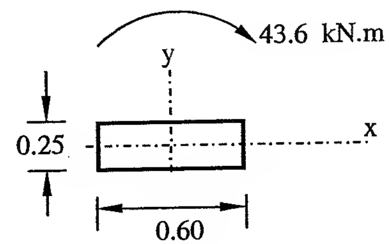
Loads and the reactions of the vertical beam

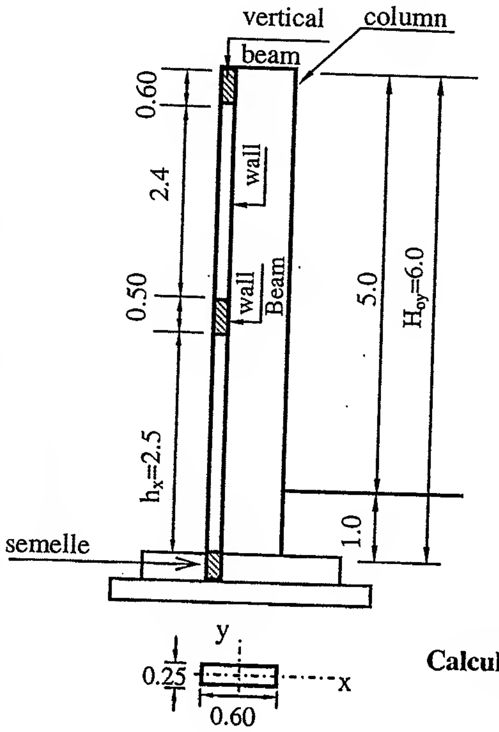
Step 8.2: Calculation of the reinforcement

The unsupported length in X- direction is 6.0 ms and the unsupported length in Y- direction is 3.4 ms. The calculation of the additional moment can be summarized in the following table.

Item	**X-direction**	**Y-direction**
bracing condition	unbraced	braced
Ultimate load P_u(kN)	568.3	568.3
Short column if	λ<10	λ<15
H_o (m)	6.0	2.5
t (m)	0.6	0.25
bracing condition at top	case 3	case 1
bracing condition at bottom	case 1	case 1
k (bracing factor)	1.6 (Code table 6-10)	0.75 (Code table 6-9)
H_e	9.6	1.875
$\lambda = k \times H_o / t$	16	7.5
Status	long(λ>10)	short (λ>15)
$\delta = \lambda^2 \times t / 2000$	0.0768	0
$M_{add} = P_u . \delta$	43.6	0
$M_{tot} = M_u + M_{add}$	43.6	0

It is clear from the previous table that the column is subjected to a uniaxial bending moment as shown in the figure.





Calculation of buckling lengths (refer to the table)

Using interaction diagram with f_y=360N/mm^2, α=1.0, ζ=0.8, calculate

$$R_b = \frac{P_u}{f_{cu} \times b \times t} = \frac{568.3 \times 1000}{25 \times 250 \times 600} = 0.152$$

$$\frac{M_u}{f_{cu} \times b \times t^2} = \frac{43.6 \times 10^6}{25 \times 250 \times 600^2} = 0.019$$

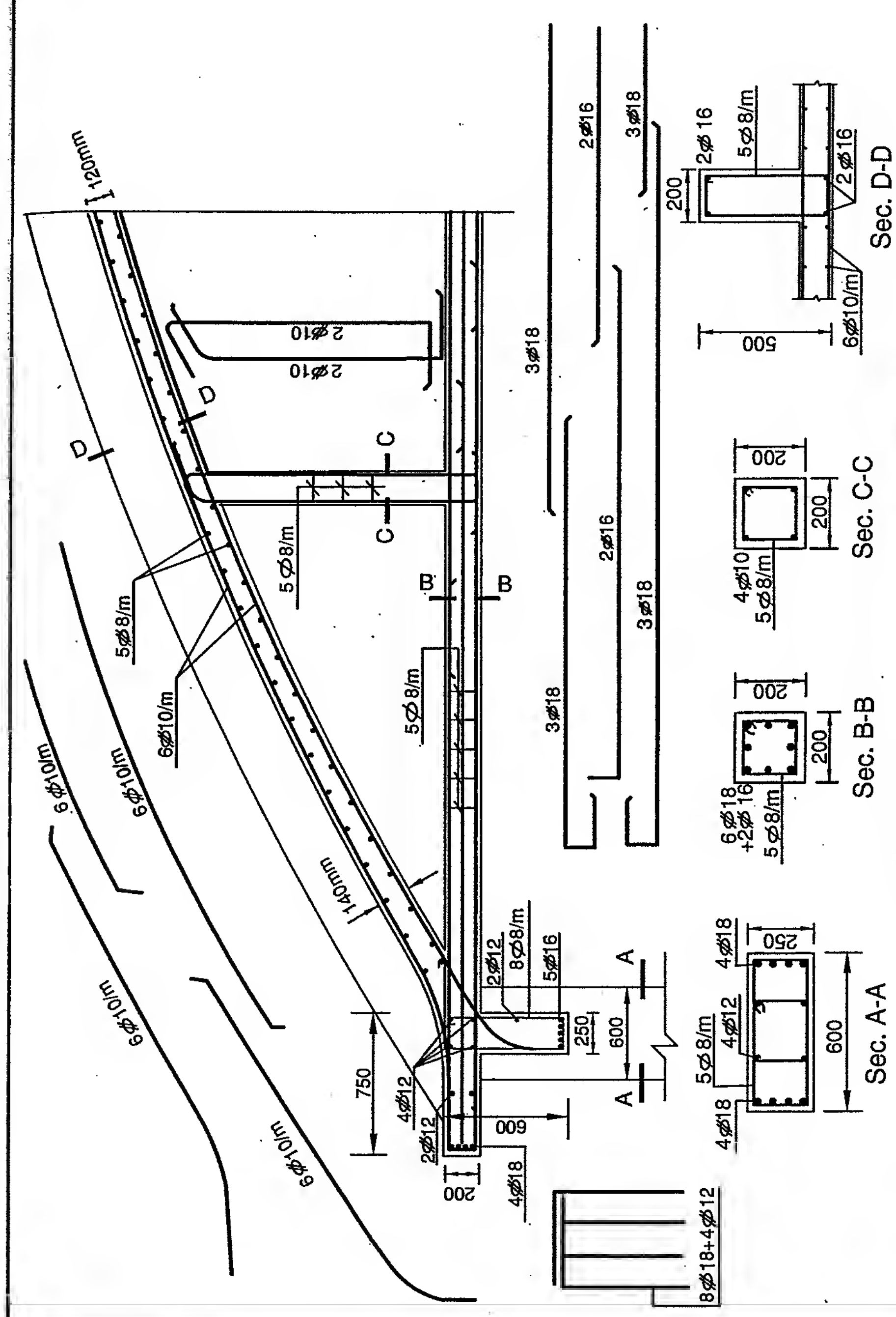
The intersection point is below the chart $\rightarrow$ use μ_{min}

Since the column is long, the minimum reinforcement ratio μ_{min} is given by:

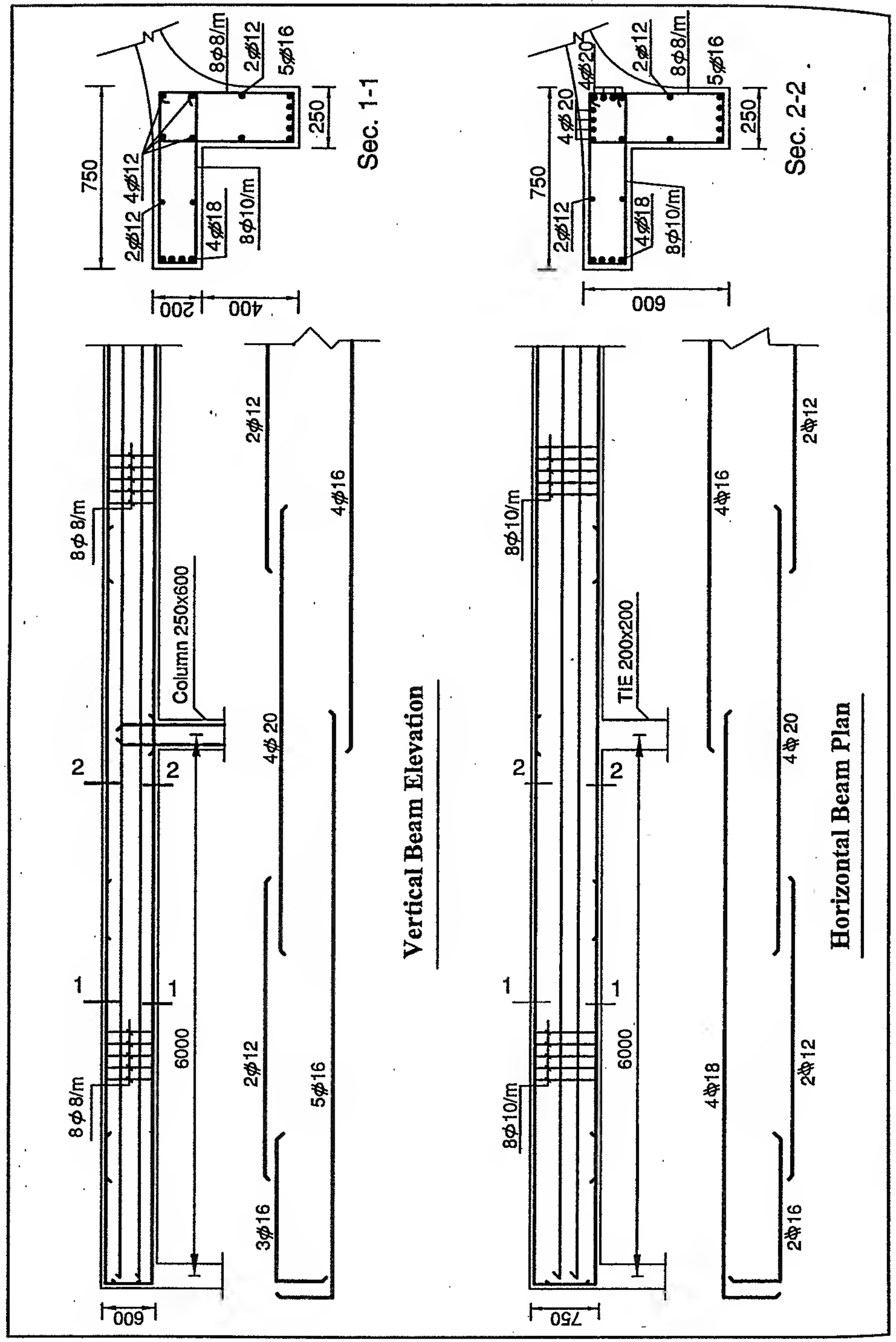
$$\mu_{min} = 0.25 + 0.052 \cdot \lambda = 0.25 + 0.052 \times 16.0 = 1.1082\,\%$$

$$A_{s,min} = \mu_{min} \times b \times t = \frac{1.1082}{100} \times 250 \times 600 = 1623\ mm^2$$

Choose (8 Φ18, 2035 mm^2).



Reinforcement Details of the Arched Slab



2

DEEP BEAMS AND CORBELS

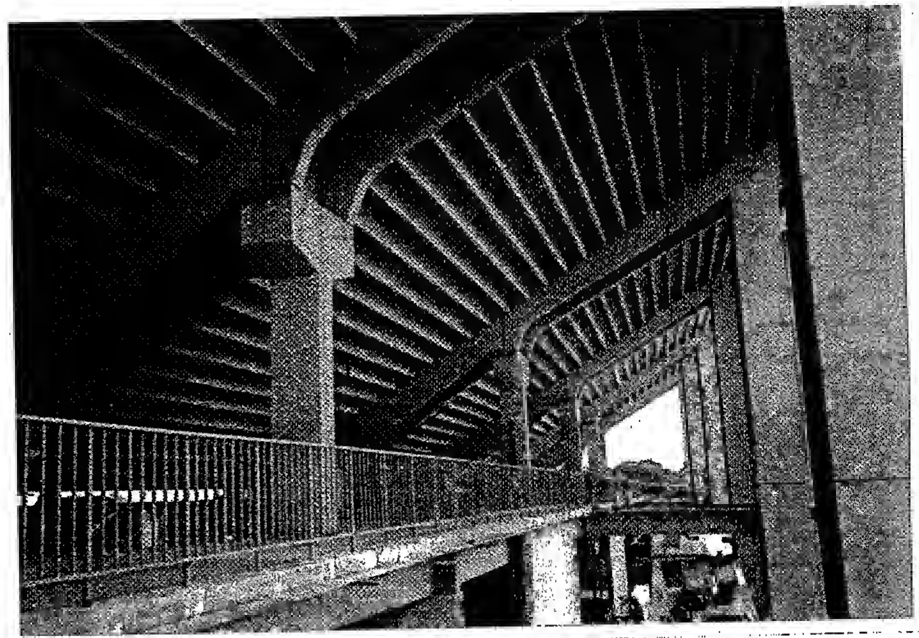


Photo 2.1 Corbels supporting beams in a stadium

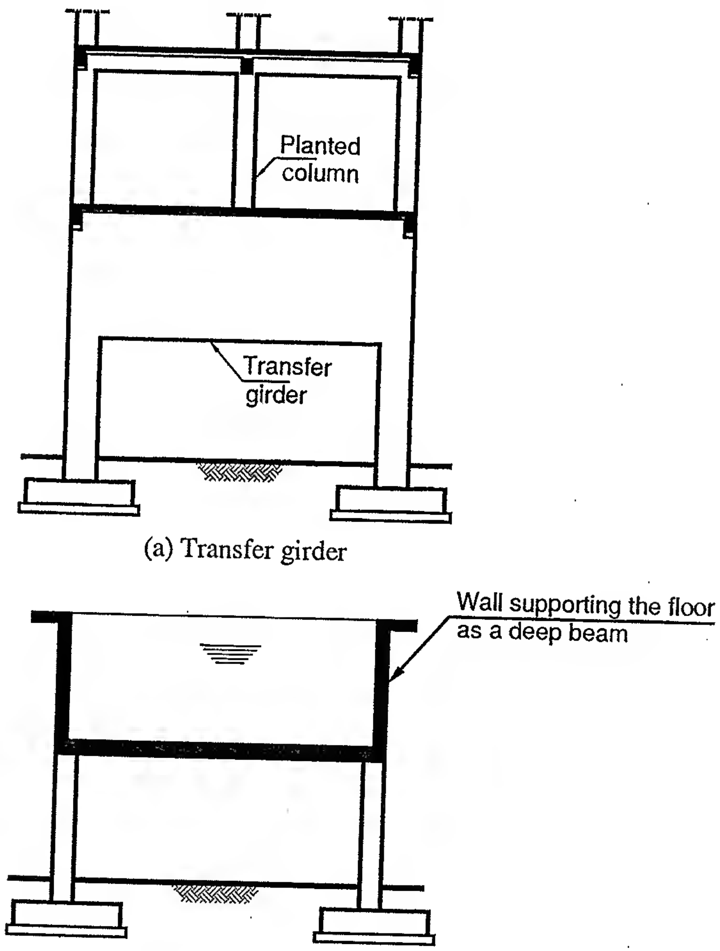
2.1 Introduction

This chapter will discuss the behavior of reinforced concrete deep beams and corbels (*short cantilevers*). The behavior of these members is different from shallow (slender beams). In deep beams and in corbels, plane sections before bending do not remain plain after bending. In order to fully understand the behavior of these members, the subject of shear friction will be presented. Another approach for designing these members is the Strut and Tie Model that will be presented in Chapter Six of this volume.

2.2 Deep beams

2.2.1 General

Deep beams are beams of relatively high depth-to-span ratio. Most typically, deep beams occur as transfer girders. A transfer girder supports the load from one or more columns, transferring it to other columns (Fig. 2.1a). Deep beams also occur in tanks and walls supported on columns (Fig.2.1b).



(a) Transfer girder

(b) Elevated water tank

Fig. 2.1 Typical examples of deep beams

Deep beams may be loaded at their top surface as in the case of a transfer girder supporting the load from one or more columns (Fig. 2.2a). The loading may take place at the bottom surface as in water tank wall loaded by the action of the suspended tank's floor (Fig. 2.2b). Loads may also act along the height of the wall as shown in Fig. (2.2c). The wall in this figure approximates the case of wall supporting successive floor slabs and transferring the loads to columns at ground floor level.

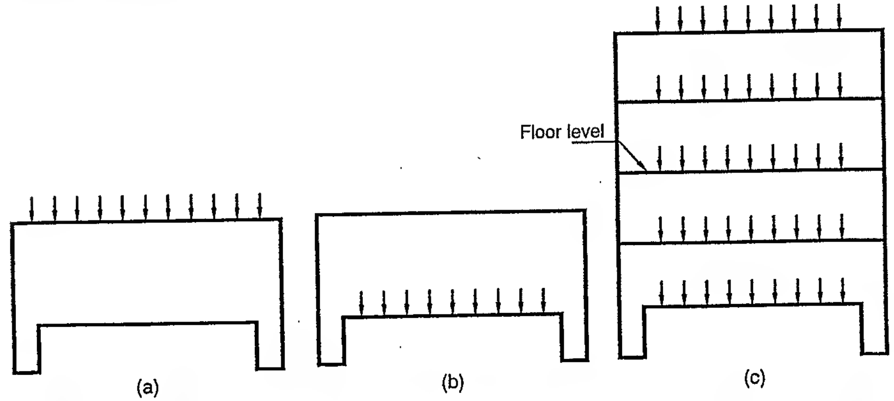


Fig. 2.2 Types of loading of deep beams

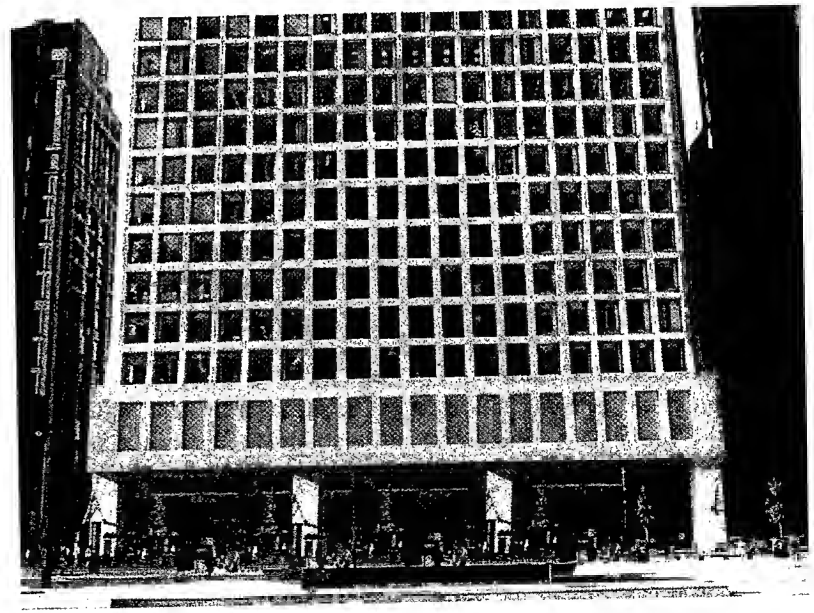
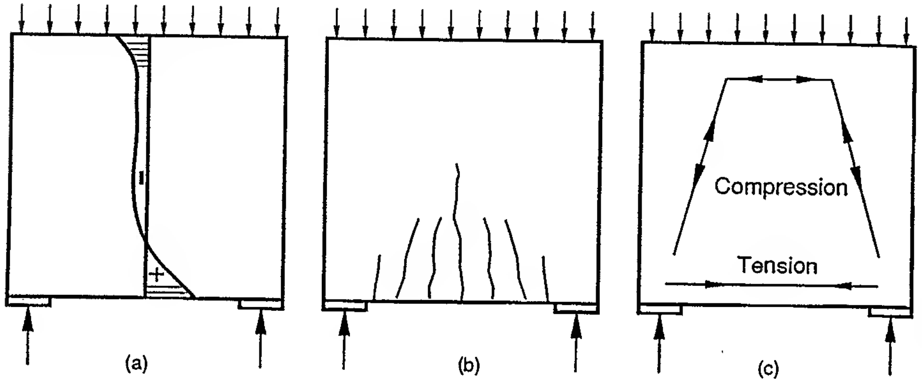


Photo 2.2 Deep beam supporting columns (Brunswick Building, Chicago)

Elastic analysis of deep beams indicates that the usual assumption that plane sections before bending remain plane after bending is not valid for such members. Thus, flexural stresses are not linearly distributed even in the elastic range. Typical stress distribution is shown in Fig. (2.3a). The cracking load of a deep beam is about 1/3 to 1/2 of the ultimate load.

Traditional principles of analysis and design of ordinary reinforced concrete beams are neither suitable nor adequate to determine the strength of reinforced concrete deep beams. The cracking pattern of a uniformly loaded deep beam is shown in Fig. (2.3b). After cracking a major redistribution occurs and the elastic analysis is no longer valid. Deep beams loaded at the top behave mainly as a tied arch as shown in Fig. (2.3c).



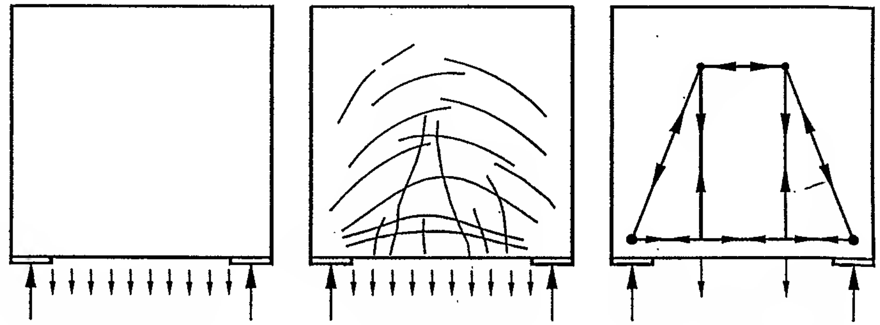
a) Normal stress **b) Cracking pattern** **c) Arch mechanism**

Fig. 2.3 Top loaded deep beams

The tied-arch mechanism, shown in Fig. (2.3c), brings designer attention to the fact that longitudinal tension reinforcement acting as a tie that is fully stressed over nearly the whole span. Therefore, sufficient anchorage at the supports and continuity of reinforcement bars without curtailment are essential requirements for top loaded deep beams.

Figure 2.4a shows a deep beam that is supporting uniformly distributed load acting at the lower face of the beam. Vertical stirrups must be provided as hangers to prevent local failure and to transfer the effective acting load to a higher level. If such a beam is provided with stirrups that are able to deliver the bottom load to the upper part of the beam, the beam will be behave nearly like a top loaded beam.

The crack pattern in Fig. 2.4b clearly shows that the load is transferred upward by reinforcement until it acts on the compression arch, which then transfers the loads down to the support as shown in Fig. 2.4c.



a) Loading pattern **b) Cracking pattern** **c) Arch mechanism**

Fig. 2.4 A bottom loaded deep beam

2.2.2 Egyptian Code's Provisions for Deep Beams

The Egyptian Code's provisions for deep beams are applied to deep beams loaded at the top or at the compression faces. If loads are applied at the bottom of a deep beam, the Egyptian Code requires using vertical reinforcement that is able to transfer the load to a height equals at least half the span. This vertical reinforcement should be added to that resulting from the design of the beam as if it is a top loaded deep beam.

In deep beams plain sections do not remain plain after bending and the design methods developed for shallow beams can not be applied. The Egyptian Code presents two methods for designing deep beams. These methods are:

- The Empirical design method
- The Strut and Tie method

2.2.2.1 The Empirical Design Method

The empirical design method applies to beams having the following ratios of the span (L) to the effective depth (d):

$$\text{Simply supported beams: } (L/d) < 1.25 \quad \text{..................} (2.1a)$$

$$\text{Continuous beams: } (L/d) < 2.50 \quad \text{..........................} (2.1b)$$

where L is defined with reference to Fig. (2.5) as the smaller value of the following:

$$L = 1.05\, L_n \quad \text{..} (2.2a)$$

$$L = L_o \quad \text{..} (2.2b)$$

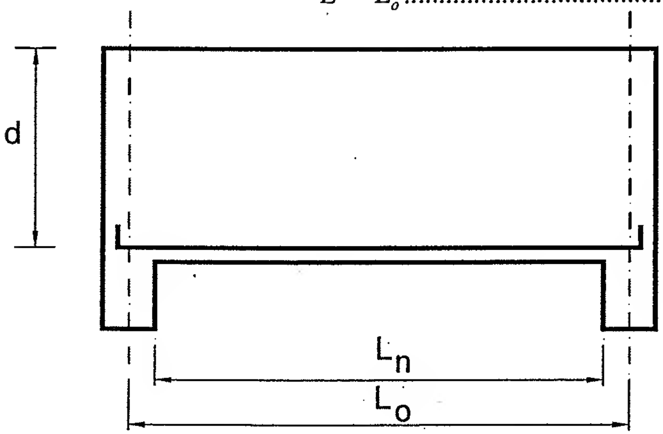


Fig. 2.5 Definition of a deep beam

A: Design for Flexure

The longitudinal reinforcement should be provided to resist the tension force that resulting from the applied bending moment. The tension force at any section is given by:

$$T_u = \frac{M_u}{y_{ct}} \quad \text{..} (2.3)$$

Where M_u is the applied ultimate moment, T_u is the developed tension force at the critical section, and y_{ct} is the lever arm and is given by:

$y_{ct} = 0.86\ L \le 0.87d$	For simply supported beams.
$y_{ct} = 0.43\ L \le 0.87d$	For continuous beams at mid-span.
$y_{ct} = 0.37\ L \le 0.87d$	For continuous beams at interior support.

The reinforcement can be obtained by dividing the developed tension force by the steel yield stress as follows:

$$A_s = \frac{T_u}{f_y/1.15} \quad \text{..} (2.4)$$

The distribution of this reinforcement differs from that of the slender beams. The flexural reinforcement is placed near the tension edges. Because of the greater depth of the tension zone, it is required to distribute such steel over a certain height of the cross-section (See Figs. 2.7 and 2.8).

The tied-arch mechanism of deep beams dictates that longitudinal tension reinforcement acting as a tie is fully stressed over nearly the whole span of simply supported deep beams. Therefore, sufficient anchorage at the supports and continuity of reinforcement bars without curtailment are essential requirements. Recommendations for the detailing of deep beams are given in Figs. 2.7 to 2.10.

The Egyptian code requires that the actual area of steel A_s in any section should be greater than A_{smin} given by:

$$A_{s\,min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} \geq \dfrac{1.1}{f_y} b\ d \\ 1.3 A_s \end{cases} \quad \text{........................ (2.5)}$$

$$\text{but not less than} \begin{Bmatrix} \dfrac{0.25}{100} b\ d\ (mild\ steel) \\ \dfrac{0.15}{100} b\ d\ (high\ grade) \end{Bmatrix}$$

B: Design for Shear

The design for shear in deep beams is of special importance. The amount and spacing of both the vertical and horizontal web reinforcement differ than those used in shallow beams, as well as the expressions that to be used in design.
The critical section for shallow beams is taken at a distance d/2 from the face of the support, and the shear plane is inclined more and closer to the support. However, in deep beams, the critical section for shear is to be taken as:

Uniformly distributed $\rightarrow x = 0.15\ L_n$ .... (2.6a)

Concentrated load $\rightarrow x = 0.5\ a$ .... (2.6b)

In either case, the distance x should not exceed the distance d/2 as shown in Fig 2.6. If both uniform and concentrated load exist on the beam, design the most critical one.

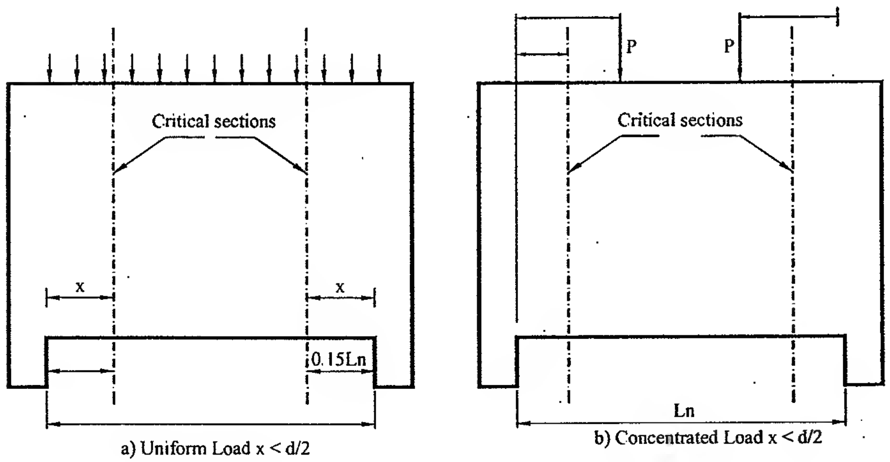


Fig. 2.6 Critical sections for shear design

Nominal Ultimate Shear Stress

The applied shear stress is given by:

$$q_u = \frac{Q_u}{b \times g} \leq q_{u\,max} \quad \text{.. (2.7)}$$

Where b is width of the beam, and g = the smaller of d or L_n

The value of the nominal shear stress q_u should be less than q_{umax} given by

$$q_{u\,max} = \delta_d \times 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} \leq (\delta_d\ 4.0) N/mm^2 \quad \text{................. (2.8)}$$

in which

$$\delta_d = 1/3\ (2 + 0.4(L_n/d)) \leq 1 \quad \text{.............................. (2.9)}$$

If $q_u \geq q_{u\,max}$, the dimensions of the section should be increased

It interesting to note the δ_d is less than or equal to one. This could wrongly imply that the maximum shear strength is less than shallow beams. However, this requirement is intended to prevent bearing failure in deep beam rather than controlling shear failure.

Shear Strength provided by Concrete

The nominal ultimate shear provided by concrete is given as follows:

- No axial load

$$q_{cu} = \delta_{dc} \times 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} \leq 0.46\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{............................(2.10a)}$$

Where

$$\delta_{dc} = 3.5 - 2.5\frac{M_u}{Q_u d} \geq 1.0$$
$$\leq 2.5$$

M_u : is the ultimate moment at the critical section.
Q_u: is the ultimate shear at the critical section.

The factor δ_{dc} is a multiplier for q_{cu} in shallow beams to account for the higher resisting capacity of deep beams due to arching action.

- Axial compression (P_u)

$$q_{cu} = \delta_{dc} \times \delta_c \; 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{.......................... (2.10b)}$$

where $\delta_c = (1 + 0.07\frac{P_u}{A_c}) \leq 1.5$

- Axial tension (T_u)

$$q_{cu} = \delta_{dc} \times \delta_t \; 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{..........................(2.10c)}$$

where $\delta_t = (1 - 0.3\frac{T_u}{A_c})$

A limiting value is placed by the code on q_{cu} by the equation:

$$q_{cu} \leq 0.46\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{.............................. (2.10d)}$$

To design the beam for shear, two cases should be considered

Case A $q_u \leq q_{cu}$

If the value of q_u does not exceed q_{cu}, minimum shear reinforcement in the form of vertical and horizontal web reinforcement as shown in Fig. (2.7) should be provided. The minimum values are given with reference to Fig. (2.7) as follows:

$$A_v = (0.20/100) b\, s_v \quad \text{......................... for mild steel} \quad (2.11a)$$

$$A_v = (0.15/100) b\, s_v \quad \text{...................for high grade steel} \quad (2.11b)$$

$$A_h = (0.30/100) b\, s_h \quad \text{......................... for mild steel} \quad (2.11c)$$

$$A_h = (0.25/100) b\, s_h \quad \text{...................for high grade steel} \quad (2.11d)$$

s_v or s_h should be less than 200 mm

The purpose of providing minimum web reinforcement is to limit crack width along the surface area of the beam.

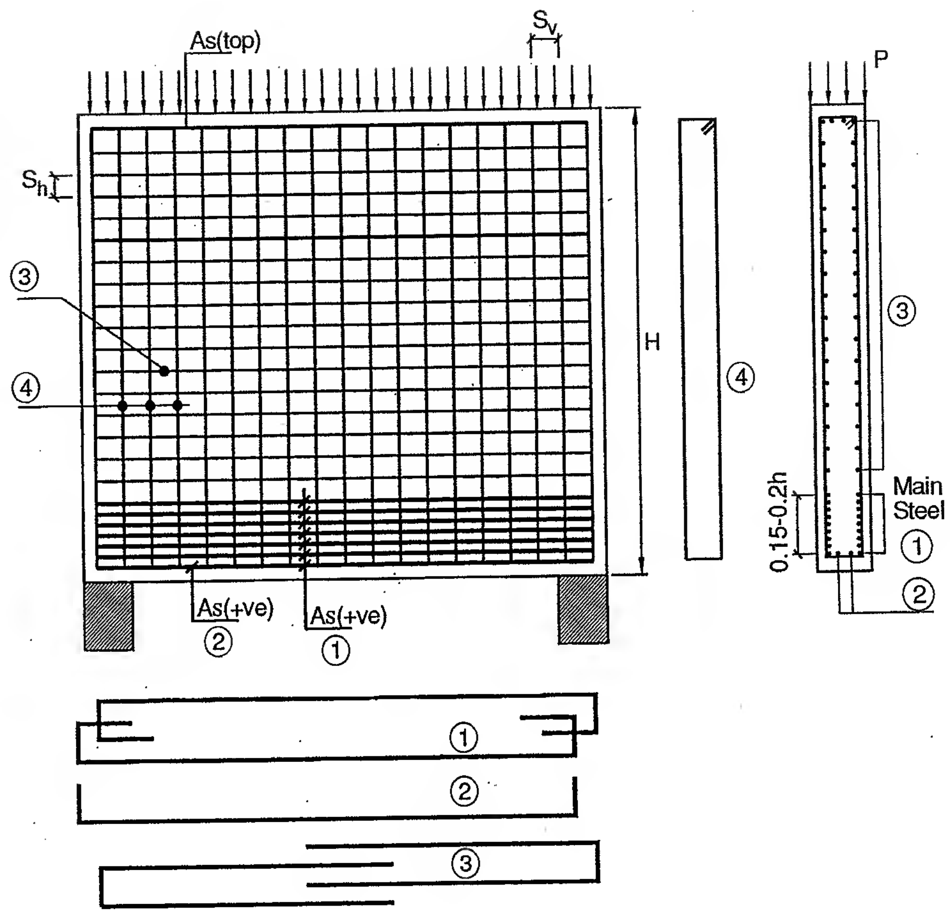


Fig. 2.7 Reinforcement detailing of a top-loaded simply supported deep beam

Case B $q_u > q_{cu}$

If, on the other hand, the value of q_u exceeds q_{cu}, web reinforcement should be provided to resist the ultimate shear stress q_{su} . For deep beams in the ranges of the *L/d* ratios considered, diagonal cracks will be at a slope steeper than 45°. Consequently, both horizontal and vertical web reinforcements are required. In fact, for such *L/d* ratios, horizontal reinforcement could be more effective than vertical reinforcement. The horizontal bars are effective because they act more nearly in the direction perpendicular to the diagonal crack.
The ECP-203 gives the following equations for calculating the web reinforcement for deep beams:

$$q_{su} = q_u - \frac{q_{cu}}{2} \quad \text{.......................................(2.12a)}$$

$$q_{su} = \delta_v \times q_{suv} + \delta_h \times q_{suh} \quad \text{....................... (2.12b)}$$

in which

$$\delta_h = \frac{11-(L_n/d)}{12} \quad \text{.................................(2.13a)}$$

$$\delta_v = \frac{1+(L_n/d)}{12} \quad \text{.................................. (2.13b)}$$

$$q_{suv} = \frac{A_v \times (f_y/\gamma_s)}{s_v \times b} \quad \text{..............................(2.13c)}$$

$$q_{suh} = \frac{A_h \times (f_y/\gamma_s)}{s_h \times b} \quad \text{.............................. (2.13d)}$$

It can be concluded from Eq. 2.12 and Eq. 2.13 (as stated above) that horizontal reinforcement is more effective than the vertical web reinforcement.

Equation 2.12b has four parameters (A_v, s_v, A_h, s_h). It is customary to assume the value of these parameters and calculate the value of the shear carried by the reinforcement q_{su}. The value of q_{su} in Eq. 2.12b should be greater than required shear stress q_{su} given by Eq. 2.12a. Thus, assume three of these parameters to obtain the fourth unknown.

Figure (2.7) shows the recommended reinforcement detailing of a simply supported top-loaded deep beam.

2.2.2.2 Design Using the Strut and Tie Method

The Egyptian Code permits the use of the Strut and Tie Model (explained in detail in Chapter 6) to design the beams in which the ratio of the effective span to depth satisfies the following conditions:

A- Simply supported beams

$$1.25 \le L/d \le 4.0$$

B- Continuous beams

$$2.5 \le L/d \le 4.0$$

The model consists of compression struts in the concrete and tension ties in the steel reinforcement and truss nodes as shown in Fig. 2.8. The detail of the application of the method for the case of deep beams is explained in Chapter 6, together with illustrative examples.

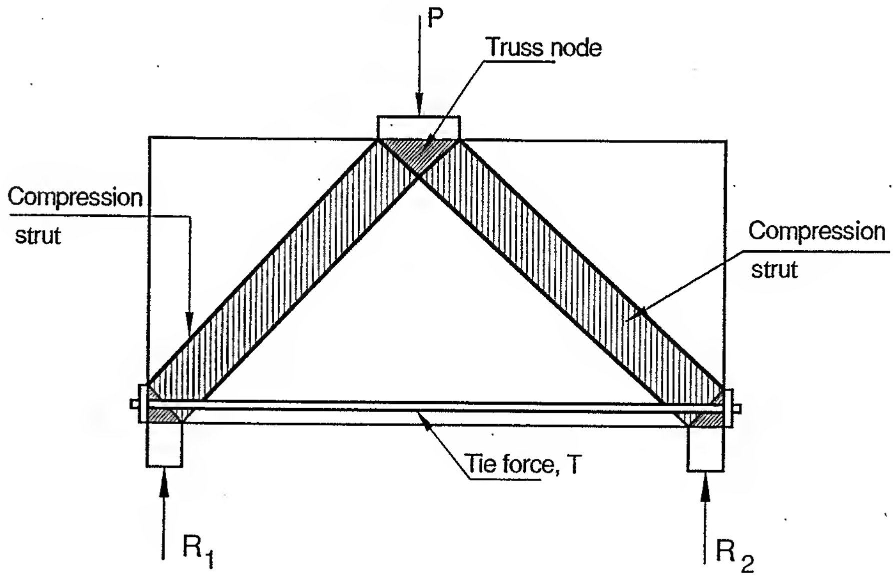


Fig. 2.8 Strut and tie model for a deep beam

2.2.3 Detailing of Other Types of Deep Beams

2.2.3.1 Bottom Loaded Deep Beam

Figure 2.9 shows the reinforcement detailing of a bottom loaded deep beam. As mentioned before, a bottom loaded deep beam could behave nearly like a top loaded one if provided with vertical stirrups that are able to deliver the bottom load to the upper part of the beam. It should be mentioned that these vertical stirrups should be added to those required as shear reinforcement. The Egyptian Code does not give special recommendations for the design of the bottom loaded deep beam. However, it implicitly recommends designing them as shallow beams.

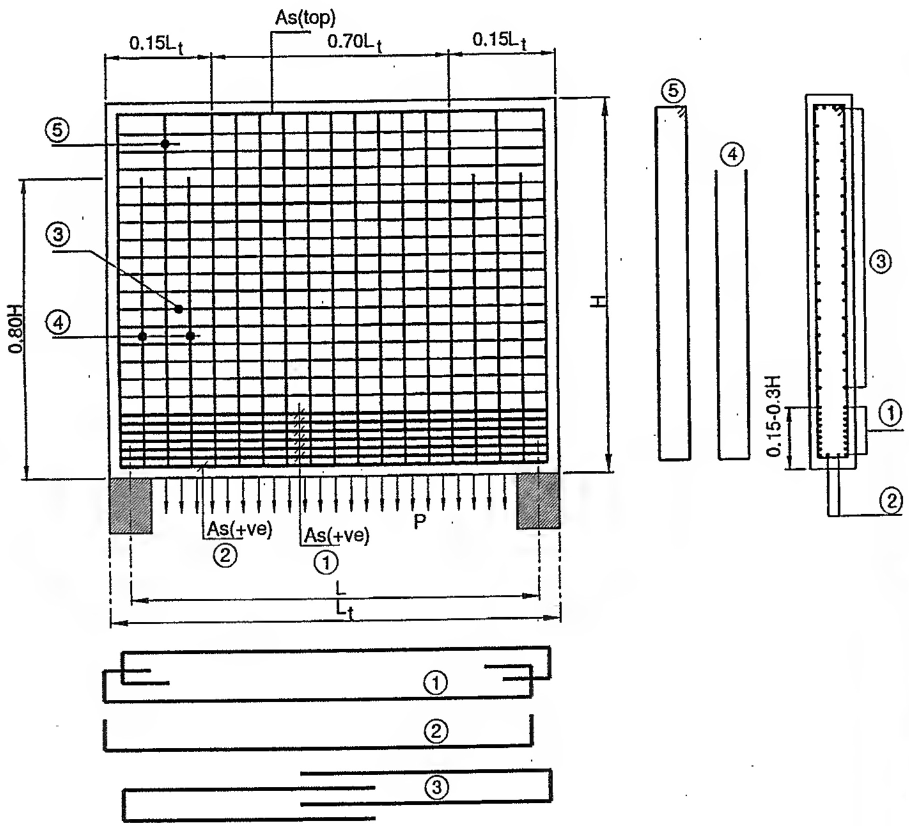


Fig. 2.9 Reinforcement detailing of a bottom-loaded simply supported deep beam

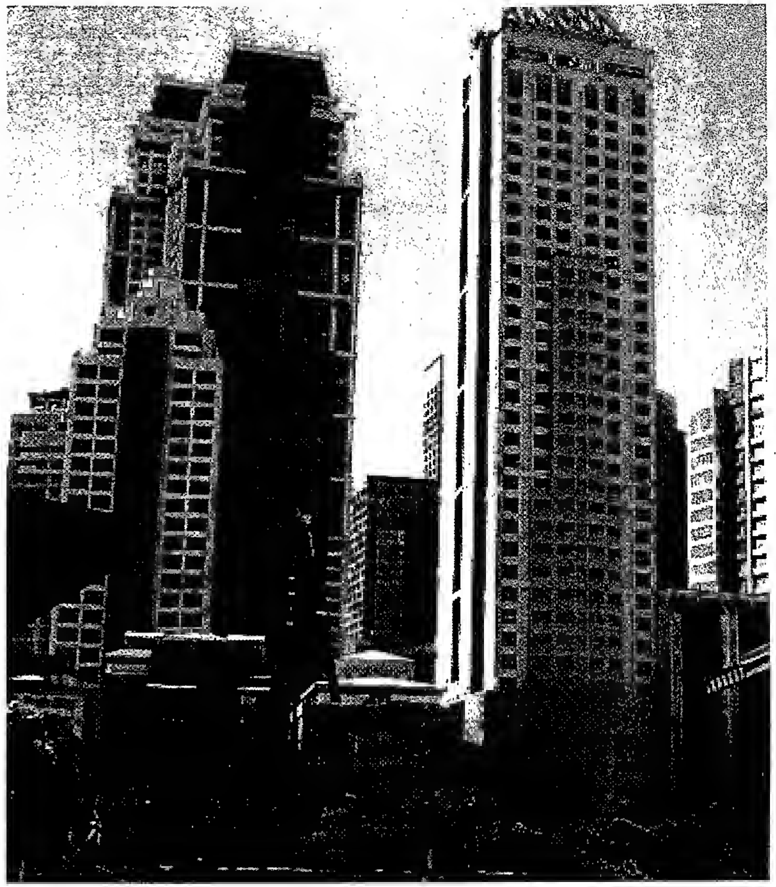
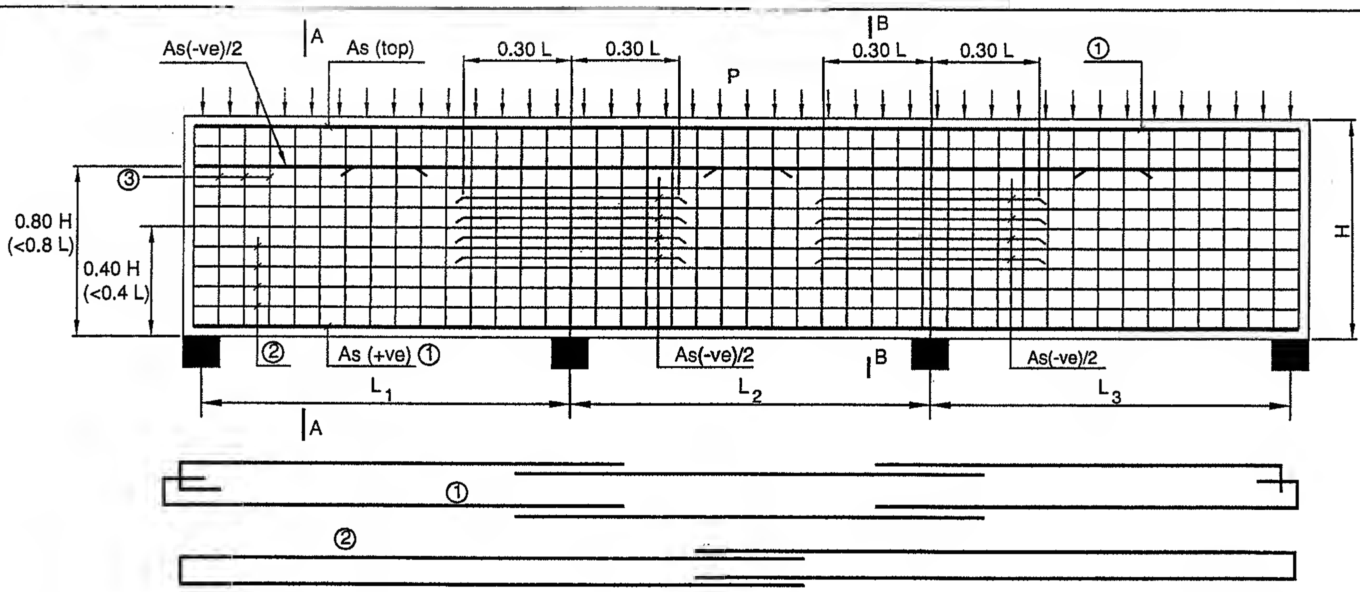


Photo 2.3 Multistory reinforced concrete buildings

2.2.3.2 Continuous Deep Beams

Continuous deep beams can be designed in the same fashion as simply supported deep beams, except that additional reinforcement is provided at the location of the middle support.

Figures 2.10a and 2.10b show the recommend detailing of the reinforcement of continuous deep beams loaded at the top and at the bottom, respectively.



1- The main bottom steel should cover the whole span.
2- One-half of the main top steel should be located at a height (0.8H< 0.8L) and should cover the total length of the beam.
3- Span variations and variations in loads should not be more than 20%.
4- In case of span variations, (L) is the bigger span.

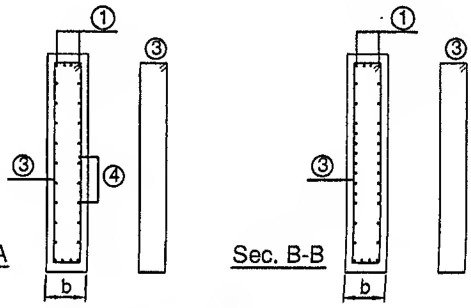
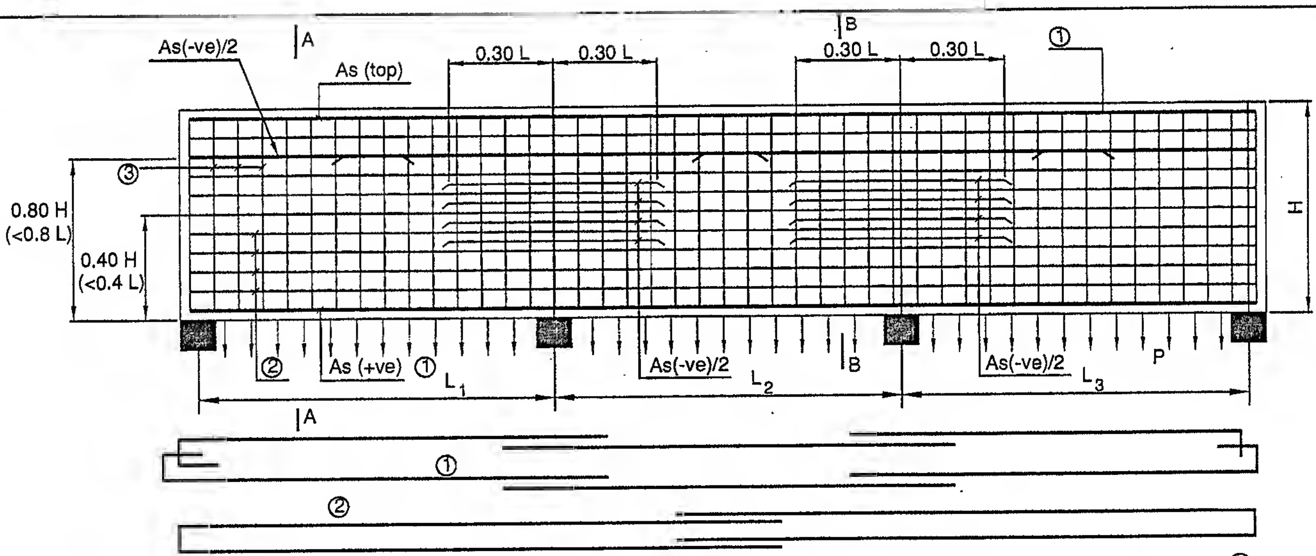


Fig. 2.10a Reinforcement of a top-loaded continuous deep beam



1- The main bottom steel should cover the whole span.
2- One-half of the main top steel should be located at a height (0.8H< 0.8L) and should cover the total length of the beam.
3- The vertical steel should be designed to deliver the bottom load to the upper part of the beam.
4- Span variations and variations in loads should not be more than 20%.
5- In case of span variations, (L) is the bigger span.

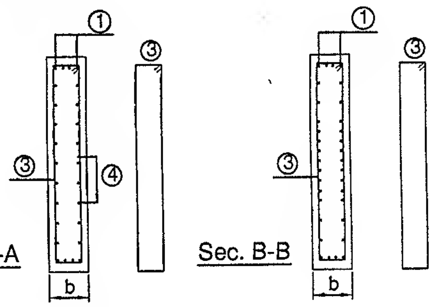


Fig. 2.10b Reinforcement of a bottom-loaded continuous deep beam

2.2.3.3 Deep Beam Supporting another Deep Beam

Special provisions are needed when loads or reactions are introduced along the full depth of a beam for example, when deep beams support each other, as illustrated in Fig. (2.11).

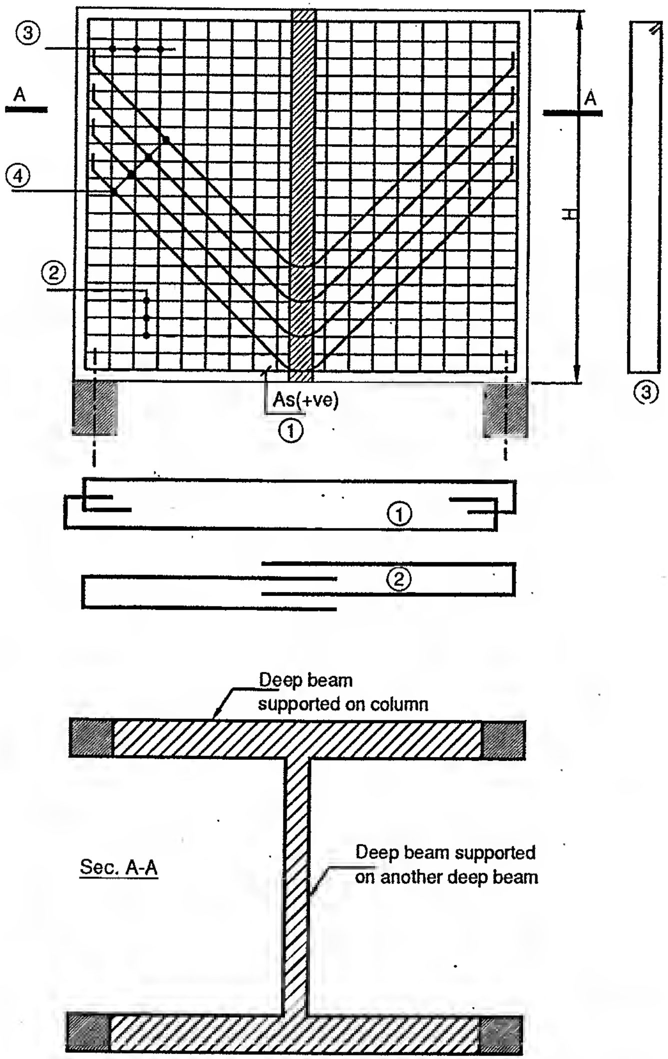
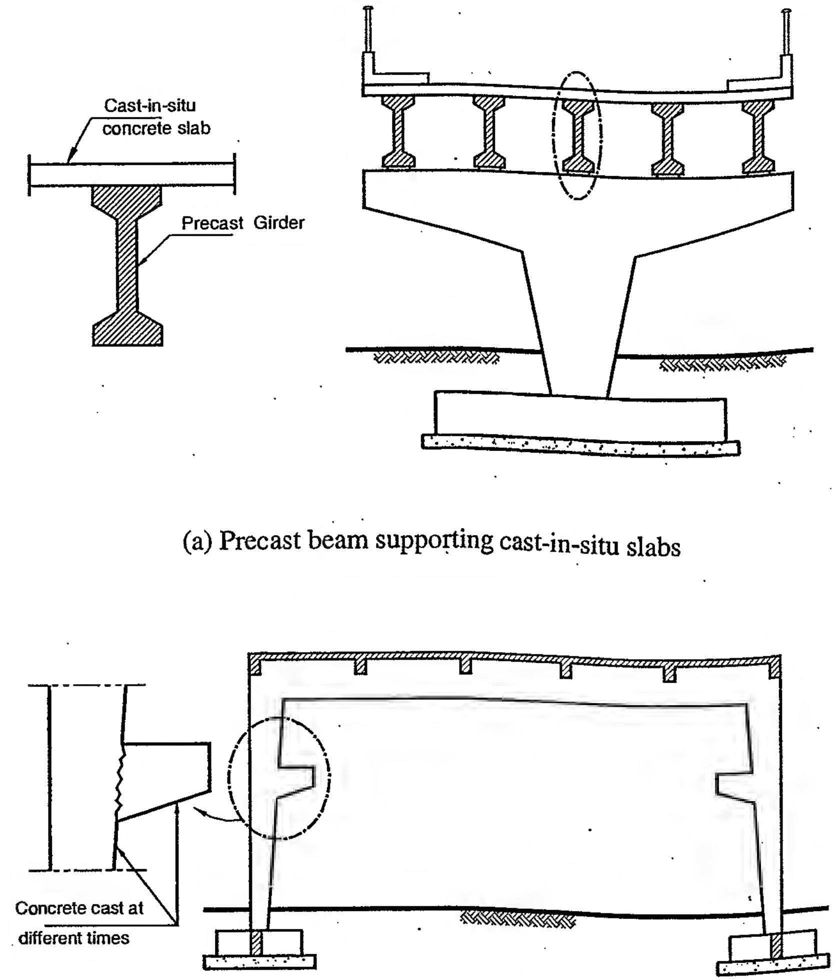


Fig. 2.11 Deep beam supporting another deep beam

2.3 Shear- Friction Concept

There are many situations in reinforced concrete structures where it is necessary to transfer shear across planes of weakness such as interface between concrete cast at different times. Shear-friction concept provides a simple but powerful model to investigate situations such as those shown in Fig. (2.12).



(a) Precast beam supporting cast-in-situ slabs

(b) Corbels

Fig. 2.12 Applications of shear friction concept

Typical examples are reinforced concrete bridges in which the deck is cast-in-situ concrete slab supported on precast girders as shown in Fig. 2.12a. Another example is corbels supporting crane girders.

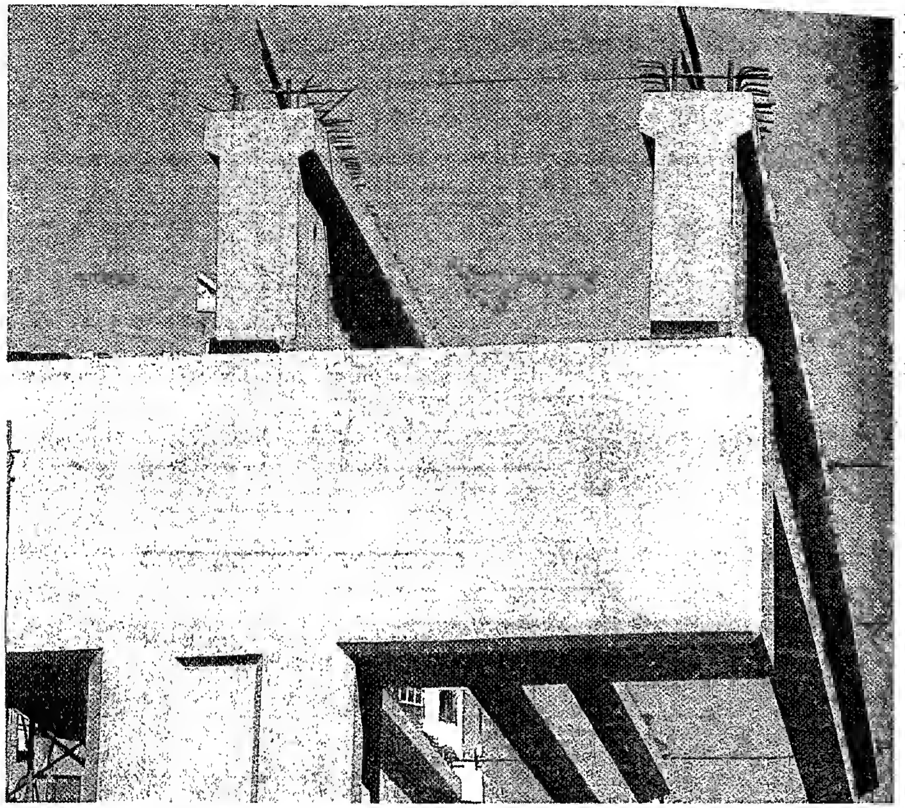


Photo 2.4 Short cantilever supporting prestressed beams

The basis of this model is explained in Fig. (2.13). When shear is applied to an initially cracked surface, or a surface formed by placing one layer of concrete on top of an existing layer of hardened concrete, relative slip of the layers causes a separation of the surfaces as shown in Fig. (2.13a). If there is reinforcement across the crack, it is elongated by the separation of the surfaces. The elongation of the reinforcement means that it is stressed in tension. For equilibrium of the free body diagram at the interface, a compressive stress is needed as shown in Fig. (2.13b). Figure 2.13c shows aggregate interlock at crack interface.

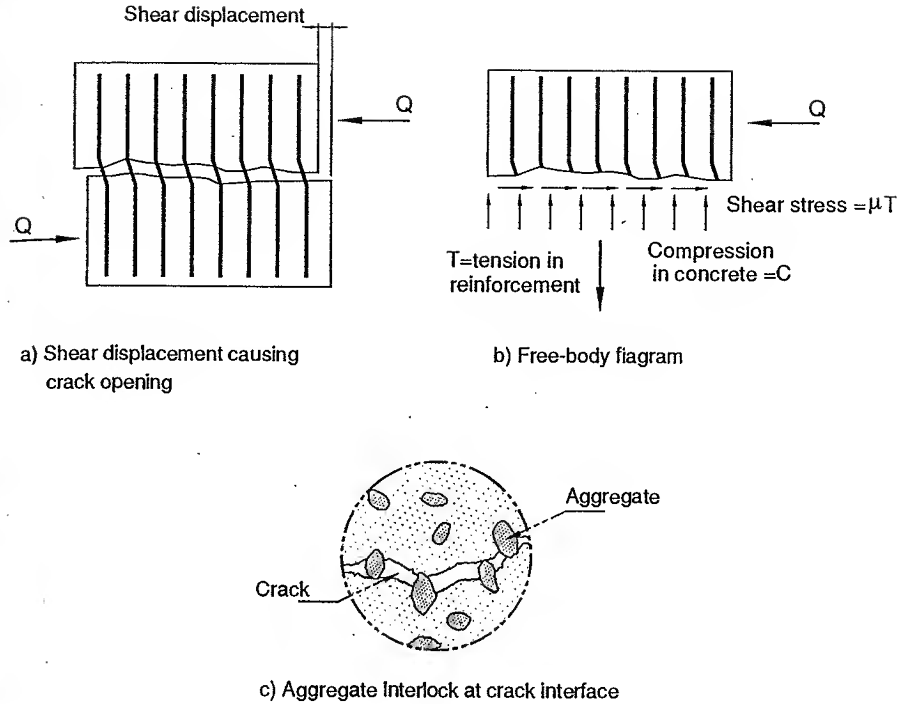


Fig. 2.13 Mechanism of shear friction

Photo 2.5 Aggregate distribution in concrete section

Shear is transmitted across the crack by:

1. Friction resulting from the compressive stress.
2. Interlocking of aggregate protrusions on the cracked surfaces combined with dowel action of the reinforcement crossing the surface.

The shear stresses on the concrete face are assumed to be related to the compressive stresses by a coefficient of friction μ. The maximum capacity is assumed to be reached when the reinforcement crossing the crack yields leading to a shear resistance of:

$$Q = A_{sf}\, f_y / \gamma_s\ \mu \qquad (2.14)$$

where A_{sf} is the area of reinforcement crossing the surface and f_y its yield strength. Equation (2.14) states that the resistance to slip is equal to the normal force times the coefficient of friction μ.

Tests have shown that shear-friction capacity is also a function of the concrete strength and the area of contact. As the concrete strength and the area of contact increase, the aggregate interlock mechanism becomes more efficient and the shear friction increases. Hence, there is an upper limit on the shear resistance due to friction:

$$Q = \text{constant}\ (f_{cu}\, A_c) \qquad (2.15)$$

where A_c is the area of contact. The area of reinforcement that crosses the crack A_{sf} (Fig. 2.14) is given by the Egyptian Code as:

$$A_{sf} = \frac{Q_u}{\mu\ f_y / \gamma_s} \qquad (2.16)$$

If the section is subjected to a tension force in addition to the shear force, additional steel should be provided as given by the following equation:

$$A_{sf} = \frac{Q_u}{\mu\ f_y / \gamma_s} + \frac{N_u}{f_y / \gamma_s} \qquad (2.17)$$

The values given by the Egyptian Code for the coefficient of friction (μ) are given in Table 2.1.

Table 2.1: Values of μ according to surface condition

	Crack Interface Condition	μ
1	Concrete cast monolithically	1.20
2	Concrete cast against hardened concrete with surface intentionally roughened	0.80
3	Concrete cast against hardened concrete not intentionally roughened or concrete anchored to structural steel by headed studs or bars.	0.50

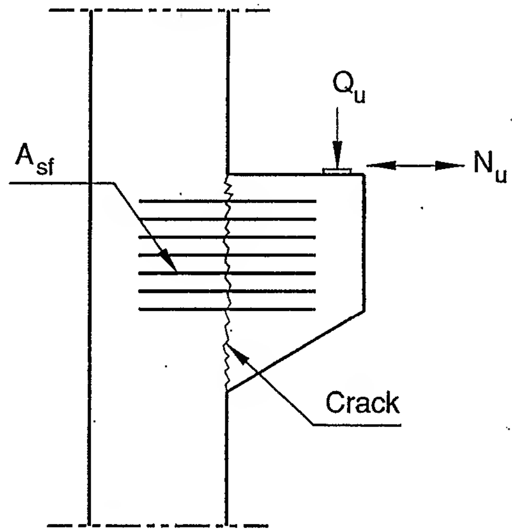


Fig. 2.14 Shear friction reinforcement

The steel must be placed approximately uniform across the shear plane so that all parts of the crack are clamped together. Each bar must be anchored on both sides of the crack to develop the yield strength.

The ultimate shear (Q_u / A_c) shall not exceed the following limits:

$$q_u = Q_u / A_c \le 0.15 f_{cu} \qquad (2.18a)$$

$$q_u = Q_u / A_c \le 4.0\ N/mm^2 \qquad (2.18b)$$

2.4 Short Cantilevers (Brackets or Corbels)

Corbels or brackets are short cantilever members that project from a column or a beam to support another beam or heavy concentrated load. The importance of these members is clear in precast buildings where corbels support beams and girders. Therefore, the total safety of these types of structures depends on the ability of the corbels and brackets to transfer the load safely to the columns. Steel bearing plates or angles are commonly used in the top surface of the brackets to provide a uniform contact surface and to distribute the reaction.

Short cantilevers are defined by the Egyptian Code as cantilevers whose shear span-to depth ratio (a/d) is 1.0 or less (See Fig. (2.15)). This small ratio changes the pattern and distribution of stresses similar to the case of deep members. In corbels, a large horizontal force develops due to shrinkage and creep of the supported elements such as beams that are connected to the corbels.

The code provisions apply to short cantilevers in which the depth at the outside edge of the bearing area is not less that $(0.5d)$ where d is the depth measured at column face. Short cantilevers are designed to support beams transferring vertical reactions Q_u. Horizontal force (N_u) caused by restrained shrinkage, creep in prestressed beams and expansion or contraction effects. Therefore, it is advisable to consider a minimum horizontal force, $N_u = 0.2Q_u$.

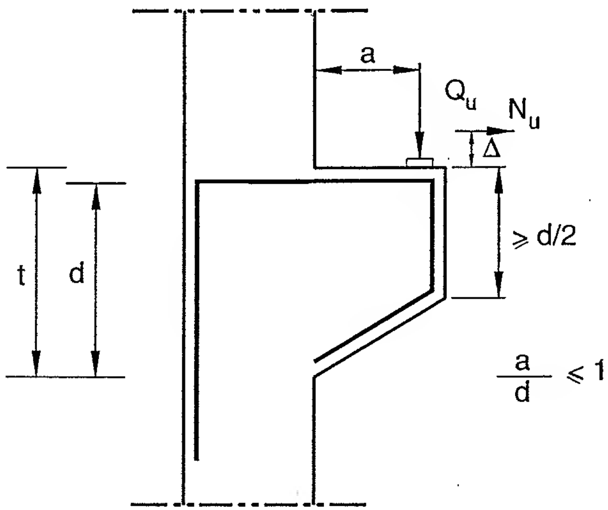


Fig. 2.15 Definition of a Corbel according to the Egyptian Code

The structural action of a short cantilever can be idealized as a truss made up of a compression strut and a tension tie as shown in Fig. (2.16a). The inclination of the strut determines the tension in the tie by a simple force polygon. Since the tension tie supports a constant tension force, sufficient anchorage of bars should be provided beyond the corbel interface with the column. Failure of the strut-and tie model could occur as a result of yielding of the tension tie, failure of the compression strut, or failure of the end anchorage of the tension tie.

A direct shear failure could also be a possible mode of failure along the face of the column as shown in Fig.(2.16b). Local failure under the bearing plate could occur. Finally, if the corbel is too shallow at the outside end, there is a danger that cracking may extend through the corbel as shown in Fig. (2.16c). For this reason, ECP 203 requires the depth of the corbel to be 0.5d at the outside edge of the bearing plate.

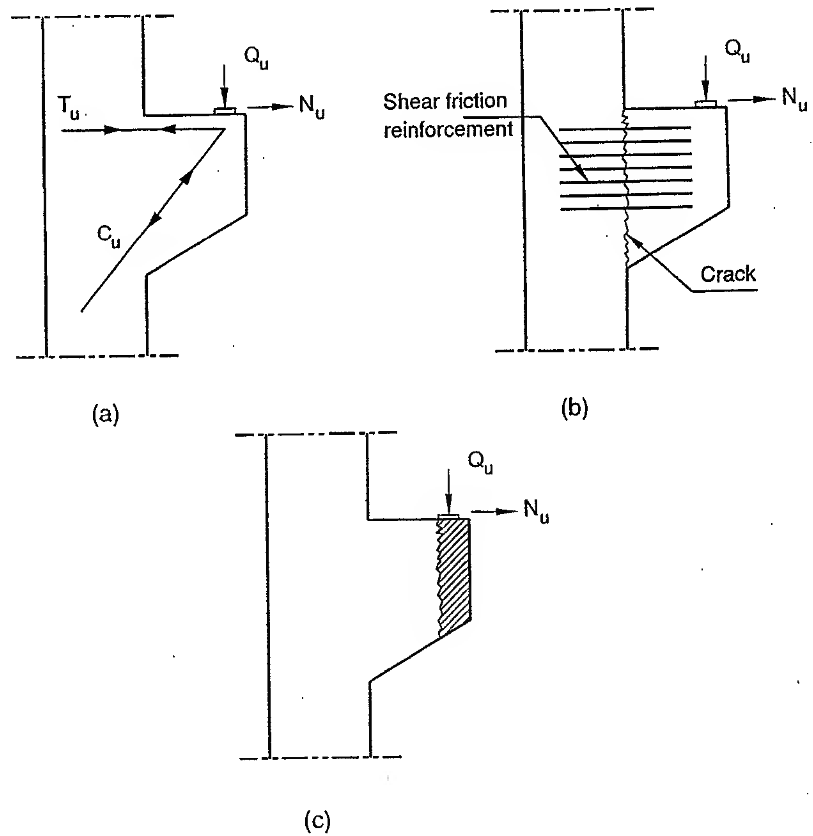


Fig. 2.16 Failure modes of corbels

The Egyptian Code requires that reinforcement be arranged as shown in Fig. (2.17). The main tension reinforcement is calculated to resist a moment (M_u) at column face and a normal force (N_u).

The area of steel required to resist the tensile force $(N_u \geq 0.2Q_u)$ is given by:

$$A_n = \frac{N_u}{f_y/\gamma_s} \quad \text{.......................................} \quad (2.20)$$

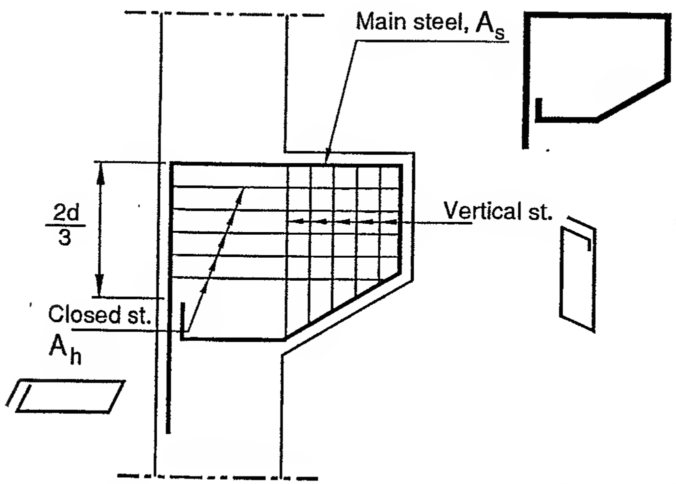


Fig. 2.17 Reinforcement of corbels

Photo 2.6 reinforced concrete buildings

The bending moment is calculated as follows (refer to Fig. 2.15). The flexural reinforcement A_f is calculated using regular sectional analysis.

$$M_u = Q_u \ a + N_u \ (t + \Delta - d) \quad \text{..........................} \quad (2.20)$$

The shear-friction reinforcement (A_{sf}) calculated using the shear-friction concept is given by:

$$A_{sf} = \frac{Q_u}{\mu (f_y/\gamma_s)} + \frac{N_u}{f_y/\gamma_s} \quad \text{..............................} \quad (2.21)$$

Corbel reinforcement consists of three types:

1. Main reinforcement.
2. Horizontal stirrups.
3. Vertical stirrups.

1. Main Reinforcement

The total main top steel A_s is the greater of the following:

1. $A_s = A_n + A_f$ ....(2.22a)
2. $A_s = A_n + 2/3 A_{sf}$ ....(2.22b)
3. $A_{s\,min} = 0.03 \frac{f_{cu}}{f_y} b \ d$ ....(2.23)

where b is the width of the corbel.

2. Horizontal Stirrups

The horizontal shear reinforcement, A_h, consists of horizontal closed stirrups uniformly distributed in the top 2/3 of the cross section. This area is given by:

$$A_h = 0.5 \ (A_s - A_n) \quad \text{..................................} \quad (2.24)$$

3. Vertical Stirrups

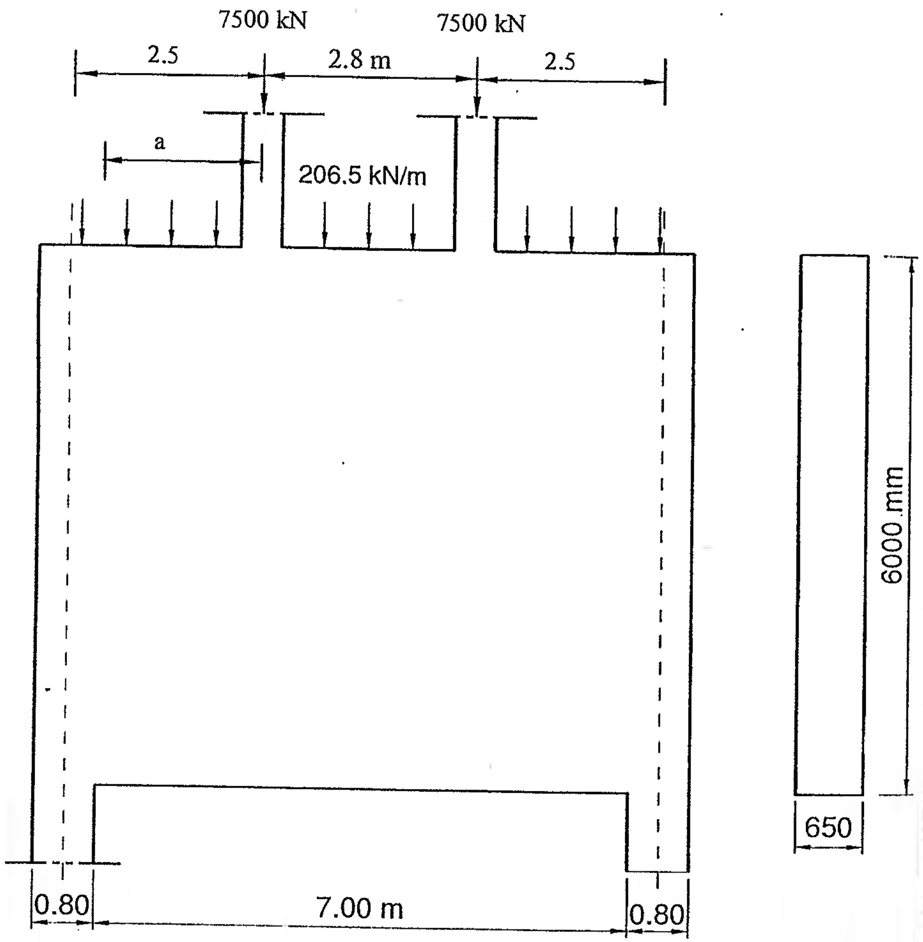
Corbels should also be provided with vertical stirrups that satisfies the minimum requirements of the ECP-203.

$$A_v = \frac{0.4}{f_y} b \ s \quad \text{..} \quad (2.25)$$

where s is the spacing of the vertical stirrups.

Example 2.1

A transfer girder is to support two columns, each having a factored load of 7500 kN as shown in the figure. Its clear span is 7.0 m. The girder has to carry also a factored uniform load, including its weight, having a value of 206.5 kN/m′. The material properties are $f_{cu} = 25\,N/mm^2$ and $f_y = 360\,N/mm^2$. Design the beam using the empirical design method presented in the ECP 203



Solution

Step 1: Check the applicability of the empirical method

$$L_{eff} = smaller\ of \begin{cases} 1.05 \times clear\ span = 1.05 \times 7.00 = 7.35\,m \\ CL.to\,CL. = 7.8\ \ m \end{cases}$$

$$\therefore L_{eff} = 7.35\,m$$

Assume the distance from the bottom fibers to the center of the tension reinforcement = 100 mm $\rightarrow$ $d = 6000 - 100 = 5900\,mm$

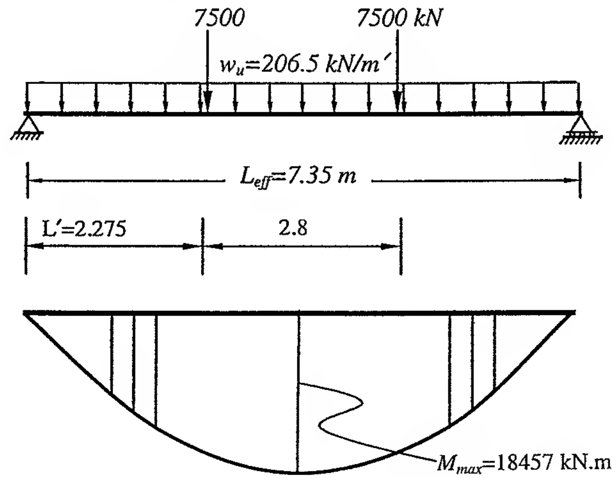
$$\frac{L_{eff}}{d} = \frac{7.35}{5.90} = 1.245 < 1.25$$

Since the $(d / L_{eff}) < 1.25$ (simple beam condition), the empirical method can be applied.

Step 2: Flexural design

$$y_{ct} = smaller\ of \begin{cases} 0.86\ L_{eff} = 0.86 \times 7.35 = 6.32\ m \\ 0.87\ d = 0.87 \times 5.9 = 5.133\ \ m \end{cases}$$

$$\therefore y_{ct} = 5.13\,m$$



Bending moment diagram

$$M_{max} \text{ (at mid-span)} = \frac{w_u \times L^2}{8} + P_u \times L' = \frac{206.5 \times 7.35^2}{8} + 7500 \times 2.275$$

$$= 18457 \text{ kN.m}$$

$$A_s = \frac{M_u}{y_{ct} \times f_y / \gamma_s} = \frac{18457 \times 10^6}{5130 \times 360 / 1.15} = 11493\, mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} = \dfrac{0.225\sqrt{25}}{360} \times 650 \times 5900 = 11984\ mm^2 \\ 1.3A_s = 1.3 \times 11493 = 14941\, mm^2 \end{cases}$$

But not less than $\frac{0.15}{100} b\ d = \frac{0.15}{100} \times 650 \times 5900 = 5752\ mm^2$

$A_s < A_{s\,min} \ldots\ldots \therefore A_s = A_{s\,min} = 11718\ mm^2$

Use 20 Φ 28 mm(=12315 mm^2), arranged in three layers.

Assume that the top steel equals 15% of the bottom steel

Use (A_s') 4 Φ 25 mm.

Step 3: Shear design

Step 3.1: Straining actions at the critical sections

- The Critical section for shear is at *0.5a* from the face of the support but not more than *d/2* from the face of support.
  - $a/2 = 0.5(2500 - 400) = 1050\, mm$
  - $d/2 = \frac{5900}{2} = 2950\, mm$

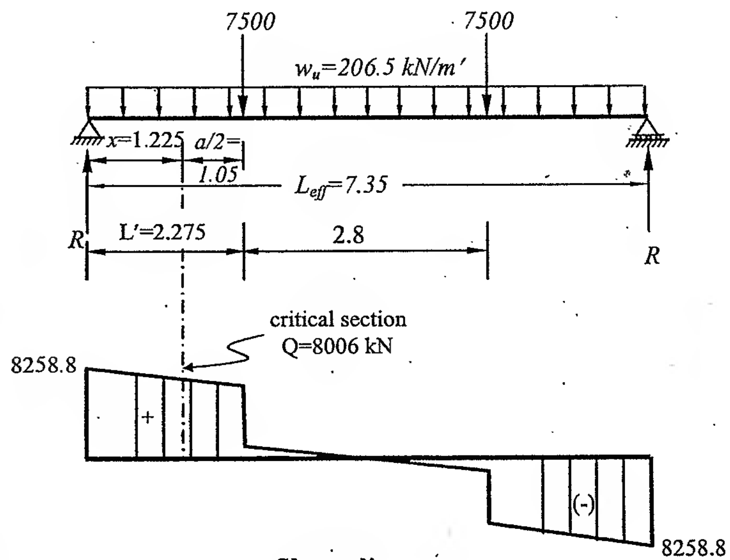
  $x = \frac{7.35 - 2.8 - 2 \times 1.05}{2} = 1.225\, m$ (CL distance)

- At the critical section for shear, the straining actions are:

$R = 206.5 \times \frac{7.35}{2} + 7500 = 8258.88\ kN$

$Q = R - w_u\ x = 8258.88 - 206.5 \times 1.225 = 8006\ kN$

$M_u = R\ x - w_u\ x^2/2 = 8258.88 \times 1.225 - 206.5 \times 1.225^2/2 = 9962\ kN.m$



Shear diagram

Step 3.2: Check the adequacy of the concrete dimensions

Average shear stress at the critical section is given by:

$q_u = \frac{Q_u}{b \times g}$ (g is the smaller of d (5900 mm) or L_{eff} (7350 mm))

$q_u = \frac{8006 \times 10^3}{650 \times 5900} = 2.09\, N/mm^2$

Maximum allowable shear stresses, $q_{u\,max} = \delta_d \times 0.7\sqrt{f_{cu}/\gamma_c} \le \delta_d \times 4$

$$\delta_d = \frac{1}{3}(2 + 0.4\frac{L_n}{d})$$

$$\delta_d = \frac{1}{3}(2 + 0.4\frac{7.0}{5.9}) = 0.825$$

$$q_{u\,max} = 0.825 \times 0.7\sqrt{25/1.5} = 2.35\, N/mm^2 < 0.825 \times 4 \ldots\ldots \rightarrow o.k$$

Since the average shear stress at the critical section is less than the maximum allowable shear stress, the concrete dimensions are adequate.

Step 3.3: Calculation of shear carried by concrete

$$\delta_{dc} = 3.5 - 2.5(M_u / Q_u d) \quad \begin{matrix} > 1.0 \\ < 2.5 \end{matrix}$$

$$\delta_{dc} = 3.5 - 2.5 \times (\frac{9962}{8006 \times 5.90}) = 2.97 > 2.5 \;\rightarrow \delta_{dc} = 2.5$$

The concrete shear strength q_{cu} is chosen as the smaller of:

1. $q_{cu} = \delta_{dc} \times 0.24\sqrt{f_{cu}/1.5} = 2.5 \times 0.24\sqrt{25/1.5} = 2.45 N/mm^2$

2. $q_{cu} = 0.46\sqrt{f_{cu}/1.5} = 0.46\sqrt{25/1.5} = 1.88 N/mm^2$

$q_{cu} = 1.88\ N/mm^2$

The average shear stress at the critical section is more than the shear carried by concrete. Web reinforcement (vertical and horizontal) needs to be designed.

Step 3.4: Design of web reinforcement

$$q_{su} = q_u - 0.5 q_{cu} = 2.09 - \frac{1.88}{2} = 1.15 N/mm^2$$

$$\delta_h = \frac{(11 - L_n/d)}{12} = \frac{(11 - \frac{7.0}{5.9})}{12} = 0.818$$

$$\delta_v = \frac{(1 + L_n/d)}{12} = \frac{(1 + \frac{7.0}{5.9})}{12} = 0.182 \qquad \text{or } \delta_v = 1 - \delta_h = 1 - 0.818 = 0.182$$

$$q_{su} = \delta_v \times q_{suv} + \delta_h \times q_{suh}$$

$$q_{su} = \frac{f_y}{b\gamma_s}\left[\delta_v(\frac{A_v}{s_v}) + \delta_h(\frac{A_h}{s_h})\right] \rightarrow \qquad q_{su} = \frac{f_y}{b\gamma_s}\left[0.182(\frac{A_v}{s_v}) + 0.818(\frac{A_h}{s_h})\right]$$

Try vertical bars of diameter 12.0 mm (2 branches) and horizontal bars of diameter 16.0 mm (2 branches).

$A_v = 226 mm^2$ and $A_h = 400 mm^2$

Assume $s_v = 200 mm$ (satisfies code requirement)

$$q_{su} = 1.13 = \frac{360}{650 \times 1.15}\left[0.182\left(\frac{226}{200}\right) + 0.818\left(\frac{400}{s_h}\right)\right] \qquad \rightarrow \quad s_h = 152.8 \text{ mm}$$

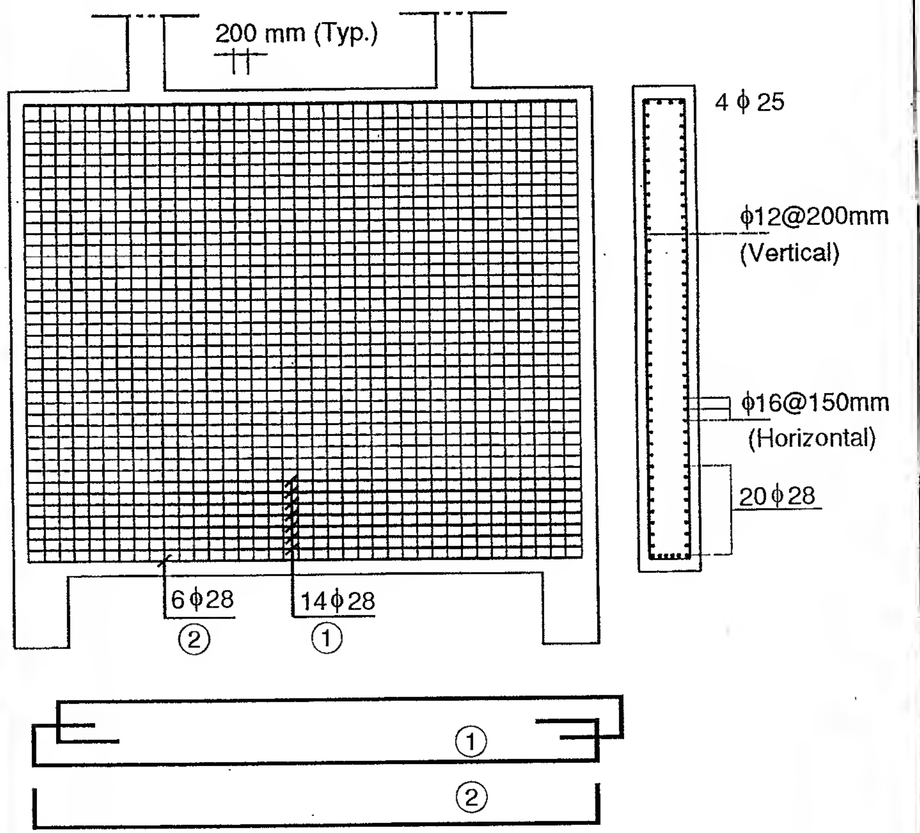
Take $s_h = 150 mm$ (satisfies code requirements)

Note: $q_{su\,(provided)} = 1.15 N/mm^2 > q_{su\,(required)} = 1.13 \quad \rightarrow$ O.K.

Check the satisfaction of the minimum web reinforcement

$A_{v,\min} = 0.0015\, b\, s_v = 0.0015 \times 650 \times 200 = 195\ mm^2 \quad < A_v$ ....ok

$A_{h,\min} = 0.0025\, b\, s_h = 0.0025 \times 650 \times 150 = 244 mm^2 \quad < A_h$ .....ok



Reinforcement details

Example 2.2

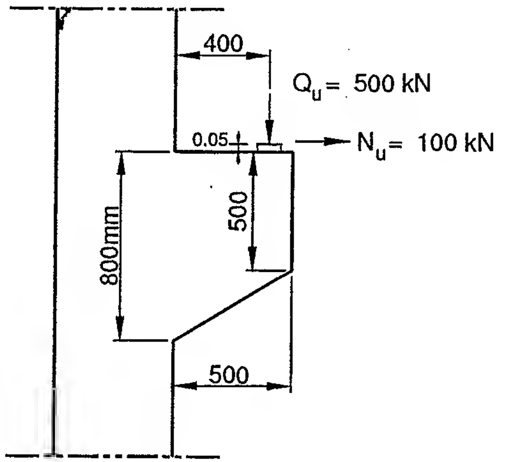
Determine the required reinforcement for the bracket shown in figure according to the following data:

Bracket dimensions (b x t) = 300 mm x 800 mm and d = 750 mm,

$f_{cu} = 25\,N/mm^2$ and $f_y = 240\,N/mm^2$

Factored vertical load Q_u = 500 kN

Factored horizontal load N_u = 100 kN



Step 1: Check the bracket dimensions:

To be classified as a corbel, the distance a should not exceed the effective depth.

d = 750 mm > a (400 mm) →→ok.

Step 2: Check the ultimate shear friction value:

$\frac{Q_u}{bd} \leq 0.15 f_{cu}$ but not more than 4 N/mm^2

$$\frac{500\times10^3}{300\times750} = 2.22 \leq 0.15x\,25 = 3.75\ N/mm^2\ O.K.$$

Step 3: Area of main reinforcement:

The bending moment acting on the bracket equals to:

$$M_u = Q_u.a + N_u(t + \Delta - d)$$

$$M_u = 500\times0.4 + 100(0.8 + 0.05 - 0.75) = 210\,kN.m$$

$$M_u = 0.67\frac{f_{cu}}{\gamma_c} b.a_f.(d - a_f/2)$$

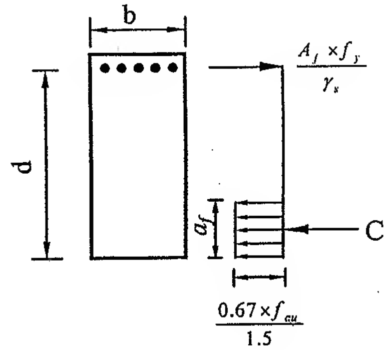
$$M_u = 210\times10^6 = 0.67\times\frac{25}{1.5}\times300\times a_f \times(750 - a_f/2)$$

$a_f = 88$ $\quad > 0.1d$

$$A_f = \frac{M_u}{(d - a_f/2) f_y/\gamma_s} = \frac{210\times10^6}{(750-88/2)\times240/1.15} = 1425\,mm^2$$

$$A_n = \frac{N_u}{f_y/\gamma_s} = \frac{100x\,10^3}{240/1.15} = 479\ mm^2$$

$$A_{sf} = \frac{Q_u}{\mu f_y/\gamma_s} + \frac{N_u}{f_y/\gamma_s}$$



For monolithically cast concrete $\mu = 1.2$

$$A_{sf} = \frac{500\times10^3}{1.2\times\frac{240}{1.15}} + \frac{100\times10^3}{240/1.15} = 2476\,mm^2$$

The area of the main reinforcement is the largest area obtained by evaluating three equations:

The first equation

$$A_s = A_n + A_f$$

Then $A_s = 479 + 1425 = 1904\,mm^2$

The second equation

$$A_s = A_n + \frac{2}{3}A_{sf}$$

then $A_s = 479 + \frac{2}{3}\times2476 = 2130\,mm^2$

The third equation

$$A_s = 0.03\frac{f_{cu}}{f_y}b\,d = 0.03\times\frac{25}{240}\times 300\times 750 = 703\,mm^2$$

Hence, the area main reinforcement is obtained from the second equation.

A_s = 2130 mm^2 Use 5 ϕ 25 (2454.369 mm^2)

Step 4: Find the area of the horizontal stirrups

$$A_h = 0.50(A_s - A_n) = 0.50(2130 - 479) = 826\ mm^2$$

This area has to be distributed over 2/3 of the effective depth, i.e. over a distance equals to (2/3x750) = 500 mm.

Choose closed stirrups (two branches) having a bar diameter = 12 mm and spaced at 166 mm.

The available area of horizontal stirrups = 113×2 ×(500/166 +1) = 906.7 mm^2 > 826 mm^2 O.K.

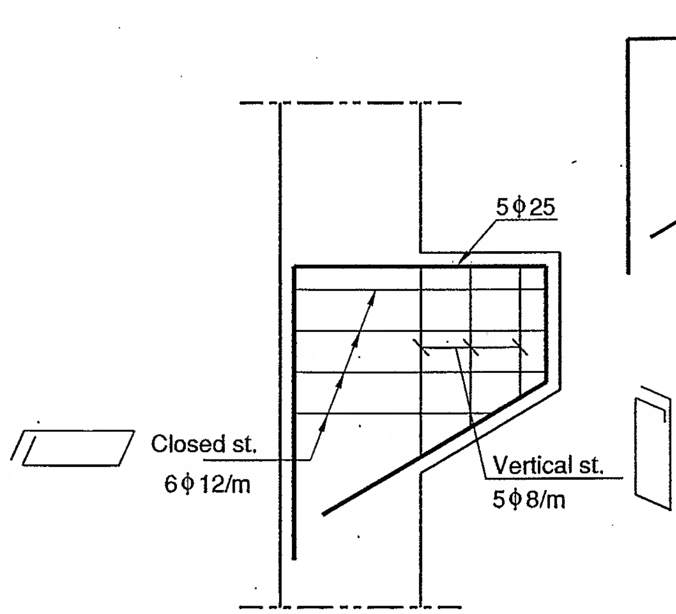
Step 5: Find the area of the vertical stirrups

Assume that the spacing of the vertical stirrups is 200 mm.

$$A_{st} = \mu_{min} b\ s = \frac{0.4}{f_y}\times b\times s = \frac{0.4}{240}\times 300\times 200 = 100\,mm^2$$

Choose vertical stirrups (two branches) having a bar diameter =8 mm and spaced at 200 mm.

The available area = 50 x 2 = 100 mm^2 O.K.



Reinforcement Details

3

CONTROL OF DEFLECTIONS

Photo 3.1 A cable-stayed bridge during construction

3.1 Introduction

The Egyptian Code is based on the limit states design method. The limit states (states at which the structure becomes unfit for its intended function) are divided into two main groups: those related to collapse and those that disrupt the use of the structures but do not cause collapse. These are referred to as ultimate limit states and serviceability limit states, respectively. The major

serviceability limit states are excessive deflections, undesirable vibrations and excessive cracking. Deflection control will be thoroughly presented in this chapter. Control of cracking will be discussed in chapter four.

The adoption of the limit states design method in recent years, accompanied by the use of higher strength concrete and high-grade steel, has permitted the use of relatively shallower members. As a result, deflection calculations gained more importance than they were few decades ago. Excessive deflections of beams and slabs may cause excessive vibrations, damage to the appearance of the structure, poor roof drainage, and uncomfortable feelings for the occupants. Also, such deflections may damage partitions and cause poor fitting of doors and windows. Therefore, it is very important to maintain control of deflections.

The Egyptian code presents the following two approaches for controlling deflection:

- *Control of deflection by limiting the span/thickness ratio of the member.*
- *Control of deflection by calculating the deflection and set limitations to its value.*

The first approach indirectly controls the deflection by setting an upper limit for span-to-thickness ratio. It is simple to follow without the need for deflection calculations. However, if smaller members are required, the second approach should be followed by calculating the deflections and comparing the computed values with specific limitations imposed by the code.

3.2 Load-Deflection Behavior of RC Beams

Figure 3.1 shows the load deflection response of a reinforced concrete beam. Initially, the beam is uncracked and is stiff. With further loads, cracking occur at mid-span when the applied moment exceeds the cracking moment of the beam. When a section cracks its moment of inertia decreases leading to a significant reduction in the stiffness of the beam. This is the start of the cracking stage. At this stage, the beam continues to carry load but with relatively large deflection. Eventually the reinforcement yields at mid-span leading to a large increase in deflections with little change in load (points C and D).

Since the service load of any member is about 65% of its ultimate load, the service load level of the beam in Fig. 3.1 can be represented by point B. Long-term application of service load (*sustained load*) results in increasing the deflection from point B to B′, due to creep of concrete. The short-term, or immediate, deflection under service load (point B) and the long-term service load deflection (point B′) are both of interest in design and will be discussed later.

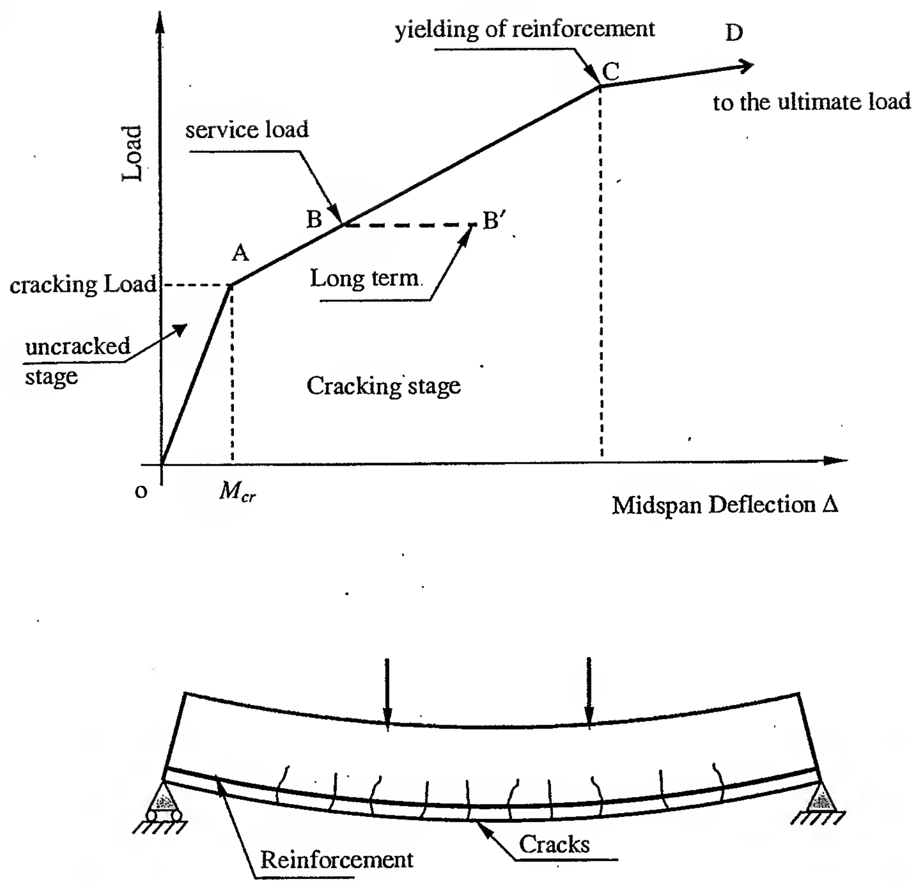


Fig. 3.1 Load-deflection response

3.3 Moment of Inertia of RC sections

3.3.1 Gross moment of inertia

As mentioned in the previous section, if the applied moment is less than the cracking moment, the section is considered uncracked. In this case, the moment of inertia of the section equals to I_g *(uncracked stage)* as shown in Fig. 3.2.

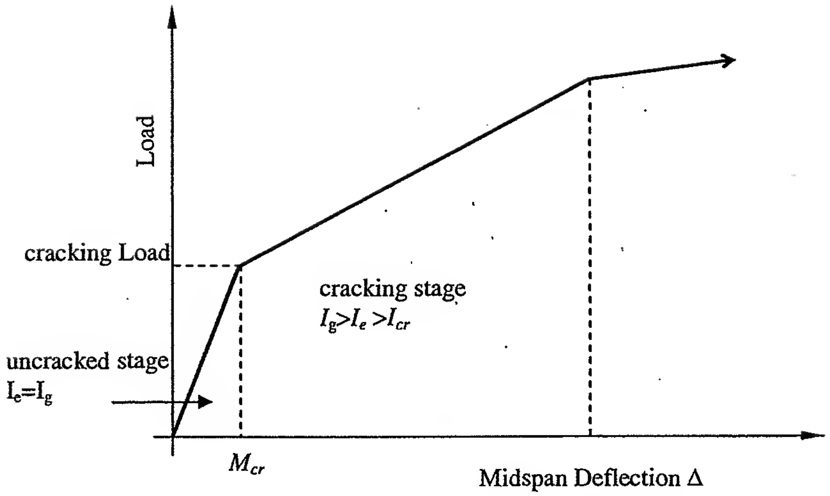


Fig. 3.2 Moment of inertia in concrete beams

For design purposes, the calculation of the gross uncracked moment of inertia, I_g, can be carried out by neglecting the cross-sectional area of steel reinforcement (e.g. I_g for rectangular sections = b $t^3/12$). For normal reinforcement ratios, the error in calculating I_g does not exceed 10%.

The ECP 203 gives the following formula for calculating the cracking moment:

$$M_{cr} = \frac{f_{ctr} . I_g}{y_t} \quad \text{.. (3.1)}$$

where f_{ctr} is the concrete tensile strength (N/mm^2), I_g is the gross moment of inertia neglecting the effect of reinforcement (mm^4), and y_t is the distance from the neutral axis to the extreme fiber in tension for the uncracked section (mm). In the ECP 203, the concrete tensile strength f_{ctr} is given by:

$$f_{ctr} = 0.6\sqrt{f_{cu}} \quad \text{.. (3.2)}$$

For rectangular sections, y_t equals to half the section thickness. For T-sections the reader should pay attention to the direction of the bending moment. Thus, for T-section in cantilever beams the distance y_t is measured from the top fibers Fig.3.3.a and for T-sections in simple beams it is measured from the bottom fibers as shown in Fig.3.3.b.

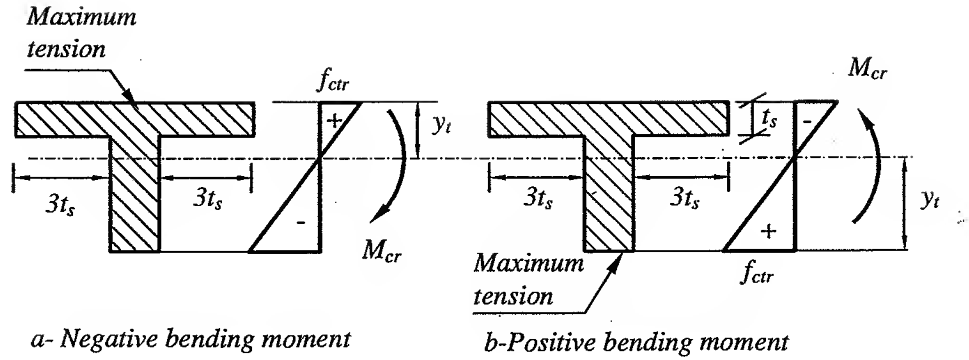
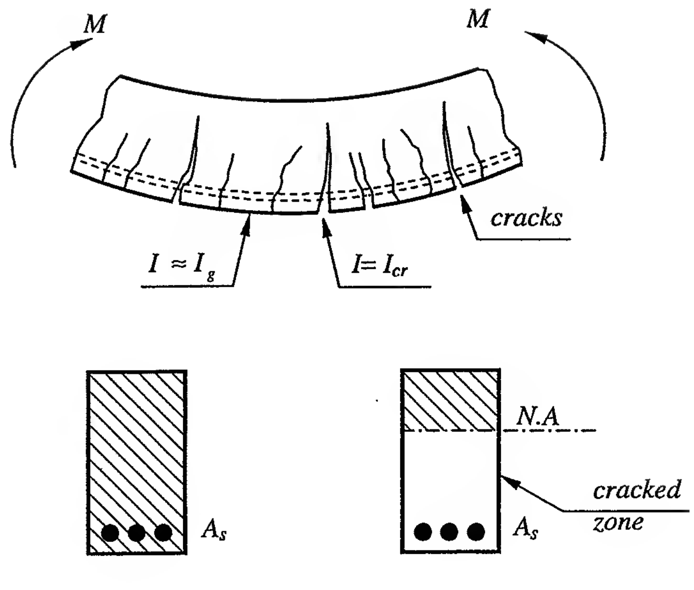


Fig. 3.3 Determination of the distance y_t in simple and cantilever T-beams

3.3.2 Cracked Transformed Moment of Inertia

When the applied moment exceeds M_{cr}, the developed tensile stress exceeds the tensile strength of concrete producing cracks as shown in Fig. 3.4. The developed cracks will cause the moment of inertia to drop to a value less than the gross moment of inertia I_g. Since concrete is weak in tension, it will crack below the neutral axis and its contribution to the rigidity and strength will be neglected. On the other hand, the concrete in the compression zone acts effectively and contribute to the section rigidity. The actual cracked section is non-homogeneous and consists of the compressed concrete above the neutral axis and the reinforcing steel bars below the neutral axis. The non-homogeneous section can be replaced by an imaginary homogenous section called the *transformed section*.

To obtain the transformed section of a reinforced concrete beam, the area of the reinforcing steel bars A_s is replaced by an equivalent area of concrete equals nA_s, in which $n= E_s/E_c$ is the modular ratio (the modulus of elasticity of steel / modulus of elasticity of concrete). The moment of inertia of this transformed section is called the *cracked transformed moment of inertia* I_{cr}.



Uncracked section Cracked section

Fig. 3.4 Cracking of concrete section under applied loads

The neutral axis is located at distance z from the compression face. The location of the neutral axis can be easily determined by taking the first moment of area about the center of gravity of the section (c.g.). It should be noted that the center of gravity coincides with neutral axis *(no normal force).*

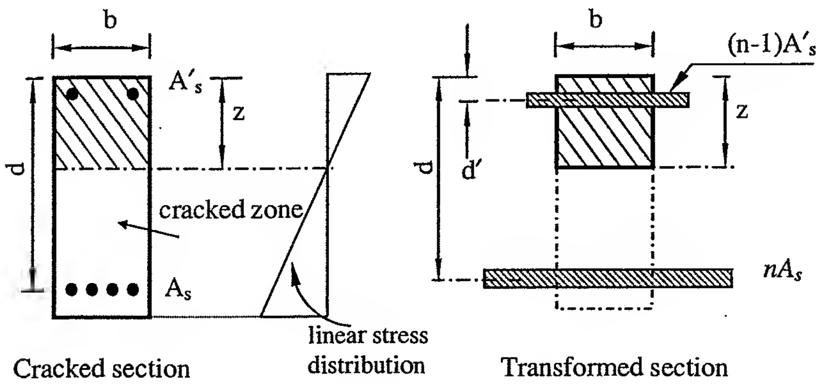


Fig. 3.5 Determination of the neutral axis and cracked transformed moment of inertia calculations

$$b \times z^2/2 - n\,A_s\,(d-z) = 0 \quad \text{.............................} (3.3)$$

Subsisting with $z{=}kd$ and $\mu = A_s/b\,d$ gives:

$$\frac{b\,(kd)^2}{2} - n\,A_s\,(d-kd) = 0 \quad \text{.............................} (3.4)$$

Dividing by bd^2, substituting with $\mu_n = (n\,\mu)$, and solving for k gives

$$k = \sqrt{2\mu_n + \mu_n^{\,2}} - \mu_n \quad \text{.............................} (3.5)$$

and, $z{=}k\,d$

Having determined the neutral axis distance z, the cracked moment of inertia I_{cr} can be computed as

$$I_{cr} = \frac{b \times z^3}{3} + n\,A_s\,(d-z)^2 \quad \text{.............................} (3.6)$$

Using the previous set of equations, design chart was prepared to facilitate the determination of the I_{cr} for singly reinforced section (refer to the Appendix).

For doubly reinforced section, the compression steel displaces the stressed concrete and has a transformed area of $(n\text{-}1)A_s$ Referring to Fig. 3.5 and taking the first moment of area about the top fibers gives:

$$b \times z^2/2 + (n-1)\,A'_s\,(z-d') - n\,A_s\,(d-z)^2 = 0 \quad \text{............} (3.7)$$

The previous equation is a quadratic equation in z and can be solved directly. Te value of z can be directly obtained from Eq. 3.8.

$$z = \frac{-b_1 + \sqrt{b_1^2 - 4\,a_1 c_1}}{2\,a_1} \quad \text{.............................} (3.8)$$

where

a_1 $= b/2$

b_1 $= n\,A_s + (n\text{-}1)\,A'_s$

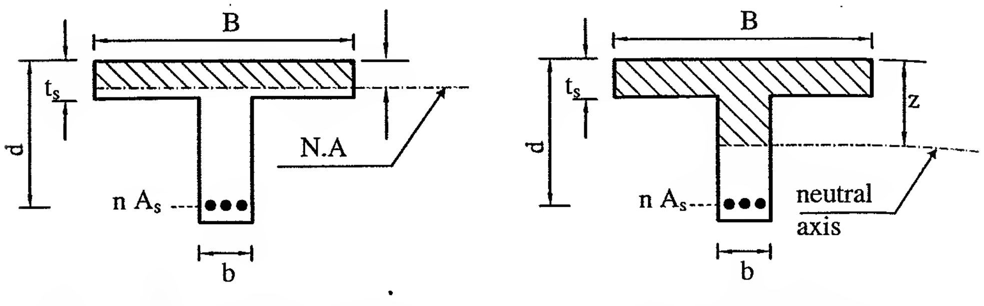
c_1 $= -[(n\text{-}1)\,A_s\,d' + n\,A_s\,d]$

The cracked moment of inertia equals

$$I_{cr} = \frac{b \times z^3}{3} + n\,A_s\,(d-z)^2 + (n-1)\,A'_s\,(z-d')^2 \quad \text{.....................} (3.9)$$

Design aids for calculating the cracked moment of inertia for rectangular sections with tension steel only are given in Appendix.

In T-sections, the neutral axis could be located inside or outside the flange as shown in Fig. 3.6. Therefore, hand calculations should be carried out as explained in the illustrative examples.



Neutral axis inside the flange $z<t_s$ Neutral axis outside the flange $z>t_s$

Fig. 3.6 Location of the neutral axis in cracked T-sections

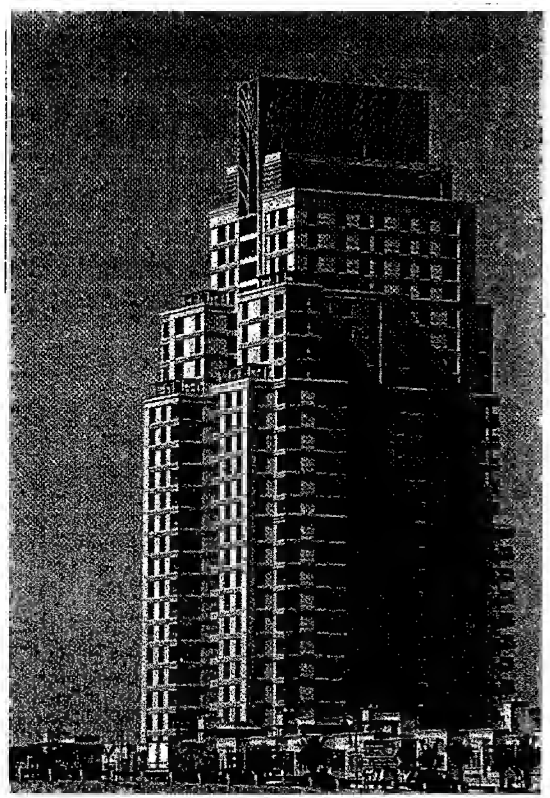


Photo 3.2 Reinforced concrete building

3.3.3 The Effective moment of inertia I_e

Sections located at tension cracks have their moment of inertia approximately equal to the transformed cracked moment of inertia I_{cr}. However, between cracks the moment of inertia could be approximately taken equals to I_g. Referring to Fig. 3.4, it is clear that a cracked reinforced concrete beam behaves as a beam with variable moment of inertia. To simplify deflection calculations, the cracked RC beam is assumed to have a constant moment of inertia (called the effective moment of inertia I_e).

The effective moment of inertia has a value less than I_g but is greater than I_{cr}. The most widely accepted formula for estimating the effective moment of inertia was developed by *Branson* and is adopted in the Egyptian code. This empirical equation, presented graphically in Fig. 3.7, was based on statistical analysis of deflections measured from test data, and is given by:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \quad \text{.......................} (3.10)$$

$$i.e \left(\frac{M_{cr}}{M_a}\right)^3 \le 1$$

The previous equation can be simplified as:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \quad \text{..} (3.11)$$

where

I_{cr} cracked transformed moment of inertia.
M_a maximum service (*unfactored*) moment in the member.
M_{cr} cracking moment calculated using Eq. 3.1.

The variability of deflection calculated using this expression, which is based on laboratory tests, is relatively high. However, considering the variety of factors that influence deflection of reinforced concrete beams, greater accuracy can be hardly expected from using such a simple equation.

Figure 3.6 shows variation of the effective moment of inertia I_e with the applied moment M_a. In this figure, the horizontal axis refers to the applied bending moment and the vertical axis refers to the moment of inertia that should be used in deflection calculations. It is clear that if the applied moment is less than cracking moment of the beam; deflection is calculated using the gross moment

of inertia I_g. On the other hand, if the applied moment is greater than the cracking moment, deflection is calculated using the effective moment of inertia I_e. It is interesting to note that the value of the effective moment of inertia approaches the cracked moment of inertia as the applied moment increases.

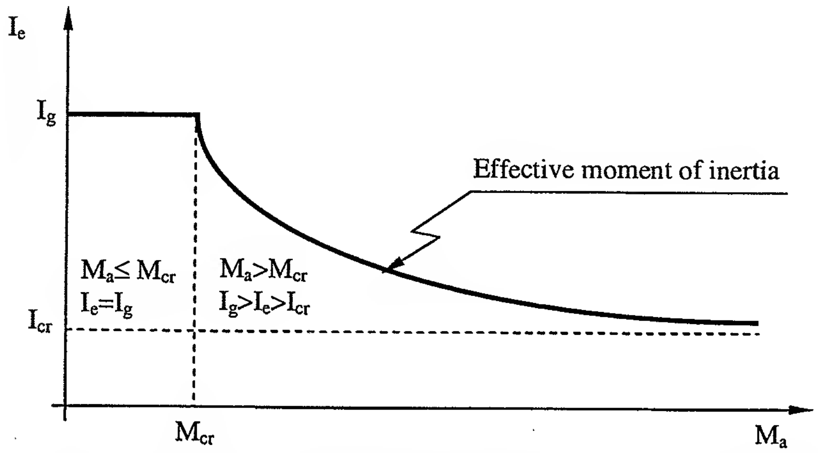


Fig. 3.7 Variation of the effective moment of inertia I_e with the applied moment M_a

In summery the effective moment of inertia equals

$$I_e = \begin{cases} I_g & \text{if } M_a \le M_{cr} \\ I_{cr} + (I_g - I_{cr})\left(\dfrac{M_{cr}}{M_a}\right)^3 & \text{if } M_a > M_{cr} \end{cases} \quad \text{..................} (3.12)$$

3.4 Code Provisions for Control of Deflections

The Egyptian code presents two approaches for controlling deflection. The first is indirect by setting an upper limit for span-to-thickness ratio. In the second approach, the computed member deflections are compared with specific limitations imposed by the code.

3.4.1 Control of Deflection by Span-to-Thickness Ratio

3.4.1.1 Beams and One-Way slabs

The Egyptian code imposes restrictions on the member thickness relative to the clear span L_n, to ensure that the member will be rigid enough so that deflections are unlikely to cause problems as given by Eq. 3.13 and Table 3.1. Table 3.1 .can not be used for beams or slabs supporting elements that are likely to be damaged due to deflection. It can not also be used in case of abnormal buildings and in case of heavy or uneven loads The code recognizes the effect of support conditions on deflection by assigning different span/thickness ratios according to the continuity conditions at both ends of the member.

$$\frac{L_n}{t} \le \text{Values listed in Table 3.1} \quad \text{..........................} (3.13)$$

Table 3.1 L_n/t ratios for members spanning less than 10 meters or cantilevers spanning less than 2m. (Deflection calculations are waived)

Element	Simply supported	One end continuous	Two end continuous	Cantilever
Solid slabs	25	30	36	10
Hidden Beams and hollow blocks	20	25	28	8
Beams	16	18	21	5

The values listed in Table (3.1) are valid when using high grade steel 400/600. In the case of using other types of reinforcing steel, the values mentioned in Table 3.1 should be divided by factor ζ, given by:

$$\xi = 0.40 + \frac{f_y}{650} \quad \text{...................................} (3.14)$$

Where f_y is the yield strength of reinforcing steel in N/mm^2.

T-sections

The limiting values listed in Table 3.1 are also valid for T-sections by multiplying the values by the reduction factor δ determined from the either Eq. 3.15 or Fig 3.8.

$$\delta = 0.71 + 0.29\left(\frac{b}{B}\right) \geq 0.8 \quad \text{.................................... (3.15)}$$

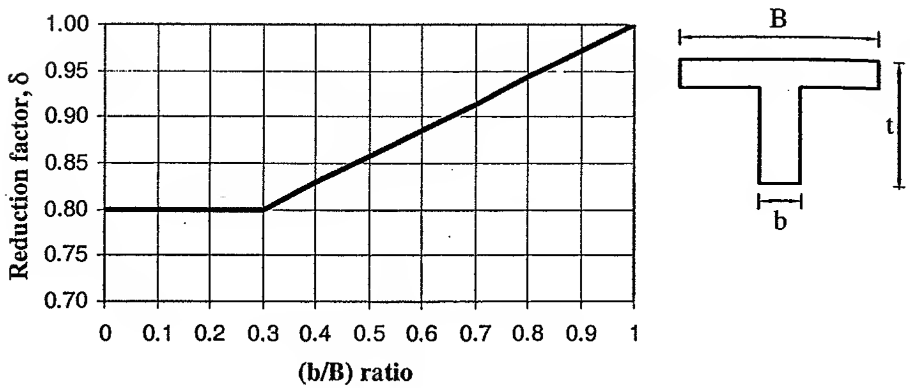


Fig. 3.8 Modification of L/t ratio for T-sections

3.4.1.2 Two-way slabs Resting on Rigid Beams

For two-way slabs resting on rigid beams, having spans of less than 10 meters, subjected to uniform loads that are not heavy and attached to non-structural elements not likely to be damaged by large deflections, the deflection calculations can be waived if the slab thickness is grater than *t* calculated using the following equation:

$$t = \frac{a\left(0.85 + \dfrac{f_y}{1600}\right)}{15 + \dfrac{20}{b/a} + 10\,\beta_p} \geq 100\ mm \quad \text{.................. (3.16)}$$

Where

a is the smaller dimension of the slab, b is longer dimension of the slab., β_p is the ratio between the length of all continuous edges to the total perimeter, and f_y is the yield strength of reinforcing steel in N/mm^2

Also the slab thickness should be greater than t_{min}

$$t_{min} = \begin{cases} \dfrac{a}{35} & \text{for simply supported slabs} \\ \dfrac{a}{40} & \text{for slab continuous from one side} \\ \dfrac{a}{45} & \text{for slab continuous from two sides} \end{cases} \quad \text{............... (3.17)}$$

where *a* is the short direction.

3.4.2 Control of Deflection by Limiting its Value

Calculations of deflections are carried out in the following cases:

- Values of span-to-thickness ratios given in Table 3-1 are not satisfied.
- Span of the beam is more than 10 ms or length of the cantilever is greater than 2 ms.
- The member is subjected to heavy or uneven loads or located in abnormal type of building.

3.4.2.1 Calculation of Immediate Deflection

Deflection of reinforced concrete members can be calculated using the simple structural analysis expressions. Examples of these expressions are given in Eq. 3.18, the rest is given in appendix A. It interesting to note that the deflection of a uniformly loaded simple beam is five times the deflection of a uniformly loaded beam with fixed ends.

$$\Delta = \begin{cases} \dfrac{w\ L^4}{384\,E_c I_e} & \text{for fixed end beam with uniform load } (w) \\ \dfrac{5w\ L^4}{384\,E_c I_e} & \text{for simple beam with uniform load } (w) \\ \dfrac{P\ L^3}{48\,E_c I_e} & \text{for simple beam with point load at midspan} \\ \dfrac{w\ L^4}{8\,E_c I_e} & \text{for cantilever beam with uniform load } (w) \\ \dfrac{P\ L^3}{3\,E_c I_e} & \text{for cantilever beam with point load at edg } (P) \end{cases} \quad \text{...... (3.18)}$$

Where I_e and L are the effective moment of inertia and the beam span, respectively. E_c is the concrete modulus of elasticity and is given by :

$$E_c = 4400\sqrt{f_{cu}} \quad \text{.......................................} (3.19)$$

The total immediate delectation Δ_i due to the existence of dead and live loads equals to:

$$\Delta_i = \Delta_{DL} + \Delta_{LL} \quad \text{..} (3.20)$$

where Δ_{DL} is the deflection due to dead loads including the own weight of the member and the weight of the finishes and Δ_{LL} is the deflection due live loads.

3.4.2.2 Long Term Deflection

Due to the combined effect of creep and shrinkage, the deflection increases with time. The factors affecting long-term deflection include humidity, temperature, curing conditions, ratio of stress to strength, the age of concrete at the time of loading and compression steel content. If the concrete is loaded at an early age, its long-term deflection will be increased. The creep deflection after about five years can range two-three times the initial deflection. It should be noted that more than 90% of the long-term deflection occurs at the first five years after the initial loading.

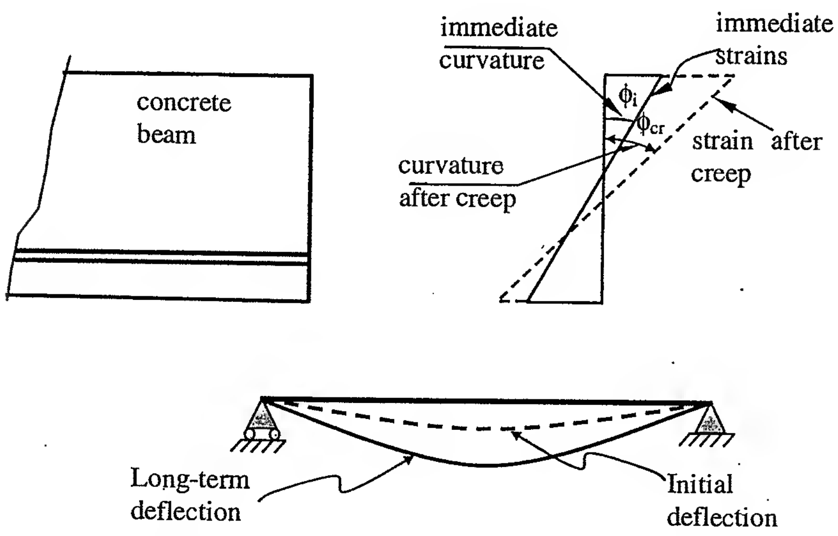


Fig. 3.9 Effect of creep on deflections, curvature and strains.

The addition of compression steel reinforcement reduces the long-term deflection significantly. Figure 3.10 presents experimental deflection versus time for beams with and without compression reinforcement. The additional deflection with time is 195% of the initial deflection for beams without compression reinforcement (A'_s=0), while it is only 100% of the initial deflection for beams with compression steel equals to the tension reinforcement (A'_s=A_s).

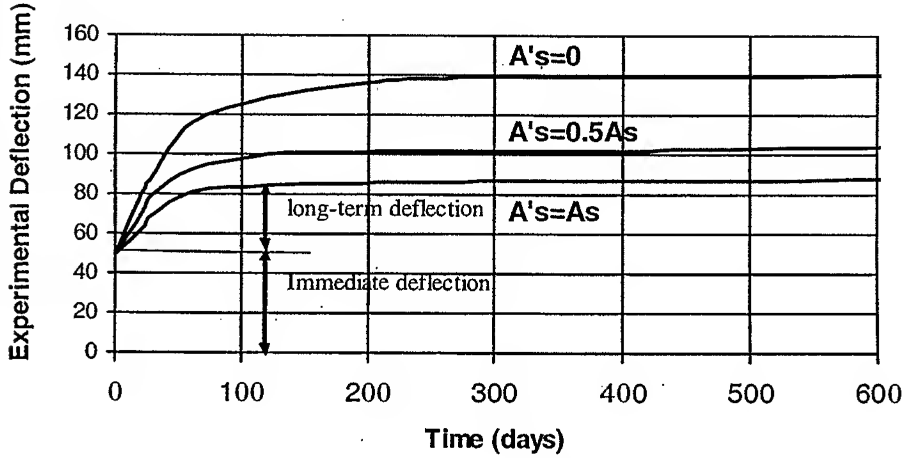


Fig. 3.10 Effect of compression reinforcement on long-term deflection

Based on experimental results, the ECP 203 specifies that additional long-term deflection due to creep Δ_{creep} is calculated by multiplying the dead load deflection Δ_{DL} by the factor α. For a singly reinforced section this factor is equal to 2. The reduction factor α for sections with compression steel can be computed from the following relation:

$$\alpha = 2 - 1.2\left(\frac{A'_s}{A_s}\right) \geq 0.6 \quad \text{............................} (3.21)$$

$$\Delta_{creep} = \alpha\, \Delta_{DL} \quad \text{...} (3.22)$$

Thus the long-term deflection including the effect of creep equals:

$$\Delta_{total} = \Delta_{creep} + \Delta_{DL} + \Delta_{LL} \quad \text{........................} (3.23)$$

$$\Delta_{total} = (1+\alpha)\,\Delta_{DL} + \Delta_{LL} \quad \text{.........................} \quad (3.24)$$

3.4.2.3 Permissible Deflections

As mentioned before, deflections of roofs and floors may cause cracking of brick walls and malfunction of doors and windows. Moreover, deflection due to accumulated water on the roof may cause additional deflections allowing it to hold more water. The ECP 203 imposes the following deflection limits:

- The total deflection of members in ordinary buildings under the effect of all loads including the effect of temperature, shrinkage and creep, measured from the support level should be limited as follows:

1- For beams, one-way labs and two-at-slabs:

$$\Delta_{total} \leq \frac{L}{250} \quad \text{...} \quad (3.25a)$$

2- For cantilevers:

$$\Delta_{total} \leq \frac{L}{450} \quad \text{...} \quad (3.25b)$$

where L is the distance between the inflection points for beams or slabs and is the cantilever length (See Fig. 3.1). The value of L is based on the short span for one-way and two-way slabs, and based on the long span for flat slabs.

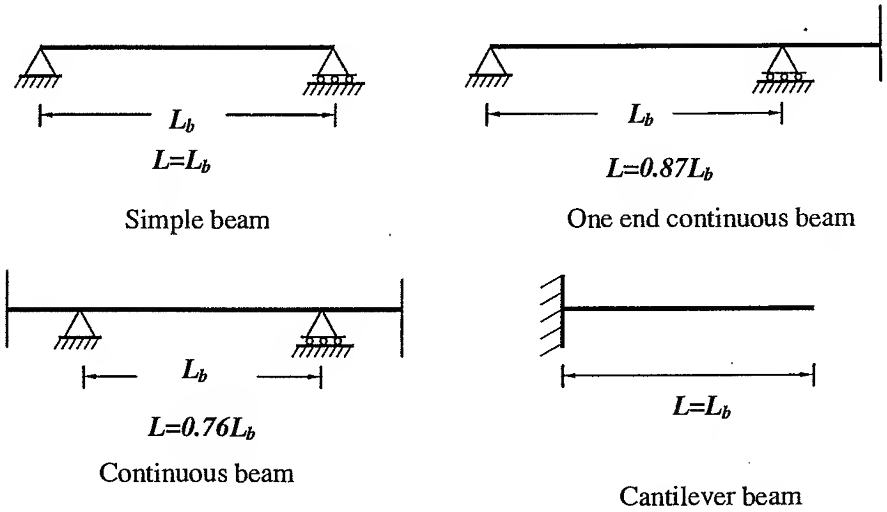


Fig. 3.11 Definition of *L* in deflection calculations

- The ECP 203 requires that the immediate deflections due to live loads only for beams and slabs supporting or attached to non-structural elements not likely to be damaged by large deflections, to be limited to:

$$\Delta_{LL} \leq \frac{L}{360} \quad \text{...} \quad (3.26)$$

- The ECP-203 requires that for beams and slabs carrying non-structural elements that are likely to be affected by deflection such as curtain walls, the part of the total deflection that occurs after the execution of the floor finishes and partitions and that results from all loads including the effect of temperature, shrinkage and creep to be limited to:

$$\Delta_p = \Delta_{LL} + \alpha\,\Delta_{sus} \leq \frac{L}{480} \quad \text{..............................} \quad (3.27)$$

where

Δ_{LL} = instantaneous deflection due to live loads (not likely to be sustained)

$\alpha\,\Delta_{sus}$ = long term deflection (creep + shrinkage) due to all dead loads applied after the installation of partitions including any sustained (permanent) live loads

Table 3.2 and Fig. 3.12 summarize the previous rules.

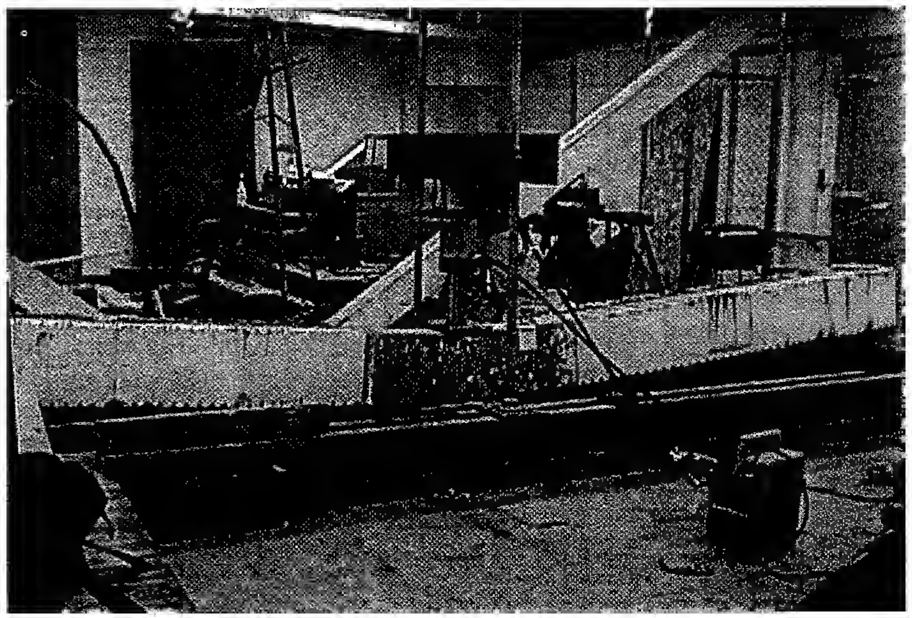


Photo 3.3 Beam deflection during testing

Table 3.2 Maximum permissible deflections

Type of member	Deflection to be considered	Deflection limit
Beams and slabs in ordinary buildings	Total deflection (measured from the level of the support) under the effect of all loads including the effect of temperature, shrinkage and creep	$L/250$ for beams & slabs $L/450$ for cantilevers
Beams and slabs supporting or attached to non-structural elements **not likely** to be damaged by large deflections	Immediate deflection due to live loads	$L/360$
Beams and slabs supporting or attached to non-structural elements **likely** to be damaged by large deflections	Immediate deflection due to live loads plus long-term deflection due to all additional loads *(applied after construction of non-structural elements)* including flooring and partitions	$L/480$

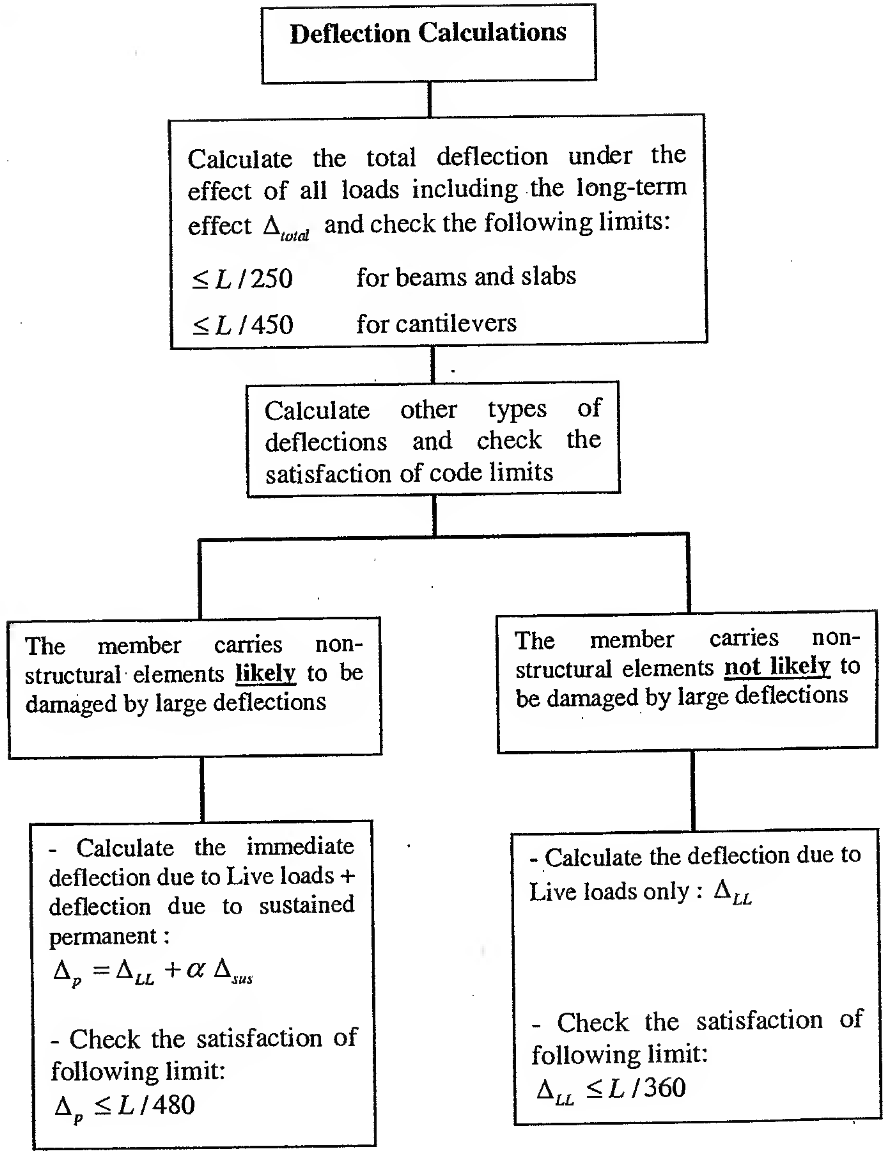


Fig. 3.12 Deflection Calculations

3.4.2.4 Deflection of Continuous Beams

For continuous spans, the ECP 203 calls for a simple average value for the effective moment of inertia obtained from Eq. 3.10 as follows:

$$I'_e = 0.50\, I_{em} + 0.25 \times (I_{e1} + I_{e2}) \quad \text{......................} (3.28)$$

where I'_e is the average effective moment of inertia, I_{em} is the effective moment of inertia at mid-span and I_{e1} and I_{e2} are the values of the effective moment of inertia calculated at the negative moment sections. Figure 3.13 shows the application of Eq. 3.28 for the calculation of the average effective moment of inertia for an interior span of a continuous beam. The value of the effective moment inertia at mid-span I_{em} is calculated form Eq. 3.10 using the maximum moment M_{am}. On the other hand, the values of I_{e1} and I_{e2} are calculated from Eq. 3.10 suing the maximum negative moments M_{a1} and M_{a2}.

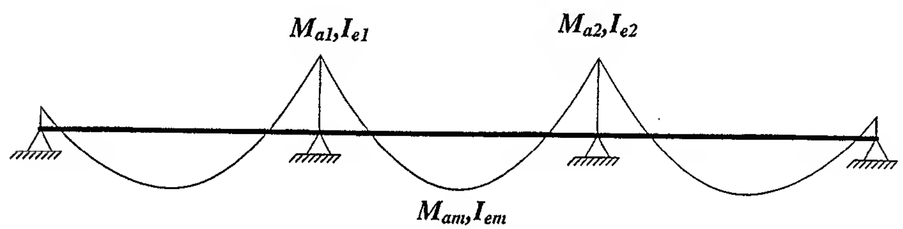


Fig. 3.13 Calculation of the effective moment of inertia for continuous beams

For continuous beams in which the exterior support does not prevent any rotation (brick wall), the effective moment of inertia can be approximated by

$$I'_e = 0.75\, I_{em} + 0.25 \times I_{e1} \quad \text{..................................} (3.29)$$

Where I_{em} is the effective moment of inertia at mid-span and I_{e1} is the value of the effective moment of inertia calculated at the first interior support.

To determine the effect of continuity on the deflection at mid-span, it is easier to express the deflection equation in terms of moment. For example, for a uniformly loaded simply supported beam the deflection can be expressed as:

$$\Delta = \frac{5 \times L^2}{48\, E_c\, I'_e} M_o \quad \text{......................................} (3.30)$$

For a beam with concentrated negative moment M_1 at beam end the deflection equals

$$\Delta_1 = \frac{3 \times L^2}{48\, E_c\, I'_e} M_1 \quad \text{...} (3.31)$$

Referring to Fig. 3.13 and by using the principle of super-position, one can concluded that the mid span deflection Δ for a continuous beam is

$$\Delta = \frac{5 \times L^2}{48 \times E_c\, I'_e} [M_m - 0.1 \times (M_1 + M_2)] \quad \text{.................} (3.32)$$

where M_1, M_m and M_2 are the bending moments at end 1, midspan, and end 2 respectively. To calculate the dead load deflection for example, one should use the dead load moment $M_{m,DL}$ at midspan and at the two ends ($M_{1,DL}$ and $M_{2,DL}$).

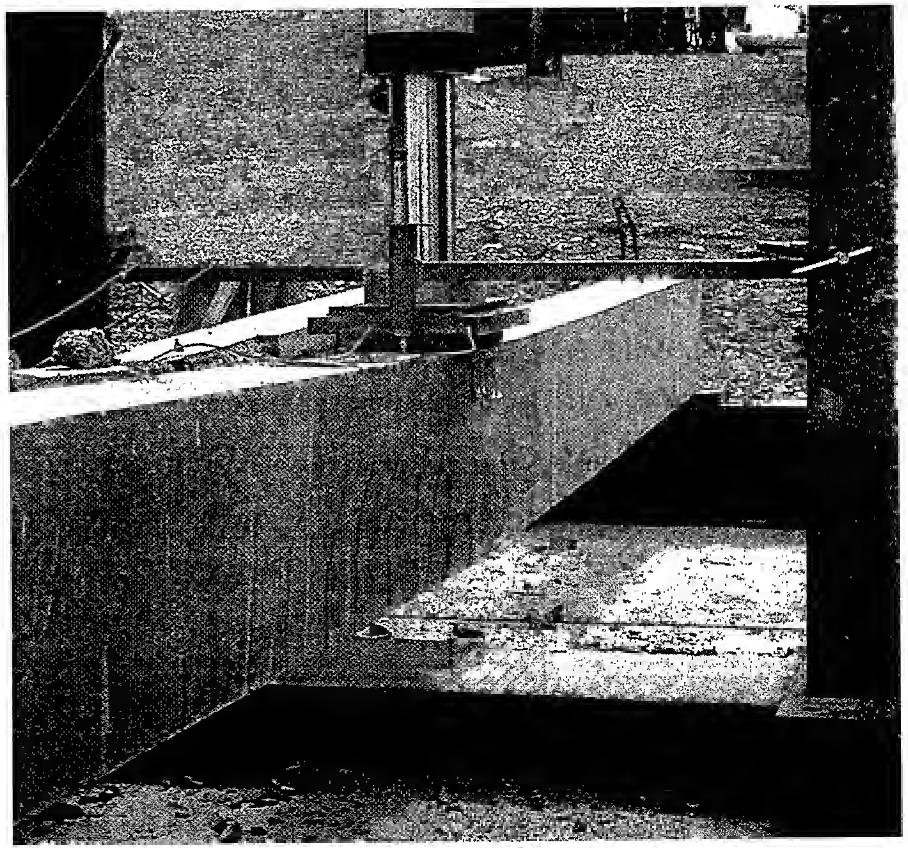


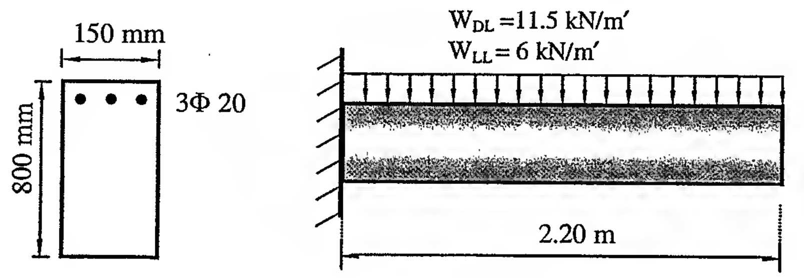
Photo 3.4 Deflection of a simply supported beam during testing

Example 3.1

The cantilever beam shown in the figure below carries an unfactored dead load of 11.5 kN/m′ and an unfactored live load of 6 kN/m′. The beam is located at a typical floor and supports walls that are not likely to be damaged by deflection. It is required to calculate the immediate and the long-term deflections. Does the beam meet ECP 203 requirements for deflections?

f_{cu} = 35 N/mm^2

n = 10



Beam Section   Beam Elevation

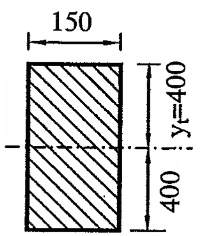
Solution

Step 1: Calculate the uncracked section properties

Neglect the reinforcement steel in calculating the gross moment of inertia.

$y_t = 400$ mm

$$I_g = \frac{b \times t^3}{12} = \frac{150 \times 800^3}{12} = 6.4 \times 10^9 \quad mm^4$$



Uncracked section

Step 2: Calculate the cracking moment and the applied moment

$$f_{ctr} = 0.6\sqrt{f_{cu}} = 0.6\sqrt{35} = 3.55 \ N/mm^2$$

$$M_{cr} = \frac{f_{ctr} \cdot I_g}{y_t} = \frac{3.55 \times 6.4 \times 10^9}{400} \times \frac{1}{10^6} = 56.79 \ kN.m$$

$$w_{total} = 11.5 + 6 = 17.5 \ kN/m'$$

The maximum negative moment in the cantilever is at the support

$$M_a = \frac{w_{total} \ L^2}{2} = \frac{17.5 \times 2.2^2}{2} = 42.35 \ kN.m \ldots < \ M_{cr} \ (not\ cracked)$$

Since $M_a < M_{cr}$ then $I_e = I_g$

Step 3: Calculate the deflection

Step 3.1: Calculate the immediate deflection

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{35} = 26030 \ N/mm^2$$

From the appendix, the maximum deflection for a cantilever beam carrying uniform load equals to:

$$\Delta = \frac{w \ L^4}{8 \ E_c \ I_e}$$

Recalling that 1 kN/m′=1 N/mm, then w_{DL}=11.5 kN/m′=11.5 N/mm′

$$\Delta_{DL} = \frac{w_{DL} \ L^4}{8 \ E_c \ I_e} = \frac{11.5 \times (2.2 \times 1000)^4}{8 \times 26030 \times 6.4 \times 10^9} = 0.20 \ mm$$

$$\Delta_{LL} = \frac{w_{LL} \ L^4}{8 \ E_c \ I_e} = \frac{6 \times (2.2 \times 1000)^4}{8 \times 26030 \times 6.4 \times 10^9} = 0.11 \ mm$$

The total immediate deflection Δ_i equals:

$$\Delta_i = \Delta_{DL} + \Delta_{LL} = 0.2 + 0.11 = 0.31 \ mm$$

Step 3.2: Calculate the long-term deflection

The total deflection due to all loads including the effect of creep equals

$$\Delta_{total} = (1+\alpha)\,\Delta_{DL} + \Delta_{LL}$$

Since A'_s=0 then α=2

$$\Delta_{total} = (1+2)\,0.2 + 0.11 = 0.71\,mm$$

Step 4: Check the code requirements

- The code maximum limit for cantilever beams = 2200/450 = 4.88 mm

Since Δ_{total} (0.71 mm) < $\Delta_{allowable}$ (4.88 mm), the code limit is satisfied.

- Since the beam is located at a floor and support walls that are not likely to be damaged by deflection, then

$$\Delta_{LL\,(allowable)} \le \frac{L}{360} \le \frac{2200}{360} = 6.11\,mm$$

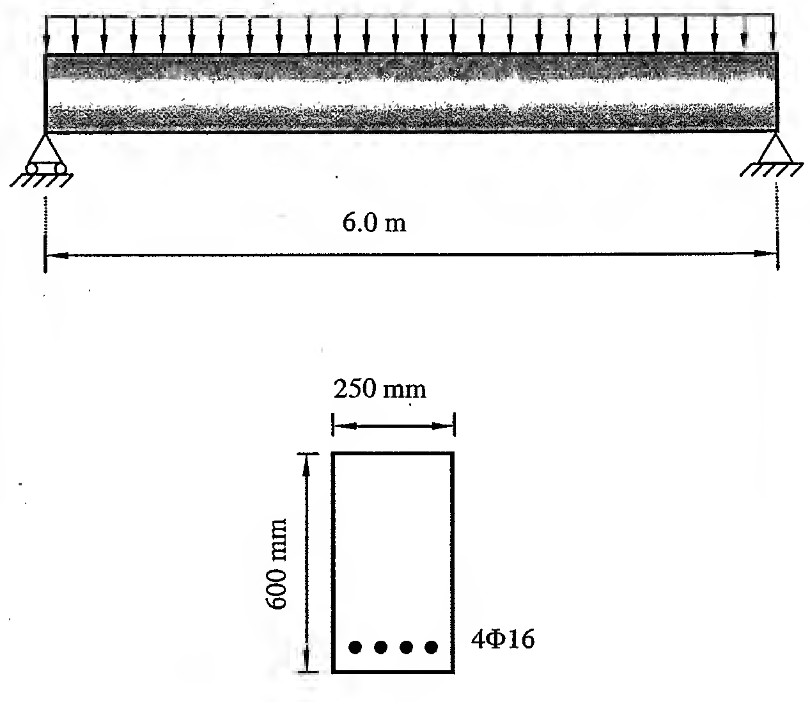
Since Δ_{LL} (0.11 mm) < $\Delta_{LL(allowable)}$ (6.11 mm), the code limit is satisfied.

Example 3.2

The simple beam shown in the figure below is located at a roof f a building and it does not support any partitions. The unfactored dead load is (including own weight) 15.0 kN/m′, and the unfactored live load is 9.0 kN/m′ . Check whether the beam meets the ECP 203 requirements for deflections.

f_{cu} = 25 N/mm^2

n = 10



Beam Section

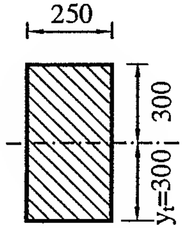
Solution

Step 1: Calculate uncracked section properties

Neglect the tension steel in calculating gross moment of inertia.

y_t=300 mm

$$I_g = \frac{b \times t^3}{12} = \frac{250 \times 600^3}{12} = 4.5 \times 10^9 \quad mm^4$$



Uncracked section

Step 2: Calculate the cracking moment and the applied moment

$$f_{ctr} = 0.6\sqrt{f_{cu}} = 0.6\sqrt{25} = 3 \ N/mm^2$$

$$M_{cr} = \frac{f_{ctr} . I_g}{y_t} = \frac{3 \times 4.5 \times 10^9}{300} \times \frac{1}{10^6} = 45 \ kN.m$$

$$w_{total} = 15 + 9 = 24 \ kN/m'$$

The maximum bending moment M_a is at mid span equals:

$$M_a = \frac{w_{total} \ L^2}{8} = \frac{24 \times 6^2}{8} = 108 \ kN.m \ldots > M_{cr} \ \text{(cracked section analysis)}$$

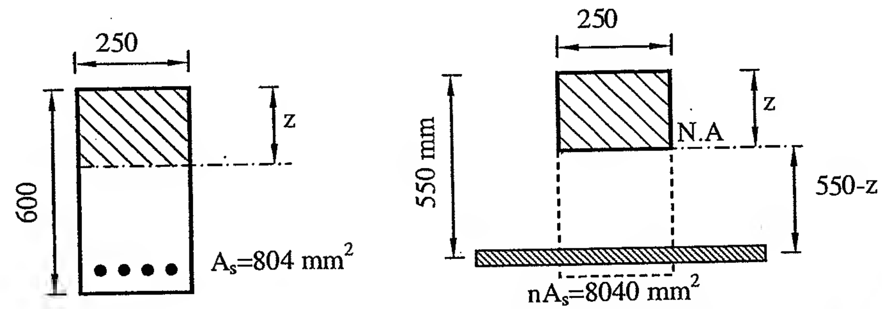
Since $M_a > M_{cr}$ then calculate I_e

Step 3: Calculate the cracked section properties

$A_s = 4\Phi\ 16 = 804\ mm^2$

Assume the neutral axis is located at a distance z from the compression force. Transform the area of steel reinforcement into an equivalent area of concrete ($n\,A_s$).

n A_s=10 x 804 = 8040 mm^2



Original section

Transformed section

Taking the first moment of area for the transformed section about the N.A., gives:

$$250 \times z \times \frac{z}{2} = 8040\,(550 - z)$$

$$125\ z^2 + 8040\ z - 4422000 = 0$$

z =158.68 mm

Calculate cracked moment of inertia I_{cr}

$$I_{cr} = \frac{b\ z^3}{3} + n\ A_s (d - z)^2$$

$$I_{cr} = \frac{250 \times 158.68^3}{3} + 10 \times 804 \times (550 - 158.68)^2 = 1.56 \times 10^9 \ mm^4$$

Step 4: Calculate the effective moment of inertia

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{25} = 22000 \ N/mm^2$$

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3$$

$$I_e = 1.56 \times 10^9 + (4.5 \times 10^9 - 1.56 \times 10^9)\left(\frac{45}{108}\right)^3 = 1.77 \times 10^9 \ mm$$

Step 5: Calculate the deflection

Step 5.1: Calculate the immediate deflection

W_{DL} = 15 kN/m′ =15 N/mm′

W_{LL} = 9 kN/m′ = 9 N/mm′

The maximum dead load deflection for simple beam at mid span equals

$$\Delta_{DL} = \frac{5\, w_{DL}\, L^4}{384\, E_c\, I_e} = \frac{5 \times 15 \times (6 \times 1000)^4}{384 \times 22000 \times 1.77 \times 10^9} = 6.5\ mm$$

Since the relation between deflection and load is linear, we can determine the deflection of other loads simply by using ratios of the applied loads as follows:

$$\Delta_{LL} = \Delta_{DL} \times \frac{w_{LL}}{w_{DL}} = 6.5 \times \frac{9}{15} = 3.9\ mm$$

$$\Delta_i = \Delta_{DL} + \Delta_{LL} = 6.5 + 3.9 = 10.4\ mm$$

Step 5.2: Calculate the long-term deflection

The total deflection due to all loads including the effect of creep equals:

$$\Delta_{total} = (1+\alpha)\,\Delta_{DL} + \Delta_{LL}$$

since A'_s=0 then α=2

$$\Delta_{total} = (1+2)\,6.5 + 3.9 = 23.4\ mm$$

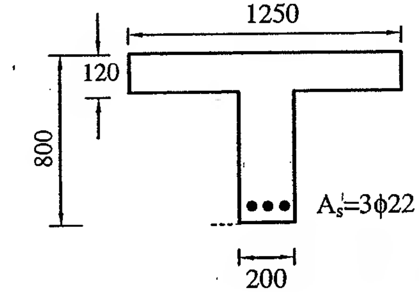
Step 6: Check the code requirements

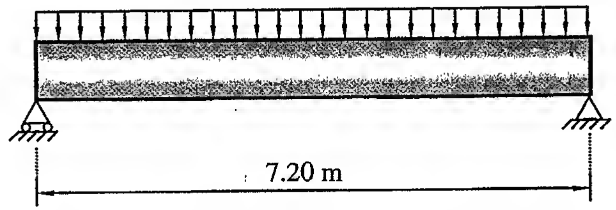
1. The code maximum limit for simple beams = 6000/250 = 24 mm, since Δ_{total} (23.4 mm) < $\Delta_{allowable}$ (24 mm), the code limit is satisfied.

2. Since the beam does not support partitions, no additional checks are needed.

Example 3.3

The T-beam shown in figure is subjected to an unfactored dead load of 20 kN/m′ and an unfactored live load of 8 kN/m′. The beam supports partitions that are sensitive to deflection. Calculate the immediate deflection and check ECP 203 requirements, knowing that 30% of the live loads are permanent loads. The concrete strength is 20 N/mm².

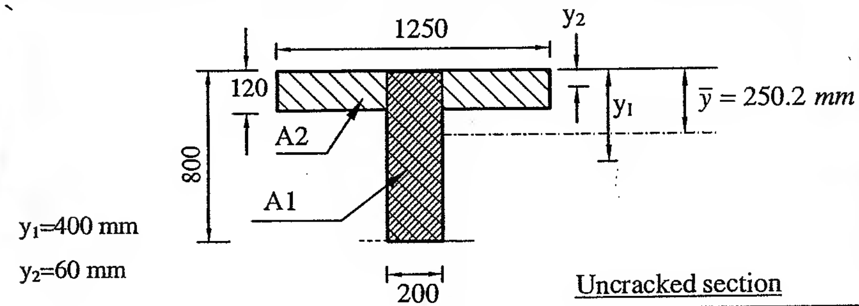




Solution

Step 1: Calculate the uncracked section properties

Neglect the tension steel in calculating the gross moment of inertia.
Since the section is not symmetrical, calculate the center of gravity (c.g.).



Uncracked section

$$A_1 = 200 \times 800 = 160000\ mm^2$$

$$A_2 = (1250 - 200) \times 120 = 126000\ mm^2$$

$$\bar{y} = \frac{160000 \times 400 + 126000 \times 60}{160000 + 126000} = 250.2\ mm$$

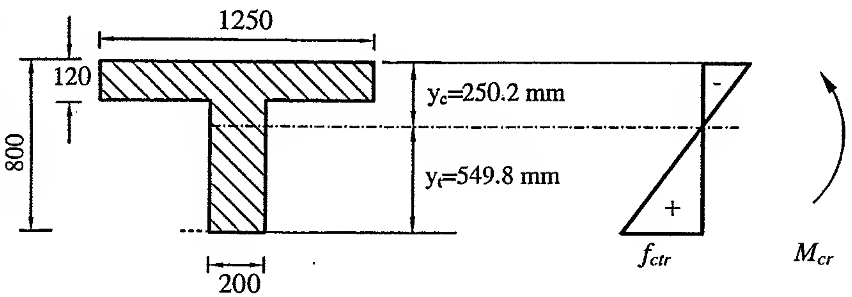
$$I_g = \frac{200 \times 800^3}{12} + 160000 \times (400 - 250.2)^2 + \frac{(1250-200) \times 120^3}{12} + 126000 \times (250.2 - 60)^2$$

$I_g = 16.83 \times 10^9\ \text{mm}^4$

Step 2: Calculate the cracking moment and the applied moment

$$f_{ctr} = 0.6\sqrt{f_{cu}} = 0.6\sqrt{20} = 2.683\ N/mm^2$$

$$y_t = 800 - \bar{y} = 800 - 250.2 = 549.8\ mm$$



$$M_{cr} = \frac{f_{ctr} \cdot I_g}{y_t} = \frac{2.683 \times 16.83 \times 10^9}{549.8} \times \frac{1}{10^6} = 82.15\ kN.m$$

$$w_{total} = 20 + 8 = 28\ kN/m'$$

$$M_a = \frac{w_{total}\ L^2}{8} = \frac{28 \times 7.2^2}{8} = 181.44\ kN.m$$

Since $M_a > M_{cr}$ then calculate I_e

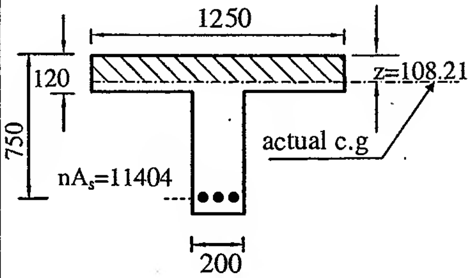
Step 3: Calculate the cracked section properties

$A_s = 3\Phi\ 22 = 1140.4\ \text{mm}^2$

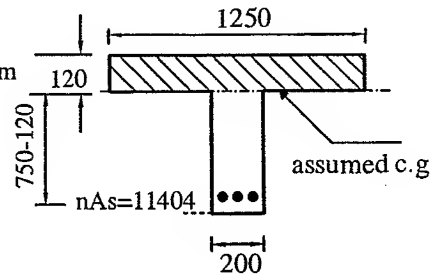
Assume that the neutral axis is located at a distance z from the compression force. Transforming the steel reinforcement into equivalent area of concrete gives:

$nA_s = 10 \times 1140.4 = 11404\ \text{mm}^2$

Assuming concrete cover of 50 mm, d=800-50 =750 mm



Exact calculation for the c.g.



Quick estimate for the c.g.

To quickly determine whether or not the c.g. is inside the flange, calculate the first moment of area at the end of the slab.

$$1250 \times 120 \times \frac{120}{2} > 11404 \times (750 - 120)$$

Hence, assume the c.g. inside the flange. Taking the first moment of area for the transformed section about the c.g.

$$1250 \times z \times \frac{z}{2} = 11404\ (750 - z)$$

$$625\ z^2 + 11404\ z - 8553000 = 0$$

z =108.21 mm < 120 mm *(inside the flange as assumed)*

Calculating cracked moment of inertia I_{cr}

$$I_{cr} = \frac{B\,z^3}{3} + n\,A_s (d - z)^2$$

$$I_{cr} = \frac{1250 \times 108.2^3}{3} + 11404 \times (750 - 108.2)^2 = 5.22 \times 10^9 \; mm^4$$

Step 4: Calculate the effective moment of inertia

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{20} = 19677 \; N/mm^2$$

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3$$

$$I_e = 5.22 \times 10^9 + (16.83 \times 10^9 - 5.22 \times 10^9)\left(\frac{82.15}{181.44}\right)^3 = 6.3 \times 10^9 \; mm$$

Step 5: Calculate deflection

Step 5.1: Calculate immediate deflection

W_{DL} = 20 kN/m′ = 20 N/mm′ $\qquad$ W_{LL} = 8 kN/m′ = 8 N/mm′

The dead load deflection for simple beam equals:

$$\Delta_{DL} = \frac{5\,w_{DL}\,L^4}{384\,E_c\,I_e} = \frac{5}{384} \times \frac{20 \times (7.2 \times 1000)^4}{19677 \times 6.3 \times 10^9} = 5.64 \; mm$$

Since the relation between deflection and load is linear, we can determine the deflection of other loads simply by using ratios as follows:

$$\Delta_{LL} = \Delta_{DL} \times \frac{w_{LL}}{w_{DL}} = 5.64 \times \frac{8}{20} = 2.26 \; mm$$

Thus the immediate deflection equals:

$$\Delta_i = \Delta_{DL} + \Delta_{LL} = 5.64 + 2.26 = 7.90 \; mm$$

Step 5.2: Calculate long-term deflection

The total deflection due to all loads including the effect of creep equals

$$\Delta_{total} = (1 + \alpha)\,\Delta_{DL} + \Delta_{LL}$$

Since $A'_s = 0$ then $\alpha = 2$

$$\Delta_{total} = (1 + \alpha)\Delta_{DL} + \Delta_{LL} = (1 + 2) \times 5.64 + 2.26 = 19.19 \; mm$$

Step 5: Check the code requirements

- The code maximum limit for simple beams = 7200/250 = 28.8 mm. Since Δ_{total} (19.19 mm) < $\Delta_{allowable}$ (28.8 mm), the code limit is satisfied.
- Since the beam is attached to partitions that are likely to be damaged by large deflections, the code also requires that:

$$\Delta_{LL} + \alpha\,\Delta_{sus} \le \frac{L}{480} \le \frac{7200}{480} \le 15 \; mm$$

$$\Delta_{sus} = \Delta_{DL} + 0.3 \times \Delta_{LL}$$

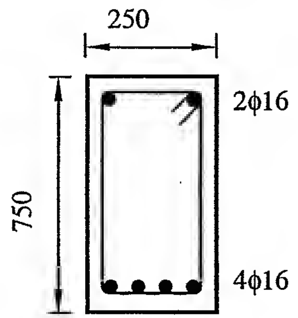
Where Δ_{sus} is the deflection due to all dead loads applied after the installation of all partitions plus the permanent live loads (given as 30%).

$$\Delta_{sus} = 5.64 + 0.3 \times 2.26 = 6.32 \; mm$$

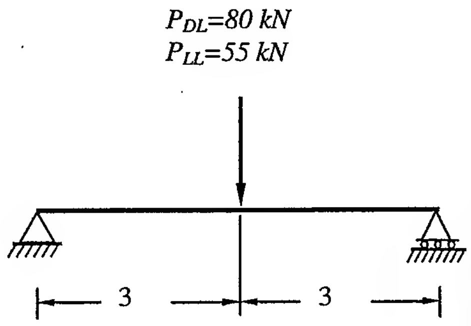
$$2.26 + 2 \times 6.32 = 14.9 \; mm < 15 \; mm \; .....o.k$$

Example 3.4

The floor beam shown in figure is subjected to an unfactored concentrated dead load of 80 kN, and an unfactored concentrated live load of 55 kN (the own weight of the beam may be neglected). The beam supports glass partitions that are likely to be damaged by large deflections. Check the satisfaction of ECP 203 limits for deflection. The concrete compressive strength is f_{cu}=40 N/mm^2



Beam Section



I_g = 8.8 x 10^9 mm^4

I_{cr} = 2.72 x 10^9 mm^4

Solution

Step 1: Calculate the cracking moment and the applied moment

$$f_{ctr} = 0.6\sqrt{f_{cu}} = 0.6\sqrt{40} = 3.79\ N/mm^2$$

$$M_{cr} = \frac{f_{ctr} \cdot I_g}{y_t} = \frac{3.79 \times 8.8 \times 10^9}{375} \times \frac{1}{10^6} = 88.94\ kN.m$$

$$P_{total} = 80 + 55 = 135\ kN$$

$$M_a = \frac{P_{total}\ L}{4} = \frac{135 \times 6}{4} = 202.5\ kN.m$$

Since $M_a > M_{cr}$, then calculate I_e

Step 2: Calculate the effective moment of inertia

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{40} = 27828 N/mm^2$$

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3$$

$$I_e = 2.72\times10^9 + \left(8.8\times10^9 - 2.72\times10^9\right)\left(\frac{88.94}{202.50}\right)^3$$

$$I_e = 3234016475 = 3.23\times10^9\ mm^4$$

Step 3: Calculate the deflection

Step 3.1: Calculate the immediate deflection

The deflection for a simple beam carrying a concentrated load equals $\frac{P\ L^3}{48\ E_c\ I_e}$

$$\Delta_{DL} = \frac{(80\times1000)\times(6000)^3}{48\times 27828\times3.23\times10^9} = 4.0\ mm$$

Since the relation between deflection and load is linear, we can determine the deflection of other loads simply by using ratios as follows:

$$\Delta_{LL} = \Delta_{DL} \times \frac{P_{LL}}{P_{DL}} = 4\times\frac{55}{80} = 2.75\ mm$$

Thus the immediate deflection equals:

$$\Delta_i = \Delta_{DL} + \Delta_{LL} = 4.0 + 2.75 = 6.75\ mm$$

Step 3.2: Calculate the long-term deflection

$$\frac{A_s'}{A_s} = \frac{2\phi16}{4\phi16} = 0.5 \rightarrow \alpha = 2 - 1.2\left(\frac{A_s'}{A_s}\right) = 2 - 1.2\,(0.5) = 1.4 \geq 0.6 \cdots o.k.$$

The total deflection due to all loads including the effect of creep equals:

$$\Delta_{total} = (1+\alpha)\,\Delta_{DL} + \Delta_{LL}$$

$$\Delta_{total} = (1+1.4)\ 4.0 + 2.75 = 12.35\ \ mm$$

Step 4: Check the code requirements

- The code maximum limit for simple beams = 6000/250 = 24.0 mm. Since Δ_{total} (12.35 mm) < $\Delta_{allowable}$ (24.0 mm), the code limit is satisfied.
- Since the beam is attached to partitions that are likely to be damaged by large deflections, the code also requires that:

$$\Delta_{LL} + \alpha\ \Delta_{sus} \leq \frac{L}{480} \leq \frac{6000}{480} \leq 12.5\ mm$$

$$\Delta_{sus} = \Delta_{DL}$$

$$\Delta_{sus} = 4.0\ mm$$

$$2.75 + 1.4 \times 4.0 = 8.35\,mm < 12.5\ mm\ .....o.k$$

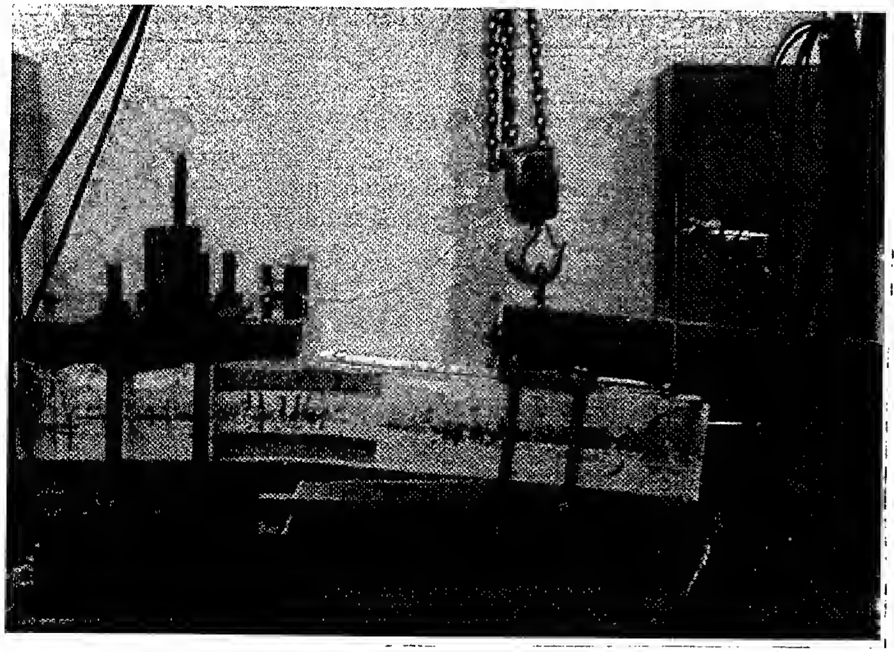
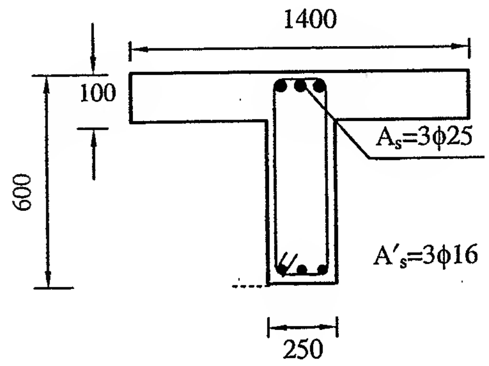
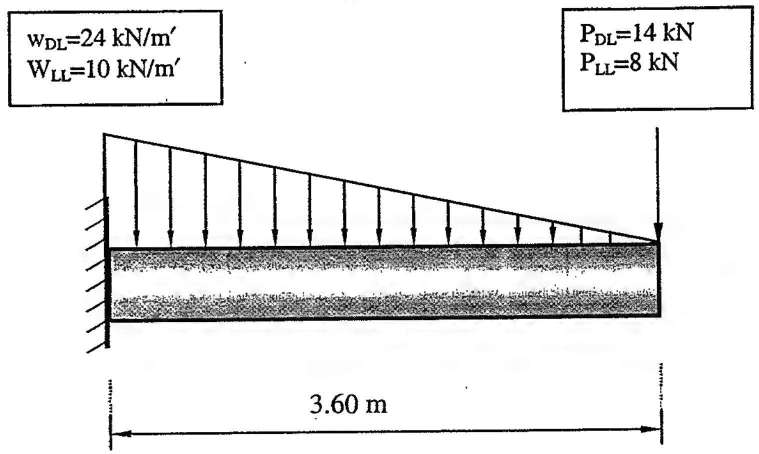


Photo 3.5 Cantilever beam during testing

Example 3.5

The T-beam shown in figure is a part of a roof and it supports a triangular load. The beam supports partitions that are not likely to be damaged by deflections. Does the beam shown in the figure below satisfy the ECP 203 requirements for deflections? The concrete strength is 25 N/mm^2. Assume of n=10. All the given loads are unfactored

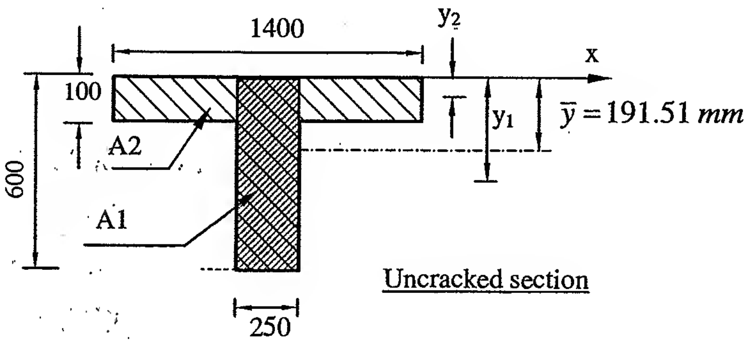




Solution

Step 1: Calculate the uncracked section properties

Since the section is not symmetrical, calculate the c.g.



Uncracked section

$A_1 = 250 \times 600 = 150000\ mm^2$ $\quad$ y_1=300 mm

$A_2 = (1400 - 250) \times 100 = 115000\ mm^2$ $\quad$ y_2=50 mm

Taking the first moment of area about x-axis:

$$\bar{y} = \frac{150000 \times 300 + 115000 \times 50}{150000 + 115000} = 191.51\ mm$$

$$I_g = \frac{250 \times 600^3}{12} + 150000 \times (300 - 191.51)^2 + \frac{(1400-250) \times 100^3}{12} + 115000 \times (191.51 - 50)^2$$

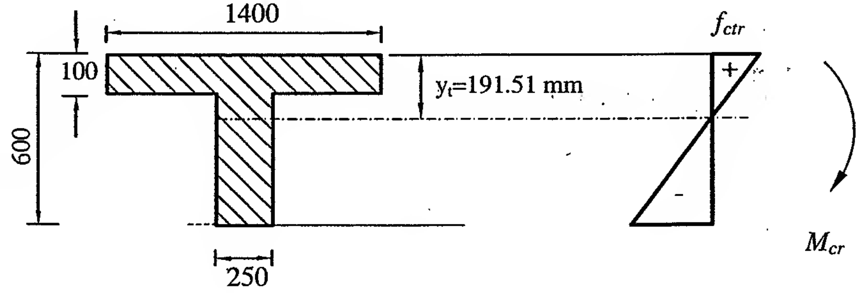
I_g=8.66 x10^9 mm^4

Step 2: Calculate the cracking moment and the applied moment

$$f_{ctr} = 0.6\sqrt{f_{cu}} = 0.6\sqrt{25} = 3\ N/mm^2$$

Noticing that the tension side for the cantilever is at the top flange, then;

$$y_t = \bar{y} = 191.51\ mm$$



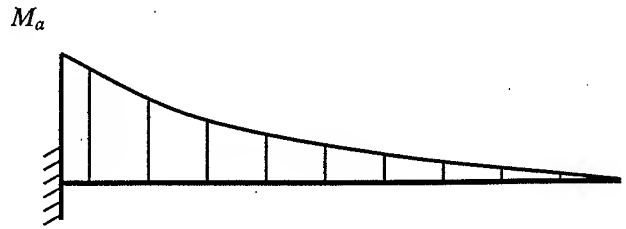
$$M_{cr} = \frac{f_{ctr} . I_g}{y_t} = \frac{3.0 \times 8.66 \times 10^9}{191.51} \times \frac{1}{10^6} = 135.73\ kN.m$$

$$w_{total} = 24 + 10 = 34\ kN/m'$$

$$P_{total} = 14 + 8 = 22\ kN$$

$$M_a = \frac{w_{total}\ L}{2} \times \frac{L}{3} + P \times L = \frac{34 \times 3.6}{2} \times \frac{3.6}{3} + 22 \times 3.6 = 152.64\ \ kN.m$$

Since $M_a > M_{cr}$, then calculate I_e.



Step 3: Calculate the cracked section properties

$A_s = 3\Phi\ 25 = 1472.6$ mm^2

$A'_s = 3\Phi\ 16 = 603.2$ mm^2

Assume that the neutral axis at a distance z from the compression force. The reader should notice that the compression part for cantilever is at the bottom of the section.

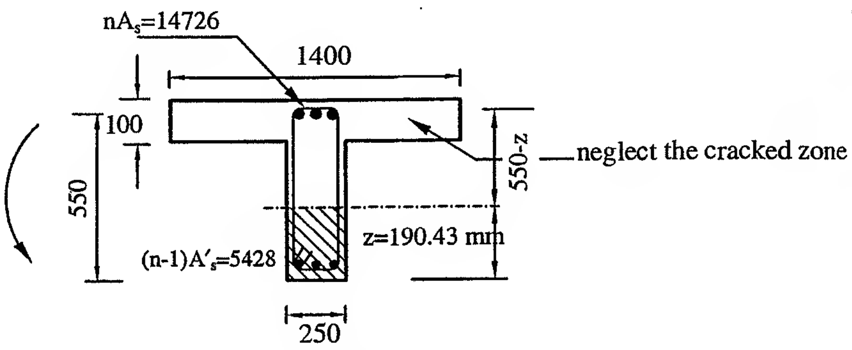
Transforming the steel reinforcement into an equivalent area of concrete, gives:

$nA_s = 10 \times 1472.6 = 14726\ mm^2$

The steel in the compression is transformed by multiplying with (n-1) to account for the concrete area.

$(n-1)\ A'_s = (10-1) \times 603.2 = 5428.8\ mm^2$

Assuming concrete cover of 50 mm, d = 800-50 =750 mm



Calculation of the c.g. for the cracked section

Taking the first moment of area for the transformed section about the c.g

$$250 \times z \times \frac{z}{2} + 5428.8 \times (z-50) = 14726\ (550-z)$$

$$125\ z^2 + 20154\ z - 8370740 = 0$$

z =190.43 mm

Calculate cracked moment of inertia I_{cr}.

$$I_{cr} = \frac{b\ z^3}{3} + (n-1)\ A'_s (z-d')^2 + n\ A_s (d-z)^2$$

$$I_{cr} = \frac{250 \times 190.43^3}{3} + 5428 \times (190.43-50)^2 + 14726 \times (550-190.43)^2$$

$$= 2.58 \times 10^9\ mm^4$$

Step 4: Calculate the effective moment of inertia

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{25} = 22000\ N/mm^2$$

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3$$

$$I_e = 2.58 \times 10^9 + (8.66 \times 10^9 - 2.58 \times 10^9)\left(\frac{135.73}{152.64}\right)^3 = 6.86 \times 10^9\ mm$$

Step 5: Calculations of the deflections

Step 5.1: Immediate deflection

The deflection for this cantilever beam is the sum of the deflection due to the concentrated load and that due to the uniform load. These deflections are given by the following equations:

$$\Delta = \begin{cases} \dfrac{w\ L^4}{30\ E_c I_e} & \text{for cantilever beam with triangular load} \\[2ex] \dfrac{P\ L^3}{3\ E_c I_e} & \text{for cantilever beam with concentrated load at the edge} \end{cases}$$

The dead load deflection equals:

$$\Delta_{DL} = \frac{w_{DL}\ L^4}{30\ E_c\ I_e} + \frac{P_{DL}\ L^3}{3\ E_c\ I_e} = \frac{24 \times (3.6 \times 1000)^4}{30 \times 22000 \times 6.86 \times 10^9} + \frac{(14 \times 1000) \times (3.6 \times 1000)^3}{3 \times 22000 \times 6.86 \times 10^9}$$

$$\Delta_{DL} = 2.33\ mm$$

The live load deflection equals:

$$\Delta_{LL} = \frac{w_{LL} L^4}{30\,E_c\,I_e} + \frac{P_{LL}\,L^3}{3\,E_c\,I_e} = \frac{10 \times (3.6 \times 1000)^4}{30 \times 22000 \times 6.86 \times 10^9} + \frac{(8 \times 1000) \times (3.6 \times 1000)^3}{3 \times 22000 \times 6.86 \times 10^9}$$

$$\Delta_{LL} = 1.20\ mm$$

Step 5.2: Long-term deflection

The total deflection due to all loads including the effect of creep equals

$$\Delta_{total} = (1+\alpha)\,\Delta_{DL} + \Delta_{LL}$$

$$\alpha = 2 - 1.2\left(\frac{A_s'}{A_s}\right) = 2 - 1.2\left(\frac{603}{1472}\right) = 1.51 \geq 0.6 \cdots o.k.$$

$$\Delta_{total} = (1+1.51)\,2.33 + 1.20 = 7.05\ mm$$

Step 6: Check the code requirements

- The code maximum limit is = 3600/450 = 8 mm

Since Δ_{total} (7.05 mm) < $\Delta_{allowable}$ (8.0 mm), the code limit is satisfied.

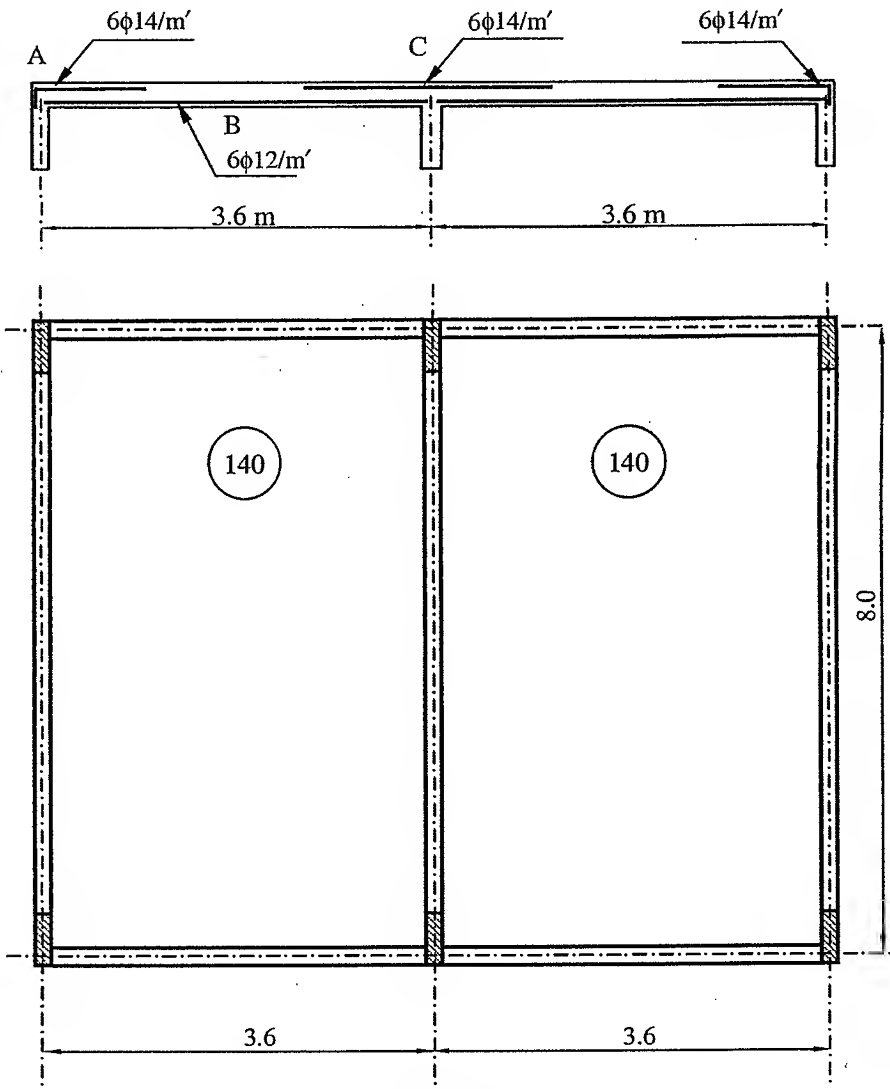
- The beam supports partitions that are not likely to be damaged by deflections, then the deflection limit is given by:

$$\Delta_{LL\,(allowable)} \leq \frac{L}{360} \leq \frac{3600}{360} = 10\ mm$$

Since Δ_{LL} (1.2 mm) < $\Delta_{LL(allowable)}$ (10 mm), the code limit is satisfied.

Example 3.6

The reinforced concrete one-way slab shown in the figure below supports an unfactored dead load of 6 kN/m^2 and an unfactored live load of 3 kN/m^2. Calculate the immediate and long-term deflections at point (B). f_{cu} equals 30 N/mm^2.



Floor plan

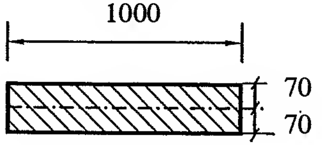
Solution

Step 1: Calculate the uncracked section properties

Taking a 1m slab width of the (b=1000 mm)

y_t=70 mm

$$I_g = \frac{b \times t^3}{12} = \frac{1000 \times 140^3}{12} = 228 \times 10^6 \quad mm^4$$



Uncracked section

Step 2: Calculate the cracking moment

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{30} = 24099.8\ N/mm^2$$

$$f_{ctr} = 0.6\sqrt{f_{cu}} = 0.6\sqrt{30} = 3.29\ N/mm^2$$

$$M_{cr} = \frac{f_{ctr} \cdot I_g}{y_t} = \frac{3.29 \times 228 \times 10^6}{70} \times \frac{1}{10^6} = 10.74\ kN.m$$

The cracking moment is valid for sections A, B and C (refer to the figure).

Step 3: Calculate cracked section properties

Since the slab is continuous, the moment of inertia of any span depends on the average values of the moment of inertia of positive and negative sections

Step 3.1: section B

$A_s = 6\Phi\ 12 = 678.5\ mm^2$

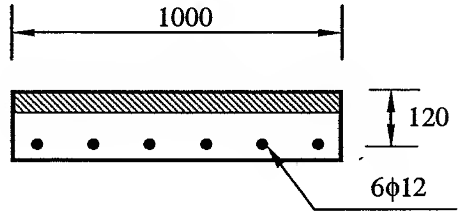
Assume concrete cover of 20 mm $\rightarrow$ d =140-20 =120 mm

Using design aids given in the Appendix

$$\mu = \frac{A_s}{b \times d} = \frac{678.5}{1000 \times 120} = 0.00565$$

From the curve with μ=0.00565, k_{II}=0.0365

$$I_{cr} = K_{II} \times b \times d^3 = 0.0365 \times 1000 \times 120^3 = 63.1 \times 10^6\ mm^4$$



Cracked section at B

Step 3.2: Sections A and C

The positive reinforcement is not developed at supports, hence it will not be considered as compression steel for sections subjected to negative moment at the supports (i.e. $A_s' = 0$).

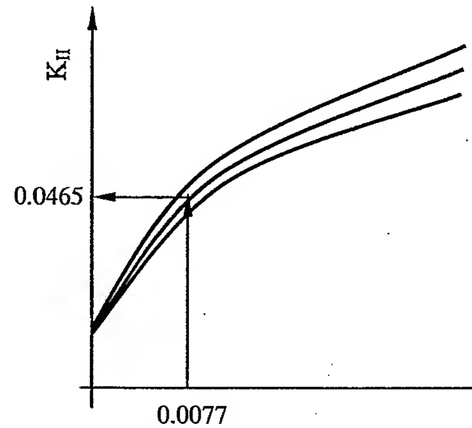
$A_s = 6\Phi\ 14 = 923.6\ mm^2$

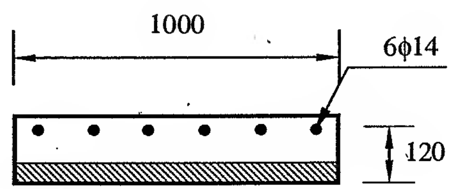
Using design aids given in the Appendix

$$\mu = \frac{A_s}{b \times d} = \frac{923.6}{1000 \times 120} = 0.0077$$

From the curve k_{II}= 0.0465

$$I_{cr} = K_{II} \times b \times d^3 = 0.0465 \times 1000 \times 120^3 = 80.3 \times 10^6\ mm^4$$





cracked section at C

Step 4: Calculate the effective moment of inertia

$$w_{total} = 6 + 3 = 9\ kN/m^2$$

Thus, for a 1m width of the slab, $w_{total} = 9$ kN/m′

Since the slab is continuous with equal spans and loading, the code coefficients for moments in slabs are used as follows:

$$M_a = \frac{w_{total} \times L^2}{k} = \frac{9 \times 3.6^2}{k}$$

The following table summarizes the calculations.

Calculation of the effective moment of inertia

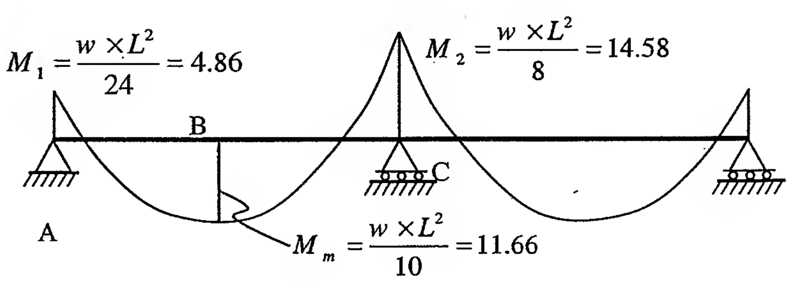
Point	(k)	M_a, kN.m	M_{cr} kN.m	I_{cr}, mm⁴	status	I_e, mm⁴
A	24	4.86	10.74	80.3 x 10^6	uncracked (I_e=I_g)	228.6 x 10^6 (I_{e1})
B	10	11.66	10.74	63.1 x 10^6	cracked	192.2 x 10^6 (I_{em})*
C	8	14.58	10.74	80.3 x 10^6	cracked	139.56 x 10^6 (I_{e2})**

Note the values of the effective moment of inertia (given in the table) for section B and C are calculated as follows:

$$^{*}I_{e,B} = I_{em} = 63.1\times10^6 + \left(228.6\times10^6 - 63.1\times10^6\right)\left(\frac{10.74}{11.66}\right)^3 = 192.2\times10^6\,mm^4$$

$$^{**}I_{e,C} = I_{e2} = 80.3\times10^6 + \left(228.6\times10^6 - 80.3\times10^6\right)\left(\frac{10.74}{14.58}\right)^3 = 139.56\times10^6\,mm^4$$

Since the slab is continuous, the average value of I'_e should be calculated.



$$I'_e = 0.50\, I_{em} + 0.25\times(I_{e1} + I_{e2})$$

$$I'_e = 0.50\times192.2\times10^6 + 0.25\times(228.6\times10^6 + 139.56\times10^6) = 188.1\times10^6\ mm^4$$

Step 5: Calculation of the deflections

W_{DL} = 6 kN/m′=6 N/mm′ & W_{LL} = 3 kN/m′ = 3 N/mm′

The deflection for a uniformly loaded continuous beam equals:

$$\Delta = \frac{5\times L^2}{48\times E_c\, I'_e}\left[M_m - 0.1\times(M_1 + M_2)\right]$$

$$\Delta_{DL} = \frac{5\times3600^2}{48\times24099.8\times188.1\times10^6}\left(\frac{6\times3.6^2}{10} - 0.1\times\left\{\frac{6\times3.6^2}{24} + \frac{6\times3.6^2}{8}\right\}\right)\times10^6$$

$$\Delta_{DL} = 1.93\ mm$$

Since the relation between deflection and load is linear, we can determine the deflection due to live load as a ratio of that due to dead load as follows:

$$\Delta_{LL} = \Delta_{DL}\times\frac{w_{LL}}{w_{DL}} = 1.93\times\frac{3}{6} = 0.96\ mm$$

The immediate deflection Δ_i equals:

$$\Delta_i = \Delta_{DL} + \Delta_{LL} = 1.93 + 0.96 = 2.89\ mm$$

The total deflection due to all loads including the effect of creep equals:

$$\Delta_{total} = (1+\alpha)\,\Delta_{DL} + \Delta_{LL}$$

Since A'_s=0, then α=2

$$\Delta_{total} = (1+2)\,1.93 + 0.96 = 6.75\ mm$$

Step 5: Check the code requirements

- The code maximum limit for one-way slabs = L/250, where L is the length between the inflection points. Since the slab is continuous from one end the length L equals 0.87 (3600) =3132 mm.

  $\Delta_{allowable} = 3132/250 = 12.53$ mm

  Since Δ_{total} (6.75 mm) < $\Delta_{allowable}$ (12.53 mm), the code limit is satisfied.

4

CONTROL OF CRACKING

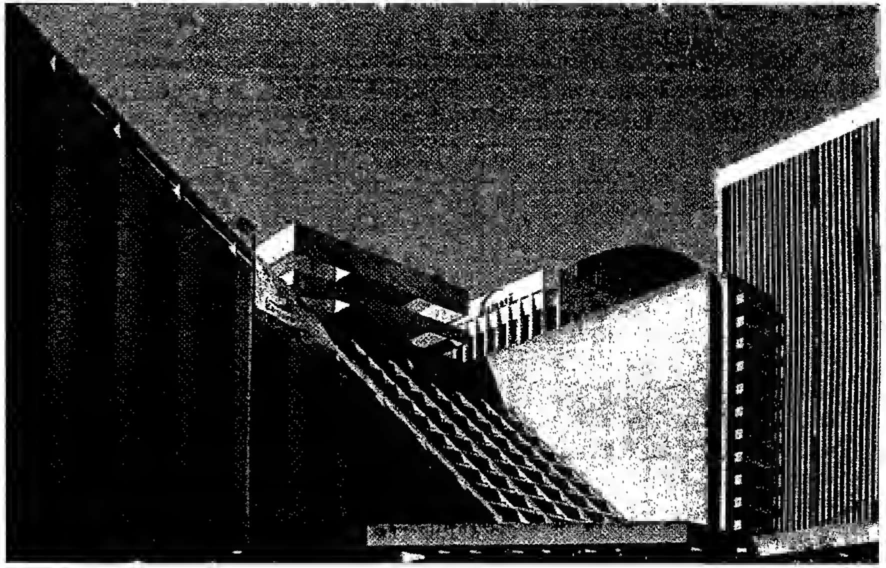


Photo 4.1 A hotel building in San Francisco, USA

4.1 Introduction

In Chapter (1) of volume (1), the concept of limit states design was discussed. The limit states (the states at which the structure becomes unfit for its intended function) are divided into two groups: those leading to collapse and those that disrupt the use of structures but do not cause collapse. These are referred to as the ultimate limit states and the serviceability limit states, respectively. The major serviceability limit states for reinforced concrete structures are: excessive deflections, and excessive cracking. This chapter presents the serviceability limit state of cracking.

4.2 Reasons for Controlling Crack Widths

Crack widths should be limited for the following reasons:

1. Wide cracks lead to concern by owners and occupants. Previous studies suggested that cracks wider than 0.25 to 0.33 mm leads to public concerns.
2. Preventing the corrosion of reinforcement. Corrosion of steel occurs if wetting and drying cycles occur such that the concrete at the level of the steel is alternatively wet and dry. It does not occur in permanently saturated concrete members because water prevents oxygen flow to the steel.
3. Preventing leakage in liquid-retaining structures.

4.3 Types of Cracks

Tensile stresses induced by loads cause distinctive crack patterns as shown in Fig. 4.1. Members loaded in direct tension develop cracks through the entire cross section (Fig. 4.1a). Slender beams subjected to bending moments develop flexural cracks as shown in Fig. 4.1b. These vertical cracks extended almost to the zero-strain axis (neutral axis) of the member. Cracks due to shear have a characteristic inclined shape as shown in Fig. 4.1c. Members subjected to pure torsion develop spiral cracks as shown in Fig. 4.1d. Cracks also develop due to imposed deformation such as differential settlement, shrinkage and temperature changes. If shrinkage is restrained, shrinkage cracks may occur. Generally, however, shrinkage simply increases the width of load-induced cracks.

Photo 4.2 Cracks in a bridge member due to rusting of the reinforcement

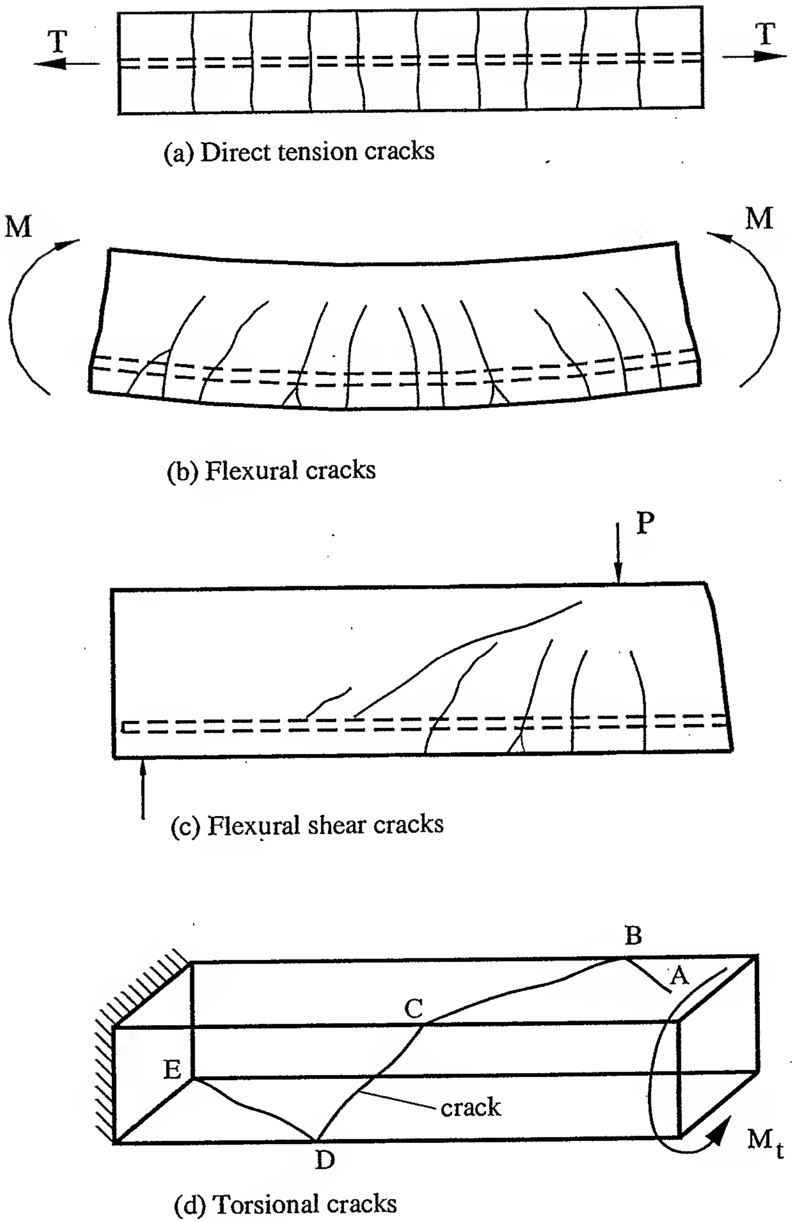


Fig. 4.1 Types of cracks

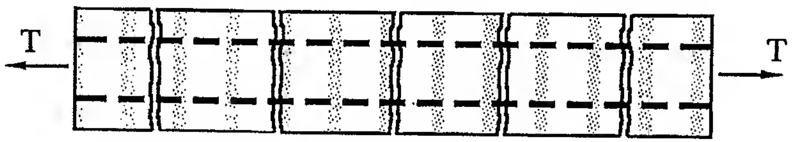
4.4 Development of Cracks due to Loads

Figure 4.2a shows an axially loaded prism. Cracking starts when the tensile stress in the concrete reaches the tensile strength of concrete at some point in the member. When this occurs, the prism cracks. Figures 4.2b and 4.2c show the variation in the steel and concrete stresses along the axially loaded prism. At the cracks, the steel stress and strain are at a maximum value. At the location of the cracks, the stresses in the concrete are equal to zero, while between the cracks, stresses start to develop in the concrete. This reaches a maximum value mid-way between two cracks.
The width of the crack, w, is the difference in the elongation of the steel and the concrete over a length A-B equal to the crack spacing:

$$w = \int_A^B (\varepsilon_s - \varepsilon_c)\, dx \quad \text{...(4.1)}$$

where ε_s and ε_c are the strains in the steel and concrete, respectively, at a given location between A and B and x is measured along the axis of the prism.

The crack spacing and the strains in the steel and concrete are difficult to determine in practice and empirical equations are usually used to compute the crack width.



(a) Cracked member

(b) Variation of steel stress along bar

(c) Variation of concrete stress along prism

Fig 4.2 Stresses in concrete and steel in a cracked element

4.5 Crack Control in the Egyptian Code

4.5.1 Categories of structures

The Egyptian Code categorizes reinforced concrete structures according to their exposure to environmental effects as given in Table 4.1.

Table (4.1) Categories of structures according to the exposure of concrete tension surface to environmental effects

Category	Degree of exposure to environmental conditions
One	Structure with protected tension sides such as: i- All protected internal members of ordinary buildings. ii- Permanently submerged members in water (without harmful materials) or members permanently dry. iii- Well insulated roofs against humidity and rains.
Two	Structures with unprotected tension sides, such as: i- Structures in open air, e.g. bridges and roofs without good insulation. ii- Structures of category one built nearby seashores. iii- members subjected to humidity such as open halls, sheds and garages.
Three	Structure with severely exposed tension sides, such as: i- Members with high exposure to humidity. ii- Members exposed to repeated saturation with moisture. iii- Water tanks. v- Structures subjected to vapour, gases or weak chemical attack.
Four	Members with tension sides very severely exposed to corrosive influences of strong chemical attack that cause rusting of steel i- Members subjected to conditions resulting in rust of steel such as gases, vapour including chemicals. ii- Other tanks, sewerage and structures subjected to sea water.

The Egyptian Code requires a minimum suitable cover for protecting the steel reinforcement. The cover should not be less than the larger value determined from Table (4.2) or the largest bar diameter.

Table (4.2) Minimum concrete cover (mm)**

Category of structure - Table (4.1)	All element except walls and slabs		Walls and Solid slabs	
	$f_{cu} \leq 25$ N/mm^2	$f_{cu} > 25$ N/mm^2	$f_{cu} \leq 25$ N/mm^2	$f_{cu} > 25$ N/mm^2
One	25	20	20	20
Two	30	25	25	20
Three	35	30	30	25
Four	45	40	40	35

** The concrete cover should not be less than the largest bar diameter

4.5.2 Satisfaction of Cracking Limit State

In order to satisfy the limit state of cracking, the Egyptian Code requires the fulfillment of the following relation:

$$w_k = \beta . S_{rm} . \varepsilon_{sm} \leq w_{k\max} \quad \text{.............................(4.2)}$$

$$S_{rm} \quad (mm) = \left(50 + 0.25 k_1 k_2 \frac{\phi}{\rho_r} \right) \quad \text{..................(4.3)}$$

$$\varepsilon_{sm} = \frac{f_s}{E_s} \left(1 - \beta_1 \beta_2 \left(\frac{f_{sr}}{f_s} \right)^2 \right) \quad \text{............................(4.4)}$$

The values of w_k calculated using Eq. 4.2 should be less than w_{kmax} values given in the Table (4.3).

where

β = Coefficient that relates the average crack width to the design crack width. It shall be taken as follows:

1.7 For cracks induced due to loading

1.3 For cracks induced due to restraining the deformation in a section having a width or depth (whichever smaller) less than 300 mm.

1.7 For cracks induced due to restraining the deformation in a section having a width or depth (whichever smaller) more than 800 mm.

For cross sections having a width or depth (whichever smaller) between value 300 mm and 800 mm, the coefficient β shall be proportionally calculated.

ϕ = Bar diameter in mm. In case of using more than one diameter, the average diameter shall be used.

β_1 = A coefficient that reflects the bond properties of the reinforcing steel. It shall be taken equal to 0.8 for deformed bars and 0.5 for smooth bars.

β_2 = Coefficient that takes into account the duration of loading. It shall be taken equal to 1.0 for short term loading and 0.50 for long term loading or cyclic loading.

k_1 = Coefficient that reflects the effect of bond between steel and concrete between cracks. It shall be taken equal to 0.8 for deformed bars and 1.6 for smooth bars.
In case of members subjected to imposed deformation, the values of k_1 shall be modified to kk_1 where the value of k is taken as follows:

k = 0.80 for the case in which the tensile stresses are induced due to restraining the deformation. For rectangular cross section, the value of k is taken as follows:

k = 0.8 for rectangular section having a thickness $\geq$ 300 mm.

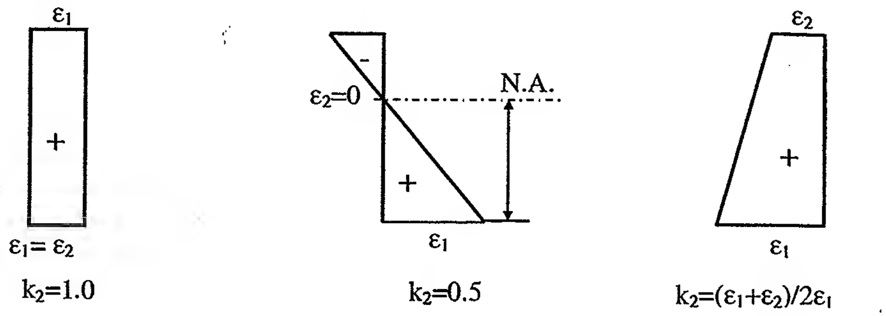
k = 0.50 for rectangular sections having a thickness $\leq$ 800 mm.
For rectangular cross sections having thickness ranging between 300-800 mm, the value of k can be calculated using linear interpolation.

k= 1.0 for cases in which the tensile stresses are induced due to restraint of extrinsic deformations.

k_2 = Coefficient that reflects the strain distribution over the cross section. It shall be taken equal to 0.5 for section subjected to pure bending and 1.0 for section subjected to pure axial tension. For section subjected to combined bending and axial tension, k_2 shall be calculated from Eq. 4.5.

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1} \quad \text{..(4.5)}$$

Where ε_1 and ε_2 are the maximum and minimum strain values, respectively, to which the section is subjected. They shall be calculated based on the analysis of a cracked section as shown in Fig. 4.3.



a– Strain distribution due to axial tension

b– Strain distribution due to B.M. or eccentric forces with big eccentricity

c– Strain distribution due to eccentric tension with small eccentricity.

Fig. 4.3 Values of the factor k_2

f_s = stress in longitudinal steel at the tension zone, calculated based on the analysis of cracked section under permanent loads.

f_{sr} = Stress in longitudinal steel at the tension zone, calculated based on the analysis of cracked section due to loads causing first cracking (M_{cr}).

In case of intrinsic imposed deformation, f_s may be taken equal to f_{sr}.

ρ_r = Ratio of effective tension reinforcement.

$$\rho_r = \frac{A_s}{A_{cef}} \quad \text{..(4.6)}$$

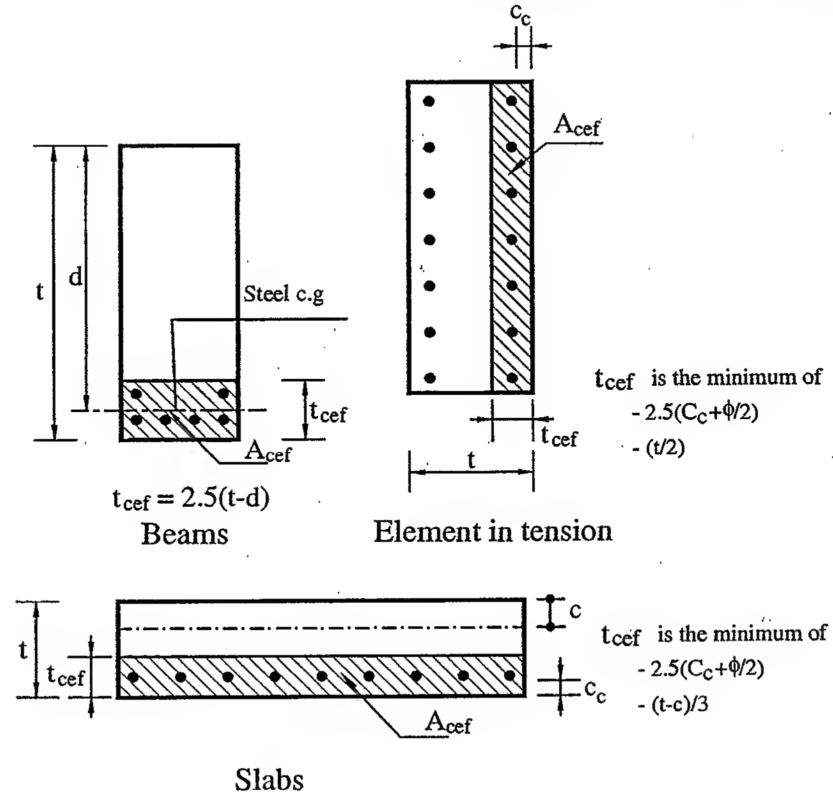
where

A_s = area of longitudinal tension steel within the effective tension area.

A_{cef} = area of effective concrete section in tension,(=width of the section × t_{cef}). The value of t_{cef} can be calculated according to Fig. 4.4.

Table (4.3) Values of w_{kmax} (mm)

Category of structure – Table (4.1)	One	Two	Three	Four
w_k	0.3	0.2	0.15	0.1



c = neutral axis depth measured from the compression fibers

c_c = clear concrete cover

4.5.3 Code Related Provisions

The Egyptian Code permits nit carrying out the calculations of the limit state of cracking in accordance with Eq. (4.2) if one of the following conditions is met:

1- In ordinary buildings classified as Class No. 1 or Class No. 2 according to Table (4.1) and where live loads do not exceed 5.0 kN/m^2 for the following two cases:

   i) Solid slabs of thickness not exceeding 160 mm.

   ii) T- and L- beams with the flange in the tension provided that the effective flange width to the web width (B/b) exceeds 3.

2- For elements subjected to bending moments and axial compressive forces exceeding $(0.2 f_{cu} A_c)$ under service load conditions.

3- For elements in which tensile steel stress f_s under service loads are equal to or less than the values given in Tables (4.4) and (4.5).

4- In case of using limit states design method, it can be considered that the limit state of cracking regarding the stresses in the reinforcing steel is satisfied by multiplying the yield strength f_y by the factor β_{cr} in Tables (4.4) and (4.5).

5- For structures classified as category 3 or 4 in which water tightness is required, the tension stresses should satisfy Eq. 4.7.

Table (4.4) Control of cracking for smooth bars by limiting steel stress under service loads or reduction of design yield stress in steel to $(\beta_{cr} f_y)$

f_s (N/mm²) W.S.D	Reduction factor $\beta_{cr\,(U.L.D)}$	Category one	Category two	Category three & four
		Largest Bar diameter (ϕ_{max}) in mm		
140	1.00	25	18	12
120	0.84	28	20	18
100	0.69	-	-	28

Table (4.5) Control of cracking for deformed bars by limiting steel stress under service loads or reduction of design yield stress in steel to $(\beta_{cr} f_y)$

f_s (N/mm²) W.S.D	Reduction factor $\beta_{cr\,(U.L.D)}$		Category one	Category two	Category three & four
	36/52	40/60	Largest Bar diameter (ϕ_{max}) in mm		
220	1.00	0.92	18	12	8
200	0.93	0.83	22	16	10
180	0.85	0.75	25	20	12
160	0.75	0.67	32	22	18
140	0.65	0.58	--	25	22
120	0.56	0.50	--	--	28

4.6 Liquid Containing Structures

Liquid containing structures should be designed as non-cracked sections. In these structures the tensile stresses induced by loading should be less than the value given by the following equation:

$$f_{ct} = \left[f_{ct(N)} + f_{ct(M)}\right] \le \frac{f_{ctr}}{\eta} \qquad \text{.............................(4.7)}$$

where

f_{ctr} = the cracking strength of concrete and is given by:

$$f_{ctr} = 0.6\sqrt{f_{cu}} \qquad \text{...(4.8)}$$

$f_{ct(N)}$ = the tensile stresses due to unfactored axial tension force (negative sign is used for compressive stresses).

$f_{ct(M)}$ = tensile stresses due to unfactored bending moment.

The coefficient (η) is determined in accordance with Table (4.6) and it depends on the "virtual" thickness t_v calculated from Eq. 4.9.

$$t_v = t\left[1 + \left(\frac{f_{ct(N)}}{f_{ct(M)}}\right)\right] \qquad \text{..(4.9)}$$

where t is the actual thickness of the cross-section.

Table (4.6) Values of the Coefficient η

Virtual thickness, t_v (mm)		**Coefficient η**
Smaller than or equal to	100	1.00
	200	1.30
	400	1.60
Greater than or equal to	600	1.70

4.7 Design Aids for Calculating w_k

The calculation of the factor w_k is complicated and time consuming. Therefore, design curves may be used to reduce the computation time.

The curves were prepared for rectangular sections reinforced with deformed bars and subjected to long term loading that result in pure bending moment.

Hence, the factors appeared in Eqs. 4.2, 4.3 and 4.4 are evaluated as follows:

$k_1 = 0.80$ → deformed bars

$k_2 = 0.50$ → section subjected to simple bending moment

$\beta_1 = 0.80$ → deformed bars

$\beta_2 = 0.50$ → long term loading

$\beta = 1.70$ → cracking due to loads

The crack width equation can then be expressed as:

$$w_k = \beta\, S_{rm}\, \varepsilon_{sm} = 1.7\, S_{rm}\, \varepsilon_{sm} \qquad \text{...................(4.10)}$$

In which

$$S_{rm} = \left(50 + 0.25\, k_1\, k_2 \frac{\phi}{\rho_r}\right)$$

$$\rho_r = \frac{A_s}{A_{cef}} = \frac{\mu \times b \times d}{A_{cef}}$$

$$A_{cef} = 2.5 \times (t - d) \times b$$

$$\rho_r = \frac{\mu \times b \times d}{2.5 \times (t-d) \times b} = \frac{\mu}{2.5\,(t/d - 1)}$$

$$S_{rm} = \left(50 + 0.25 \times 0.8 \times 0.5 \times \frac{\phi}{\rho_r}\right) = \left(50 + 0.25 \times \frac{\phi\,(t/d - 1)}{\mu}\right) \qquad \text{.......(4.11)}$$

The previous equation is a function of t/d, the bar diameter ϕ , and the reinforcement ratio μ. The values of S_m are given in the Appendix.

An example of such design aids is given in Fig. 4.6.

Similarly, the second term ε_{sm}' can be simplified as follows:

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left(1-\beta_1\beta_2\left(\frac{f_{sr}}{f_s}\right)^2\right) \quad \text{.......(4.12)}$$

To calculate the steel stresses, f_s, the cracked moment of inertia need to be computed. Referring to Fig. 4.5, the neutral axis distance is obtained by taking the first moment of area of the transformed section as follows:

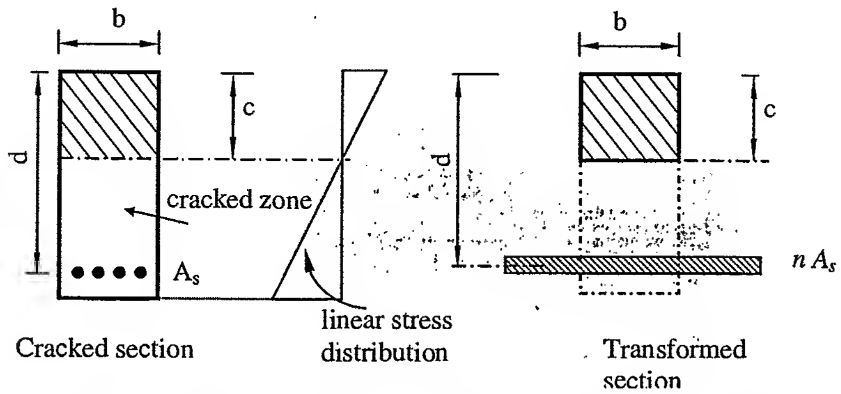


Fig. 4.5 Determination of the neutral axis and cracked transformed moment of inertia calculations

$$b \times c^2/2 - n\,A_s\,(d-c) = 0 \quad \text{......................(4.13)}$$

Subsisting with c=λd and $\mu = A_s/b\,d$ gives:

$$\frac{b\,(\lambda d)^2}{2} - n\,A_s\,(d-\lambda d) = 0 \quad \text{......................(4.14)}$$

Dividing by bd^2, substituting with μ_n= (n μ), and solving for λ gives:

$$\lambda = \sqrt{2\mu_n + \mu_n^{\,2}} - \mu_n \quad \text{................................(4.15)}$$

Having determined the neutral axis distance c, the cracked moment of inertia I_{cr} can be computed as

$$I_{cr} = \frac{b \times c^3}{3} + n\,A_s\,(d-c)^2 \quad \text{........................(4.16)}$$

The ratio f_{sr}/f_s can expressed as:

$$f_{sr} = n \times \frac{M_{cr}}{I_{cr}} \times (d-c) \quad \text{................................(4.17)}$$

$$f_s = n \times \frac{M}{I_{cr}} \times (d-c) \quad \text{................................(4.18)}$$

$$\frac{f_{sr}}{f_s} = \frac{M_{cr}}{M} \quad \text{..(4.19)}$$

$$M_{cr} = f_{ctr} \times \frac{I_g}{y} = 0.6\sqrt{f_{cu}} \times \frac{b \times t^3/12}{t/2} = 0.1\,\sqrt{f_{cu}}\;b\,t^2 \quad \text{.........(4.20)}$$

Normalizing the cracking moment by diving by (b d^2) gives;

$$\frac{M_{cr}}{b\,d^2} = 0.1\,\sqrt{f_{cu}}\,\left(\frac{t}{d}\right)^2 \quad \text{..........................(4.21)}$$

$$\varepsilon_{sm} = \frac{n \times M \times (d-c)}{2 \times 10^6 \times I_{cr}}\left(1 - 0.80 \times 0.5\left(\frac{M_{cr}}{M}\right)^2\right) \quad \text{.............(4.22)}$$

$$\varepsilon_{sm} = \frac{n \times M/b\,d^2 \times (1-\lambda)}{2 \times 10^6 \times I_{cr}/b\,d^2}\left(1 - 0.80 \times 0.5\left(\frac{M_{cr}/b\,d^2}{M/b\,d^2}\right)^2\right) \quad \text{...........(4.23)}$$

The previous equation is a function of M/d b^2, t/d, and the reinforcement ratio μ. For each value of the concrete strength f_{cu}, design curves are plotted and given in the Appendix.

An example of such curves is given in Fig. 4.6 and the rest are in the Appendix

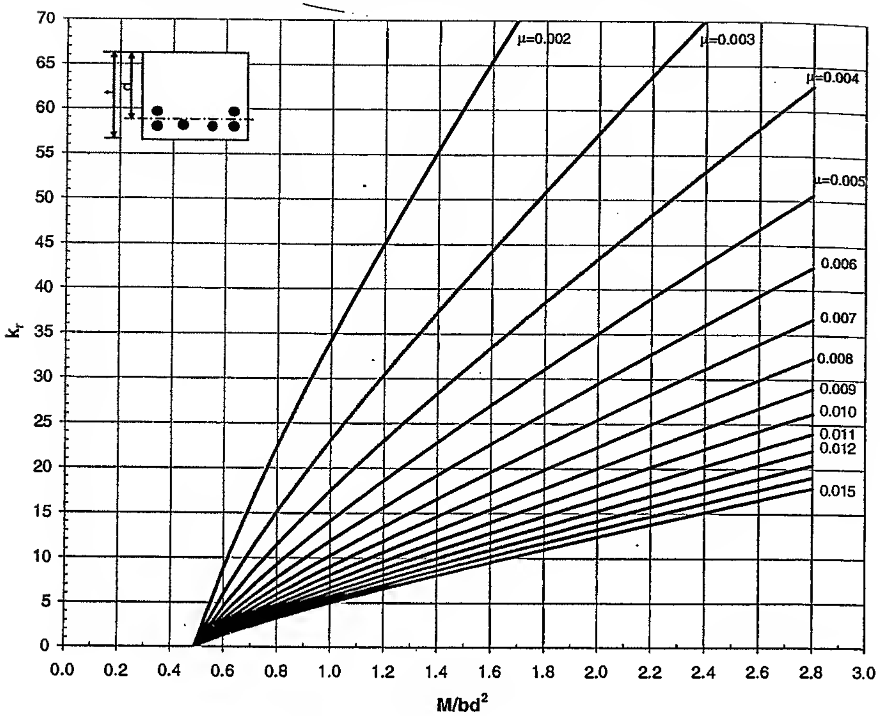
Take $k_r = 1.7\,\varepsilon_{sm} \times 10^4$. Knowing M/d b^2, t/d, and the reinforcement ratio μ, the value of k_r is obtained using the design aids. The value of w_k can be obtained using the following equation:

$$w_k = S_m \times k_r \times 10^{-4} \quad \text{................................(4.24)}$$

The use of these curves is illustrated in Example 4.3.

w_k factor for sections subjected to bending only

f_{cu}=50 N/mm^2, t/d=1.05, ribbed bars, n=10



Values of S_m

μ	Bar Diameter									
	10	12	14	18	18	20	22	25	28	32
0.001	175	200	225	250	275	300	325	363	400	450
0.002	113	125	138	150	183	175	188	206	225	250
0.003	92	100	108	117	125	133	142	154	167	183
0.004	81	88	94	100	106	113	119	128	138	150
0.005	75	80	85	90	95	100	105	113	120	130
0.006	71	75	79	83	88	92	96	102	108	117
0.007	68	71	75	79	82	86	89	95	100	107
0.008	68	69	72	75	78	81	84	89	94	100
0.009	64	67	69	72	75	78	81	85	89	94
0.010	63	65	68	70	73	75	78	81	85	90
0.011	61	64	66	68	70	73	75	78	82	86
0.012	60	63	65	67	89	71	73	76	79	83
0.013	60	62	63	65	67	69	71	74	77	81
0.014	59	61	63	64	66	68	70	72	75	79
0.015	58	60	62	63	65	67	68	71	73	77

$$w_k = S_m \times k_r \times 10^{-4}$$

Fig. 4.6 Example of design curves for calculating the factor w_k

Example 4.1

It is required to design the cross-section of a wall comprising a part of an elevated reinforced concrete water tank. The section is subjected to an unfactored bending moment of 85 kN.m/m′ and an unfactored tension force of 110 kN/m′. The material properties are f_{cu}=30 N/mm^2 and f_y =360 N/mm^2

Solution

In water containing structures, the Egyptian code requires the satisfaction of two conditions:

1- The concrete dimensions of the cross-section must be chosen such that the tensile stresses developed due to the unfactored straining actions are less than the tensile strength of concrete.

2- The steel reinforcement should be designed to resist the tensile forces developed at ultimate stage. The stresses developed in the steel reinforcement at this stage should not exceed $\beta_{cr}\, f_y$ where β_{cr} is a factor less than one and depends on the bar diameter.

Step 1: Uncracked section analysis

The concrete dimensions of the cross-section are determined such that the tensile stresses developed in the section when subjected to the unfactored straining actions are less than the tensile strength of concrete.

Assuming, $t = 550\,mm$

The tensile stresses in the section are calculated according to the following equation:

$$f_{ct} = f_{ct}(N) + f_{ct}(M) \le f_{ctr} / \eta$$

$$f_{ct}(N) = \frac{N}{A_c}$$

$$f_{ct}(N) = \frac{110\times10^3}{1000\times550} = 0.20\, N/mm^2$$

$$f_{ct}(M) = \frac{6M}{bt^2}$$

$$f_{ct}(M)=\frac{6\times85\times10^6}{1000\times550^2}=1.69\,N/mm^2$$

$$f_{ct}=0.2+1.69=1.89\,N/mm^2$$

$$f_{ctr}=0.6\sqrt{f_{cu}}$$

$$f_{ctr}=0.6\sqrt{30}=3.28\,N/mm^2$$

η = factor that is determined from Table (4.6) according to the following equation:

$$t_v=t\times\left\{1+\frac{f_{ct}(N)}{f_{ct}(M)}\right\}$$

$$t_v=550\times\left\{1+\frac{0.20}{1.69}\right\}=615\,mm \quad\rightarrow\quad \eta=1.7$$

$$\frac{f_{ctr}}{\eta}=\frac{3.28}{1.7}=1.93\,N/mm^2>f_{ct}=1.89\,N/mm^2 \text{..................Ok.}$$

Step 2: Cracked section analysis

According to ECP 203, the load factor for liquid containing structures is 1.4

$$M_u=1.4\times85=119\,kN.m/m \qquad N_u=1.4\times110=154\,kN.m/m$$

$$d=t-(clear\,cover+\phi/2)$$

According to Table 4.1 the structure is classified as class 3. For such a class, Table 4.2 gives a minimum concrete cover of 25 mm.

Assume the reinforcing bars used are of 16 mm diameter.

$$d=550-(25+16/2)=517\,mm$$

$$e=\frac{M_u}{N_u}$$

$$e=\frac{119}{154}=0.77m=770\,mm$$

Since e > t/2, the section is subjected to normal force with big eccentricity and M_{us} approach can be used.

$$e_s=e-t/2+\text{cover}$$

$$e_s=770-550/2+(25+16/2)$$

$$e_s=528\,mm$$

$$M_{us}=N_u\times e_s=154\times0.528=81.3\,kN.m/m$$

For f_y=360 N/mm², Table 4.5 is used. The value of β_{cr} for 16 mm diameter bar can be taken as the average between ϕ=12 mm and ϕ=18mm.

$$\beta_{cr}=\frac{0.85+0.75}{2}=0.80$$

f_s (N/mm²) W.S.D	Reduction factor $\beta_{cr\,(U.L.D)}$		Category one	Category two	Category three & four
	36/52	40/60	Largest Bar diameter (ϕ_{max}) in mm		
220	1.00	0.92	18	12	8
200	0.93	0.83	22	16	10
180	0.85	0.75	25	20	12
160	0.75	0.67	32	22	18
140	0.65	0.58	--	25	22
120	0.56	0.50	--	--	28

$$A_s=\frac{M_{su}}{\beta_{cr}\times f_y\times j\times d}+\frac{N_u}{\beta_{cr}\times f_y/\gamma_s}$$

$$d=c_1\sqrt{\frac{M_{us}}{b\times f_{cu}}} \qquad 517=c_1\sqrt{\frac{81.3\times10^6}{1000\times30}}$$

$$C1=9.93 \qquad \frac{c}{d}<\left(\frac{c}{d}\right)_{min} \qquad \text{take } \frac{c}{d}=0.125 \quad \text{and} \quad j=0.826$$

$$A_s=\frac{81.3\times10^6}{0.80\times360\times0.826\times517}+\frac{154\times10^3}{0.80\times360/1.15}=1276\,mm^2\text{........}Use\,7\phi16/m$$

Example 4.2

A reinforced concrete raft (categorized as category one) of a thickness 900 mm and is subjected to an unfactored bending moment of 700 kN.m/m. The material properties are f_{cu}=30 N/mm^2 and f_y =360 N/mm^2.

It is required to design the steel reinforcement to resist the applied moment and to check the satisfaction of cracking limit state in the Egyptian Code.

Solution

In order to design the steel reinforcement satisfying the limit states of cracking, the Egyptian Code gives two options to the designer:

1- Design the steel reinforcement such that the stress developed at the ultimate stage is $\beta_{cr} f_y$. The reduction in the stresses developed in the steel is intended to guarantee a limited crack width at service loads. This is a simple straightforward approach that usually leads to uneconomic design

2- Design the steel reinforcement such that the stress developed at the ultimate stage is f_y. However, the designer should check the satisfaction of the Egyptian Code (Eq. 4.2) in order to guarantee a limited crack width at service loads. This approach needs an extensive amount of calculations but usually results in an economic design.

Approach 1

As mentioned above, this approach is based on designing the steel reinforcement based on usable stresses of $=\beta_{cr} \times f_y$ at the ultimate stage.

-Minimum clear cover =40 mm

-Ultimate Moment = Ultimate Factor x Bending Moment

Assuming the factor = 1.5

Ultimate Moment $=1.5 \times 700 = 1050\, kN.m$

- The effective depth (d) = total thickness – clear cover - $\phi/2$

Assume the use of reinforcing bars of diameter of 32 mm. From Table (4-5) for Category II and assuming $\phi = 32 \rightarrow \beta_{cr} = 0.75$

$$d = 900 - 40 - 32/2 = 844.0\, mm$$

$$d = C1\sqrt{\frac{M_{us}}{b \times f_{cu}}}$$

$$844 = C1\sqrt{\frac{1050 \times 10^6}{1000 \times 30}} \qquad C1 = 4.51 \;\; \& \;\; j = 0.817$$

$$A_s = \frac{M_u}{\beta_{cr} \times f_y \times j \times d}$$

For category one structures and deformed bars of diameter 32 mm (decided by the designer), the value of β_{cr} can be obtained from Table 4-5 as 0.75.

$$A_s = \frac{1050 \times 10^6}{0.75 \times 360 \times 0.817 \times 844} = 5640\, mm^2 \quad \text{Use } 7\Phi 32$$

Approach 2

This approach is based on designing the steel reinforcement to develop the full yield strength at ultimate stage and to check cracking status using Eq. 4.2.

Step 1: *Cracked section analysis*

-Minimum clear cover = 40 mm

-Ultimate Moment = Ultimate Factor x Bending Moment

Ultimate Moment $=1.5 \times 700 = 1050\, kN.m$

- The effective depth (d) = total thickness – clear cover – $\phi/2$

Assume the use of reinforcing bar of diameter of 32 mm.

$$d = 900 - 40 - 32/2 = 844\ mm$$

$$d = C1\sqrt{\frac{M_u}{b \times f_{cu}}}$$

$$844 = C1\sqrt{\frac{1050 \times 10^6}{1000 \times 30}} \qquad C1 = 4.51 \;\; \& \;\; j = 0.817$$

$$A_s = \frac{M_u}{f_y \times j \times d}$$

$$A_s = \frac{1050 \times 10^6}{360 \times 0.817 \times 844} = 4230\, mm^2 \quad \text{Use } 6\Phi 32$$

Step 2: Calculation of the value of w_k

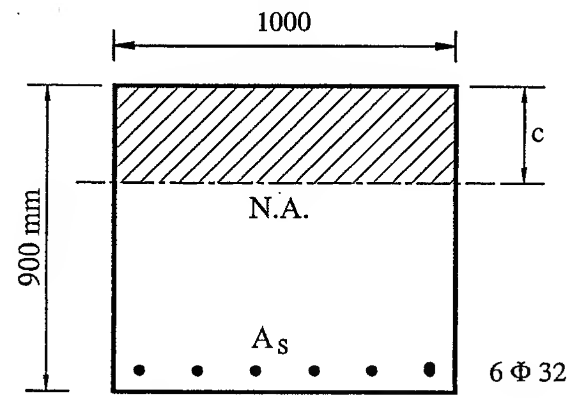
Step 2.1: Calculation of the depth of the neutral axis, c

The first moment of area of the cross-section about the neutral axis must be equal to zero.

$$b \times \frac{c^2}{2} = n \times A_s \times (d - c)$$

$$1000 \times \frac{c^2}{2} = 10 \times (6 \times 804) \times (844 - c)$$

Solving quadratic equation for c, c = 241.17 mm.



Cracked cross section

Step 2.2: Calculation of I_{cr}

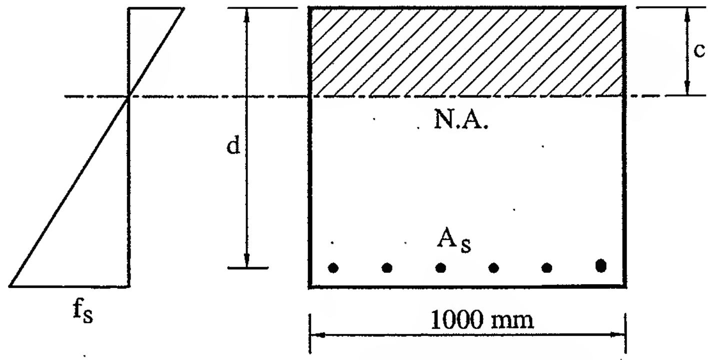
$$I_{cr} = \frac{B \times c^3}{3} + n\ A_s\ (d - c)^2$$

$$I_{cr} = \frac{1000 \times 241.17^3}{3} + 10 \times (6 \times 804)\ (844 - 241.17)^2 = 22.21 \times 10^9\ mm^4$$

Step 2.3: Calculation of steel stresses, f_s

$$f_s = n \times \frac{M}{I_{cr}} \times (d - c)$$

$$= 10 \times \frac{700 \times 10^6}{22.21 \times 10^9} \times (844.0 - 241.17) = 190.03\, N\ /\ mm^2$$



Cracked cross section

Step 2.4: Calculation of cracking moment, M_{cr}

$$M_{cr} = f_{ctr} \times \frac{I_g}{y}$$

$$f_{ctr} = 0.60 \times \sqrt{f_{cu}}$$

$$= 0.60 \times \sqrt{30} = 3.286\, N\ /\ mm^2$$

$$I_g = \frac{1}{12} \times b \times t^3$$

$$= \frac{1}{12} \times 1000 \times 900^3 = 60.75 \times 10^9\ mm^4$$

$$M_{cr} = 3.286 \times \frac{60.75 \times 10^9}{900/2} \times \frac{1}{10^6} = 443.7\, kN.m$$

Step 2.5: Calculation of steel stress f_{sr}

For n=10, c= 241.17 mm, $I_{cr} = 22.21 \times 10^9$ mm^4

$$f_{sr} = n \times \frac{M_{cr}}{I_{cr}} \times (d - c)$$

$$= 10 \times \frac{443.7 \times 10^6}{22.21 \times 10^9} \times (844.0 - 241.17) = 120.44\, N/mm^2$$

Step 2.6: Calculation of ρ_r

$$\rho_r = \frac{A_s}{A_{cef}}$$

$$A_{cef} = b \times t_{cef}$$

$$t_{cef} = 2.5 \times (clear\ cover + \phi/2)$$

$$t_{cef} = 2.5 \times \left(40 + \frac{32}{2}\right) = 140.0\, mm$$

$$\rho_r = \frac{6 \times 804.0}{1000 \times 140.0}$$
$$= 0.0345$$

It can be determined that:

$k_1 = 0.80$ → deformed bars
$k_2 = 0.50$ → simple bending moment applied
$\beta_1 = 0.80$ → deformed bars
$\beta_2 = 0.50$ → long term loading
$\beta = 1.70$ → cracking due to loads

Step 2.7: Check the value of w_k

$$w_k = \beta . s_{rm} . \varepsilon_{sm}$$

$$s_{rm} = \left(50 + 0.25 k_1 k_2 \frac{\phi}{\rho_r}\right)$$

$$s_{rm} = \left(50 + 0.25 \times 0.80 \times 0.50 \times \frac{32}{0.0345}\right)$$

$$s_{rm} = 142.75$$

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left(1 - \beta_1 \beta_2 \left(\frac{f_{sr}}{f_s}\right)^2\right)$$

$$\varepsilon_{sm} = \frac{190.03}{2 \times 10^5}\left(1 - 0.80 \times 0.50 \times \left(\frac{120.44}{190.03}\right)^2\right)$$
$$= 0.000797$$

$$w_k = 1.70 \times 142.75 \times 0.000797 = 0.1935\ mm$$

From Table 4.3, w_{kmax} for category one is equal to 0.3.

Since $w_k = 0.1935 < 0.30$ the structure stratifies the limit state of cracking.

Note;

It can be noted that the calculations needed in Approach 2 are lengthy and cumbersome. However, it results in economic design when compared to Approach 1 as noted in the amount of steel reinforcement resulted from each design.

Example 4.3

It is required to calculate the factor w_k for the raft given in example 4.2 (Approach 2) using the design aids given in the Appendix.

Solution

Step1: Calculate curve parameters

$$d = 900 - 40 - \frac{32}{2} = 844\ mm \qquad \frac{t}{d} = \frac{900}{844} = 1.066$$

The computed reinforcement from example 4.2 (Approach 2) is 6Φ32

$$\mu = \frac{A_s}{b \times d} = \frac{6 \times 804}{1000 \times 844^2} = 0.00572 = 0.572\%$$

η= 1.0, (ribbed bars)

Step 2: Calculate w_k

Step 2.1: Determine k_r using the design charts

$$\frac{M}{b \times d^2} = \frac{700 \times 10^6}{1000 \times 844} = 0.983$$

Since no chart is available for (t/d=1.066), interpolation should be made between the charts available (t/d=1.05 and t/d=1.15), (refer to Fig. EX 4.3 given below). Using charts with t/d=1.05 gives k_r = 13.7, and t/d=1.15 gives k_r = 12.6. Interpolating, one gets k_r = 13.5.

Step 2.2: Determine S_m

Using the previous design charts.

For t/d=1.05 → Sm = 120, and for t/d=1.15 → $Sm = 260$.

Interpolating, one gets S_m=142, (refer to Fig. EX 4.3).

Step 2.3: Calculate the factor w_k

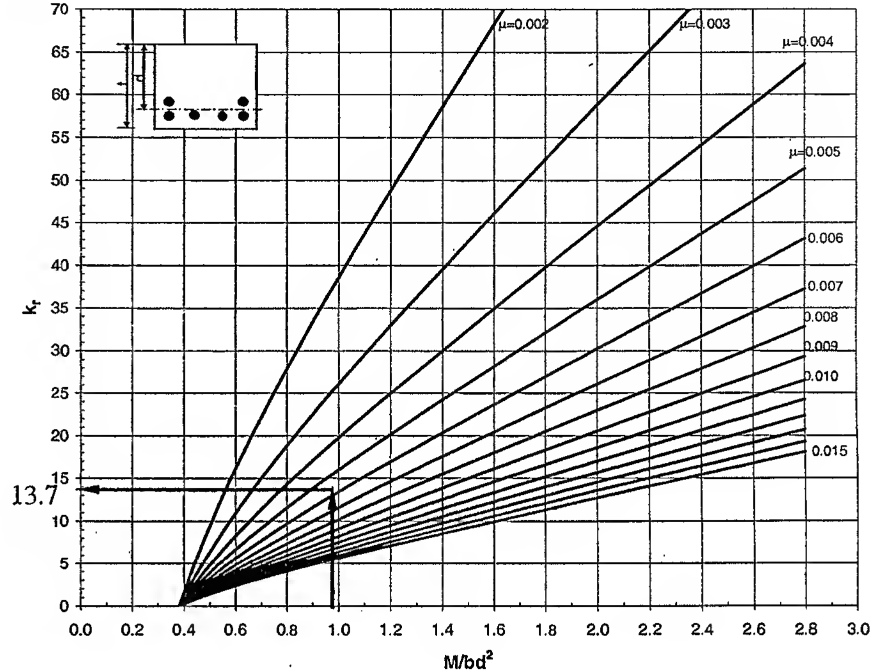
$$w_k = k_r \times S_m \times 10^{-4}$$

$$w_k = 13.5 \times 142 \times 10^{-4} = 0.192 \text{ mm}$$

The systematic application of Eq. (4.2) results in w_k=0.193 mm. Such a close agreement with the value obtained from the use of the design aids confirms their accuracy.

w_k factor for sections subjected to bending only

f_{cu}=30 N/mm², t/d=1.05, ribbed bars, n=10



Values of S_m

μ	Bar Diameter									
	10	12	14	16	18	20	22	25	28	32
0.001	175	200	225	250	275	300	325	363	400	[illegible]
0.002	113	125	138	150	163	175	188	206	225	[illegible]
0.003	92	100	108	117	125	133	142	154	167	[illegible]
0.004	81	88	94	100	106	113	119	128	138	[illegible]
0.005	75	80	85	90	95	100	105	113	120	130
0.006	71	75	79	83	88	92	96	102	108	117
0.007	68	71	75	79	82	86	89	95	100	107
0.008	66	69	72	75	78	81	84	89	94	100
0.009	64	67	69	72	75	78	81	85	89	94
0.010	63	65	68	70	73	75	78	81	85	90
0.011	61	64	66	68	70	73	75	78	82	86
0.012	60	63	65	67	69	71	73	76	79	83
0.013	60	62	63	65	67	69	71	74	77	81
0.014	59	61	63	64	66	68	70	72	75	79
0.015	58	60	62	63	65	67	68	71	73	77

120

$$w_k = S_m \times k_r \times 10^{-4}$$

Fig. EX 4.3 Using design curves for w_k factor

Example 4.4

The critical cross section of a reinforced concrete member that is a part of a structure with unprotected tension side (categorized as category II) is subjected to an unfactored bending moment of 100 kN.m and an unfactored tension force of 400 kN. The concrete dimensions of the member (b x t) are (350 mm x 900 mm). It is required to design the steel reinforcement of the section satisfying the requirements of the limit states of cracking in of the Egyptian Code. The material properties are f_{cu}=25 N/mm^2 and f_y =360 N/mm^2.

Solution

In order to design the steel reinforcement satisfying the limit states of cracking, the Egyptian Code gives two options to the designer:

1- Design the steel reinforcement such that the stress developed at the ultimate stage are $\beta_{cr} f_y$. The reduction in the stresses developed in the steel is intended to guarantee a limited crack width at service loads. This is a simple straightforward approach that usually leads to uneconomic design.

2- Design the steel reinforcement such that the stress developed at the ultimate stage is f_y. However, the designer should check the satisfaction of Eq. 4.2 in order to guarantee a limited crack width at service loads. This approach needs an extensive amount of calculations but usually results in economic design.

Approach 1

As mentioned above, this approach is based on designing the steel reinforcement based on usable stresses of $\beta_{cr} f_y$ at the ultimate stage.

Cross-section of beam $= 350\,mm \times 900\,mm$

$N_u = 1.5 \times 400 = 600\ kN$

$M_u = 1.5 \times 100 = 150\ kN.m$

$$e = \frac{M_u}{N_u}$$

$e = \frac{150}{600} = 0.25\,m = 250\,mm < \frac{t}{2} = 450\,mm$ ....small eccentric tension force

$$e_{s1} = \frac{t}{2} - e - cover$$

Assuming the cover = $40\,mm$

$$e_{s1} = 450 - 250 - 40 = 160\,mm$$

$$e_{s2} = \frac{t}{2} + e - cover$$

$$e_{s2} = 450 + 250 - 40 = 660\,mm$$

$$A_{s1} = \frac{N_u \times e_{s2}}{d - d'} / (\beta_{cr} f_y / \gamma_s)$$

From Table (4-5), for Category II and assuming $\phi = 22 \rightarrow \beta_{cr} = 0.75$

$A_{s1} = \frac{600 \times 10^3 \times 660}{860 - 40} / (0.75 \times 360 / 1.15) = 2057\,mm^2$ .... Use $6\Phi22$

$$A_{s2} = \frac{N_u \times e_{s1}}{d - d'} / (\beta_{cr} \times f_y / \gamma_s)$$

From Table (4-5), for Category II and assuming $\phi = 12 \rightarrow \beta_{cr} = 1.00$

$A_{s2} = \frac{600 \times 10^3 \times 160}{860 - 40} / (1.0 \times 360 / 1.15) = 374\,mm^2$.........Use $4\Phi12$

Approach 2

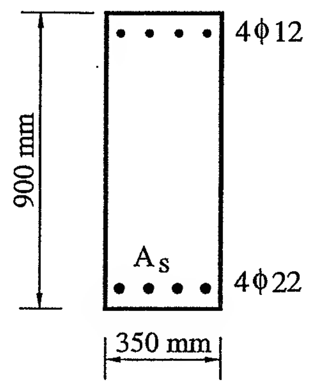
This approach is based on designing the steel reinforcement to develop the yield strength at ultimate stage and to check cracking status using Eq. 4.2.

Step 1: Cracked section analysis

Cross-section of beam $= 350 \times 900$ mm

$$e = \frac{M_u}{N_u}$$

$e = \frac{150}{600} = 0.25\,m = 250\,mm < \frac{t}{2} = 450\,mm$ ....eccentric tension force



$$e_{s1} = \frac{t}{2} - e - cover$$

Assuming the cover = 40 mm

$$e_{s1} = 450 - 250 - 40 = 160\,mm$$

$$e_{s2} = \frac{t}{2} + e - cover$$

$$e_{s2} = 450 + 250 - 40 = 660\,mm$$

$$A_{s1} = \frac{N_u \times e_{s2}}{d - d'} / (f_y / \gamma_s)$$

$$A_{s1} = \frac{600.0 \times 10^3 \times 660}{860 - 40} / (360/1.15) = 1543\,mm^2 \text{ .... Use } 4\phi 22$$

$$A_{s2} = \frac{N_u \times e_{s1}}{d - d'} / (f_y / \gamma_s)$$

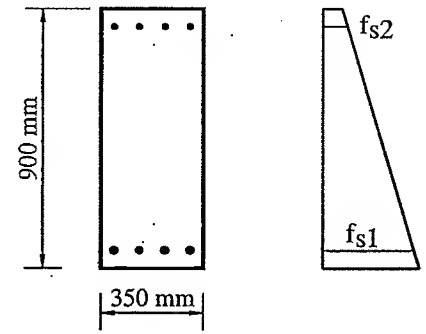
$$A_{s2} = \frac{600.0 \times 10^3 \times 160.0}{860 - 40} / (360/1.15) = 374\,mm^2 \text{ ....Use } 4\phi 12$$

Step 2: Calculation of w_k

Step 2.1 Calculation of the steel stress f_s

$$f_{s1} = \frac{N \times e_{s2}}{d - d'} / A_{s1}$$

$$= \frac{400.0 \times 10^3 \times 660}{860 - 40} / 1520 = 211.8\,N/mm^2$$



Step 2.2: Calculation of steel stress f_{sr}

In order to find the steel stress f_{str}, one has to calculate the combination M_{cr} and N_{cr} that result in first cracking of the section. It should be clear that one has to assume that the eccentricity of the tension force will be unchanged during the history of loading.

$$f_{ctr} = 0.60 \times \sqrt{f_{cu}} / \eta$$

$$f_{ctr} = 0.60 \times \sqrt{25} / 1.7 = 1.76\,N/mm^2$$

$$f_{ctr} = \frac{N_{cr}}{A} + \frac{6 \times M_{cr}}{b \times t^2}$$

$$f_{ctr} = N_{cr} \left(\frac{1}{b \times t} + \frac{6 \times e}{b \times t^2}\right)$$

$$1.76 = N_{cr} \times 10^3 \left(\frac{1}{350 \times 900} + \frac{6 \times 250}{350 \times 900^2} \right)$$

$$N_{cr} = 207.9\,kN$$

$$f_{sr} = \frac{N_{cr} \times e_{s2}}{d - d'} / A_{s1}$$

$$f_{sr} = \frac{207.90 \times 10^3 \times 660}{860 - 40} / 1520.0 = 110.1\,N/mm^2$$

Step 2.3: Calculation of ρ_{eff}

$$\rho_r = \frac{A_s}{A_{cef}}$$

$$A_{cef} = b \times t_{cef}$$

$$t_{cef} = 2.5 \times (clear\,cover + \phi/2)$$

$$t_{cef} = 2.5 \times (30 + 22/2) = 102.5\,mm$$

$$\rho_{eff} = \frac{1520.0}{350 \times 102.5} = 0.042$$

Step 2.4: Calculation of k_2

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1}$$

The strains ε_1 and ε_2 are calculated through the analysis of the transformed section.

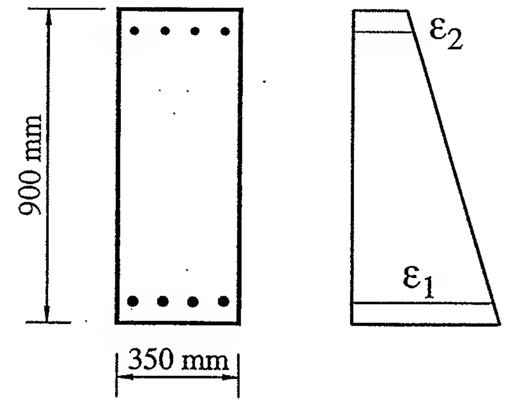
Assuming $\varepsilon_2 = \dfrac{f_y / \gamma_s}{E_s}$

$$\varepsilon_2 = \frac{360/1.15}{2 \times 10^5} = 0.00156$$

$$\varepsilon_1 = \frac{e_{s2}}{e_{s1}} \times \varepsilon_2$$

$$\varepsilon_1 = \frac{660}{160} \times 0.00156 = 0.00643$$

$$k_2 = \frac{0.00643 + 0.00156}{2 \times 0.00643} = 0.621$$



Step 2.5: Check the value of w_k

$k_1 = 0.80$ → deformed bars

$\beta_1 = 0.80$ → deformed bars

$\beta_2 = 1.0$ → short term loading

$\beta = 1.70$ → case that includes loads

$$w_k = \beta . s_{rm} . \varepsilon_{sm}$$

$$s_{rm} = \left(50 + 0.25 k_1 k_2 \frac{\phi}{\rho_r} \right)$$

$$s_{rm} = \left(50 + 0.25 \times 0.80 \times 0.621 \times \frac{22}{0.042} \right)$$

$$s_{rm} = 115.06$$

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left(1 - \beta_1 \beta_2 \left(\frac{f_{sr}}{f_s}\right)^2\right)$$

$$\varepsilon_{sm} = \frac{211.8}{2\times10^5}\left(1 - 0.8\times1.0\times\left(\frac{110.1}{211.8}\right)^2\right)$$

$$\varepsilon_{sm} = 0.00083$$

$$w_k = \beta \times s_{rm} \times \varepsilon_{sm} = 1.70\times115.06\times0.00083 = 0.162\ mm$$

From Table 4.3 w_{kmax} for category one is 0.3

Since $w_k = 0.162\ mm < 0.30$ the structure stratifies the limit state of cracking.

Note:
It can be noted that the calculations needed in Approach 2 are lengthy and cumbersome. However, it results in economic design when compared to Approach 1 as noted in the amount of steel reinforcement resulted from each design.

5

DESIGN OF FOUNDATIONS

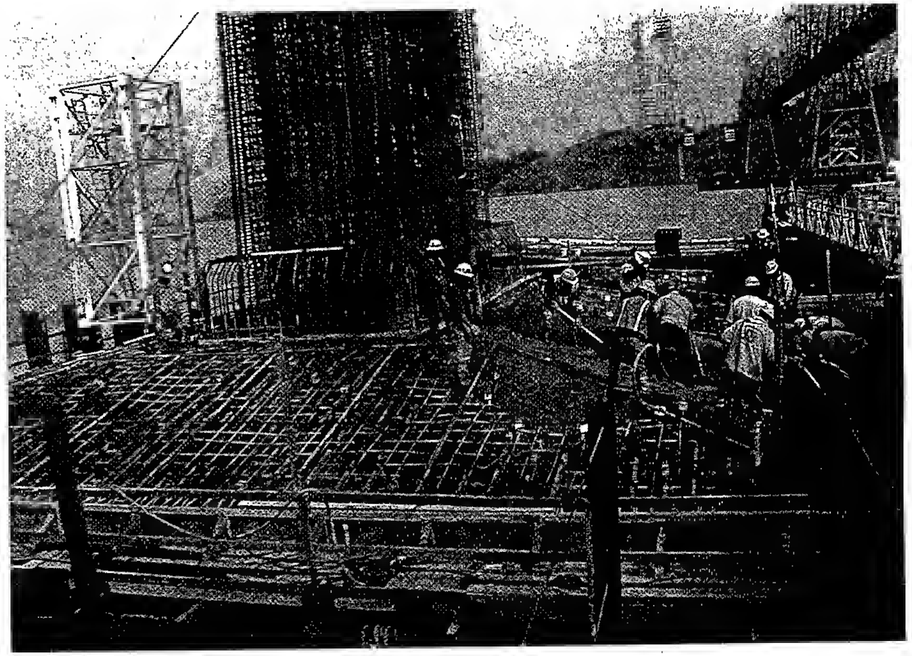


Photo 5.1 Foundation of a bridge column.

5.1 Introduction

The main purpose of footings and other foundation systems is to transfer column loads safely to the soil. Since, the soil bearing capacity is much lower than the concrete columns; the loads need to be transferred safely to the soil by using larger areas usually called *shallow foundations*. If the soil has low bearing capacity, or the applied loads are very large, it may be necessary to transfer the load to a deeper soil through the use of piles or caissons usually called *deep foundations*.

Foundation design requires both a *soil investigation*; to determine the most suitable type of foundation, and a *structural design*; to establish the depth and reinforcement of the different foundation elements. It is customary for the *geotechnical engineers* to carry out the soil investigation, and propose the best foundation system that fits a particular location. It is the responsibility of the *structural engineer* to establish the size and amount of reinforcement for each component of the proposed foundation system.

This chapter addresses the structural design of shallow foundations and piled foundations. Traditional analysis and design procedures are explained. The chapter also explains the use of the finite element method for the analysis and design of complicated foundation systems such as shallow rafts and rafts on piles.

5.2 Types of Foundations

The choice of a particular type of foundation depends on a number of factors, such as the soil bearing capacity, the water table, the magnitude of the loads that needs to be transferred to the soil, and site constraints such as the existence of a property line.

Generally, foundations may be classified as follows:

- Shallow foundations: This type includes strip footing, isolated footings combined footing, strap footing, and raft foundation
- Deep foundations: This type includes isolated pile caps or raft on piles

Fig. 5.1 shows some types of foundations that are usually used in structures.

A *strip footing* is used under reinforced concrete walls to distribute the vertical loads over the soil as shown in Fig. 5.1a. The load is transferred mainly in one direction perpendicular to the center line of the wall. *Isolated footings* are the most common type of foundations in ordinary structures. They used to distribute column load on relatively large area of soil as shown in Fig. 5.1b. They transfer the load in two directions. If two columns or more are closely spaced or the required footing sizes overlap each other, the two footings are combined in one big footing called *combined footing* (Fig 5.1c). If one of the footings is very close to a property line, then a stiff beam is used to connect this column to an interior column. This type is called *strap footing* as shown in Fig. 5.1d. If the applied loads are heavy or more than 60% of the isolated footings overlap, a *raft foundation* is used to support the entire structure as shown in Fig. 5.1e. This is similar to an inverted flat slab in which it contains column and field strips. Finally, if the applied loads are large or weak soil encountered, it may be practical to support the structure into deeper more stiff soil through the use of *pile foundations*. Pile caps are used to distribute column loads to a group of piles as illustrated in Fig. 5.1f.

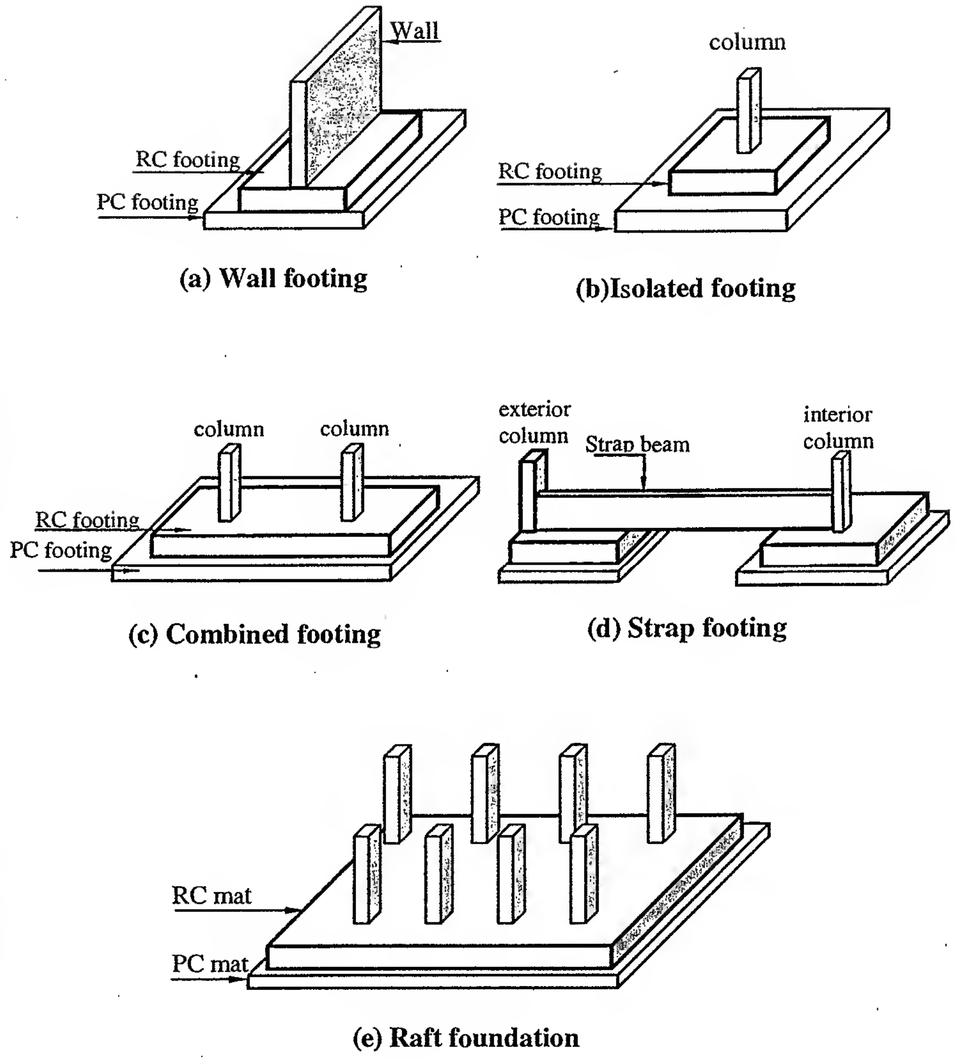
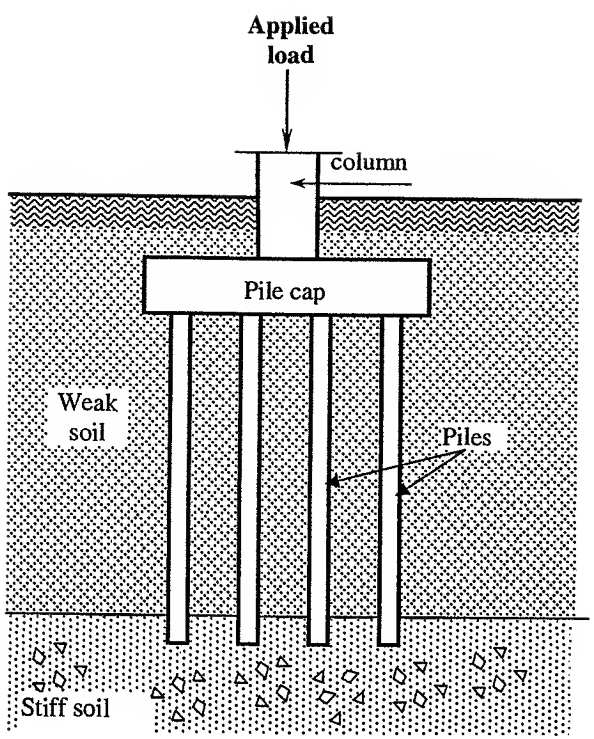
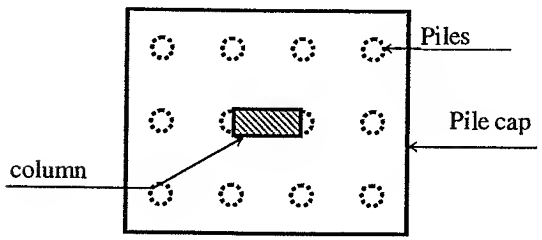


Fig. 5.1 Types of foundations



Section



Plan

(f) Pile foundation

Fig. 5.1 Types of foundations (contd.)

5.3 Soil Pressure under concentrically Loaded Footings

The actual bearing soil pressure differs significantly according to the type of soil, stiffness of the footing and loading conditions. In general, the distribution under the base of the footing is non-uniform. Assuming the loading is concentric and the footing is rigid, the soil pressure distribution under sandy soil (*cohesionless soil*) may take the parabolic shape shown in Fig. 5.2a. The part of the soil under the column is likely to be pressured more than the part at the edges. The soil particles near the edges escapes from under the footing providing less support and producing less pressure. In contrast, in a clayey soil (*cohesive soil*), the stresses near the edges are larger than those at the middle as shown in Fig. 5.2b. This is attributed to the shear stresses developed near the unloaded portion (surrounding the footing) of the soil near the edges. This additional support results in producing high stresses near the edge than those developed at the center of the footing.

In addition to the variation of soil distribution under different types of soils, the stiffness of the footing itself adds more complexity to the problem. For design purposes, the bearing soil distribution is assumed uniform regardless of the type of soil or the stiffness of the footing as shown in Fig. 5.2c. The assumption of uniform pressure simplifies the calculation of the acting forces and speeds up the design process. Experimental tests and the performance of the existing buildings indicate that this assumption results in conservative designs.

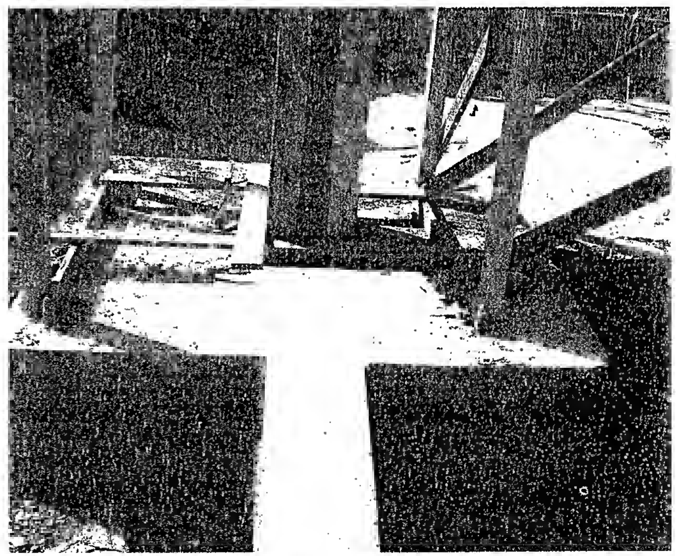
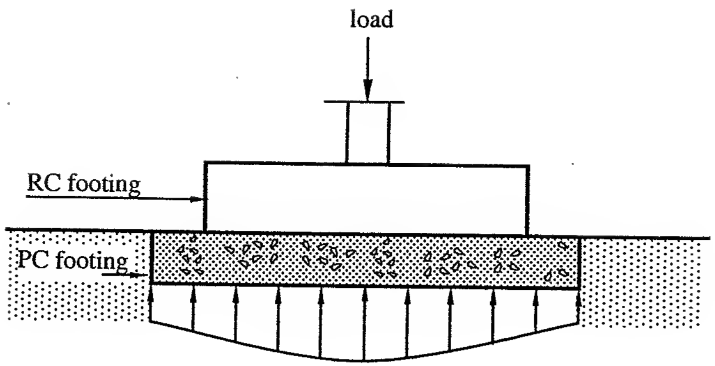
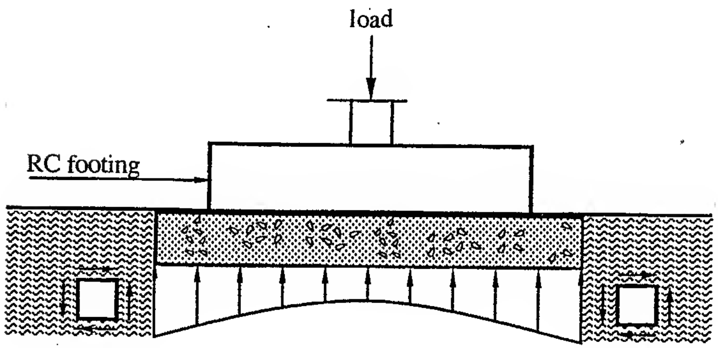


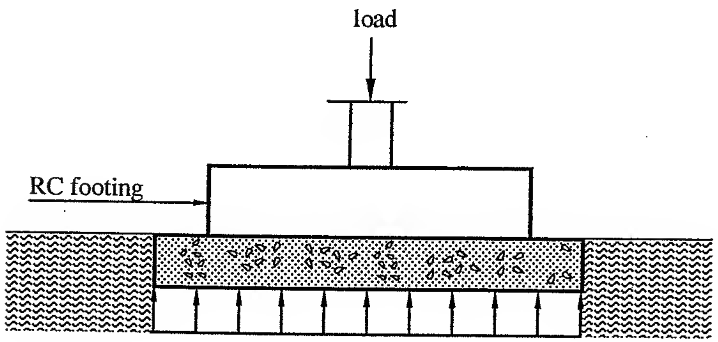
Photo 5.2 An isolated footing during construction



a- Footing on sandy soil



b- Footing on clayey soil



c- Assumed uniform pressure

Fig. 5.2 Soil pressure distribution under footings.

5.4 Soil Pressure under Eccentrically Loaded Footings

Foundations may be frequently subjected to eccentric loading resulting from lateral forces due to wind or earthquake. The moments developed at the base of the footing produce a non-uniform soil pressure that needs to be taken into account.

If the eccentricity of the load is small, compression stresses develop across the contact between the footing and the soil as shown in Fig. 5.3a. The maximum stress f_{max} should be checked against the allowable bearing capacity. As the eccentricity of the load increases, the difference between the maximum f_{max} and minimum f_{min} stresses increases. The classical stress equation is used to determine the distribution of the soil pressure as follows:

$$f_{\substack{max \\ min}} = \frac{P}{A} \pm \frac{M}{I}\, y \quad \text{.. (5.1)}$$

Where P and M are the unfactored axial load and moment, respectively, and y is the distance from the c.g of the footing.

For rectangular footings (L×b) in which $A = b \times L$, $I = b \times L^3/12$ and $y = L/2$, the previous equation may be written in the following form

$$f_{\substack{max \\ min}} = \frac{P}{b \times L} \pm \frac{6\,M}{b \times L^2} \quad \text{.................................. (5.2)}$$

$$f_{\substack{max \\ min}} = \frac{P}{b \times L}\left(1 \pm \frac{6\,e}{L}\right) \quad \text{.................................. (5.3)}$$

At a certain limit (e=L/6), the minimum stress f_{min} becomes zero at the edge as shown in Fig. 5.3b. Any further increase in the eccentricity of the load will result in negative pressure (tension). However, the previous equations are only valid when no tensile stresses are developed. This is because tensile stresses cannot be transmitted between the soil and the concrete, and redistribution of stresses should occur. For a rectangular footing of length *L*, if the eccentricity *e* exceeds L/6, a triangular stress distribution will develop over part of the base as shown in Fig. 5.3c. For equilibrium to occur, the centroid of the soil pressure must coincide with the applied load P_u. If we denote the distance from the applied load P_u to the footing edge a, then the length of the base on which the triangular stress distribution developed is *3a*. Applying the equilibrium equation gives:

$$\frac{1}{2} f_{max} \times b \times (3a) = P_u \quad \text{.................................. (5.4)}$$

$$f_{\max} = \frac{2P_u}{3 \times a \times b} \quad \text{.. (5.5)}$$

Where $\frac{L}{2} + e + a = L \quad or \quad a = \frac{L}{2} - e$

The maximum developed pressure f_{max} should not exceed the soil bearing capacity. The assumed pressure distribution is expected to deviate from the reality because of the non linear stress-strain relationship of the soil. The amount of deviation increases as the amount of eccentricity increases. However, experience over the years showed that this simplified analysis gives a safe design.

Footings subjected to high moments tend to tilt and undesirable differential settlement develops. Therefore, it is recommended to minimize the eccentricity of the applied load as much as possible.

In some other cases the footing may be subjected to eccentricities in both directions. This produces biaxial moments on the footing. Only one corner point is subjected to the maximum stress. The soil stresses may be obtained using the stress equation as follows:

$$f_{\substack{\max \\ \min}} = \frac{P}{A} \pm \frac{M_x}{I_x} y + \frac{M_y}{I_y} x \quad \text{.............................. (5.6)}$$

Hand calculations of such problems are difficult, and computer programs are usually used to determine soil distribution and the acting forces.

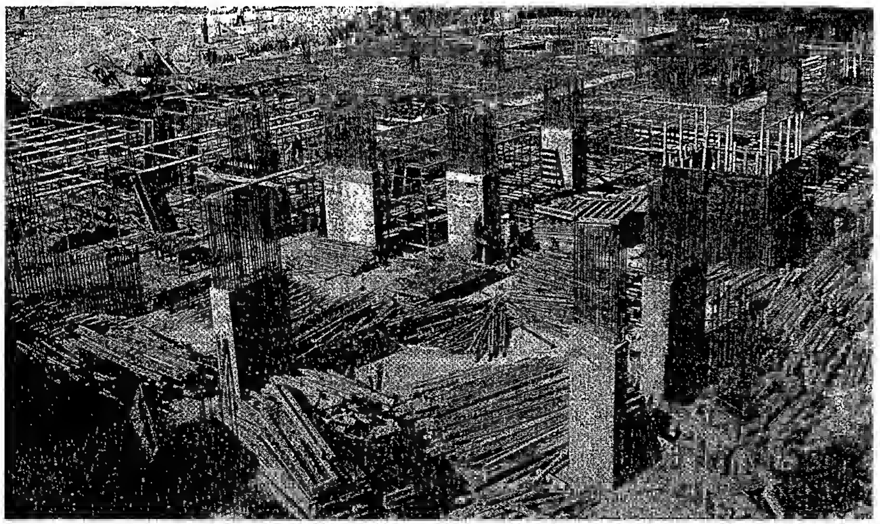


Photo 5.3 Foundations of a high-rise building during construction

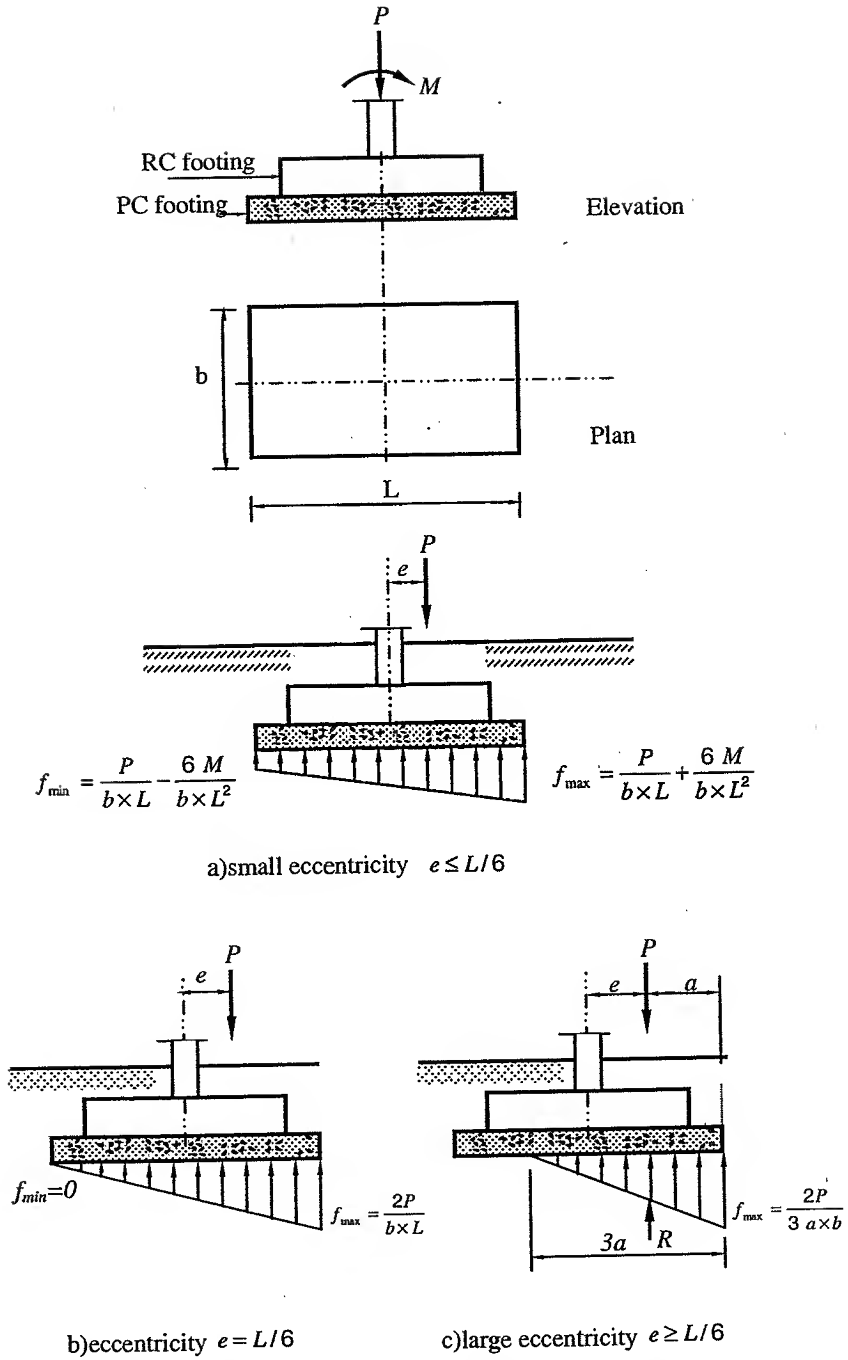
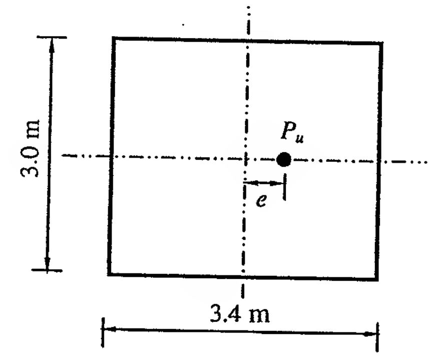


Fig. 5.3 Soil stress distribution for eccentrically loaded footings

Example 5.1

The concrete footing shown in figure is designed to support a dead load of 1260 kN and a live load of 820 kN. The allowable soil pressure is 320 kN/m^2. Determine and check safety of the developed soil pressure if the eccentricity equals:

a) e=0
b) e=0.3m
c) e=0.8 m



Solution

The total applied working loads (unfactored) equals:

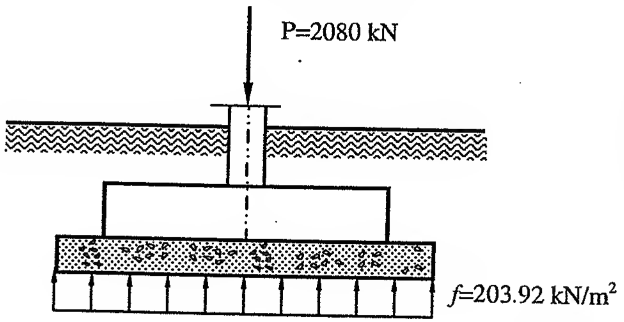
$$P = P_{DL} + P_{LL} = 1260 + 820 = 2080 \ kN$$

Case a

For a concentrically applied load (e=0), the soil pressure simply equals the load over the area of the footing.

$$f = \frac{P}{A} = \frac{2080}{3.4 \times 3.0} = 203.92 \ kN/m^2$$

Since the applied pressure (203.92 kN/m^2) is less than the allowable soil pressure (320 kN/m^2), the footing is considered safe.



Case b

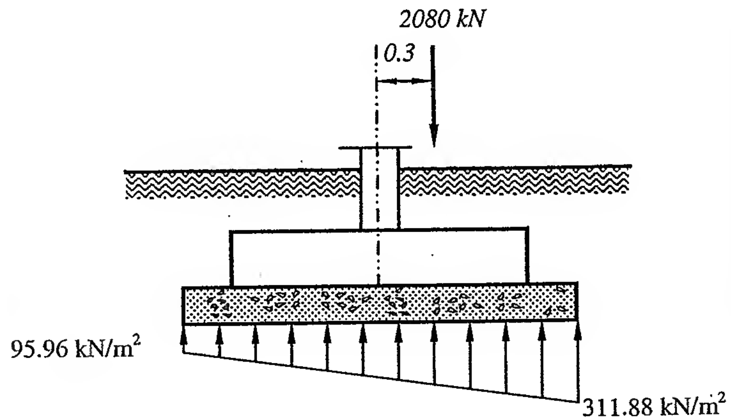
Since e = 0.3 m < L/6 = 0.567m, the soil bearing pressure can be obtained using the following equation:

$$f_{\substack{max \\ min}} = \frac{P}{b \times L}\left(1 \pm \frac{6\,e}{L}\right)$$

$$f_{max} = \frac{2080}{3.0 \times 3.4}\left(1 + \frac{6 \times 0.3}{3.4}\right) = 311.88 \text{ kN/m}^2$$

$$f_{min} = \frac{2080}{3.0 \times 3.4}\left(1 - \frac{6 \times 0.3}{3.4}\right) = 95.96 \text{ kN/m}^2$$

Since the maximum applied pressure (f_{max}=311.88 kN/m^2) is less than the allowable soil pressure (320 kN/m^2), the footing is considered safe.



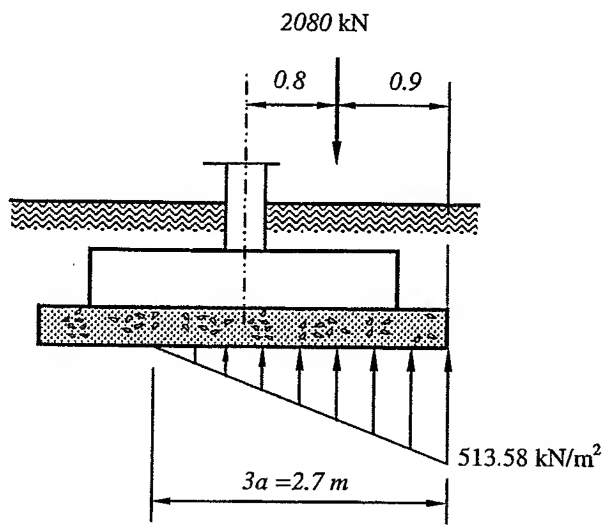
Case c

Since e = 0.8 m > L/6 = 0.567m, the soil bearing pressure can be obtained using the following equation:

$$a = \frac{L}{2} - e = \frac{3.4}{2} - 0.8 = 0.9\ m$$

$$f_{max} = \frac{2P_u}{3 \times a \times b} = \frac{2 \times 2080}{3 \times 0.9 \times 3} = 513.58\ \text{kN/m}^2$$

Since the maximum applied pressure (f_{max}=513.58 kN/m^2) is larger than the allowable soil pressure (320 kN/m^2), the footing is considered **unsafe**.



5.5 Gross and Net Soil Pressures

The soil pressure may be expressed in terms of gross or net pressure at the foundation level. The *gross soil pressure* is the total soil pressure produced by all loads above the foundation level. These loads consist of (a) the service column load at the ground surface, (b) the weight of the plain and reinforced concrete footings, and (c) the weight of the soil from the foundation level to the ground level. On the other hand, the *net soil pressure* does not include either the weight of the soil above the base of the footing or the weight of the footing. It can be simply obtained by subtracting from the gross soil pressure the weight of 1-m square of soil with a height from the foundation level to the ground level.

If a concrete footing is located at the foundation level without any column load as shown in Fig. 5.4a, the total downward pressure from the footing and the soil above is 51 kN/m^2. This is balanced by an equal and opposite (upward) soil pressure of 51 kN/m^2. Therefore, the net effect on the footing is zero and neither moments nor shear develops in the footing.

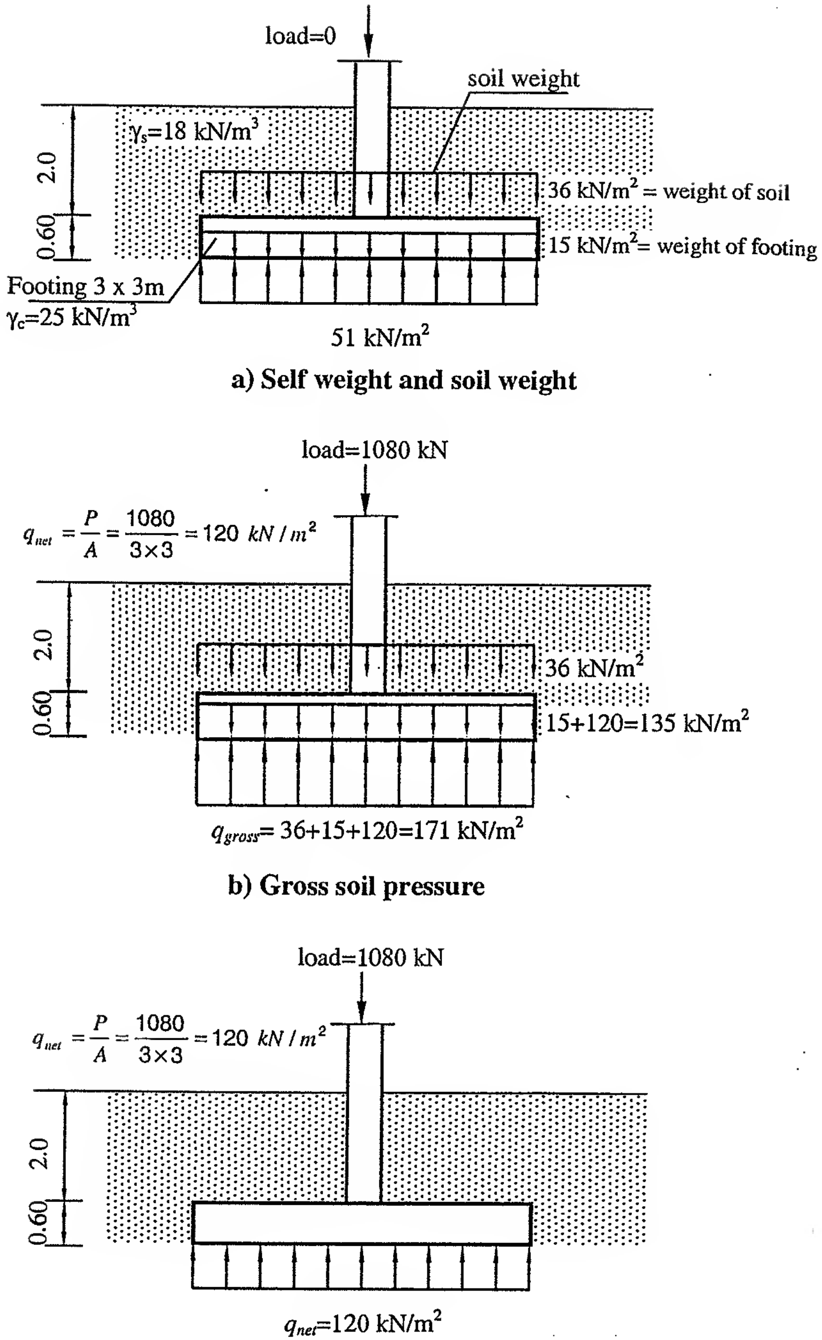
When the column load column is applied, the pressure under the footing is increased by 120 kN/m^2 as illustrated in Fig. 5.4b. Thus the total pressure on the soil becomes 171 kN/m^2. This is the gross soil pressure and must not exceed the allowable soil pressure $q_{allowable}$. When the bending moments and shear forces are computed, the upward pressure and downward pressure of 51 kN/m^2 cancel each other leaving only the net soil pressure of 120 kN/m^2 to produce the internal straining actions on the footing as shown in Fig. 5.4c.

In design, the area of the footing is chosen such that the applied gross pressure does not exceed the allowable soil pressure. The net soil pressure is used to calculate the reinforcement and to check the shear strength of the footing. The area of the plain concrete footing is calculated as follows:

$$Area = \frac{P_{gross(column\ load + footing + soil)}}{q_{allowable}} \qquad (5.7)$$

The pervious equation can be further simplified by assuming the weight of the footing and the soil above is about 5%-10% of the column load. Assuming this ratio to be 10% the area of the footing can be obtained using Eq. 5.8 as follows:

$$Area = \frac{1.1 \times P_{column}}{q_{allowable}} \qquad (5.8)$$



a) Self weight and soil weight

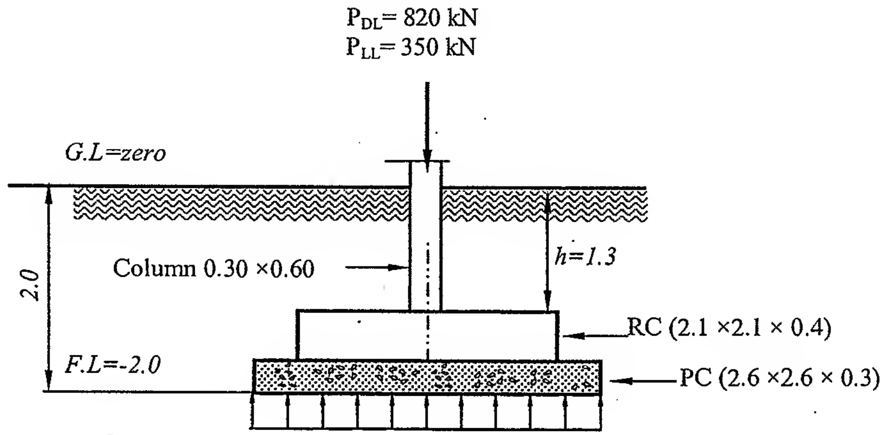
b) Gross soil pressure

c) Net soil pressure

Fig. 5.4 Gross and net soil bearing pressures.

Example 5.2

For the footing shown in the figure below, calculate the gross and the net soil pressures at the base of the footing. The densities of the plain concrete, the reinforced concrete and the soil may be taken as 22 kN/m³ , 25 kN/m³ , and soil=18 kN/m³, respectively.



Solution

The total loads above the foundation level are calculated as follows:

1.	Column load	$820 + 350 = 1170\ kN$
2.	weight of the PC footing	$22 \times 2.6 \times 2.6 \times 0.3 = 44.6\ kN$
3.	weight of the RC footing	$25 \times 2.1 \times 2.1 \times 0.4 = 44.1\ kN$
4.	weight of the Soil	$18 \times 2.1 \times 2.1 \times 1.3 = 103.19\ kN$

$$P_{total} = 1170 + 44.6 + 44.1 + 103.19 = 1361.91\ kN$$

The gross soil pressure equals:

$$f_{gross} = \frac{P}{A} = \frac{1361.91}{2.6 \times 2.6} = 201.46\ kN/m^2$$

The added (*net*) soil pressure (due to adding the footing and column load)

$$f_{net} = f_{gross} - weight\ of\ 2m\ of\ soil = 201.466 - 18 \times 2 = 165.466\ kN/m^2$$

5.6 Design of Isolated Footings

5.6.1 Introduction

The design of the isolated footings must consider bending, development of the reinforcement, one-way shear and punching shear. Since shear reinforcement is not permitted by the ECP 203 for one-way shear and two-way shear. Accordingly, shear rather the bending moment normally controls the depth of the footing.

One-way shear reinforcement is not allowed in the footings because (1) determining of the effective pattern of shear reinforcement is difficult to establish when the footing is bending in two directions, and (2) the depth of the compression zone may be not sufficient to anchor the shear reinforcement that is intended to reach the yielding stress at failure.

Punching shear reinforcement (two-way shear) is permitted by some international codes. Because of the difficulty of placing such reinforcement, the Egyptian Code insists in depending on concrete only in resisting two-way shear.

The soil pressure causes the footing to deflect upward, producing tension in two directions at its bottom fibers. Therefore the reinforcement is placed at its bottom of the footing in two perpendicular directions without the need of top reinforcement.

It is common in Egypt to construct a plain concrete footing above which a reinforced concrete footing of smaller dimensions is resting. Such an arrangement proves to be more economical than using a reinforced concrete footing resting directly on soil.

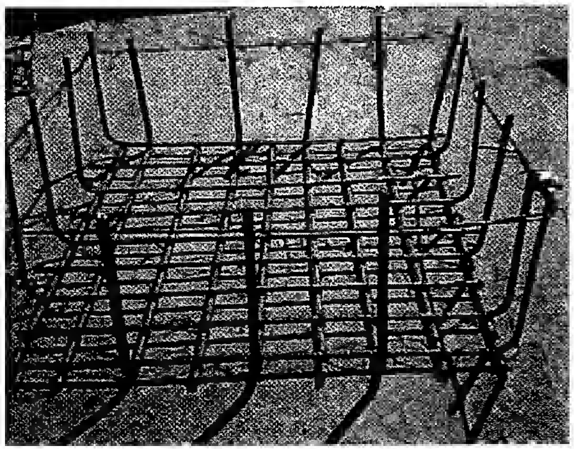


Photo 5.4 Example of isolated footing reinforcement

5.6.2 Design Steps

Step 1: Dimensioning of the Plain Concrete Footing

The plain concrete footing size is computed suing the allowable soil pressure. It is customary to assume that the weight of the soil and the footing equal to 5-10% of the column load. The loads used in the calculations are the working loads (unfactored). Thus, the area of the plain concrete footing (A) equals:

$$A = \frac{1.1\times(P_{DL}+P_{LL})}{q_{allowable}} \quad \text{...(5.9a)}$$

The dimensions of the footing are chosen such that an equal amount is projecting all around the column. Referring to Fig. 5.5, the dimensions of the footing are taken as

$$L\times B = L\times[L+(b_c-a_c)] = A \quad \text{............................ (5.9b)}$$

Dimensioning the footing in such a way will ensure producing the same bending moment in all four sides. Thus, the reinforcement in the reinforced concrete footing will approximately be equal in both directions.

The thickness of the plain concrete footing is usually assumed from 250-500 mm depending on the soil type and the magnitude of the applied loads.

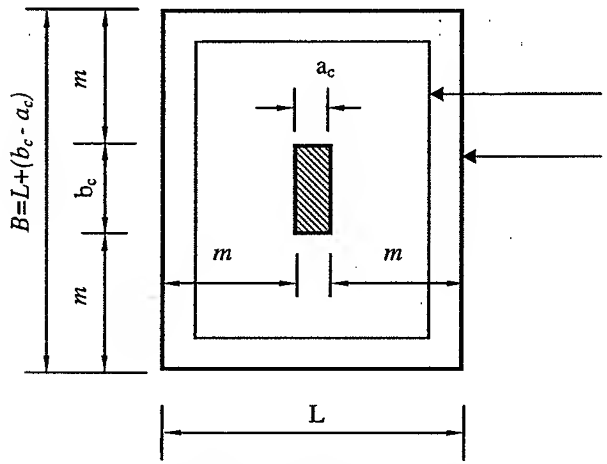


Fig. 5.5 Dimensioning of the plain concrete footing

Step 2: Dimensioning of the Reinforced Concrete Footing

The plan dimensions of the reinforced concrete footing are determined by subtracting a distance x from each side dimension of the plain concrete footing. The value of x depends on the soil bearing capacity and the thickness of the plain concrete footing. The value of x is usually assumed 0.8-1.0 the thickness of the plain concrete footing.

$$A_r = L - 2x \quad \text{and } B_r = B - 2x$$

More refined analysis can be obtained by equating the tensile strength of concrete to the tension developed in the plain concrete footing at sec 1-1 as shown in Fig. 5.6.

$$f_{ct} = f_{ct(M)} = \frac{6\,M}{b\,t^2} = \frac{6\,(P_a\,x^2/2)}{1\times t^2} = 3.0\,P_a\left(\frac{x}{t}\right)^2 \le 0.6\,\sqrt{f_{cu}}\,/\eta$$

Where P_a is the allowable soil pressure and η is a coefficient that depends on the thickness (can be assumed=1.7) and t is the thickness of the plain concrete footing obtained from step 1.

The previous equation can be solved to obtain the distance x. A factor of safety of 3 applied to the allowable tensile strength of concrete is assumed to obtain the values listed in Table 5.1. Knowing f_{cu} and the allowable soil pressure, one can get the value of x/t from table 5.1 and hence x is known.

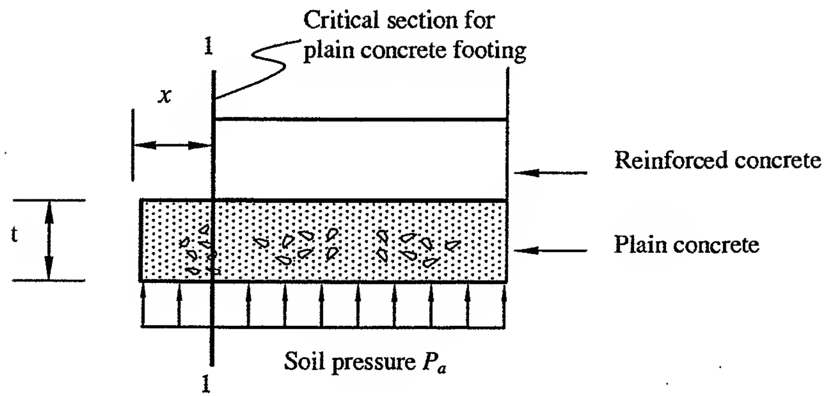


Fig. 5.6 Dimensioning of a reinforced concrete footing

Table 5.1: x/t values

f_{cu}	Allowable soil pressure P_a (kN/m^2)							
N/mm^2	100	125	150	175	200	225	250	300
15.0	1.23	1.10	1.01	0.93	0.87	0.82	0.78	0.71
17.5	1.28	1.15	1.05	0.97	0.91	0.85	0.81	0.74
20.0	1.32	1.18	1.08	1.00	0.94	0.88	0.84	0.76
22.5	1.36	1.22	1.11	1.03	0.96	0.91	0.86	0.79
25.0	1.40	1.25	1.14	1.06	0.99	0.93	0.89	0.81
30.0	1.47	1.31	1.20	1.11	1.04	0.98	0.93	0.85

The thickness of the reinforced concrete footing should not be less than 300 mm or the smallest column dimension which ever is greater.

Step 3: Design for Punching Shear

The factored pressure q_{su} at the bottom of the reinforced concrete footing is obtained using the factored load as follows:

$$q_{su} = \frac{P_u}{A_r \times B_r} = \frac{1.4\,P_{DL} + 1.6\,P_{LL}}{A_r \times B_r} \quad \text{.......................} (5.10)$$

Punching shear failure is referred as two-way shear. In the ECP 203, the critical perimeter for punching shear is at a distance d/2 from the face of the column as shown in Fig 5.7. The critical shear perimeter is given by

$$b_o = 2\,(a+b) = 2\,(a_c + d) + 2\,(b_c + d) \quad \text{..............} (5.11)$$

Where a_c and b_c are the dimensions of the column, and d is the average effective depth in the two directions.

The punching shear load is obtained by subtracting the factored pressure multiplied by the punching area from the column load as follows

$$Q_{up} = P_u - q_{su}\,(a \times b) \quad \text{..} (5.12)$$

The punching stresses

$$q_{up} = \frac{Q_{up}}{b_o \times d} \le q_{cup} \quad \text{..} (5.13)$$

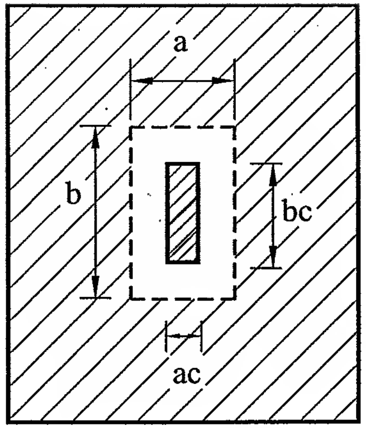


Fig. 5.7 Punching loading area and perimeter.

Since punching shear reinforcement is not allowed by the ECP 203, the developed shear q_{up} should be less than concrete strength q_{cu} given by the least of the following three values:

1. $$q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} \le 1.6\,N/mm^2 \quad \text{.........................(5.14a)}$$

2. $$q_{cup} = 0.316\,(0.50 + \frac{a}{b})\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{.........................(5.14b)}$$

3. $$q_{cup} = 0.8\,(0.20 + \frac{\alpha\, d}{b_o})\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{.........................(5.14c)}$$

Where α is a factor equals to 2, 3 ,4 for corner, exterior, internal footings respectively and b_o is the critical punching shear perimeter.

If the applied punching shear stresses are less than the concrete strength q_{cu}, the footing is considered safe, otherwise the footing depth has to be increased.

Step 4: Design for One-Way Shear

According to the ECP 203, the critical section for one-way shear is at d/2 from the face of the column as shown in Fig. 5.8. The shear stress developed in the footing is obtained from the factored soil pressure. Referring to Fig. 5.8, the total shear force at sec 1-1 equals:

$$Q_u = q_{su} \cdot (A_r - \frac{a}{2}) \cdot B_r \quad \text{.............................. (5.15)}$$

The corresponding shear stress is calculated as follows:

$$q_u = \frac{Q_u}{B_r \times d} \quad \text{.............................. (5.16)}$$

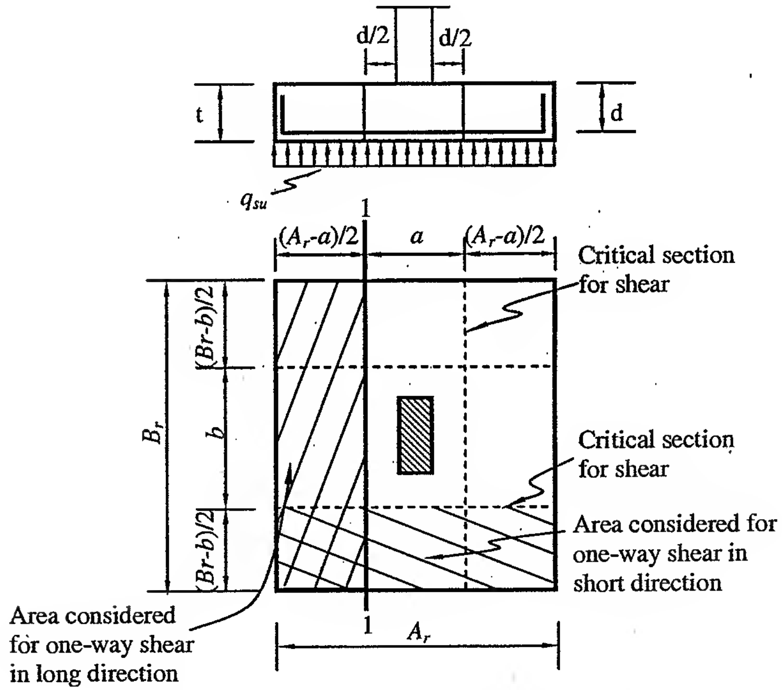


Fig. 5.8 Calculation of one-way shear stress

The ECP 203 states that the one-way shear stress should be resisted without any shear reinforcement. The concrete shear strength is given by:

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{.............................. (5.17)}$$

Step 5: Design for Flexure

The ultimate soil pressure induces moment into two perpendicular directions. Frequently, the minimum reinforcement requirement controls the design.

$$A_{s\,min} = \frac{0.6}{f_y}\, b\, d \qquad (5.18)$$

The critical section for moment is taken at the face of the column. A 1m strip is usually used to calculate the reinforcement per meter. Referring to Fig. 5.9, the moment per meter equals

$$M_u = q_{su} \frac{(A_r - a_c)^2}{8} \qquad (5.19)$$

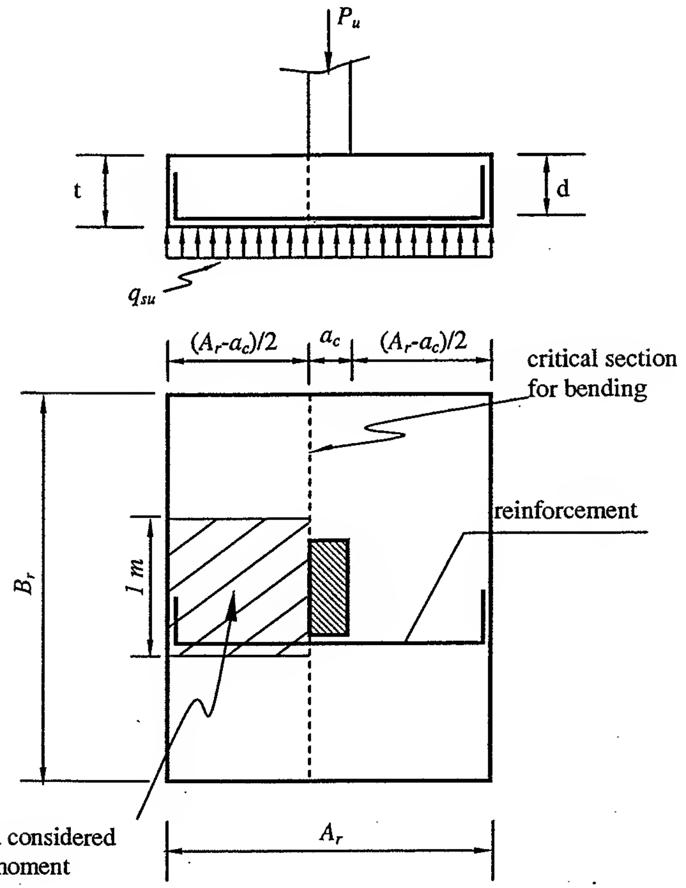


Fig. 5.9 Critical section for bending

Column reinforcement should be well anchored in the footing using column dowels. The length of these dowels inside the column should not be less than 40 the largest bar diameter as shown in Fig. 5.10.

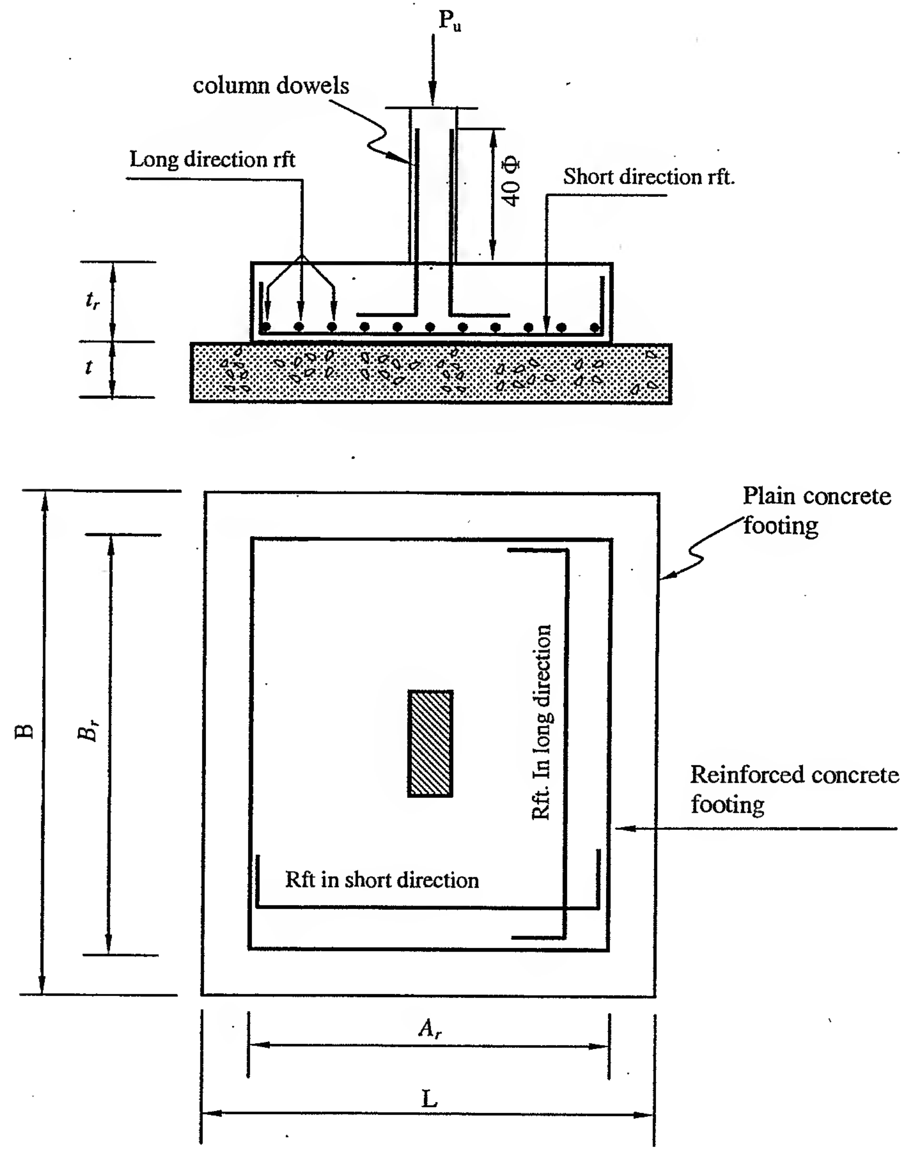


Fig. 5.10 Typical reinforcement and dimensions of isolated footings

Example 5.3

Design an isolated footing for a rectangular column (0.25 x 0.8 m) that carries unfactored dead of 780 kN and unfactored live load of 440 kN, respectively.

Design data:

Allowable soil pressure =125 kN/m^2 (1.25 kg/cm^2)

f_{cu} =30 N/mm^2 (reinforced concrete)

f_{cu} =20 N/mm^2 (plain concrete)

f_y =360 N/mm^2

Solution

Step1: Dimensioning of the plain concrete footing

The total working load equals

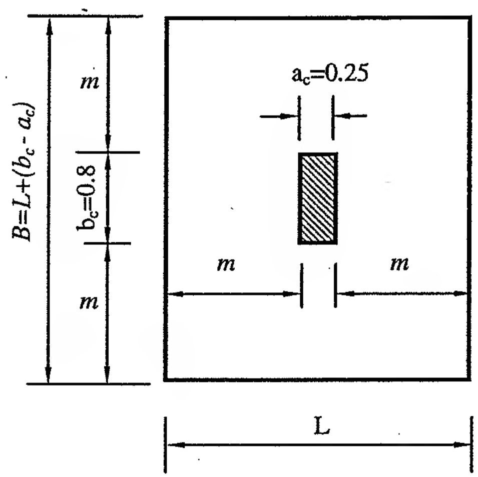
$$P_w = 780 + 440 = 1220\ kN$$

The weight of the footing and the soil are assumed (5-10% of the acting working loads) say 8%

The required area of the plain concrete footing equals

$$Area = \frac{1.08 \times P_w}{q_{allowable}} = \frac{1.08 \times 1220}{125} = 10.54\,m^2$$

To have uniform soil pressure and economic design, the dimensions of the footing are taken such that the cantilever distance (m) is same on all sides of the column.



Thus, the dimension (B) must be greater than (L) by the difference in column dimension as follows

$$B = L + (b_c - a_c) = L + (0.8 - 0.25) = L + 0.55$$

The area of the plain concrete equals

$$L\ (L + 0.55) = 10.6$$

Solving the previous equation for L gives

$$L = \frac{-0.55 + \sqrt{0.55^2 + 4 \times 10.6}}{2} = 2.99\ \text{m} \qquad \text{say L=3.0m}$$

$$B = \frac{10.54}{3} = 3.513\ m \qquad \text{say B=3.55 m}$$

The plain concrete dimension is chosen (3 × 3.55 ms)

The thickness of the plain concrete is chosen equal to 350 mm.

Step 2: Dimensioning of the reinforced concrete footing

From Table 5.1, x/t=1.18 → Assume x=t=0.35 m

$$A_r = L - 2 \times 0.35 = 3 - 0.7 = 2.3\ m$$

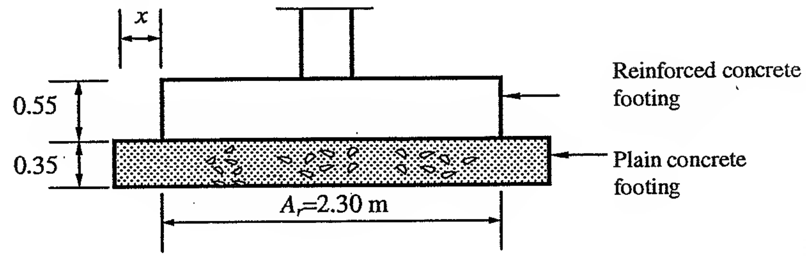
$$B_r = B - 2 \times 0.35 = 3.55 - 0.7 = 2.85\ m$$

$$P_u = 1.4\,P_{DL} + 1.6\,P_{LL} = 1.4 \times 780 + 1.6 \times 440 = 1796\ kN$$

The factored net soil pressure q_{su} equals

$$q_{su} = \frac{P_u}{A_r \times B_r} = \frac{1796}{2.3 \times 2.85} \cong 274\ kN/m^2$$

Assuming that the thickness of the RC footing is 0.55 m, the chosen dimensions are (2.3 x 2.85 x 0.55 ms).



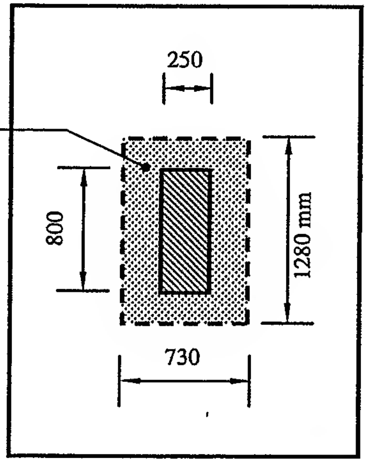
Step 3: Design for punching shear

Generally, the thickness of the isolated footing is governed by punching shear. The critical section for punching shear is at d/2 from the face of the column. Assuming 70 mm concrete cover, the effective depth d equals 480 mm

$$a = a_c + d = 250 + 480 = 730\,mm$$

$$b = b_c + d = 800 + 480 = 1280\,mm$$

$$U = 2\,(a+b) = 2\,(730+1280) = 4020\ mm$$



The punching load = column load - the load acting on the punching area

$$Q_{up} = P_u - q_{su}\,(a \times b) = 1796 - 274\,(0.73 \times 1.28) = 1540\text{ kN}$$

$$q_{up} = \frac{Q_{up}}{U \times d} = \frac{1540 \times 1000}{4020 \times 480} = 0.798\ N/mm^2$$

The concrete strength for punching is the least of the three values

$$q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{30}{1.5}} = 1.41\ N/mm^2$$

$$q_{cup} = 0.316\,(0.50 + \frac{a_c}{b_c})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\,(0.50 + \frac{0.25}{0.8})\sqrt{\frac{30}{1.5}} = 1.15\ N/mm^2$$

$$q_{cup} = 0.8\,(0.20 + \frac{\alpha\, d}{U})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\left(0.20 + \frac{4 \times 0.48}{4.02}\right)\sqrt{\frac{30}{1.5}} = 2.42\,N/mm^2$$

q_{cup}= 1.15 N/mm^2

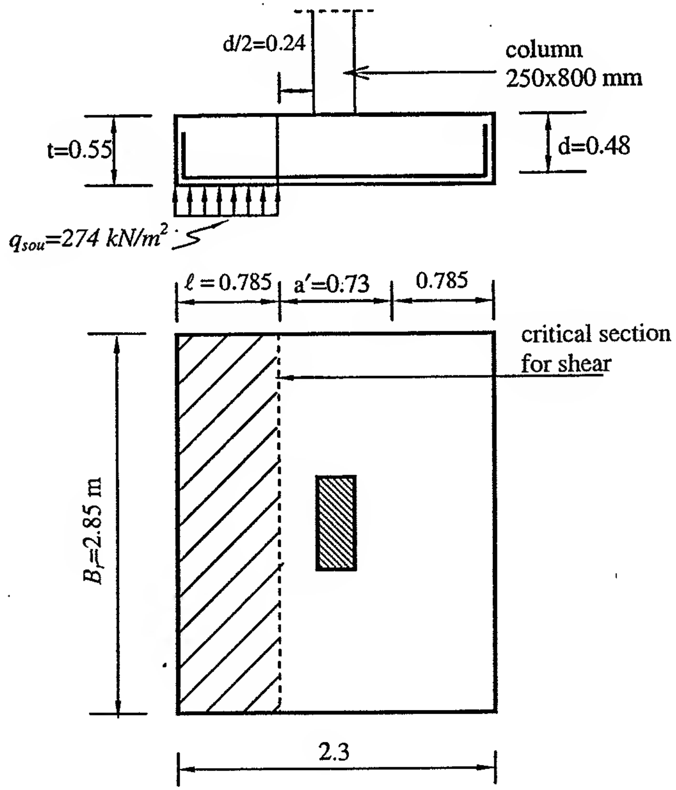
Since the applied shear stress (0.798) is less than concrete shear strength (1.15) , the footing is considered safe

Step 4: Design for one-way shear

The critical section for one-way shear is taken at d/2 from the face of the column as shown in figure.

$$a = 0.25 + d/2 + d/2 = 0.25 + d = 0.25 + 0.48 = 0.73\ m$$

$$\ell = \frac{A_r - a}{2} = \frac{2.3 - 0.73}{2} = 0.785\ m$$



$$Q_u = q_{su} \cdot \ell . B_r = 274 \times 0.785 \times 2.85 = 613 \ kN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{613 \times 1000}{2850 \times 480} = 0.45 \ N/mm^2$$

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{1.5}} = 0.16\sqrt{\frac{30}{1.5}} = 0.715 \ N/mm^2$$

Since q_u is less than q_{cu} the footing is considered safe in shear

Step 5: Flexural design

Step 5.1: Reinforcement in the short-direction

The critical section is at the face of the column, and taking a strip of 1m

$$M_u / m' = q_{su} \frac{(A_r - a_c)^2 \times 1.0}{8} = 274 \frac{(2.3 - 0.25)^2}{8} = 143.93 \ kN.m/m'$$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{143.93 \times 10^6}{30 \times 1000 \times 480^2} = 0.021$$

From the chart with R=0.021, the reinforcement index ω=0.0245

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.0245 \times \frac{30}{360} \times 1000 \times 480 = 980 \, mm^2$$

$$A_{s,\min} = \frac{0.60}{f_y} \times b \times d = \frac{0.60}{360} \times 1000 \times 480 = 800 \ mm^2 < A_s$$

A_s=980 mm^2

Use 5Φ16/m′ (1000 mm^2)

Step 5.2: Reinforcement in the long direction

The critical section is at the face of the column, and taking a strip of 1m

$$M_u = q_{su} \frac{(B_r - b_c)^2}{8} = 274 \frac{(2.85 - 0.80)^2}{8} = 143.93 \ kN.m/m'$$

Since the moment in the long direction is the same as the moment in the short direction, the reinforcement is taken identical (*same*) to that of the short direction.

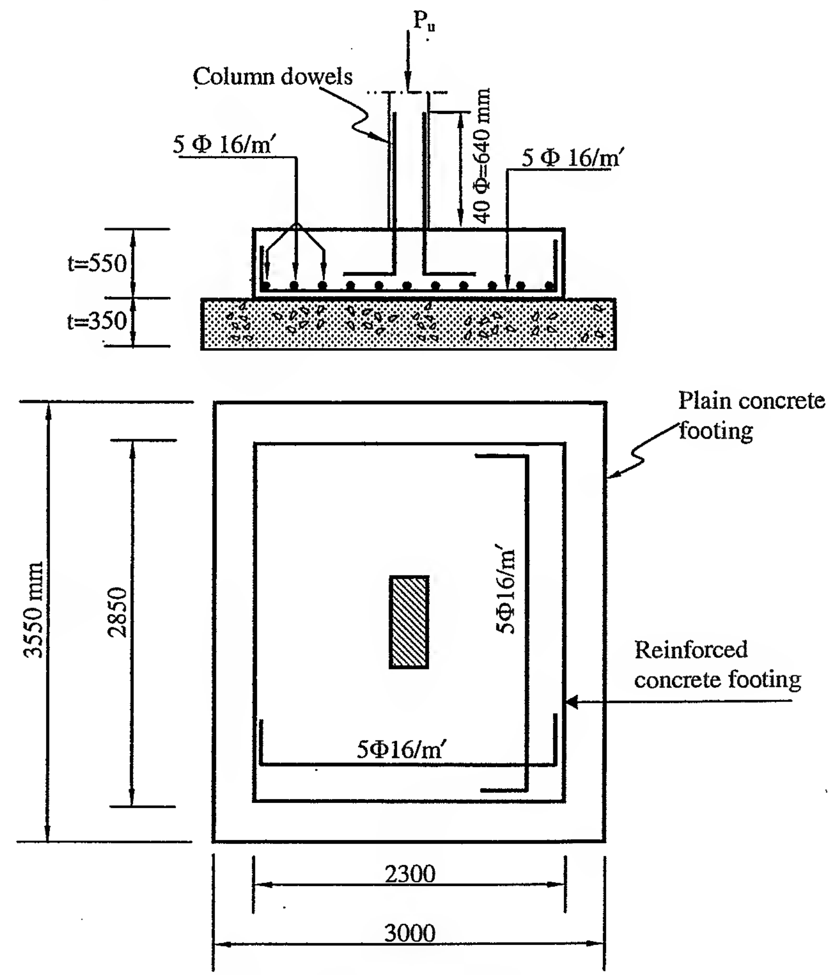
Step 5.3: Check the development of the reinforcement

For simplicity, the values listed in the Egyptian code to determine the development length is used. For high grade steel without hooks the development length L_d=60 Φ.

For 16 mm diameter, L_d=960 mm

Bar extension past the face of the column equals

$$\frac{A_r - a_c}{2} = \frac{2300 - 250}{2} = 1025 \ mm > L_d .....o.k$$

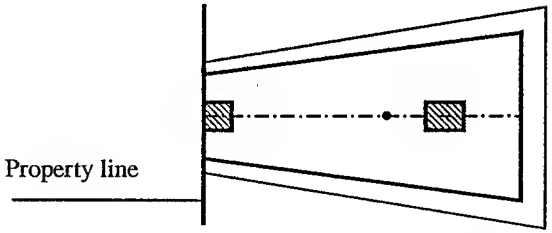


Footing reinforcement details

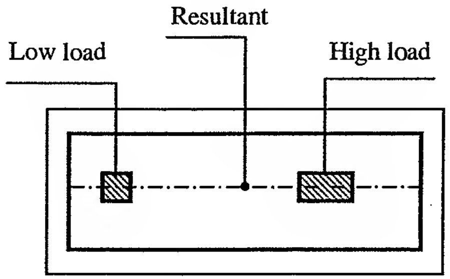
5.7 Combined Footings

Combined footings are used when either one of the columns falls on a property line or when the two columns are close to each other such that the footings overlaps. The geometry of the footing is chosen such that the resultant of the two columns coincides with the centroid of the footing. This can be achieved by using trapezoidal footing (Fig. 5.11a) or by adjusting the center of the footing at the resultant as shown in Fig. 5.11b. The resulting pressure is uniform under the footings and help to prevent differential settlement. It is common to place the reinforced concrete footing above a plain concrete footing to reduce cost as shown in Fig. 5.11c. In some cases the resulting moments between the columns may become large and it may be economical to use an inverted T-beam to increase the effective depth and reduce the reinforcement as shown in Fig. 5.11d.

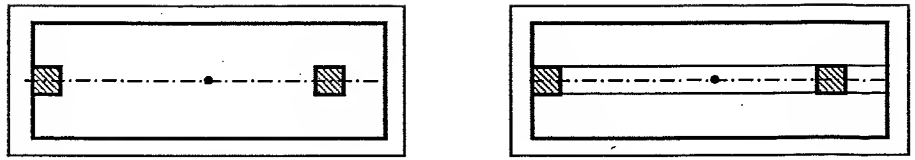
The basic assumption for the design of a combined footing is to assume that the footing is rigid and is subjected to a linear soil pressure. In actual practice, it requires very big thickness to make rigid footing. However, the assumption of rigid footing has been used successfully over the years. The combined footing can be designed as beam on elastic foundation that usually leads to more economic solutions. However, this method is time consuming and is not suitable for design office calculations.



a) Trapezoidal combined footing



b) Rectangular combined footing



c)Rectangular combined footing with PC base

d)Rectangular combined footing with PC base and T-beam

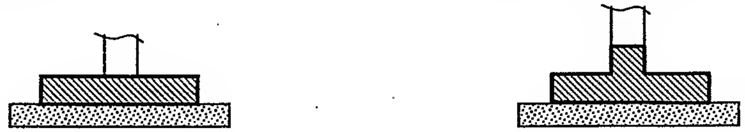


Fig. 5.11 Type of combined footings (cont.)

In combined footings, soil pressure is resisted by a series of strips running in the longitudinal direction as shown in Fig 5.12. The load is then transmitted to the cross beams AB and CD, which transmit the pressure to the columns. The cross (*hidden*) beams are assumed to extend d/2 from the face of the column. The main top longitudinal reinforcement is placed between the two columns, while the main bottom longitudinal reinforcement is placed under the columns. Main transverse reinforcement is placed at the bottom at locations of the cross beams.

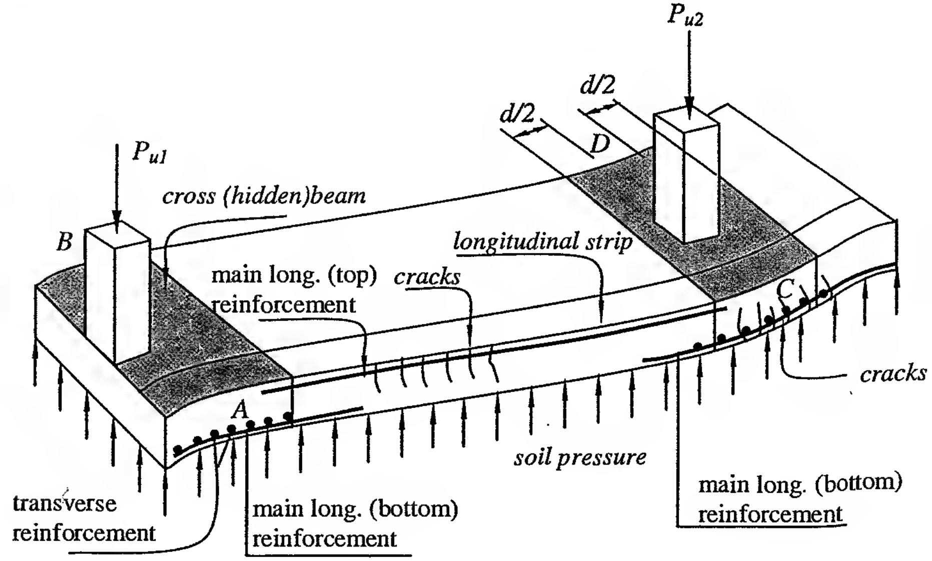
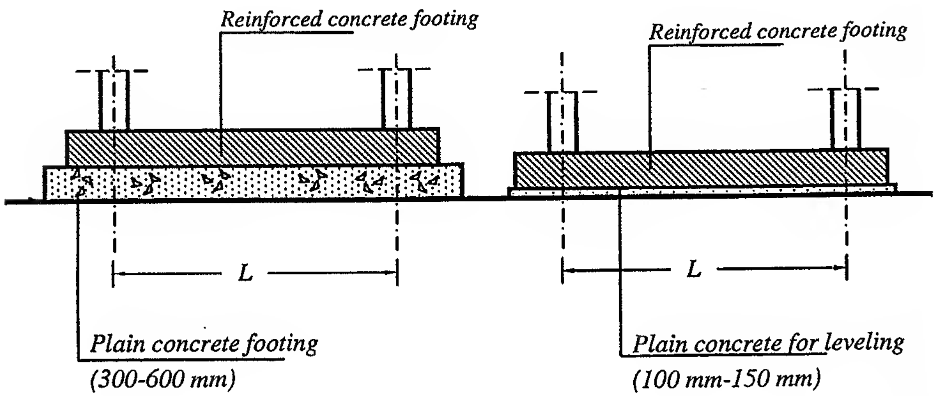


Fig. 5.12 Analysis of combined footings

It is customary in Egypt to construct reinforced concrete footings on top of plain concrete footings mainly for economical reasons as shown in Fig. 5.13a. However, it is also popular around the world to use reinforced concrete footings directly resting on soil after providing a thin layer of plain concrete (100 mm) for leveling as shown in Fig. 5.13b. Such a design approach is also adopted in Egypt in some few projects.

The analysis is carried out in a similar manner to that to that explained before with the exception of ignoring any contribution of the plain concrete. Therefore, the dimensions of the reinforced concrete footing should be chosen to distribute the applied loads safely to the soil.



a) Plain concrete is used in the design calculations.

b) Plain concrete is not used in the design calculations.

Fig. 5.13 Reinforced concrete combined footings with or without plain concrete footings

Example 5.4 illustrates the design of a combined footing that is resting on a plain concrete footing, while Example 5.5 illustrates the design of a combined footing that is resting directly on the soil.

Example 5.4: Combined footing with PC

Design a combined footing to support the two columns shown in Fig. EX 5.4. Column C_1 has a cross section of (0.3 m x 0.4 m) and supports a working load of 1320 kN. Column C_2 has a cross section of (0.3 m x 0.7 m) and supports a working load of 1960 kN. Assume that the allowable soil pressure is 175 kN/m^2, and the material properties are f_{cu}=25 N/mm^2, and f_y=400 N/mm^2.

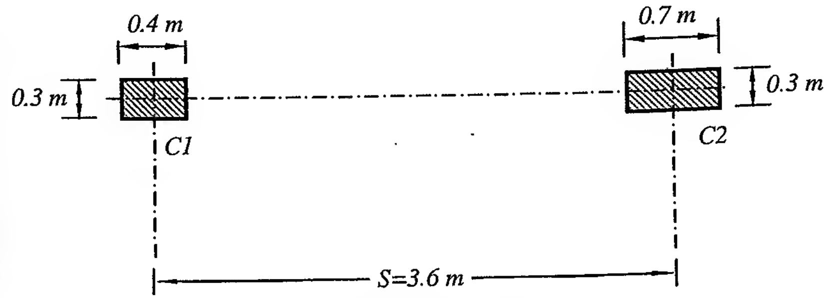
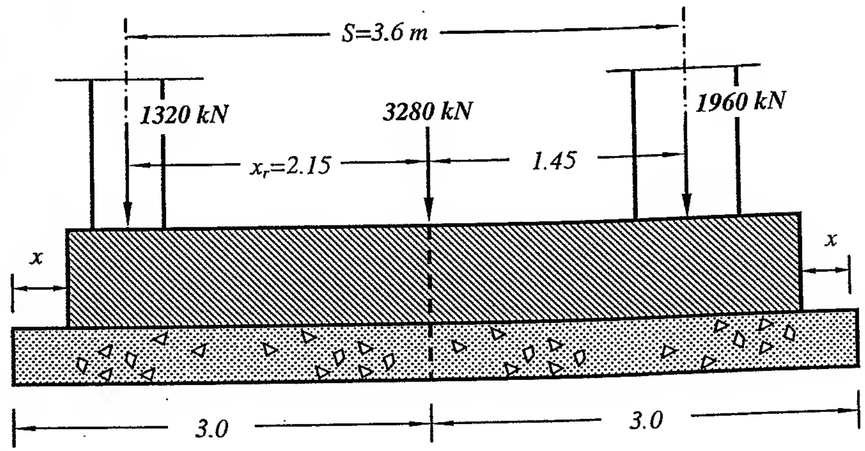


Fig. EX 5.4 Layout of Columns

Solution

Step 1: Dimensions of the plain concrete footing

The location of the resultant force is determined by taking moments of all forces about any point. Taking moment about the c.g. of column $C1$, one gets:



$$x_r = \frac{P_{C2} \times S}{P_{C1} + P_{C2}} = \frac{1960 \times (3.6)}{(1320 + 1960)} = 2.15\ m$$

To ensure uniform pressure throughout the footing, the centroid of the footing must coincide with the resultant. Assume that the length of the footing is L.

$$L = 2x_r + thickness\ of\ C1/2 + thickness\ of\ C2/2 + (1-2m) \approx 6.0 m$$

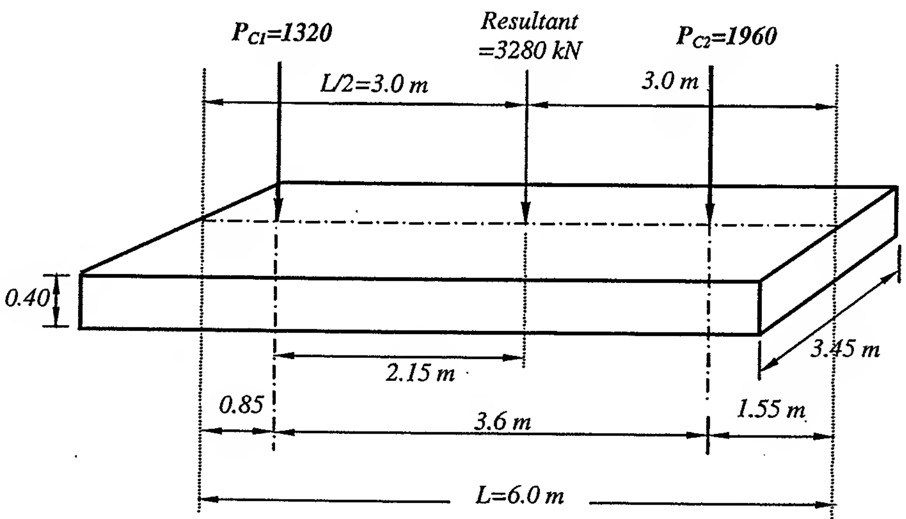
The width of the footing is determined from the allowable soil pressure. Assuming that the weight of the footing is about 10% of the total applied loads, the width of the footing equals:

$$B = \frac{1.1\ (P_{C1} + P_{C2})}{\sigma_{all} \times L} = \frac{1.1 \times (1320 + 1960)}{175 \times 6} = 3.44\ m$$

Rounding B to the nearest 50 mm $\rightarrow$ B=3.45 m.

The plain concrete footing dimensions are (6.0 m x 3.45m) and its thickness is assumed 400 mm.

$$The\ pressure\ (\sigma_{act}) = \frac{1.1\ (1320 + 1960)}{6.0 \times 3.45} = 174.3\ kN/m^2 < 175 ......o.k$$



Dimensions of the plain concrete footing

Step 2: Dimensions of the reinforced concrete footing

To ensure that a uniform pressure is acting under the reinforced concrete footing, the centroid of the footing must coincide with the resultant. Assume that the distance x = thickness of the plain concrete = 0.4 m.

$$L_1 = L - 2x = 6 - 2 \times 0.4 = 5.2\ m$$

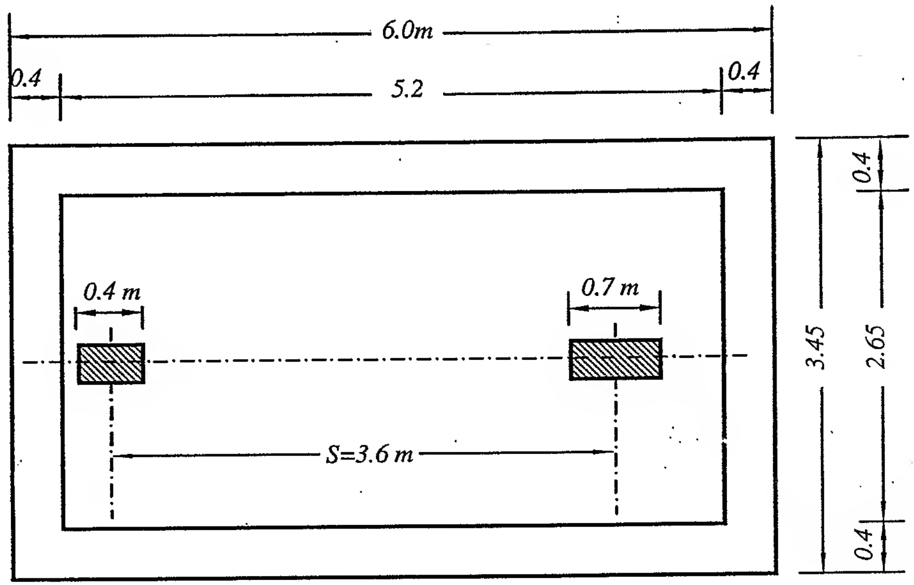
$$B_1 = B - 2x = 3.45 - 2 \times 0.4 = 2.65\ m$$

Assume the thickness of the RC footing is 0.8 m. Hence, the dimensions of the RC footing are (5.2 m x 2.65 m x 0.8 m).

The ultimate pressure is used to calculate the moments and shear forces. Assuming that the live loads are less than 75% of the dead loads (the usual case), the ultimate loads equal:

$$P_{u(C1)} = 1.5 \times P = 1.5 \times 1320 = 1980\ kN$$

$$P_{u(C2)} = 1.5 \times P = 1.5 \times 1960 = 2940\ kN$$



Step 3: Design the footing for flexure

Step 3.1: Longitudinal direction

The ultimate pressure under the reinforced concrete footing is calculated for the total width B_1, thus the load acting on 2.65 m width equals:

$$f = \frac{P_{u1} + P_{u2}}{L_1} = \frac{1980 + 2940}{5.2} = 946.1538 \ kN/m'$$

The computation of shear and moment may be carried out in a normal fashion. For example, for the location at (x=3.0 m), the forces equal:

$$Q_u = f \cdot x - P_{u1} = 946.15 \times 3 - 1980 = 858 \ kN$$

$$M_u = f \cdot x^2/2 - P_{u1} \ (x - 0.45)$$

$$M_u = 946.15 \times 3^2/2 - 1980 \times (3 - 0.45) = -791.3 \ kN.m$$

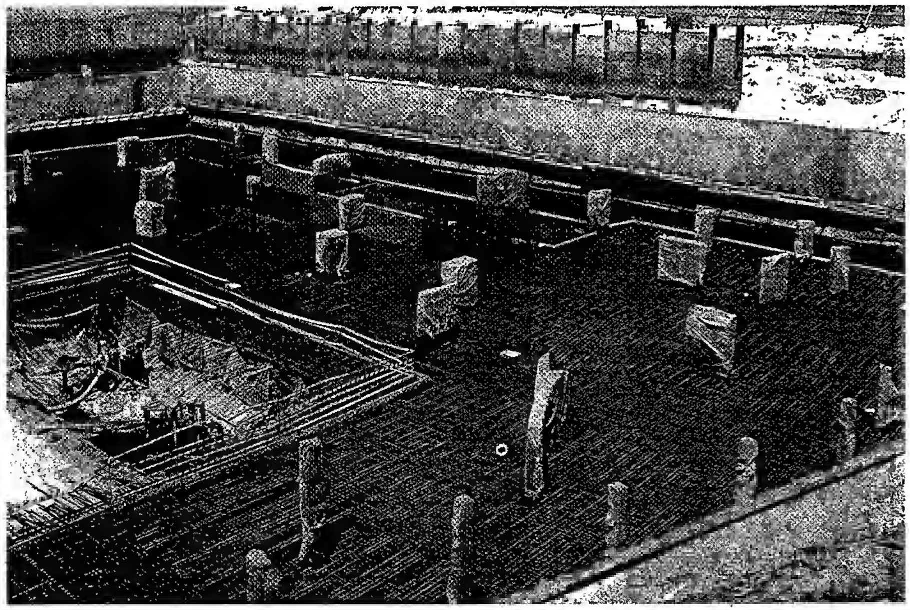
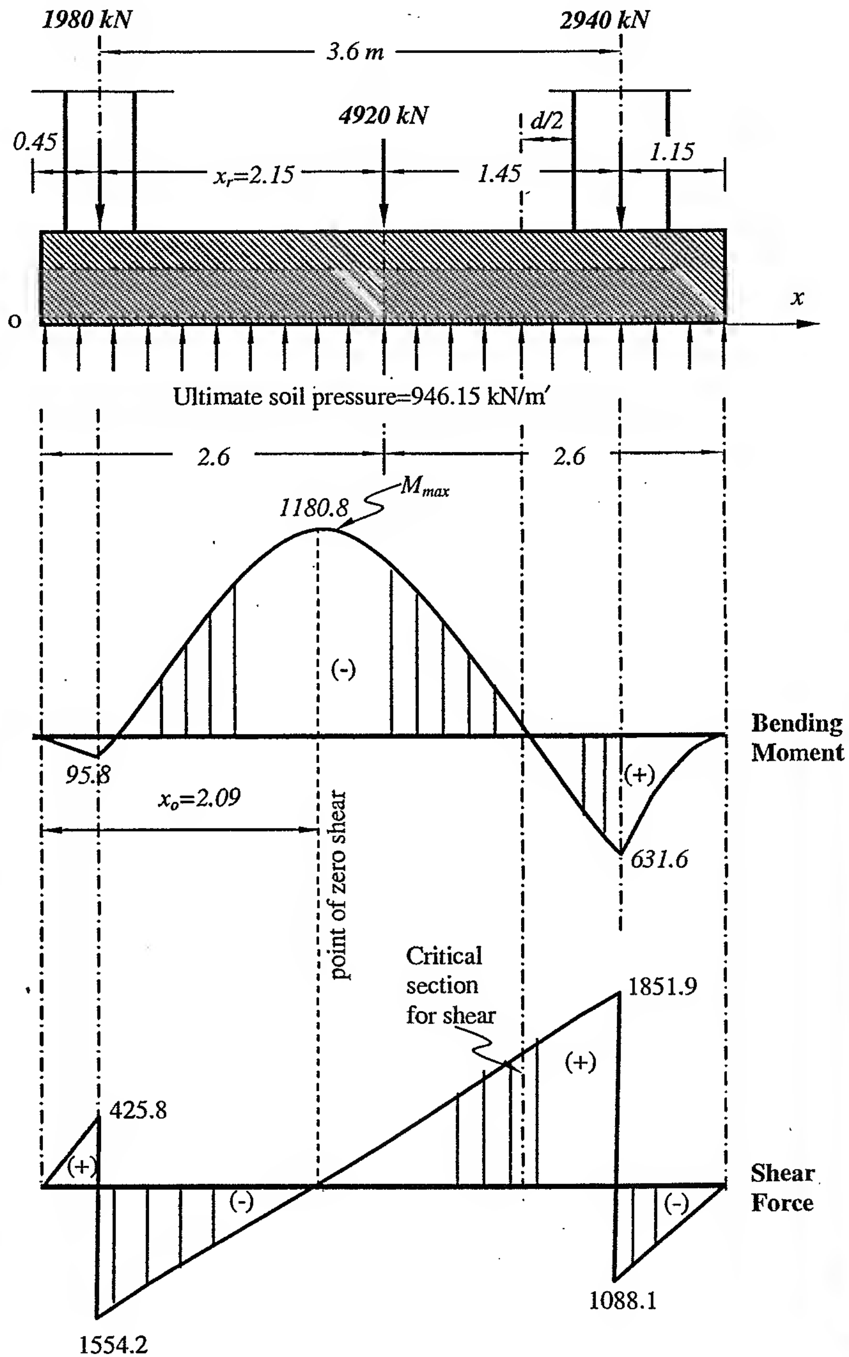


Photo 5.5 Foundations during construction



The calculation of the shear and moment may become tedious; therefore a computer program was used to generate the straining actions at different points as shown in the table below. Plots of the shear and moment are also given in the following figure.

Program Foundation: output file: combined

Location (x)(m)	Shear kN	Moment kN.m	Notes
0.00	0	0	
0.25	236.5	29.6	face of left column
0.45	425.8	95.8	C.L. of left column
0.45	-1554.2	95.8	C.L. of left column
0.65	-1365.0	-196.1	face of left column
1.00	-1033.8	-615.9	
1.50	-560.8	-1014.6	
2.09	0.0	-1180.8	point of zero shear, M_{max}
2.50	385.4	-1102.3	
3.00	858.5	-791.3	
3.70	1520.8	41.4	face of right column
4.05	1851.9	631.6	C.L. of right column
4.05	-1088.1	631.6	C.L. of right column
4.40	-756.9	302.6	face of right column
5.20	0.0	0	

To determine the maximum moment, the point of zero shear force is calculated as follows:

$946.15x_o - 1980 = 0$

x_o=2.093 m

Thus, the value of the maximum moment equals:

$$M_{max} = 946.15 \times 2.093^2/2 - 1980 \times (2.093 - 0.45) = 1180.8\ kN.m$$

Design the section of maximum negative bending moment

The section of maximum negative bending moment requires top reinforcement. Since the maximum moment is calculated for the full width of the footing, its value shall be divided by the footing width to get the moment per meter.

$$M_{max}/m' = \frac{M_{max}}{B_1} = \frac{1180.8}{2.65} \cong 446\ kN.m/m'$$

Assume the effective depth $d = t - 70\,mm = 800 - 70 = 730\ mm$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{446 \times 10^6}{25 \times 1000 \times 730^2} = 0.0335$$

From the chart with R=0.0335, the reinforcement index ω=0.040

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.040 \times \frac{25}{400} \times 1000 \times 730 = 1825\ mm^2/m'$$

$$A_{s\,min} = \frac{0.6}{f_y} \times b \times d = \frac{0.6}{400} \times 1000 \times 730 = 1095\ mm^2/m'$$

A_{smin} =1095 mm^2

Use 6Φ20/m′ (1885 mm^2/m′)

Secondary bottom reinforcement in the transversal direction should be provided with an area of at least 20% of the main reinforcement. Therefore, provide 5Φ12/m′.

Design the section of the maximum positive bending moment

The section of maximum positive bending moment requires bottom reinforcement. The critical section for the maximum **positive** bending is at the face of the right support.

$$M = 946.15 \times \frac{0.8^2}{2} = 302.8\,kN.m$$ (Refer also to the output table)

$$M/m' = \frac{M}{B_1} = \frac{302.8}{2.65} = 114.25\ kN.m/m'$$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{114.25 \times 10^6}{25 \times 1000 \times 730^2} = 0.0086$$

Since the intersection point is below the chart, the factor ω can be approximately evaluated as $\omega \cong 1.2\ R = 0.0103$.

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.0103 \times \frac{25}{400} \times 1000 \times 730 = 471 \; mm^2/m'$$

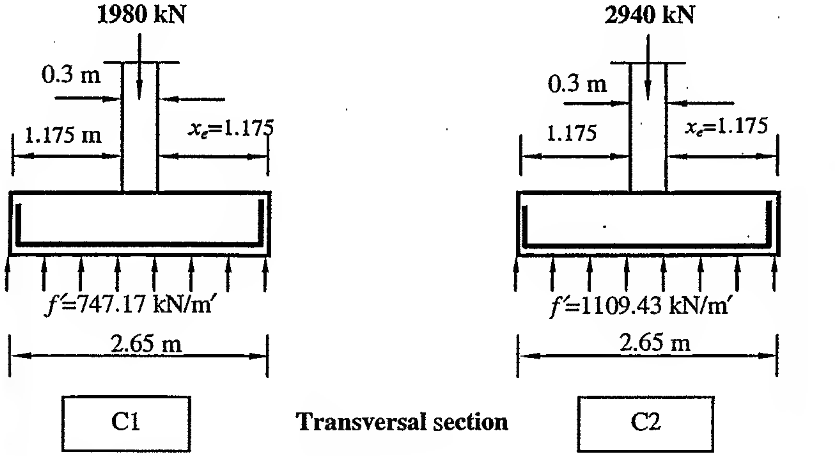
$$A_{s\,min} = \frac{0.60}{f_y} \times b \times d = \frac{0.60}{400} \times 1000 \times 730 = 1095 \; mm^2/m'$$

Use 6Φ16/m′ (1206 mm²/m′)

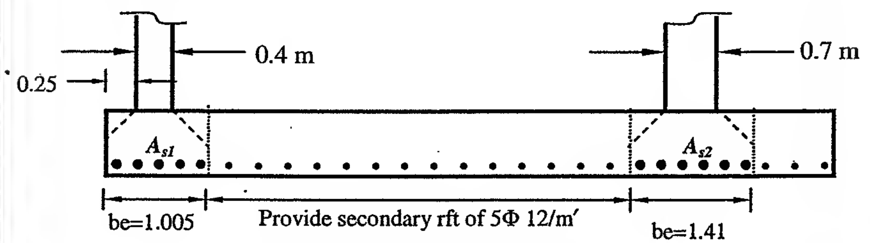
Step 3.2: Design of the footing for flexure (transversal)

To obtain the pressure (f') under each cross-beam (hidden beam), column load is divided by the footing width (2.65 m). The breadth of the cross beam (b_e) is assumed at d/2 from the column face (in the perpendicular direction) as shown in figure. The critical section for moment is at the face of the support.

The transverse bottom reinforcement is placed on the top of the longitudinal reinforcement, thus the effective depth is=730-20=710 mm



Transversal section



Effective width of cross beams

The calculations of the reinforcement are summarized in the following table

Item	C1	C2
Load P_u, kN	1980	2940
pressure $f' = P_u / 2.65$	747.17	1109.43
$M = f' x_e^2 / 2$	515.8	765.856
d (mm)	710	710
be (mm)	$= 0.25 + 0.4 + \frac{0.71}{2} = 1.005$	$= 0.7 + \frac{0.71}{2} + \frac{0.71}{2} = 1.41$
$R = \frac{M_u}{f_{cu}\, b_e\, d^2}$	0.041	0.043
ω	0.0492	0.0522
$A_s = \omega f_{cu} / f_y \; b \; d$	2194	3267
$A_{s\,min} = (0.6/fy) \times b_e \; d$	1070	1502
$A_{s,required}$ (mm²)	2194	3267
$A_{s,chosen}$ (mm²)	A_{s1}=6Φ22 (2281 mm²)	A_{s2}=9Φ22 (3421 mm²)

Step 4: Design for shear

The critical section for shear is at d/2 from the face of the column. Referring to the shear force diagram and the computer output table, the maximum shear at the face of column C_2 is (1520.8 kN). Hence, at a distance of d/2 from the left face of the support Q_u equals:

$$Q_u = Q - f'\left(\frac{d}{2}\right) = 1520.8 - 946.15\left(\frac{0.73}{2}\right) = 1176 \; kN$$

This shearing force is resisted by the full width of the footing (B=2650 mm), hence the nominal shear stress is given by:

$$q_u = \frac{Q_u}{B \times d} = \frac{1176 \times 1000}{2650 \times 730} = 0.61 \; N/mm^2$$

This shear stress must be resisted by the shear resistance of concrete, which is given by the following equation:

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{1.5}} = 0.16\sqrt{\frac{25}{1.5}} = 0.65\,N/mm^2$$

Since q_u is less than q_{cu}, the design for one-way shear is considered adequate.

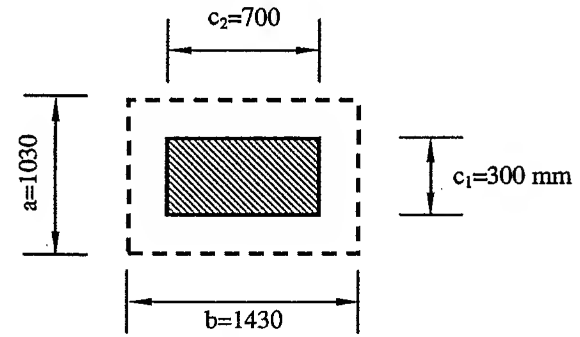
Step 5: Design for punching shear

The critical perimeter is at d/2 from the face of the column. For the interior column, the critical perimeter equals:

$$a = c_1 + d = 300 + 730 = 1030\ mm$$

$$b = c_2 + d = 700 + 730 = 1430\ mm$$

$$U = 2\,(a+b) = 2\,(1030+1430) = 4920\,mm$$



The pressure acting on the footing is given by:

$$f_2 = \frac{f}{B} = \frac{946.15}{2.65} = 357.04\ \text{kN/m}^2$$

The punching load equals:

$$Q_{up} = P_u - f_2\,(a \times b) = 2940 - 357.04\,(1.03 \times 1.43) = 2414\ \text{kN}$$

$$q_{up} = \frac{Q_{up}}{U \times d} = \frac{2414 \times 1000}{4920 \times 730} = 0.67\ N/mm^2$$

The concrete strength for punching is the least of the following three values:

1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{25}{1.5}} = 1.29\ N/mm^2$

2. $q_{cup} = 0.316\,(0.50 + \frac{a}{b})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\,(0.50 + \frac{0.3}{0.7})\sqrt{\frac{25}{1.5}} = 1.20\,N/mm^2$

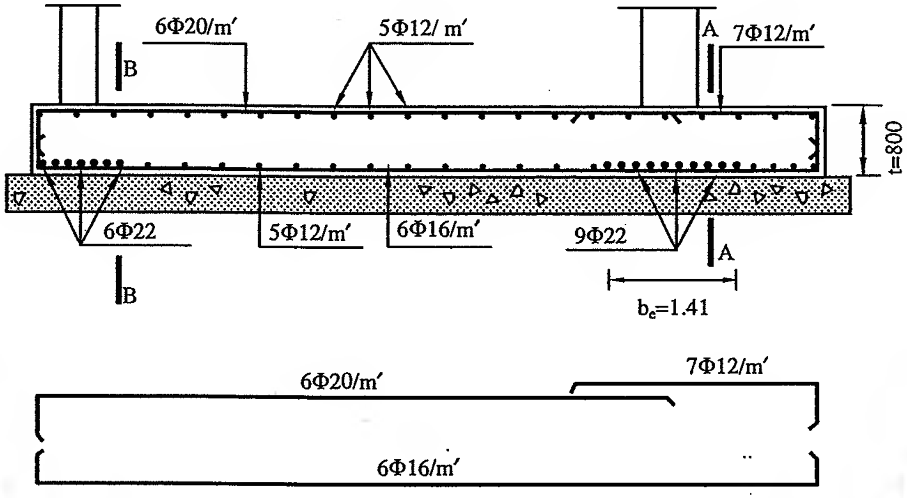
3. $q_{cup} = 0.8\,(0.20 + \frac{\alpha\ d}{U})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\left(0.20 + \frac{4 \times 0.73}{4.92}\right)\sqrt{\frac{25}{1.5}} = 2.59\ N/mm^2$

q_{cup}= 1.20 N/mm^2

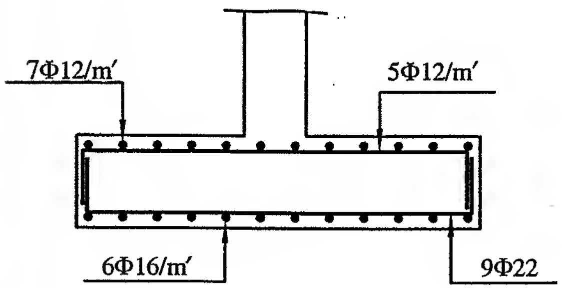
Since the applied punching shear stress (0.67) is less than concrete shear strength (1.2), the footing is considered safe.

The exterior column should also be checked for punching. The ultimate punching shear stress equals 0.54 N/mm^2 which is less than the concrete strength (*calculation not shown*).

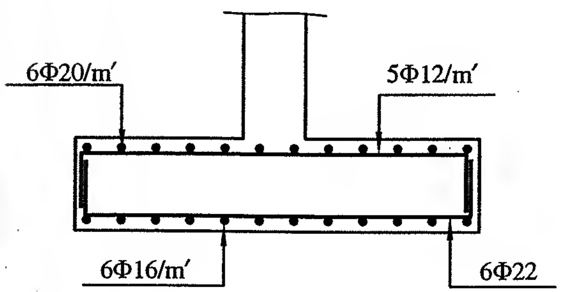
Photo 5.6 Portal bridge spanning 146 m (pont du bonhomme, France)



Longitudinal reinforcement



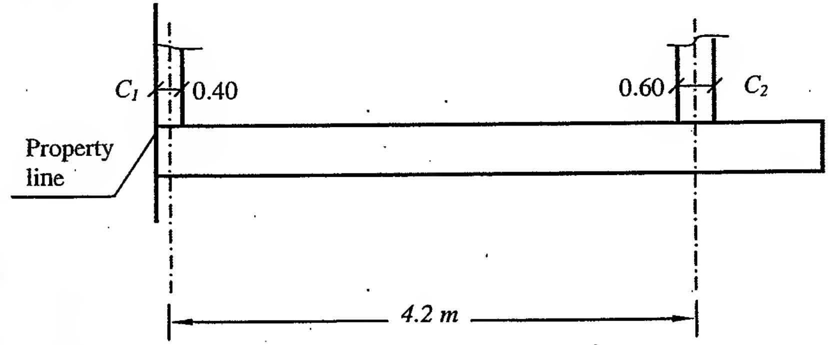
Transverse reinforcement (Sec A-A)



Transverse reinforcement (Sec B-B)

Example 5.5: RC combined footing resting directly on soil

Design a combined footing to support an exterior column C_1 (0.3 m x 0.4 m) carrying a total service load of 1100 kN and an interior column C_2 (0.3 m x 0.6 m) carrying a total service load of 1600 kN. The plain concrete is used only for leveling the reinforced concrete footing.
Assume that the allowable soil pressure is 185 kN/m^2, f_{cu}=30 N/mm^2 and f_y=400 N/mm^2



Solution

Step 1: Estimate the dimensions of the RC footing

The location of the resultant force is determined by taking moment about point A (see the figure below):

$$x_r = \frac{1100\times0.2+1600\times(4.2+0.2)}{(1100+1600)} = 2.69\ m$$

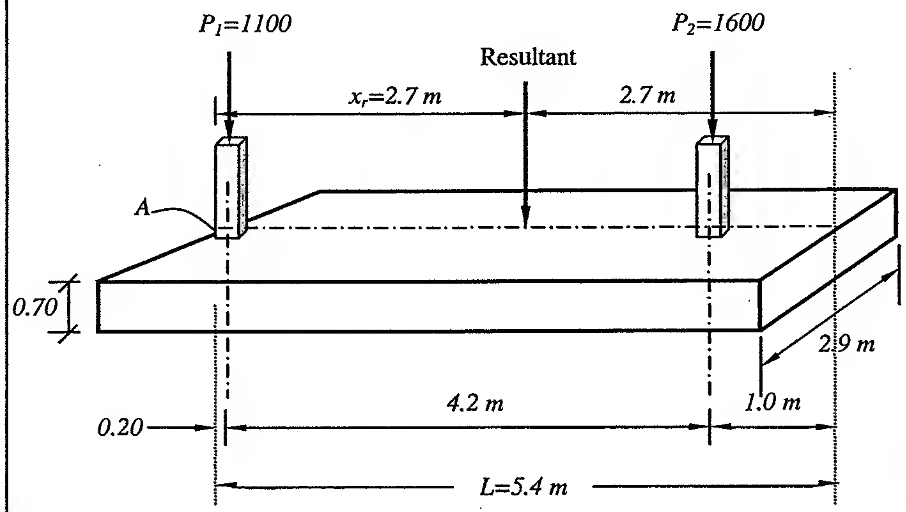
To ensure uniform pressure throughout the footing, the centroid of the footing must coincide with the resultant of the loads @ 2.69 m. Thus the length of the footing will be 5.38 m, say L=5.40 m.

Assume that the weight of the footing is about 10% of the total applied loads. The width of the footing is determined from the allowable soil pressure as follows:

$$B = \frac{1.05\times(1100+1600)}{185\times5.4} = 2.84\,m \quad \rightarrow Take\ B = 2.9\,m$$

$$The\ pressure\ (\sigma_{act}) = \frac{1.05\times(1100+1600)}{5.4\times2.9} = 181\ kN/m^2 < 185\ kN/m^2\ \ .....ok$$

Assume the thickness of the RC footing is 700 mm.



Step 2: Calculate the bending moments and the shear forces

The ultimate pressure is now used to calculate the moment and shear force. Assuming that the live loads are less than 75% of the dead loads (the usual case), the ultimate loads equal:

$$P_{u1} = 1.5 \times P = 1.5 \times 1100 = 1650\ kN$$

$$P_{u2} = 1.5 \times P = 1.5 \times 1600 = 2400\ kN$$

The load for the full 2.9 meters equals:

$$f = \frac{P_{u1} + P_{u2}}{L} = \frac{1650 + 2400}{5.4} = 750\ kN\ / m'$$

The computation of shear and moment may be carried out in a normal fashion. For example, for the location at 3.0 m, the forces equal:

$$Q = 750 \times 3 - 1650 = 600\ kN$$

$$M = 750 \times 3 \times 1.5 - 1650 \times (3 - 0.2) = -1245\ kN.m$$

The calculation of the shear and moment may become tedious; therefore a computer program was used to generate the straining actions at different locations as shown in the table below. Plots of the values of the shear forces and bending moments are also given in the following figure.

Program Combined Foundation: output file: combined

Location (m)	Shear force (kN)	Bending moment (kN.m)
0.00	0	0
0.20	150	15
0.20	-1500	15
0.40	-1350	-270
1.00	-900	-945
2.20	0	-1485
3.00	600	-1245
4.10	1425	-131
4.40	1650	375
4.40	-750	375
4.70	-525	184
5.40	0	0

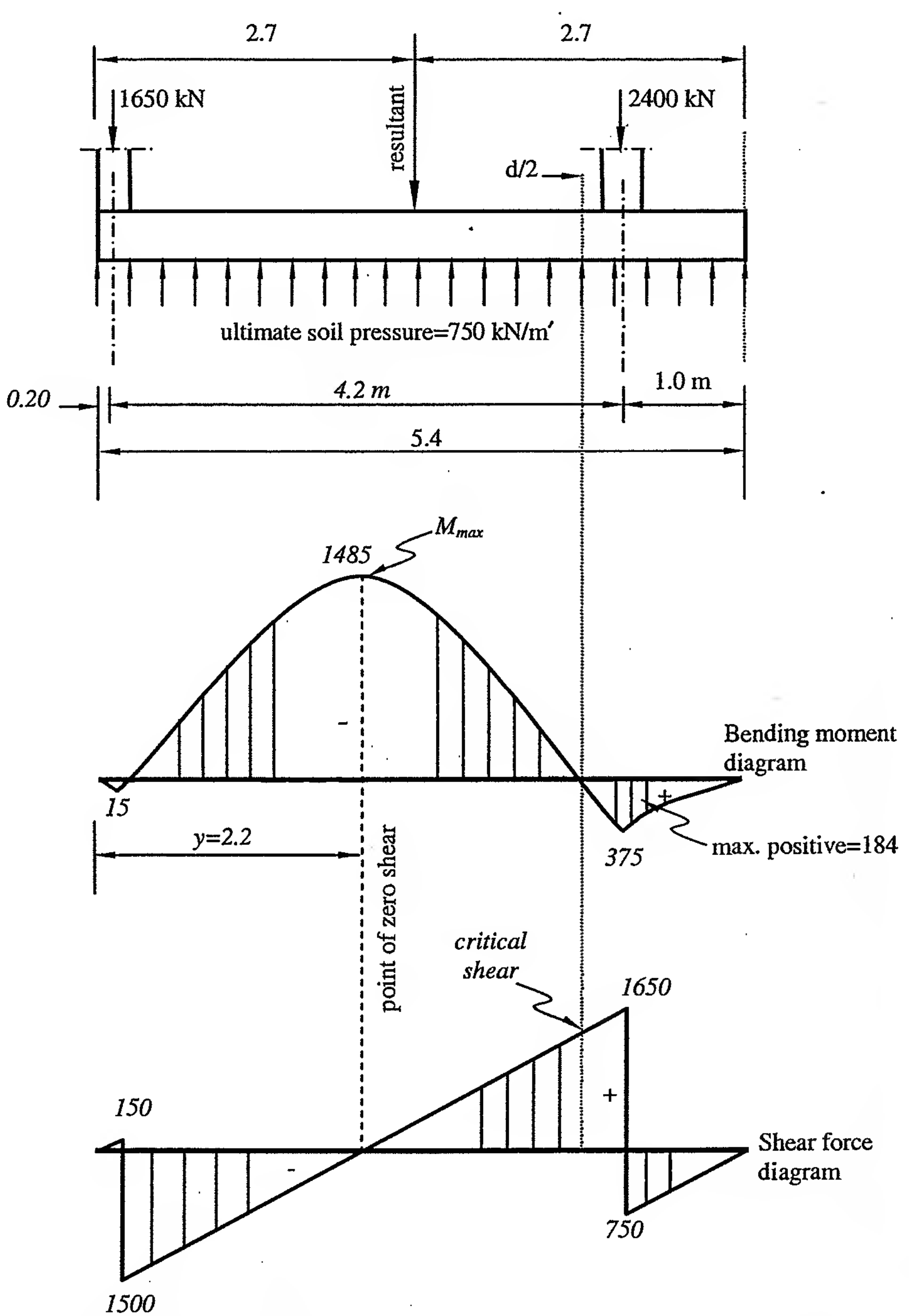
To determine the maximum moment, the point of zero shear is calculated as follows:

$$750\,y - 1650 = 0$$

y =2.2 m

The value of the maximum moment is given as:

$$M_{max} = 750 \times 2.2^2 / 2 - 1650 \times (2.2 - 0.2) = -1485\ kN.m$$



Values of the shear forces and bending moments for the combined footing

Step 3: Design the footing for flexure

Step 3.1: Longitudinal direction

Section of maximum negative moment

The section of maximum negative moment requires top reinforcement. The maximum negative moment per meter is given by:

$$M_{max}/m' = \frac{M_{max}}{B} = \frac{1485}{2.9} = 512\ kN.m/m'$$

Assuming that the distance from the c.g. of the reinforcing steel to the concrete surface is 70 mm, the effective depth equals:

$$d = t - 70\,mm = 700 - 70 = 630\ mm$$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{512 \times 10^6}{30 \times 1000 \times 630^2} = 0.043$$

From the chart with R=0.043, the reinforcement index ω=0.052

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.052 \times \frac{30}{400} \times 1000 \times 630 = 2457\ mm^2/m'$$

$$A_{s\,min} = \frac{0.60}{f_y} \times b \times d = \frac{0.60}{400} \times 1000 \times 630 = 945\ mm^2/m'$$

Use 7Φ22/m′ (2660 mm^2)

Section of maximum positive moment

The critical section for the maximum positive bending is at the face of the support, from the output table this moment equals 184 kN.m′

$$M/m' = \frac{M}{B} = \frac{184}{2.9} = 63.4\ kN.m$$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{63.4 \times 10^6}{30 \times 1000 \times 630^2} = 0.0053$$

The point is below the chart, use $\omega \approx 1.2\ R \cong 0.0064$

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.0064 \times \frac{30}{400} \times 1000 \times 630 = 302\ mm^2/m'$$

$$A_{s\,min} = \frac{0.60}{f_y} \times b \times d = \frac{0.60}{400} \times 1000 \times 630 = 945\ mm^2/m'$$

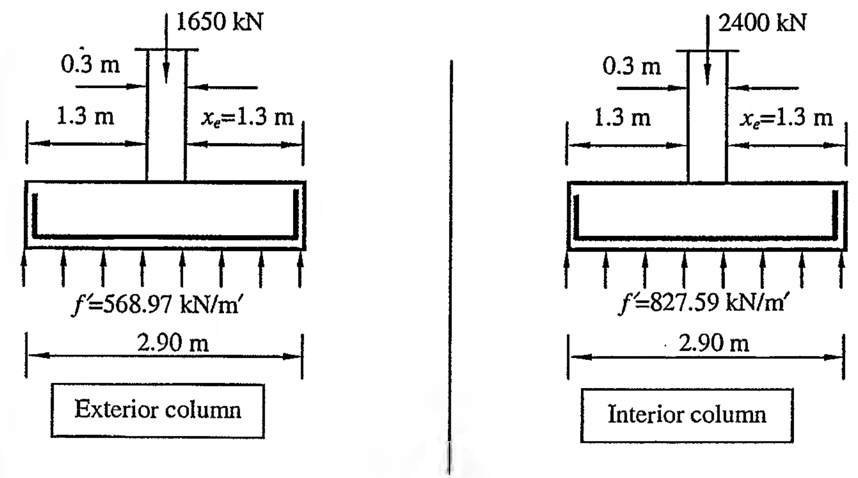
Since $A_s < A_{smin}$, use A_{smin}

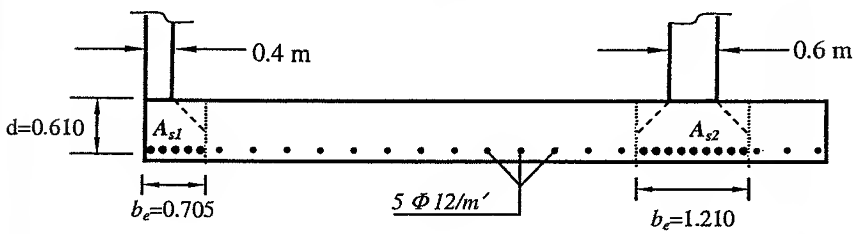
Use 5Φ16/m′ (1000 mm^2)

Step 4.2: Design the footing for flexure (hidden beams)

Transverse strip under each column will be assumed to transmit the load from the longitudinal direction to the column. The load under each column is divided by the footing width (2.9 m) t get the load per meter for the hidden beam. The breadth of the beam is assumed at d/2 from the column face. The critical section for moment is at the face of the support.

The reinforcement of the hidden beam is place on top of that of the footing. Hence, the effective depth is=700 -70 -20=610 mm





The calculations of reinforcement are shown in the following table

Item	Interior column	Exterior column
Load P_u, kN	1650	2400
pressure $f'=P_u/2.9$	568.97	827.59
$M = f'\,x_e^2/2$	480.8	699.310
b (mm)	705	1210
d (mm)	610	610
$R = \frac{M_u}{f_{cu}\,b_e\,d^2}$	0.06	0.05
ω	0.0759	0.0635
$A_s = \omega\,f_{cu}/f_y\ b\ d$	2447	3513
$A_{s,min}$	645	1107
$A_{s,required}$ (mm^2)	2447	3513
$A_{s,chosen}$ (mm^2)	A_{s1}=7Φ22 (2661 mm^2)	A_{s2}=10Φ22 (3810 mm^2)

Step 4: Design for shear

The critical section for shear is at d/2 from the face of the column. The maximum shear at the centerline of the interior column is equal to (1650 kN), thus at a distance of d/2 from the face of the support Q_u equals:

$$Q_u = Q - f\left(\frac{c}{2} + \frac{d}{2}\right) = 1650 - 750\left(\frac{0.6}{2} + \frac{0.63}{2}\right) = 1188.75\ kN$$

This shearing force is resisted by the full width of the footing (B=2900 mm).

$$q_u = \frac{Q_u}{B \times d} = \frac{1188.75 \times 1000}{2900 \times 630} = 0.65\ N/mm^2$$

This shear stress must be resisted by the concrete shear strength, which is given by the following equation:

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{1.5}} = 0.16\sqrt{\frac{30}{1.5}} = 0.72\ N/mm^2$$

Since q_u is less than q_{cu}, the design for one-way shear is considered adequate.

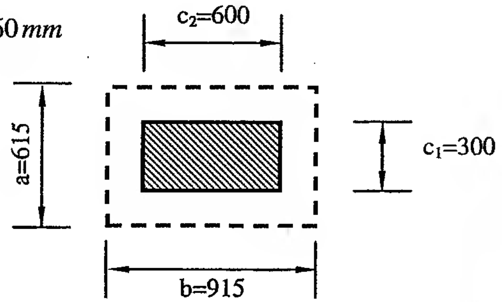
Step 5: Design for punching shear

The critical perimeter is at d/2 from the face of the column. For the interior column, the critical perimeter equals:

$$a = c_1 + \frac{d}{2} = 300 + \frac{630}{2} = 615\ mm$$

$$b = c_2 + \frac{d}{2} = 600 + \frac{630}{2} = 915\ mm$$

$$U = 2\ (a+b) = 2\ (615+915) = 3060\,mm$$



The acting pressure underneath the footing is given by:

$$f_2 = \frac{f}{B} = \frac{750}{2.9} = 258.62\ \text{kN/m}^2$$

The punching load equals column load minus the load inside the punching area.

$$Q_{up} = P_u - f_2\,(a \times b) = 2400 - 258.62\ (0.615 \times 0.915) = 2254.5\ \text{kN}$$

$$q_{up} = \frac{Q_{up}}{U \times d} = \frac{2254.5 \times 1000}{3060 \times 630} = 1.17\ N/mm^2$$

The concrete strength for punching the least of the three values

1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{30}{1.5}} = 1.41\ N/mm^2$

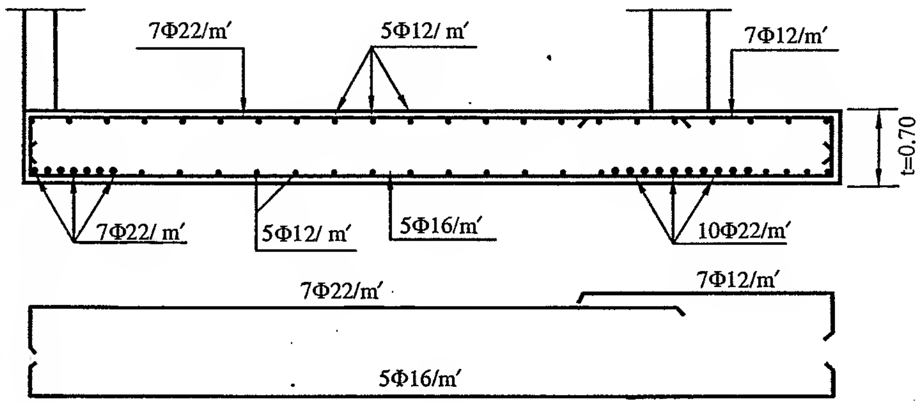
2. $q_{cup} = 0.316\ (0.50 + \frac{c_1}{c_2})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\ (0.50 + \frac{0.3}{0.6})\sqrt{\frac{30}{1.5}} = 1.41\ N/mm^2$

3. $q_{cup} = 0.8\ (0.20 + \frac{\alpha\ d}{U})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\left(0.20 + \frac{4 \times 0.63}{3.06}\right)\sqrt{\frac{30}{1.5}} = 3.66\ N/mm^2$

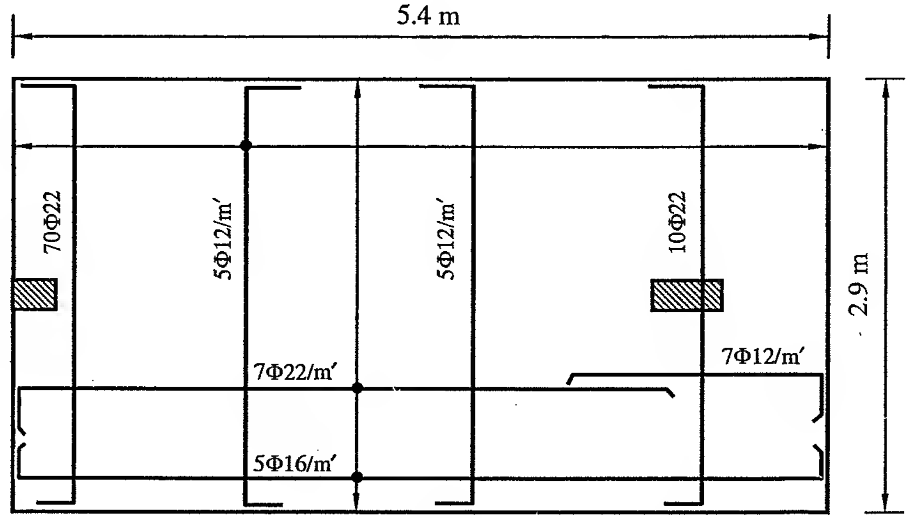
q_{cup}= 1.41 N/mm^2

Since the applied shear stress is less than concrete shear strength, the footing is considered safe.

The exterior column should also be checked for punching because its perimeter is only from three sides. The ultimate punching shear stress equals 1.08 N/mm^2 which is less than the concrete strength (*calculations are not shown*).



Section



Plan

5.8 Strap Footings

If one of the columns in a building is constructed near the property line, the column will be eccentric with respect to the center of gravity of the footing as shown in Fig. 5.14. The eccentric position of the footing causes uneven soil pressure distribution, which could leads to tilting of the footing. To avoid such a tilting, the exterior footing is connected to the interior footing with a massive beam called *strap beam.*

The dimensions of the strap are chosen such that it is very rigid compared to the footing. *J. E. Bowles* recommended that the rigidity of the strap beam is at least twice that of the footing ($I_{strap} / I_{footing} > 2$).

The dimensions of the footing are chosen such that the bearing pressures are uniform and equal under both bases. Therefore, the centroid of the combined area of the two footings must coincide with the resultant of the two loads. The strap beam joining the footings should not bear against the soil.

It is common to neglect the strap weight in the design. The strap should be adequately attached to the both the column and the footing by the use of dowels such that the footing and the strap act as one unit. The footing is subjected to one-way bending. The strap beam is reinforced with main reinforcement at the top between the columns and at bottom under the interior footing.

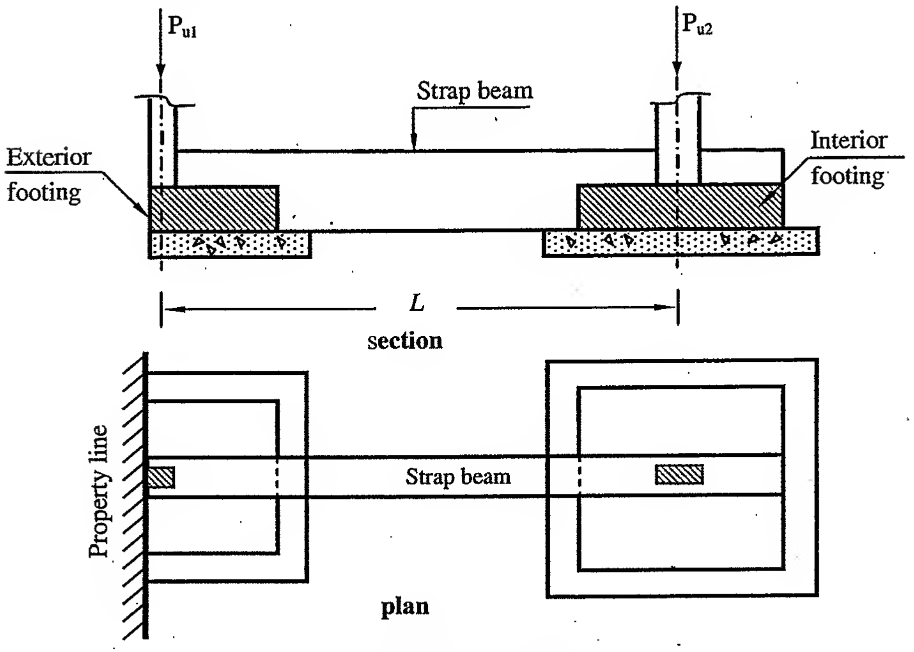
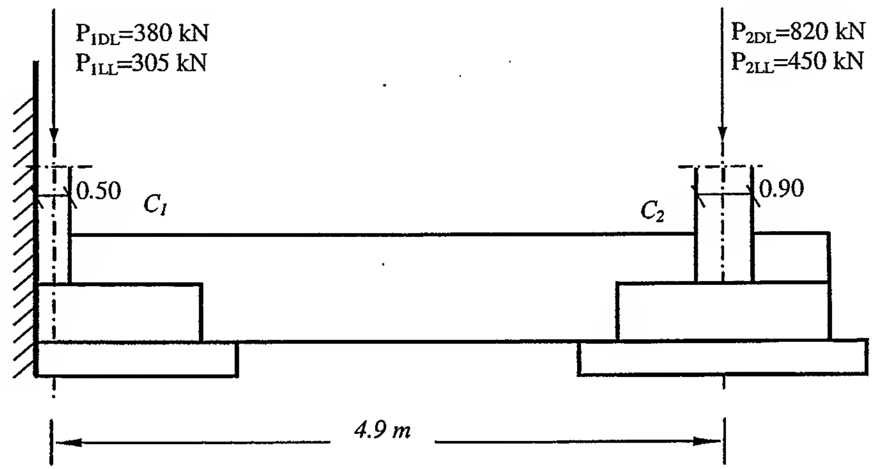


Fig. 5.14 Strap footing

Example 5.6

Design a strap footing to support an exterior column (0.30 m x 0.50 m) and an interior column (0.30 m x 0.90 m). The unfactored dead and live loads carried by each column are shown in the figure below. Assume that the allowable soil pressure is 150 kN/m^2, f_{cu}=25 N/mm^2 and f_y=360 N/mm^2



Solution

Step 1: Estimate the dimensions of the plain concrete footing

The working loads are calculated as:

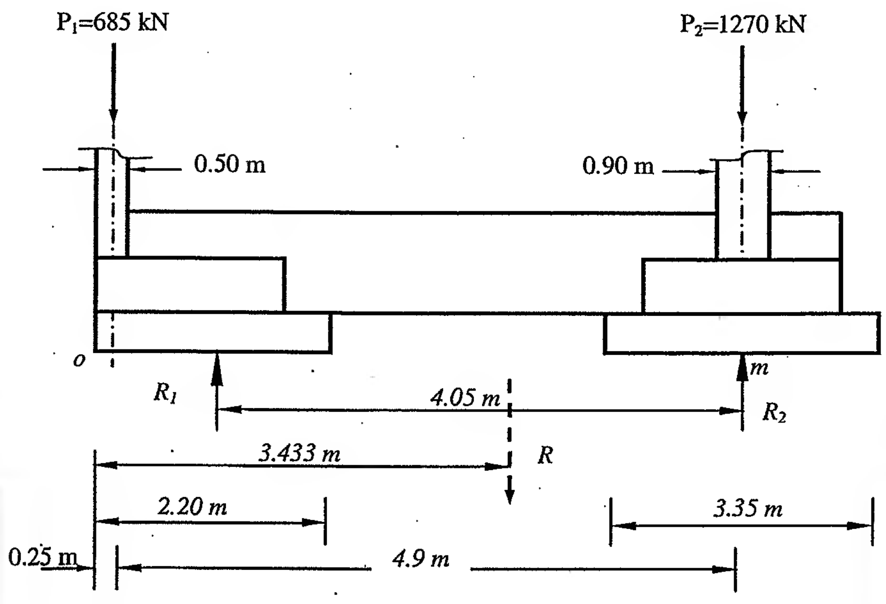
$$P_1 = 380 + 305 = 685 \ kN$$

$$P_2 = 820 + 450 = 1270 \ kN$$

The location of the resultant of the loads may be determined by taking moment about point o.

$$y = \frac{685 \times 0.25 + 1270 \times 5.15}{685 + 1270} = 3.433 \ m$$

The length of the exterior and interior footings should be assumed such that the pressures under the two footings are almost the same. This is achieved by having the resultant of the loads coincided with the c.g. of the footing.



The weight of the strap beam, the footings, and the soil above may be estimated as 10% of the total loads.

$$P_{total} = 1.1\,(P_1 + P_2) = 1.1\,(685 + 1270) = 2150.5\,kN$$

The total required area of the plain concrete footing under C1 and C2 equals to:

$$Area = \frac{P_{total}}{q_{allowable}} = \frac{2150.5}{150} = 14.34\ m^2$$

The thickness of the plain concrete is assumed to be 400 mm. Assume that the length of the exterior footing is 2.2 m and the length of the interior footing is 3.35 m.

To reasonably determine the width of the footings, the reactions R_1 and R_2 are calculated by taking moments about R_2.

$$R_1 = \frac{(1.1\times685)\times4.9}{(4.9+0.25-2.2/2)} = 911.64\ kN \qquad B_1 = \frac{911.64}{150\times2.2} = 2.76\ m \rightarrow 2.8\,m$$

$$R_2 = 2150.5 - 911.64 = 1238.9\ kN \qquad B_2 = \frac{1238.9}{150\times3.35} = 2.46\ m \rightarrow 2.6\,m$$

The final chosen dimensions of the plain concrete footings are

Item	L (m)	B (m)	Area (m^2)
Exterior footing	2.2	2.8	6.16
Interior footing	3.35	2.6	8.71
Total			14.87> 14.34

The c.g. of the footings can be obtained by taking moment of area about point o.

$$y = \frac{6.16\times1.1 + 8.71\times5.15}{14.87} = 3.47\ m$$

Note that the center of gravity of the footings (3.47 m) is very close to the location of the resultant of the loads (3.43m).

Step 2: Dimensions of the reinforced concrete footings

The dimensions of the reinforced concrete footing can be determined as shown in the following table.

Item	L_1 (m)	B_1 (m)	Area (m^2)
Exterior footing	=2.2-0.4=1.8	=2.8-0.8=2.0	3.6
Interior footing	=3.35-0.8=2.55	=2.6-0.8=1.80	4.59
Total			8.19

To ensure the uniform stress distribution, the c.g. of the reinforced concrete footings should also coincide with the resultant as much as possible (usually within 10% is acceptable).. The distance measured from the c.g. to point o equals:

$$y = \frac{3.6\times1.8/2 + 4.59\times5.15}{8.19} = 3.28\ m$$

The location of the c.g. is close enough (to the location of the resultant.

Step 3: Calculate the ultimate pressure

To ensure that the strap beam will distribute the pressure uniformly, the concrete dimensions are taken 0.4 m x1.3 m. The own weight of the strap beam is usually neglected; however, it can be approximately added to the dead load of each column as follows:

$$w_{beam} = \gamma_c \times b \times t \times L = 25 \times 0.4 \times 1.3 \times 4.9 \cong 63\ kN$$

P_{1DL}=380+63/2= 411.5 kN

P_{2DL}=820+63/2= 851.5 kN

The ultimate loads for the columns are calculated in order to calculate the ultimate moment and shear.

$$P_{u1} = 1.4 \times P_{1DL} + 1.6 \times P_{1LL} = 1.4 \times 411.5 + 1.6 \times 305 \approx 1064\ kN$$

$$P_{u2} = 1.4 \times P_{2DL} + 1.6 \times P_{2LL} = 1.4 \times 851.5 + 1.6 \times 450 \approx 1912\ kN$$

To determine the magnitude of R_{u1} ,take the moment about R_{u2}.

$$R_{u1} = \frac{1064 \times 4.9}{(4.9 + 0.25 - 1.8/2)} = 1226.7\ kN$$

$$R_{u2} = P_{u1} + P_{u2} - R_{u1} = 1064 + 1912 - 1226.7 = 1749.3\ kN$$

The pressure under the exterior footing equals $\sigma_1 = \dfrac{1226.7}{1.80} = 681.5\ kN/m^2$

The pressure under the interior footing equals $\sigma_2 = \dfrac{1749.3}{2.55} = 686\ kN/m^2$

The resulting pressures are slightly different under each footing (681.5, 686). More uniform pressures can be attained by adjusting the dimensions of the footings. However, the attained accuracy is quite satisfactory (1% difference).

Step 4: Design of the strap beam

Step 4.1: Draw bending moment and shear force diagrams

The computation of shear and moment may be carried out in a normal fashion. For example, at a distance of 1.5 m from the left edge, the forces equal:

$$Q = 681.5 \times 1.5 - 1064 = -41.8\ kN$$

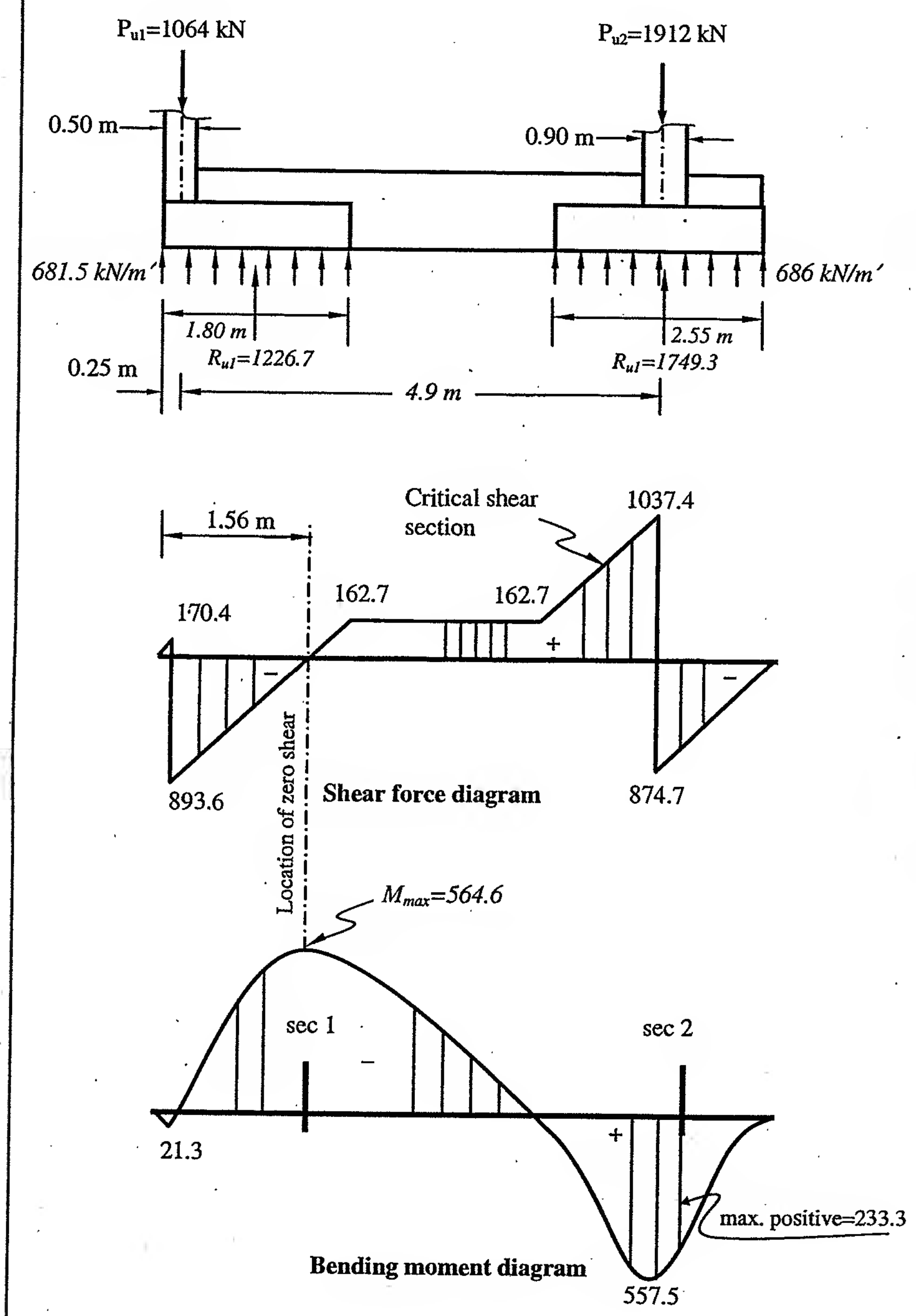
$$M = 681.5 \times \frac{1.5^2}{2} - 1064 \times (1.5 - 0.25) = -563.3\ kN.m$$

Point of zero shear $= \dfrac{1064}{681.5} = 1.56\ m$

A computer program was prepared to generate the straining actions at different locations as shown in the table below. Plots of the shear and moment is also in the following figure

Program Foundation: output file: strap

Location	Shear force (kN)	Bending moment (kN.m)	Notes
0.00	0.0	0.0	
0.25	170.4	21.3	C.L. of left column
0.25	-893.6	21.3	C.L. of left column
0.50	-723.3	-180.8	face of left column
1.00	-382.5	-457.3	intermediate point
1.50	-41.8	-563.3	intermediate point
1.56	0.0	-564.6	point of zero shear, M_{max}
1.80	162.7	-545.2	
3.00	162.7	-349.9	intermediate point
3.50	162.7	-268.6	intermediate point
3.88	162.7	-207.6	face of right column
4.70	728.7	160.1	
5.15	1037.4	557.5	C.L. of right column
5.15	-874.7	557.5	C.L. of right column
5.60	-566.0	233.3	face of right column
6.43	0.0	0.0	



Step 4.2: Design for flexure

Design of section 1

Assuming that the distance from the c.g. of the reinforcing steel to the concrete surface is 70 mm, the effective depth equals

$$d = t - 70mm = 1300 - 70 = 1230\ mm$$

The maximum moment equals 564.6 kN.m

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{564.6 \times 10^6}{25 \times 400 \times 1230^2} = 0.037$$

From the chart with R=0.037, the reinforcement index ω=0.045

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.045 \times \frac{25}{360} \times 400 \times 1230 = 1537\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{25}}{360} \times 400 \times 1230 = 1536\,mm^2 \\ 1.3 A_s = 1.3 \times 1537 = 1998\ mm^2 \end{cases}$$

Use 8Φ16/m′ (1608 mm^2)

Design of section 2

The critical section is at the face of the column, from the output table the maximum moment equals = 233.3 kN.m

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{233.3 \times 10^6}{25 \times 400 \times 1230^2} = 0.0154$$

From the chart with R=0.0154, the reinforcement index ω=0.019

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.019 \times \frac{25}{360} \times 400 \times 1230 = 649\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{25}}{360} \times 400 \times 1230 = 1536\ mm^2 \\ 1.3 A_s = 1.3 \times 649 = 844\ mm^2 \quad \lrcorner > A_s \quad \ldots use\ A_{s,min} \end{cases}$$

$$But\ not\ less\ than\ \frac{0.15}{100} \times 400 \times 1230 = 738\ mm^2$$

A_s=844 mm^2

Use 4Φ18/m′ (1017 mm^2)

Step 4.3: Design the strap beam for Shear

The critical section for shear is at the free span of the strap beam.

Q_u=162.7 kN

$$q_u = \frac{Q_u}{b \times d} = \frac{162.7 \times 1000}{400 \times 1230} = 0.33\ N/mm^2$$

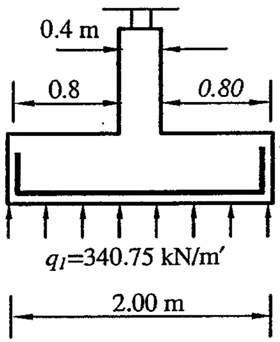
$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.98\ N/mm^2$$

Since $q_u < q_{cu}$, provide minimum stirrups. In addition, since the width of the beam equals 400 mm, stirrups with four branches shall be used. Assume spacing of 200 mm.

$$A_{st,\min} = \frac{0.4}{f_y} \times b \times s = \frac{0.4}{240} \times 400 \times 200 = 133.3$$

Try four branches 5ϕ 8/m′

$$A_{st} = 4 \times 50 = 200\ mm^2 > A_{st,\min}\ \ldots\ldots o.k$$



Step 5: Design of the footings

Step 5.1: Design for flexure

A strip of 1m width is taken to determine the area of steel for the footings.

$$q_1 = \frac{\sigma_1}{B_1} = \frac{681.5}{2} = 340.75\ kN/m^2$$

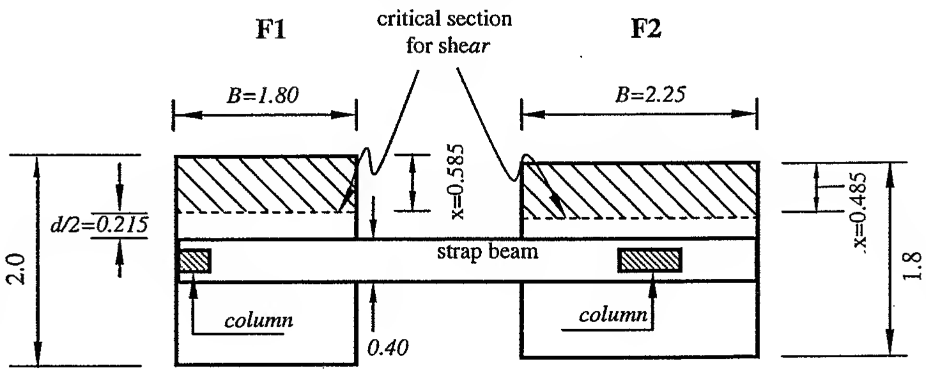
The moment is taken at the face of the strap beam as follows:

$$M_1 = \frac{q_1 \times (B_1 - b_{strap})^2}{8} = \frac{340.75 \times (2 - 0.40)^2}{8} = 109.04\ kN.m/m'$$

Assume that the depth of the footings is 500. the calculations may summarized in the following table

Item	Exterior footing	Interior footing
pressure σ (kN/m′)	681.5	686
Footing width B′ (m)	2.00	1.80
Pressure (f) $= \sigma / B'$ (kN/m^2)	340.8	381.1
Moment $= f \times (B' - b_{strap})^2 / 8$ (kN.m)	109.06	93.37
d (mm)	430	430
B(mm)	1000	1000
$R = M_u / (f_{cu}\ B\ d^2)$	0.024	0.02
ω	0.0284	0.0235
$A_{s,required}$(mm^2)$= \omega \times (f_{cu} / f_y) \times B \times d$	848.1	701.7
$A_{s,min}$(mm^2)=0.6/f_y B × d	716.66	716.6
$A_{s,chosen}$(mm^2) Max of(A_{smin}, $A_{srequired}$)	848.1	716
Reinforcement	1005.31 (5□16)/m′	1005.31 (5□16)/m′

Step 5.2 Design the footings for shear



Critical section for shear

The critical section for shear is at d/2 from the face of the strap beam. Noting that the depth d=0.430 m, the distance x equals to:

$$x = \frac{B - b_{strap}}{2} - \frac{d}{2} = \frac{1.8 - 0.4}{2} - \frac{0.43}{2} = 0.485\ m$$

Noting that the pressure under footing F_2 equals 381.1 kN/m^2, the shear force Q_u equals

$$Q_u = f \cdot x \cdot b = 381.1\,(0.485)2.55 = 471.30\ kN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{471.30 \times 1000}{(2.55 \times 1000) \times 430} = 0.43\ N/mm^2$$

The shear stress should be less than the concrete shear strength given by the following equation:

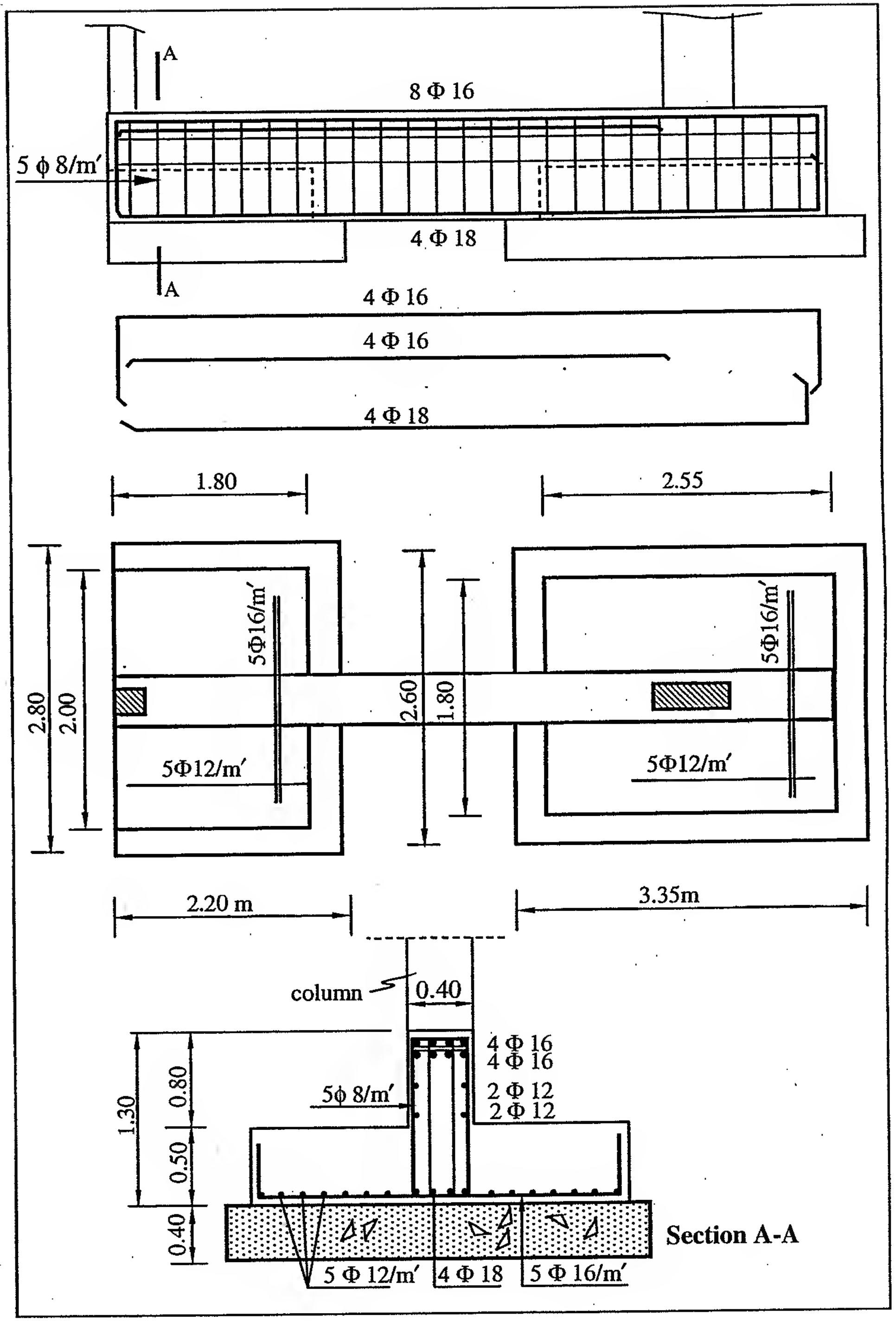
$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{1.5}} = 0.16\sqrt{\frac{25}{1.5}} = 0.65\ N/mm^2$$

Since q_u is less than q_{cu}, the footing is considered safe for shear

The design of F1 for shear is summarized in the following table

Item	F1
pressure kN/m^2	340.8
b(m)	1.8
x (m)	0.585
Q_u (kN)	358.90
q_u (N/mm^2)	0.46

It should be noted that the presence of the strap beam eliminates the need for calculating punching shear stresses for the footings.



Section A-A

5.9 Raft Foundations

5.9.1 Introduction

When the bearing capacity of the soil is low, isolated footings are replaced by a raft foundation. In such a case, a solid reinforced concrete rigid slab is constructed under the entire building as shown in Fig. 5.15. Structurally, raft foundations resting directly on soil act as a flat slab or a flat plate, upside down, i.e., loaded upward by the bearing pressure and downward by the concentrated column reactions. The raft foundation develops the maximum available bearing area under the building. If the bearing capacity of the soil is so low that even this large bearing capacity is insufficient, deep foundations such as piles must be used. Apart from developing large bearing areas, another advantage of raft foundations is that their continuity and rigidity that helps in reducing differential settlement of individual columns relative to each other, which might be caused by local variations in the quality of subsoil, or other causes.

The design of raft foundations may be carried out by one of two methods:

- The *conventional rigid method* and;
- The finite element method utilizing *computer programs.*

The conventional method is easy to apply and the computations can be carried out using hand calculations. However, the application of the conventional method is limited to rafts with relatively regular arrangement of columns.

In contrast, the finite element method can be used for the analysis of raft regardless of the column arrangements, loading conditions, and existence of cores and shear walls. Commercially available computer programs can be used. The user should, however, have sufficient background and experience.

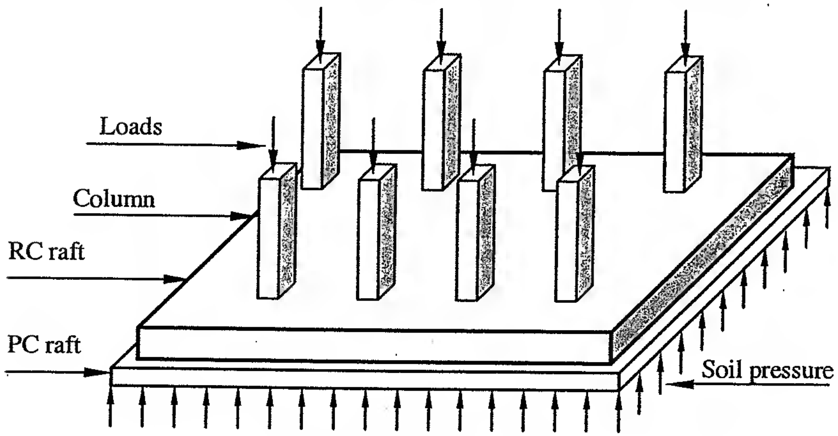


Fig. 5.15 Raft foundation

5.9.2 Conventional Rigid Method

The raft foundation shown in Fig. 5.16 has dimensions (B x L). Columns' working loads are indicated as P_1, P_2, P_3, ... etc. The application of the conventional method can be summarized as follows:

Step 1: Check soil pressure

The resultant of columns working loads equals:

$$P_{total} = P_1 + P_2 + P_3 + ... = \sum_{i=1}^{i=n} P_i \quad .... (5.20)$$

Assuming that the raft foundation is rigid, the soil pressure at any point can be obtained using the classical stress equation as follows:

$$q = \frac{P_{total}}{A} + \frac{M_x}{I_x}\, y + \frac{M_y}{I_y}\, x \le q_{allowable} \quad .... (5.21)$$

Where

A = area of the raft (B × L)

I_x = moment of inertia of the raft about x-axis = $B\ L^3/12$

I_y = moment of inertia of the raft about y-axis = $L\ B^3/12$

M_x = moment of the applied loads about the x-axis = $P_{total}\ e_y + M_{x\,(lateral\ load)}$

M_y = moment of the applied loads about the y-axis = $P_{total}\ e_x + M_{y\,(lateral\ load)}$

Where e_x and e_y are the eccentricities of the resultant from the c.g. of the raft.

The coordinates of the eccentricities are given by:

$$X' = \frac{P_1\ x_1 + P_2\ x_2 + P_3\ x_3 + ...}{P_{total}} \quad .... (5.22)$$

Where x_1, x_2, x_3 are the X-coordinates of P_1, P_2, P_3, P_i,......, P_n.

$$e_x = X' - \frac{B}{2} \quad .... (5.23)$$

$$Y' = \frac{P_1\ y_1 + P_2\ y_2 + P_3\ y_3 + ...}{P_{total}} \quad .... (5.24)$$

Where y_1, y_2, y_3 are the y-coordinates of P_1, P_2, P_3, P_i,......, P_n.

$$e_y = Y' - \frac{L}{2} \quad .... (5.25)$$

Compare the maximum soil pressures value with net allowable soil pressure.

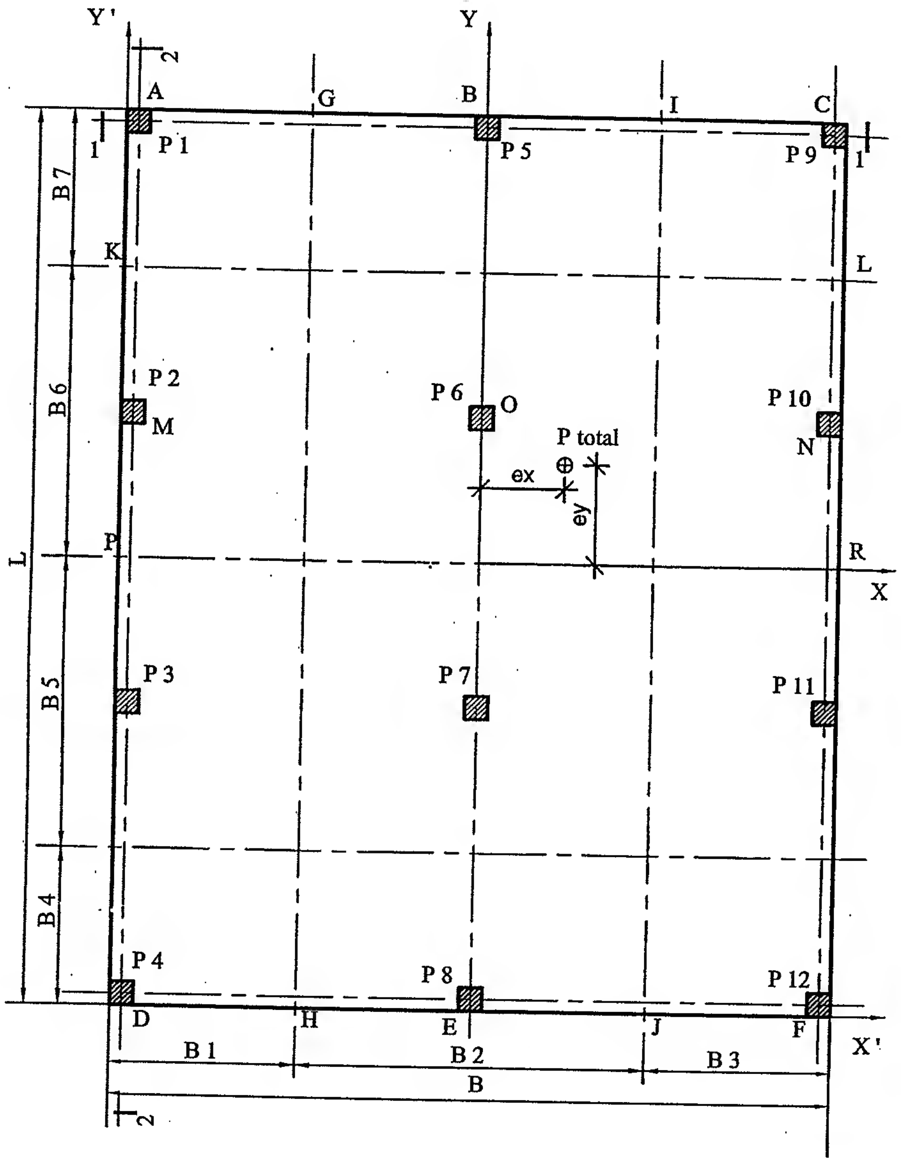


Fig. 5.16 Eccentricity of the raft

Step 2: Draw the shear force and bending moment diagrams

Divide the raft into several strips in the X-direction (B_1, B_2, B_3) and in the Y-direction (B_4, B_5, B_6, B_7) as shown in Fig. 5.16. Referring to Fig. 5.16, the interior strip GBIHEJ is used as an example for illustrating the procedure for drawing the shear force and the bending moment diagrams for the strips. The procedure may be summarized in the following steps:

1. The soil pressure at the center-line of the strip is assumed constant along the width of the strip. Referring to Fig. 5.17, the distribution of the soil pressure at the center-line of strip GBIHEJ is determined by calculating the pressures at points B (0,L/2) and E (0,-L/2) as follows:

$$q_B = \frac{P_{total}}{A} + \frac{M_x}{I_x}\frac{L}{2} \qquad (5.26)$$

$$q_E = \frac{P_{total}}{A} - \frac{M_x}{I_x}\frac{L}{2} \qquad (5.27)$$

The average pressure equals:

$$q_{avg} = \frac{q_B + q_E}{2} \qquad (5.28)$$

This value shall be used in the analysis of the strip.

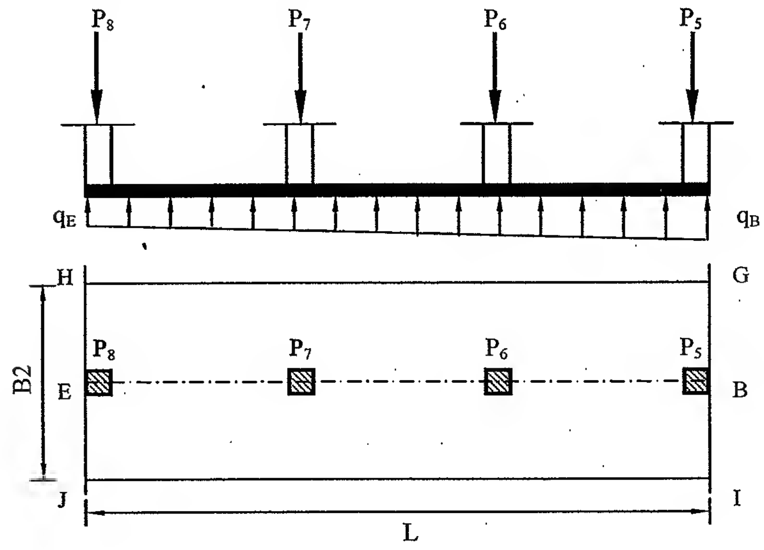


Fig. 5.17 Soil pressure distribution at the center BE

2. The total soil reaction (R_{B-E}) for the strip B-E is equal to:

$$R_{B-E} = q_{avg} \times B_2 \times L \quad \text{(5.29)}$$

Where B_2 is the width of strip B-E.

The total applied load acting on this strip equals:

$$P_{B-E} = P_5 + P_6 + P_7 + P_8 \quad \text{(5.30)}$$

3. To achieve equilibrium, columns' loads and soil reaction must be modified such that the sum of the forces is equal to zero. This is achieved by obtaining the average load on the strip P_{avg}.

$$P_{avg} = \frac{R_{B-E} + P_{B-E}}{2} \quad \text{(5.31)}$$

4. The modified soil pressure equals:

$$q_{mod} = \frac{P_{avg}}{L} \quad \text{(5.32)}$$

5. The modified columns' loads are obtained by multiplying each of the applied loads by the factor α given by:

$$\alpha = \frac{P_{avg}}{P_{B-E}} \quad \text{(5.33)}$$

Thus the modified columns' loads are αP_5, αP_6, αP_7, and αP_8. This modified loading is shown in Fig. 5.18.

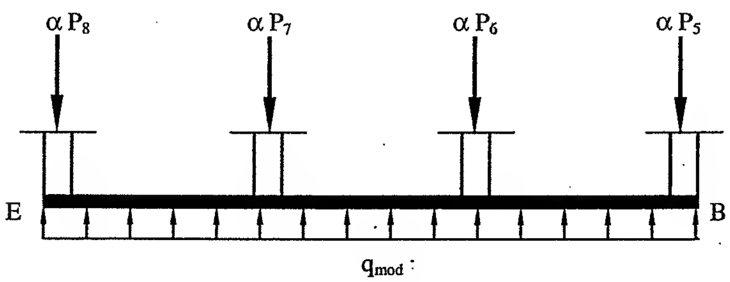


Fig. 5.18 Modified soil pressure for strip B-E

6. The shear and bending moment can be computed using regular structural analysis.

The same process should be carried out for all the strips in the raft foundation.

Step 3: Design for flexure

For each strip the maximum positive and negative moments can be obtained. It should be clear that negative moments need top reinforcement and positive moment needs bottom reinforcement.

The moment per meter is obtained by diving the moment by the strip width:

$$M' = \frac{M}{B_2} \quad \text{(5.34)}$$

The ultimate moment is obtained by multiplying the working moment by a load factor of 1.5.

$$M_u = 1.5\, M' \quad \text{(5.35)}$$

The design of different sections can be carried using design curves such as R-ω.

Step 4: Design for punching shear

The punching load for each column is calculated by multiplying the applied working load with the load factor.

$$P_u = 1.5 P_i \quad \text{(5.36)}$$

The critical perimeter is at d/2 from the face of the column. The critical shear perimeter U is calculated as shown in Fig. 5.19, and the applied punching load Q_u is obtained after subtracting the load of the punching area (a x b) by the ultimate pressure at this point q_{su}. Thus:

$$Q_{up} = P_u - q_{su}\,(a \times b) \quad \text{(5.37)}$$

The applied punching shear stress q_{up} equals:

$$q_{up} = \frac{Q_{up}}{U \times d} \quad \text{(5.38)}$$

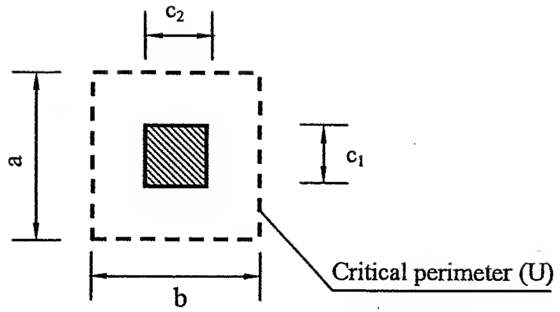


Fig. 5.19 Critical punching shear perimeter

The concrete strength for punching is the least of the following three values:

1. $$q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} \leq 1.6 N/mm^2 \quad \text{.....................(5.39a)}$$

2. $$q_{cup} = 0.316\ (0.50 + \frac{c_1}{c_2})\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{..........................(5.39b)}$$

3. $$q_{cup} = 0.8\ (0.20 + \frac{\alpha\ d}{U})\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{..........................(5.39c)}$$

$\alpha = 2$ for corner columns

$\alpha = 3$ for exterior columns

$\alpha = 4$ for interior columns

The raft thickness is considered adequate if $q_{up} < q_{cu}$, otherwise increase the thickness of the raft.

Step 5: Reinforcement Arrangement

The bending moment distribution is similar to upside down flat slab. Thus, at the locations of the columns in a raft foundation the bending moment is positive and requires bottom reinforcement shown in Fig. 5.20. (*compare to negative bending moment and top reinforcement in flat slabs*). Moreover, at a location between columns in a raft foundation the bending moment is negative and requires top reinforcement as (*compare to positive bending moment and bottom reinforcement in flat slabs*).

It is customary to reinforce the raft with a bottom basic reinforcing mesh and a basic top reinforcing mesh. Additional reinforcement is provided at locations where the capacity is exceeded.

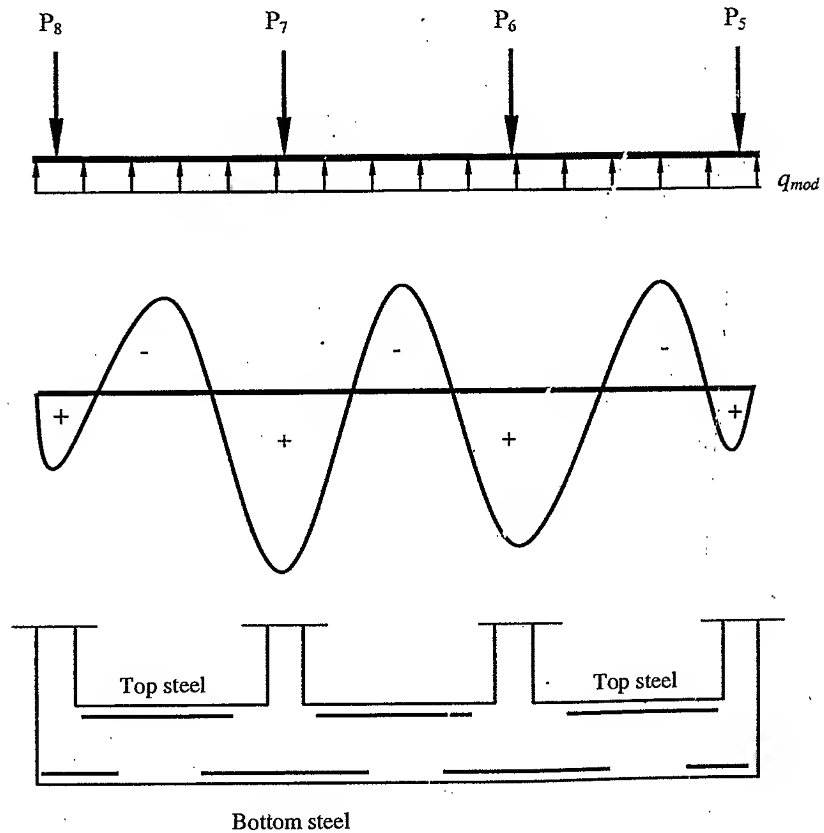


Fig. 5.20 Reinforcement arrangement

5.9.3 Analysis of the Raft Using Computer Programs

Raft foundations can be analyzed using commercially available computer programs. Such programs are based on the finite element method.

5.9.3.1 Modeling of the Raft

The raft is divided into finite plate bending elements or shell element as shown in Fig. 5.21. The practical dimensions of each element range from 0.5 m to 1 m. It is recommended that the aspect ratio of each element not to exceed 3.

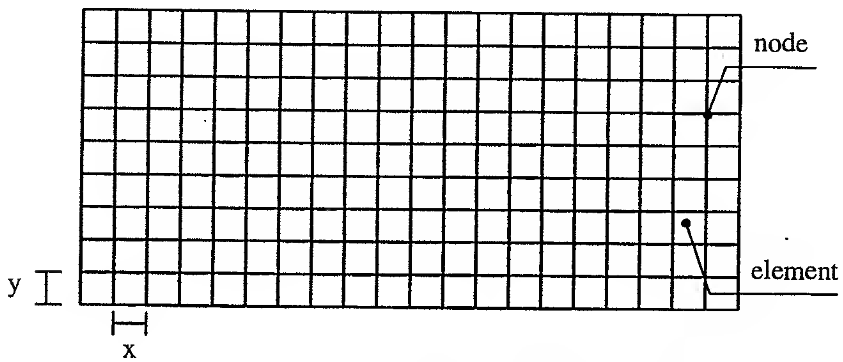


Fig. 5.21 Finite element model for the raft foundation

5.9.3.2 Modeling of the soil

The soil is represented by elastic springs located at the nodes as shown in Fig. 5.22. The elastic constant of these springs is named the *spring stiffness* (K_1, K_2, ...) (kN/m).

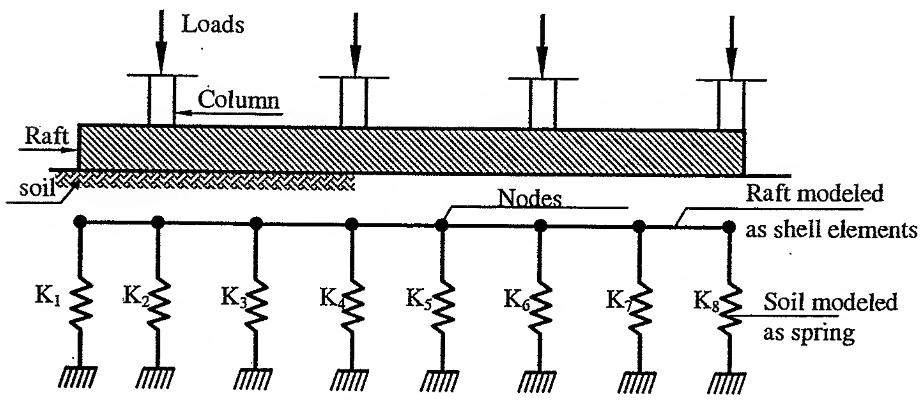


Fig. 5.22 Modeling of the soil

Calculation of spring stiffness

The spring stiffness = Coefficient of sub grade reaction (k_s) x area served.

The coefficient of subgrade reaction is a relationship between the soil pressure and its settement. If a foundation of width B is subjected to a load per unit area q, it will undergo a settlement D. Then, the coefficient of subgrade reaction k_s can be defined as:

$$k_s = \frac{q}{\Delta} \quad \text{.......................................} (5.40)$$

The unit of the coefficient of the subgrade reaction is kN/m^3. The value of the coefficient of subgrade reaction differs according to the type of soil. In general, the higher the bearing capacity, the higher the coefficient is. Its value depends on several factors, such as the type of soil, the length L, the width B of the foundation, and the foundation level of raft.

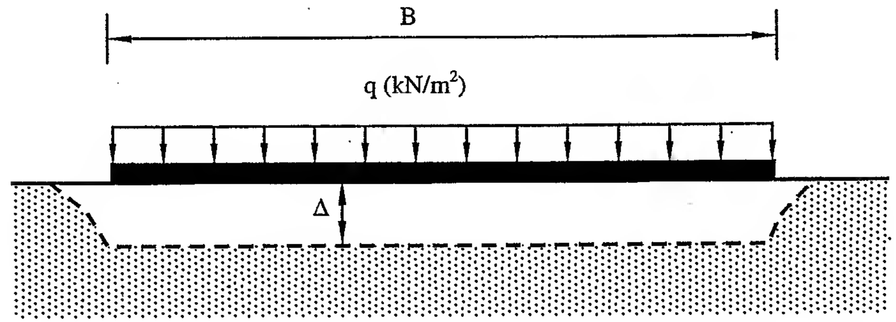


Fig. 5.23 Definition of coefficient of subgrade reaction

To determine the value of the coefficient of subgrade reaction, a field test may be performed. In such a test, the load is applied to a square plate of dimensions (0.3 m x 0.3 m) and the corresponding settlement is recorded. The value of coefficient for a large foundation of dimensions (B × B) can be obtained in the light of the value obtained for the small plate $k_{0.3}$ as follows:

$$k_s = k_{0.3} \frac{(B+0.3)^2}{2B} \quad \text{for sandy soil ....} (5.41a)$$

$$k_s = k_{0.3} \frac{(0.3)}{B} \text{ for clayey soil} \ldots\ldots (5.41b)$$

For rectangular foundation having dimensions of B x L

$$k_s = \frac{k_{B\times B}(1 + B/L)}{1.5} \ldots\ldots (5.42)$$

Where $K_{B\times B}$ is the coefficient of subgrade reaction for a square foundation with dimensions (B×B)

Typical values for the coefficient of subgrade reaction $k_{0.3}$ for sandy and clayey soils are given in Table (5.3)

Table 5.3 Values of the coefficient of subgrade reaction

Soil	Type	$k_{0.3}$ (MN/m^3)
Sand (dry or moist)	Loose	8-25
	Medium	25-125
	Dense	125-375
Sand (saturated)	Loose	10-15
	Medium	35-40
	Dense	130-150
Clay	Stiff (q=100-200 kN/m^2)	12-15
	Very stiff(q=200-400 kN/m^2)	25-50
	Hard (q>400 kN/m^2)	>50

An approximate estimate of the coefficient of subgrade reaction is obtained as follows:

$$k_s(kN/m^3) = (100 \rightarrow 120) \times \textit{soil bearing capacity } (kN/m^2) \ldots\ldots (5.43)$$

Figure 5.24 shows a plan of a raft foundation that is divided into plate bending elements of dimensions (0.50 m x 0.50 m). These elements are intersected at joints or nodes. The soil at each joint is modeled as a spring with stiffness K. The stiffness of each spring is obtained by multiplying the coefficient of subgrade reaction by the area served of each node as follows:

$$K_A = k_s \frac{x\ y}{4} = 0.0625\ k_s$$

$$K_B = k_s \frac{x\ y}{2} = 0.125\ k_s$$

$$K_C = k_s\ x\ y = 0.25\ k_s$$

The loads are applied to the raft at the columns' locations. The structure is analyzed as plane grid system, in which only z, R_X, R_Y are allowed.

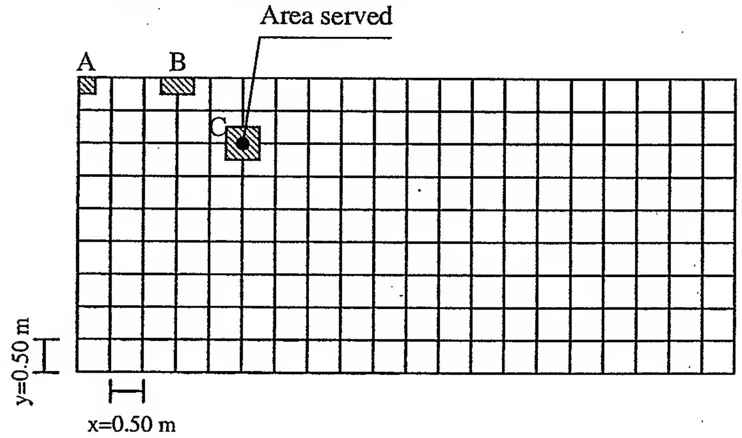


Fig. 5.24 Finite element model for the raft foundation

Photo 5.7 A raft on piles during construction

5.9.3.3 Analysis of the Computer Output

The computer output of the raft foundation consists of bending moments acting in the two directions M11 and M22. Most of the available commercial programs represent the output in graphical forms. The graphical representation is usually in the form of contour lines, in which each contour line represents a certain bending moment value. It should be mentioned that closely spaced contour lines indicate concentration of stresses. This usually occurs at the locations of the columns.

Typical output for M_{11} is shown in Fig (5-25). This bending moment requires reinforcement in the direction 1 of the shell. Basic top and bottom reinforcement meshes are usually provided throughout the raft and additional bottom reinforcement is usually provided under the columns.

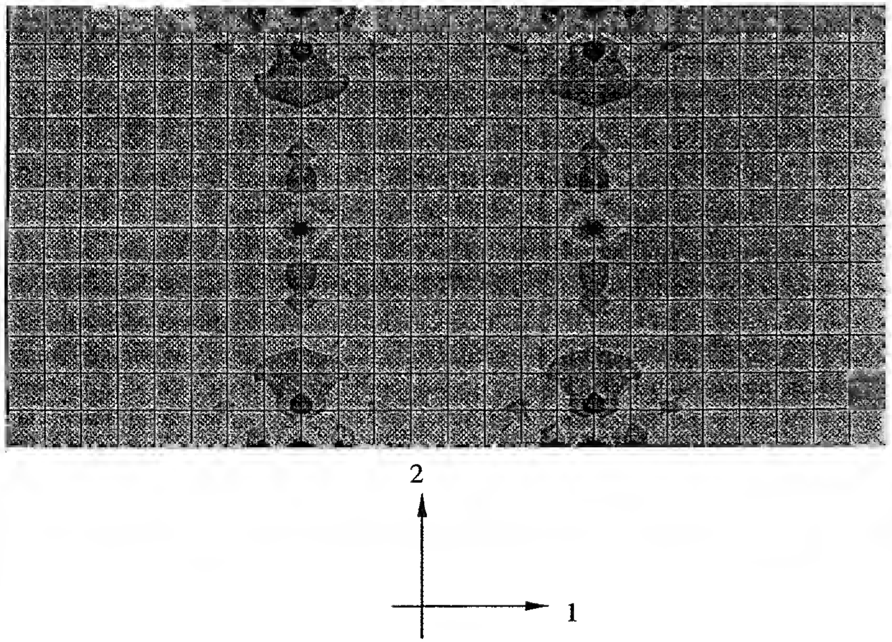


Fig. 5.25 Computer output

When designing the bottom reinforcement of the raft one should use the value of the bending moment at the face of the column (Sec. 1-1 and Sec. 2-2) as shown in Fig. 5.26. In other words, the contour line located inside the columns should be ignored.

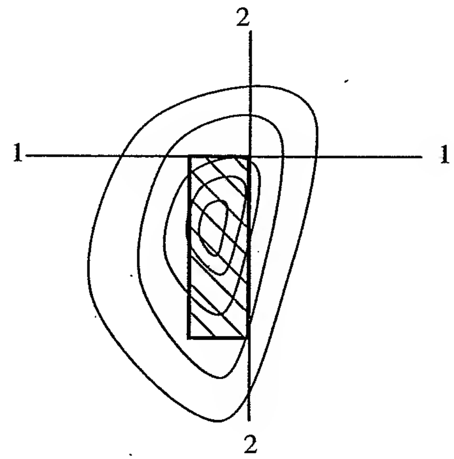


Fig. 5.26 Contours at column location

The reaction at each joint spring can be determined from the computer output. However, it is important to note that the soil capacity should be checked using the classical stress equation (Eq. 5.21) and not using the spring reactions. This is attributed to the concentration of forces at the location of the columns.

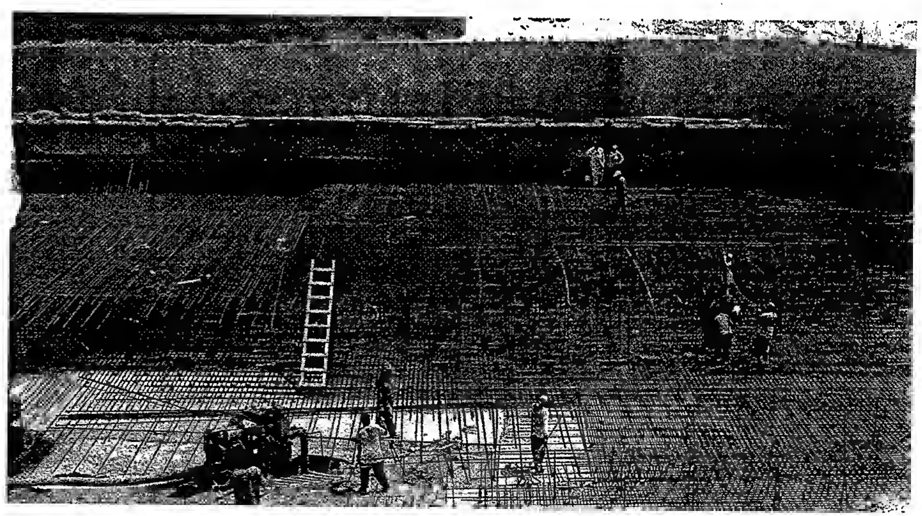


Photo 5.8 Placing the reinforcement of a raft foundation

Example 5.8: Raft using the Conventional Method

Figure EX 5.8.1 shows a raft foundation for an office building. It is required to design the reinforced concrete raft foundations. The cross section of all columns is 400 x 400 mm. The allowable soil pressure is 125 kN/m^2. The material properties for concrete and reinforcing steel are 25 N/mm^2 and 360 N/mm^2, respectively. Columns working loads (unfactored) are also shown in figure.

Solution

Step 1: Check soil pressure

$$q = \frac{P}{A} + \frac{M_x}{I_x} y + \frac{M_y}{I_y} x$$

Where

A = area of the raft $= 15.4 \times 12.4 = 190.96\ m^2$

$$I_x = \frac{B\ L^3}{12} = \frac{12.4 \times 15.4^3}{12} = 3774\ m^4$$

$$I_y = \frac{L\ B^3}{12} = \frac{15.4 \times 12.4^3}{12} = 2446.8\ m^4$$

The total vertical unfactored loads = 440 + (1360 x 2) + 370 + (1150 x 2) + (2880 x 2) + 500 + 1360 +1440 + 440 = 15330 kN.

The center of gravity of the applied loads can be obtained by taking moment of the loads about point D.

$$\bar{X} = \frac{1}{15330}[0.2\ (440 + 1360 \times 2 + 370) + 6.2\ (1150 \times 2 + 2280 \times 2)$$
$$+ 12.2 \times (500 + 1360 + 1440 + 440)]$$

$$\bar{X} = 6.282\ m$$

$$e_x = \bar{X} - \frac{B}{2} = 6.282 - \frac{12.4}{2} = 0.082\ m$$

Similarly, in the y-direction, one can get:

$$e_y = \bar{Y} - \frac{L}{2}$$

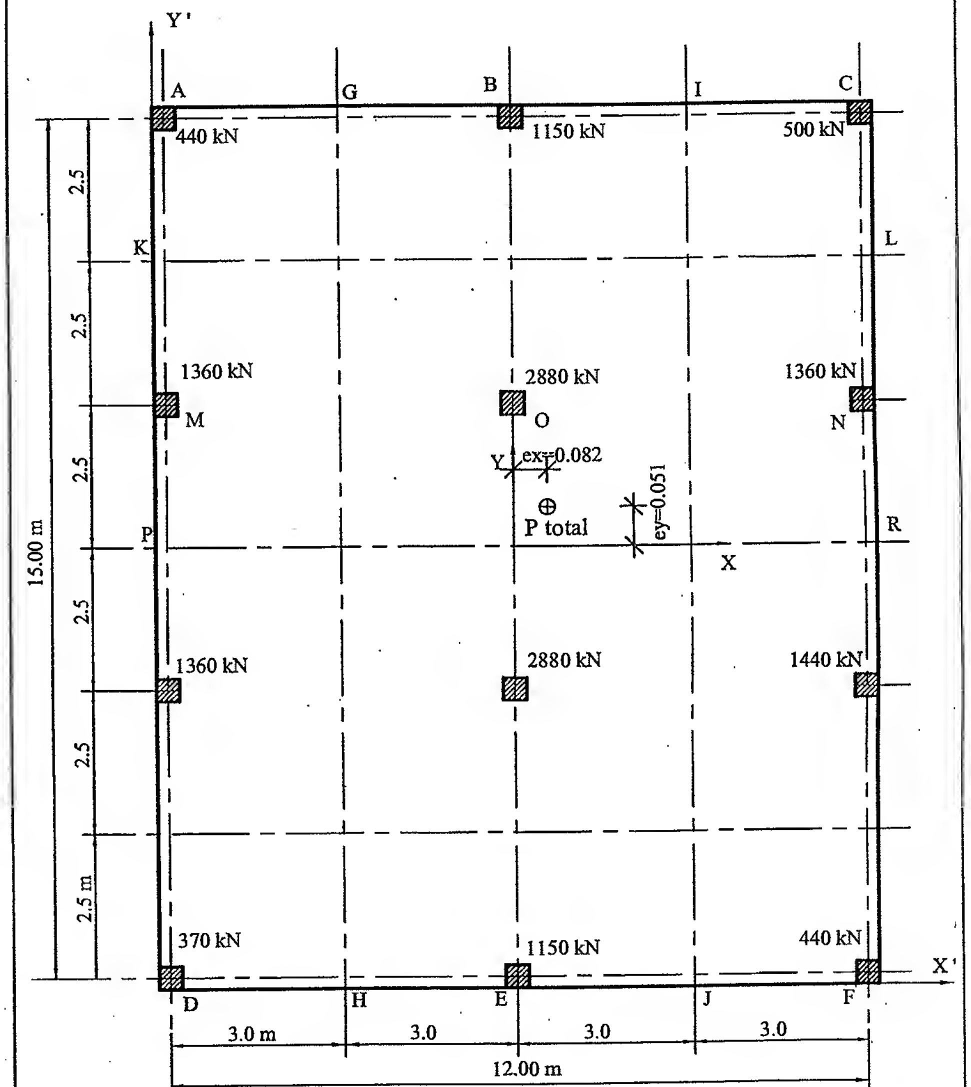


Fig. EX. 5.8 Layout of the raft

$$\bar{Y} = \frac{1}{15330}[0.2\,(370+1150+440)+5.2\,(1360+2880+1440)$$
$$+10.2\times(1360\times2+2880)+15.2(440+1150+500)]$$

$\bar{Y} = 7.751\ m$

$$e_y = 7.751 - \frac{15.4}{2} = 0.051\ m$$

The resultant applied moments are given by:

$M_x = P_{total}\ e_y = 15330\times0.051 = 781.83\ \ kN.m$

$M_y = P_{total}\ e_x = 15330\times0.082 = 1257.06\ \ kN.m$

The soil bearing pressure can be obtained by applying the following equation:

$$q = \frac{P}{A} + \frac{M_x}{I_x}\,y + \frac{M_y}{I_y}\,x = \frac{15330}{190.96} + \frac{781.83}{3774}\,y + \frac{1257.06}{2446}\,x$$

The results are summarized in the following table

Point	x (m)	y(m)	q(kN/m^2)
A	-6.2	7.7	78.69
B	0	7.7	81.87
C	6.2	7.7	85.06
D	-6.2	-7.7	75.50
E	0	-7.7	78.68
F	6.2	-7.7	81.87
M	-6.2	2.5	77.61
O	0	2.5	80.80
N	6.2	2.5	83.98

The maximum soil pressure (85.06 kN/m^2) is less than the allowable soil pressure (safe).

Step 2: Calculation of the shear forces and bending moments

Step 2.1: Strip ADHG (width= 3.2 m)

The average soil pressure for the strip can be obtained by taking the average values of the pressures at points A and D.

$$q_{avg} = \frac{78.69+75.50}{2} = 77.09\ kN/m^2$$

The total soil reaction $R_{ADHG} = q_{avg}\ B_1\ \ L = 77.09\times3.2\times15.4 = 3799\ kN$

On the other hand the total vertical on this strip equals:

$P_{ADHG} = 440+1360+1360+370 = 3530\ kN$

Now, we shall use the average value o of the total reaction and P_{ADHG}

$$\text{Average load} = P_{avg} = \frac{R_{ADHG}+P_{ADHG}}{2} = \frac{3799+3530}{2} = 3664.5\ kN$$

$$\text{The modified soil pressure (per the strip)} = \frac{P_{avg}}{L} = \frac{3664.5}{15.4} = 237.95\ kN/m'$$

The column loads are modified in the same manner by multiplying the load of each column by the ratio (P_{avg}/P_{ADHG}).

$$\alpha = \frac{P_{avg}}{P_{ADHG}} = \frac{3664.5}{3530} = 1.0381$$

Item	P_{actual} (kN)	P_{mod}(kN)
1	440	456.77
2	1360	1411.82
3	1360	1411.82
4	370	384.1

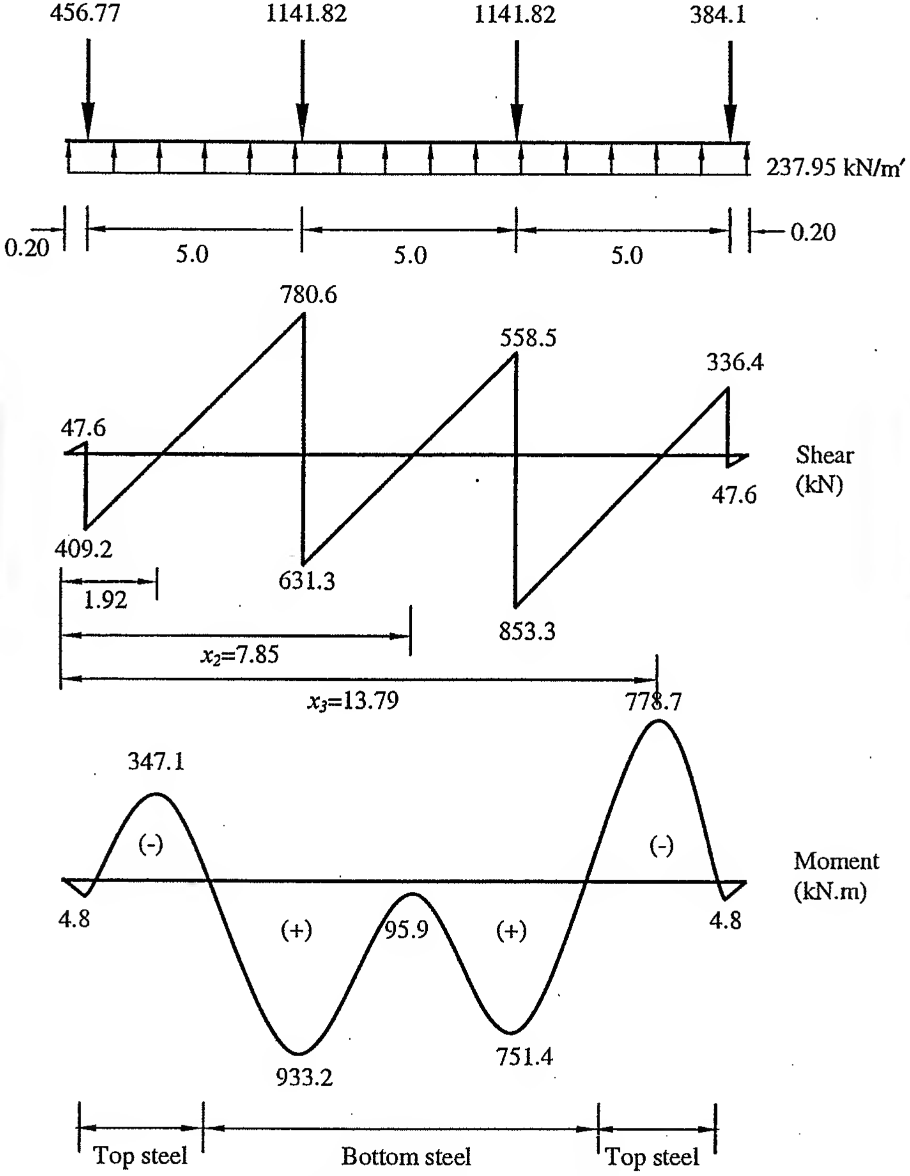
The shear force and the bending moment diagrams are shown in the figure given below. Three points of zero shears exist. They are calculated as follows:

$$x_1 = \frac{456.77}{237.95} = 1.92\ m \qquad x_2 = \frac{456.77+1411.82}{237.95} = 7.85\ m$$

$$x_3 = \frac{456.77+1411.82+1411.82}{237.95} = 13.79\ m$$

The maximum negative moment equals:

$$= 237.95 \times \frac{1.92^2}{2} - 456.77(1.92 - 0.20) = -347.1 \; kN.m$$



Shear and moment for strip ADHG

Step 2.2: Strip GHJI (width =6.0 m)

The average soil pressure for the strip can be obtained by taking the average value of pressures at points B and E.

$$q_{avg} = \frac{81.87 + 78.68}{2} = 80.28 \; kN/m^2$$

The total soil reaction $R_{GHJI} = q_{avg} \; B_1 \; L = 80.28 \times 6.0 \times 15.4 = 7417.74 \; kN$

On the other hand the total vertical on this strip equals:

$P_{GHJI} = 1150 + 2880 + 2880 + 1150 = 8060 \; kN$

Now, we shall use the average value of the total reaction and P_{ADHG}.

$$\text{Average load} = P_{avg} = \frac{R_{GHJI} + P_{GHJI}}{2} = \frac{7417.74 + 8060}{2} = 7738.87 \; kN$$

$$\text{The modified soil pressure (per the strip)} = \frac{P_{avg}}{L} = \frac{7738.87}{15.4} = 502.52 \; kN/m'$$

The column loads are modified in the same manner by multiplying each column load by the ratio (P_{avg}/P_{GHJI})

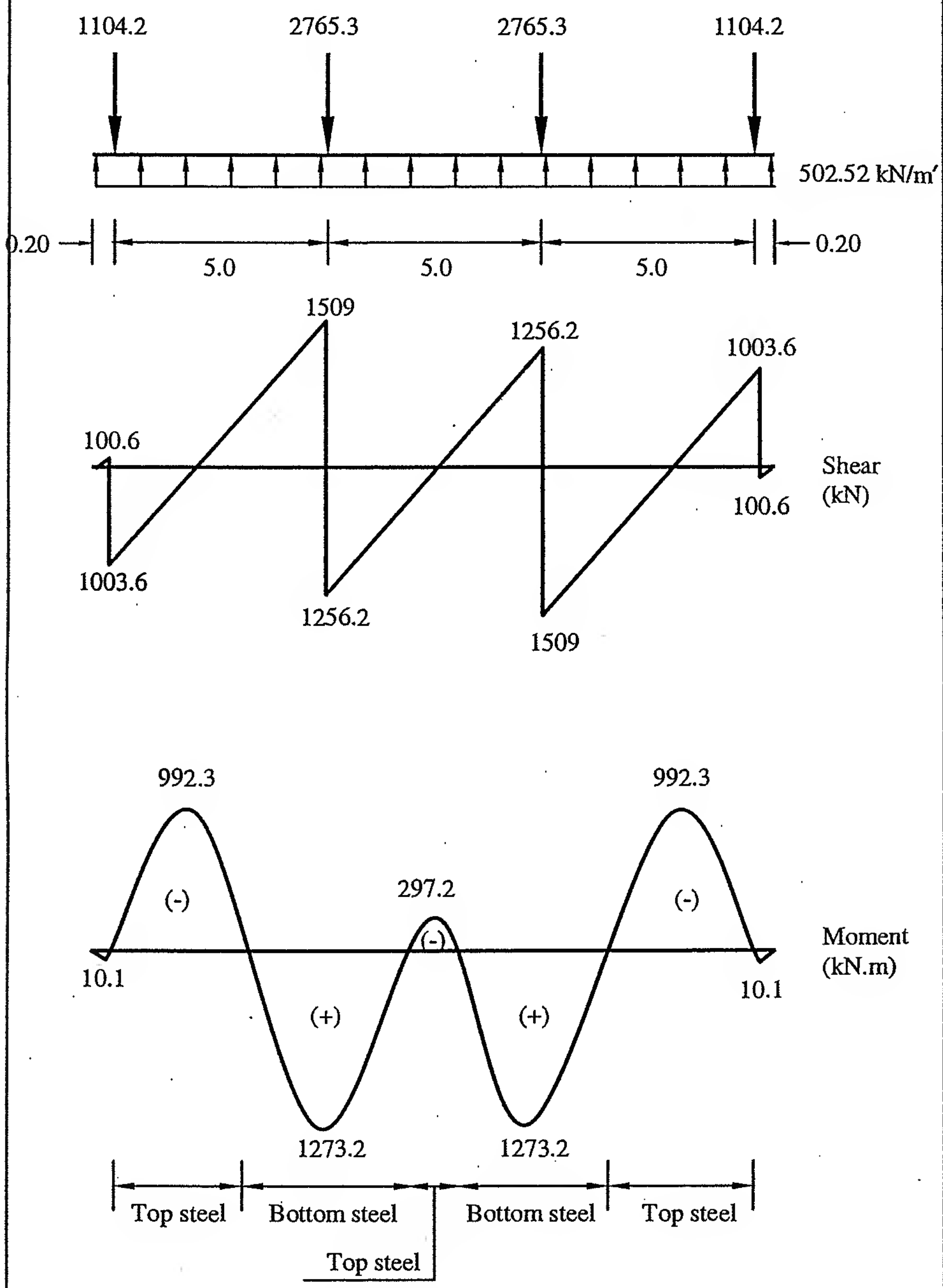
$$\alpha = \frac{P_{avg}}{P_{ADHG}} = \frac{7738.87}{8060} = 0.9602$$

Item	P_{actual} (kN)	P_{mod}(kN)
1	1150	1104.18
2	2880	2765.25
3	2880	2765.25
4	1150	1104.18

The shear force and bending moment diagrams are shown in figure. Three points of zero shears exist. They are calculated as follows:

$$x_1 = \frac{1104.18}{502.52} = 2.20 \; m \;, \quad x_2 = \frac{1104.18 + 2765.25}{502.52} = 7.70 \; m$$

$$x_3 = \frac{1104.18 + 2765.25 + 2765.25}{502.52} = 13.20 \; m$$



Shear and moment for strip GHJI

Step 2.3: Strip ACKL (width = 2.70 m)

The average soil pressure for the strip can be obtained by taking the average value of the pressures at points A and C.

$$q_{avg} = \frac{878.69 + 85.06}{2} = 81.87\ kN/m^2$$

The total soil reaction $R_{ACKL} = q_{avg}\ L_1\ B = 81.87 \times 2.70 \times 12.4 = 2741\ kN$

On the other hand, the total vertical on this strip equals:

$$P_{GHJI} = 440 + 1150 + 500 = 2090\ kN$$

Now, we shall use the average value of the total reaction and P_{ADHG}.

$$\text{Average load} = P_{avg} = \frac{R_{ACKL} + P_{ACKL}}{2} = \frac{2741 + 2090}{2} = 2415.5\ kN$$

$$\text{The modified soil pressure (per the strip)} = \frac{P_{avg}}{B} = \frac{2415.5}{12.4} = 194.8\ kN/m'$$

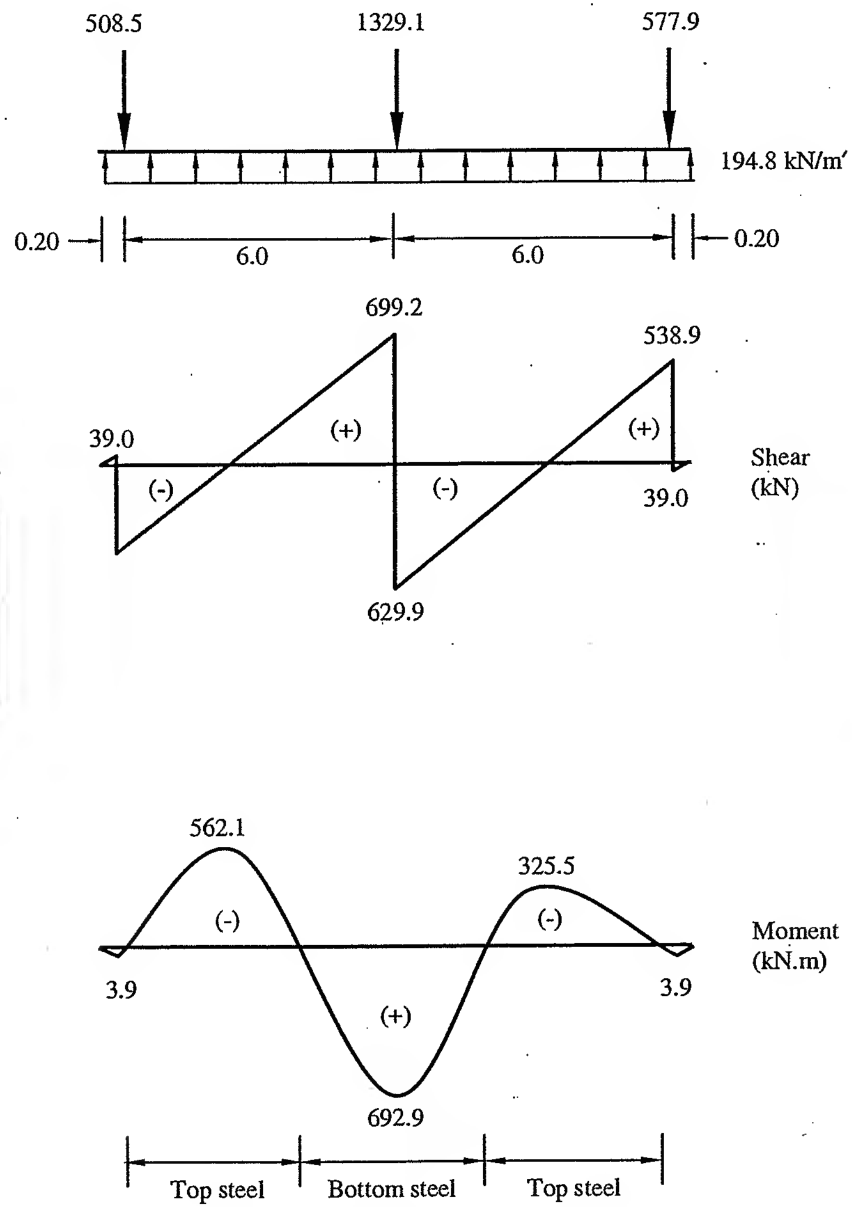
The column loads are modified in the same manner by multiplying each column load by the ratio (P_{avg}/P_{GHJI})

$$\alpha = \frac{P_{avg}}{P_{ACKL}} = \frac{2415.15}{2090} = 1.156$$

Item	P_{actual} (kN)	P_{mod}(kN)
1	440	508.54
2	1150	1329.14
3	500	577.89

The shear force and bending moment diagrams are shown in figure. Two points of zero shears exist. They are calculated as follows:

$$x_1 = \frac{508.54}{194.8} = 2.61\ m\ ,\quad x_2 = \frac{508.54 + 1329.14}{194.8} = 9.43\ m$$



Shear and moment for strip ACKL

Step 2.4: Strip KLPR (width =5.0 m)

The average soil pressure for the strip can be obtained by taking the average value of the pressures at points M and N.

$$q_{avg} = \frac{77.61+83.98}{2} = 80.80\ kN/m^2$$

The total soil reaction $R_{KLPR} = q_{avg}\ L_1\ B = 81.87 \times 5.0 \times 12.4 = 5009.4\ kN$

On the other hand, the total vertical on this strip equals:

$P_{GHJI} = 1360 + 2880 + 1360 = 5600\ kN$

Now, we shall use the average value of the total reaction and P_{ADHG}.

$$\text{Average load} = P_{avg} = \frac{R_{ACKL} + P_{ACKL}}{2} = \frac{5009.4+5600}{2} = 5304.7\ kN$$

$$\text{The modified soil pressure (per the strip)} = \frac{P_{avg}}{B} = \frac{5304.7}{12.4} = 427.80\ kN/m'$$

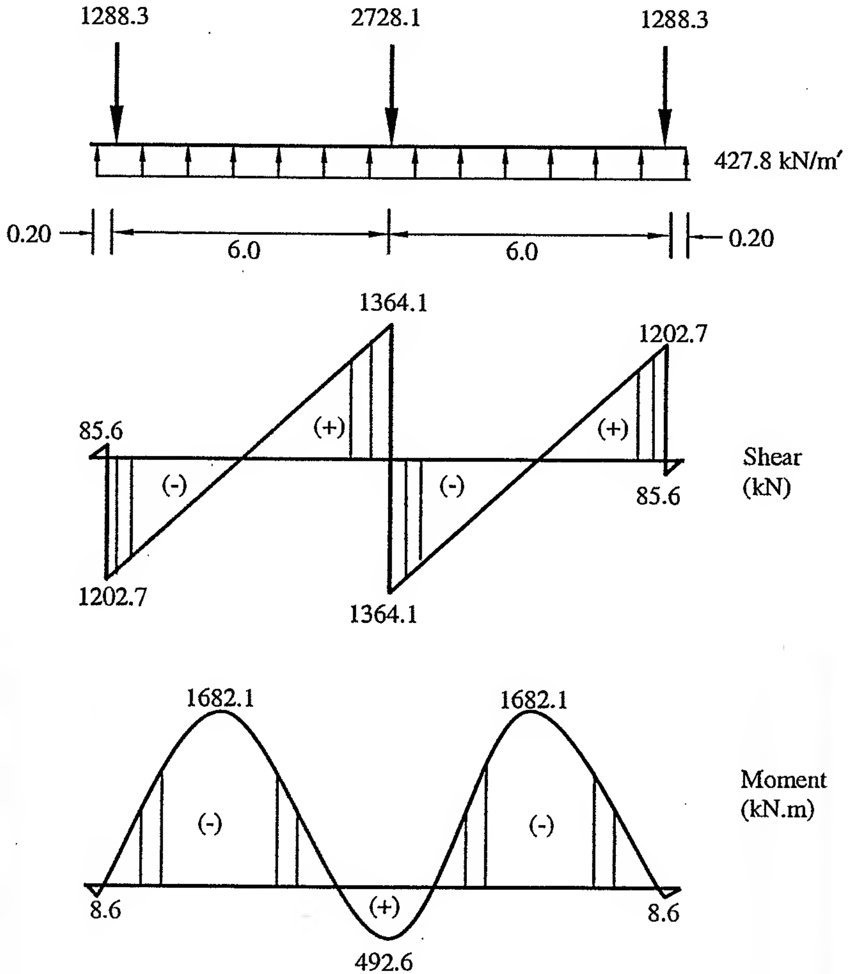
The column loads are modified in the same manner by multiplying each column load by the ratio (P_{avg}/P_{GHJI})

$$\alpha = \frac{P_{avg}}{P_{ACKL}} = \frac{5304.7}{5600} = 0.947$$

Item	P_{actual} (kN)	P_{mod}(kN)
1	1360	1288.28
2	2880	2728.13
3	1360	1288.28

The shear and moment are shown in figure. Two points of zero shears exist. They are calculated as follows:

$$x_1 = \frac{1288.28}{427.8} = 3.01\ m \quad , x_2 = \frac{1288.28+2728.13}{427.8} = 9.39\ m$$



Shear and moment for strip KLMN

Step 3: Design for flexure

Step 3.1: Reinforcement for strip ADHG

The maximum positive moment is equal to 933.2 kN.m. This positive bending moment needs bottom reinforcement.

To obtain the reinforcement per meter divide the above value by the width of strip (B=3.2 m) → $M' = \dfrac{933.2}{3.20} = 291.63\ kN.m/m'$

To design this critical section, calculate the ultimate moment by multiplying M' by the load factor 1.5.

$$M_u = 1.5\ M' = 1.5 \times 291.63 = 437.45\ kN.m$$

Assuming that the distance from the c.g. of the reinforcing steel to the concrete surface is 70 mm and the total thickness is 750 mm. The effective depth equals:

$$d = t - 70\,mm = 750 - 70 = 680\ mm$$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{437.45 \times 10^6}{250 \times 1000 \times 680^2} = 0.0378$$

From the chart with R=0.0378, the reinforcement index ω=0.046

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.046 \times \frac{25}{360} \times 1000 \times 680 = 2172\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.6}{f_y} b\ d = \dfrac{0.6}{360} \times 1000 \times 680 = 1133\ mm^2 \\ 1.3 A_s = 1.3 \times 2172 = 2823\ mm^2 \end{cases}$$

Use 7Φ20/m' (2199 mm^2) (Bottom)

Similarly, the maximum negative moment is equal to 778.7 kN.m.

$$M' = \frac{778.70}{3.20} = 243.34\ kN.m/m'$$

To design this critical section, calculate the ultimate moment by multiplying M' by the load factor 1.5.

$$M_u = 1.5\ M' = 1.5 \times 243.34 = 365.02\ kN.m$$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{365.02 \times 10^6}{250 \times 1000 \times 680^2} = 0.0315$$

From the chart with R=0.0315, the reinforcement index ω=0.038

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.038 \times \frac{25}{360} \times 1000 \times 680 = 1794\ mm^2$$

$$A_{s\,\min} = smaller\ \ of \begin{cases} \frac{0.6}{f_y} b\ d = \frac{0.6}{360} \times 1000 \times 680 = 1133\ mm^2 \\ 1.3 A_s = 1.3 \times 1794 = 2332\ mm^2 \end{cases}$$

Use 7Φ18/m′ (1781 mm^2) with additional (3.5 Φ 16/m′) (Top)

Thus in this direction use a bottom mesh 7Φ20/m′ and a top mesh 7Φ18 /m′

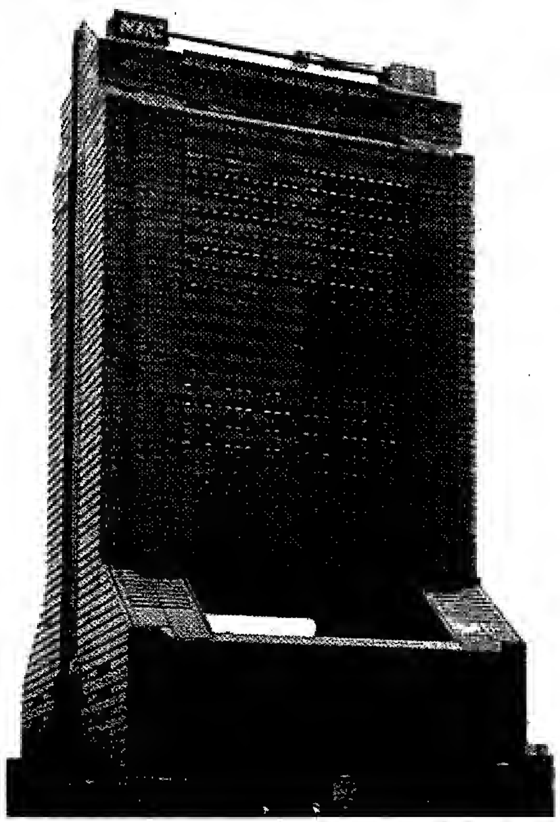


Photo 5.9 Reinforced Concrete building

Step 3.2: Reinforcement for strips GHJI, ACKL and KLPR

To avoid lengthy calculations, the following table illustrates the required steps to obtain the reinforcement.

Strip	Strip GHJI		Strip ACKL		Strip KLPR	
Reinforcement	Bottom	Top	Bottom	Top	Bottom	Top
M (kN.m)	1273.2	992.3	692.9	562.1	492.6	1682.1
B (m)	6.0	6.0	2.7	2.7	5.0	5.0
M′ (kN.m/m)	212.2	165.4	256.6	208.2	98.5	336.4
M_u (kN.m/m)	318.3	248.1	384.9	312.3	147.8	504.6
b (mm)	1000	1000	1000	1000	1000	1000
d (mm)	680	680	680	680	680	680
R	0.0275	0.0215	0.0333	0.0270	0.0128	0.0437
ω	0.0330	0.0250	0.0400	0.0320	0.0150	0.0530
As (mm^2/m)	1558	1181	1889	1511	708	2503
A_{smin} (mm^2/m)	2026	1535	2078	1964	1020	2078
$A_{s,req.}$ (mm^2/m)	2026	1535	2078	1964	1020	2503
Rft **	7Φ20/m′	7Φ18/m′	7Φ20/m′	7Φ18/m′	7Φ20/m′	7Φ18/m′
Additional	-	-	-	3.5Φ16/m′	-	3.5□18/m′

** A bottom mesh of 7 Φ 20/m′ (2199 mm^2) and a top mesh of 7 Φ 18 /m′ (1718 mm^2) are provided (Refer to Fig. EX. 5.8.2). Additional reinforcement may be placed at the location of the larger capacity.

Step 4: Design for punching shear

The maximum vertical load occurs at the column that carries 2880 kN. Thus, the ultimate load is obtained by multiplying this load by the load factor of 1.5.

$P_u = 1.5 P_{max} = 1.5 \times 2880 = 4320\ kN$

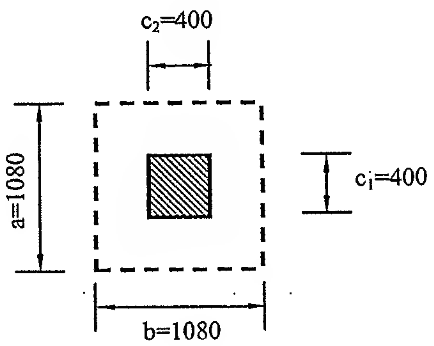
The critical perimeter is at d/2 from the face of the column. For the interior column, the critical perimeter equals:

$d = 680\ mm$

$a = c_1 + d = 400 + 680 = 1080\ mm$

$b = c_2 + d = 400 + 680 = 1080\ mm$

$U = 2\ (a + b) = 2\ (1080 + 1080) = 4320\ mm$



The pressure at point O (refer to the table) is equal to 80.8 kN/m^2

Thus the ultimate soil pressure $q_{su} = 1.5 \times 80.8 = 121.19\ kN/m^2$

The punching load equals:

$Q_{up} = P_u - q_{su}\ (a \times b) = 4320 - 121.19\ (1.08 \times 1.08) = 4178.6$ kN

$$q_{up} = \frac{Q_{up}}{U \times d} = \frac{4178.6 \times 1000}{4320 \times 680} = 1.42\ N/mm^2$$

The concrete strength for punching the least of the three values:

1. $q_{cup} = 0.316 \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316 \sqrt{\frac{25}{1.5}} = 1.29\ N/mm^2 < 1.6 ..... o.k.$

2. $q_{cup} = 0.316\ (0.50 + \frac{a}{b}) \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\ (0.50 + \frac{0.4}{0.4}) \sqrt{\frac{25}{1.5}} = 1.94 N/mm^2$

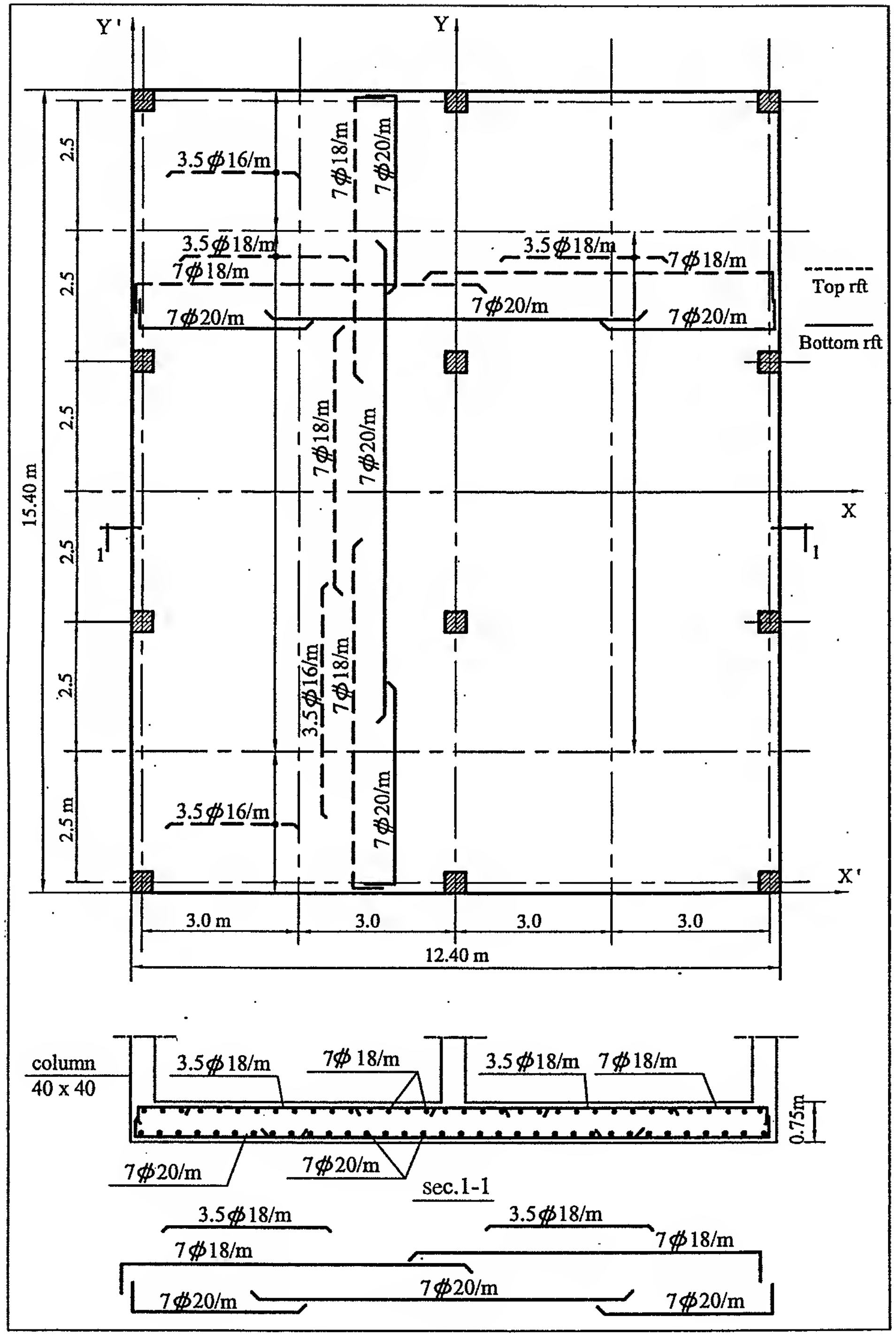
3. $q_{cup} = 0.8\ (0.20 + \frac{\alpha\ d}{U}) \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8 \left(0.20 + \frac{4 \times 0.68}{4.32} \right) \sqrt{\frac{25}{1.5}} = 2.71\ N/mm^2$

q_{cup}= 1.29 N/mm^2

Since the applied punching shear is larger than concrete punching shear strength, the raft is considered **unsafe** against punching failure. The designer may use one of two solutions:

1- Increase raft thickness to 800 mm to decrease the punching stress to 1.26 N/mm^2. The reinforcement may be redesigned for more economic solution.

2- Increase the concrete compressive strength to 31 N/mm^2 to increase q_{cup} to 1.42 N/mm^2.



Example 5.9: Raft design using computer analysis

Figure EX. 5.9 shows the axes and columns of a twelve-story building. The bearing capacity of the soil equals 200 kN/m^2 at F.L. The material properties are f_{cu} =35 N/mm^2 and f_y = 360 N/mm^2. Based on the recommendations of the geotechnical report, it is decided to use a rigid raft as a foundation system.

The building is provided with relatively rigid shear walls in the two orthogonal directions to resist the lateral loads. Consequently, analysis of the building under lateral loads could be carried out in each direction independent of the other. The following data are available from the analysis of the building in the X-direction:

1- The total unfactored moment (M_y) due to earthquake =16000.0 kN.m (reversible). The resultant of the unfactored gravity load at the foundation level = 68000.0 kN and is located as shown in Fig. EX 5.9a.

2- Structural analysis of the building under the case of the earthquake acting in the X-direction and under the critical load combination has resulted in the following straining actions at the foundation level:

Column	Ultimate M_y (kN.m)	Ultimater load (kN)
(A-1)	0	950
(W1)	8000	4200
(A-4)	0	1350
(B-1)	0	1850
(B-2)	0	4050
(B-3)	0	4950
(B-4)	0	2750
(C-1)	0	2150
(W3)	0	6500
(W4)	0	8900
(C-4)	0	3200
(D-1)	0	2150
(D-4)	0	3200
(E-1)	0	2150
(E-2)	0	4750
(E-3)	0	5800
(E-4)	0	3200
(F-1)	0	1300
(W2)	12000	7000
(F-4)	0	1900

Analysis for the building for the case of the earthquake acting in the in Y-direction provides straining actions that are not given since the example will be worked out only for the case of the earthquake acting in the in X-direction.

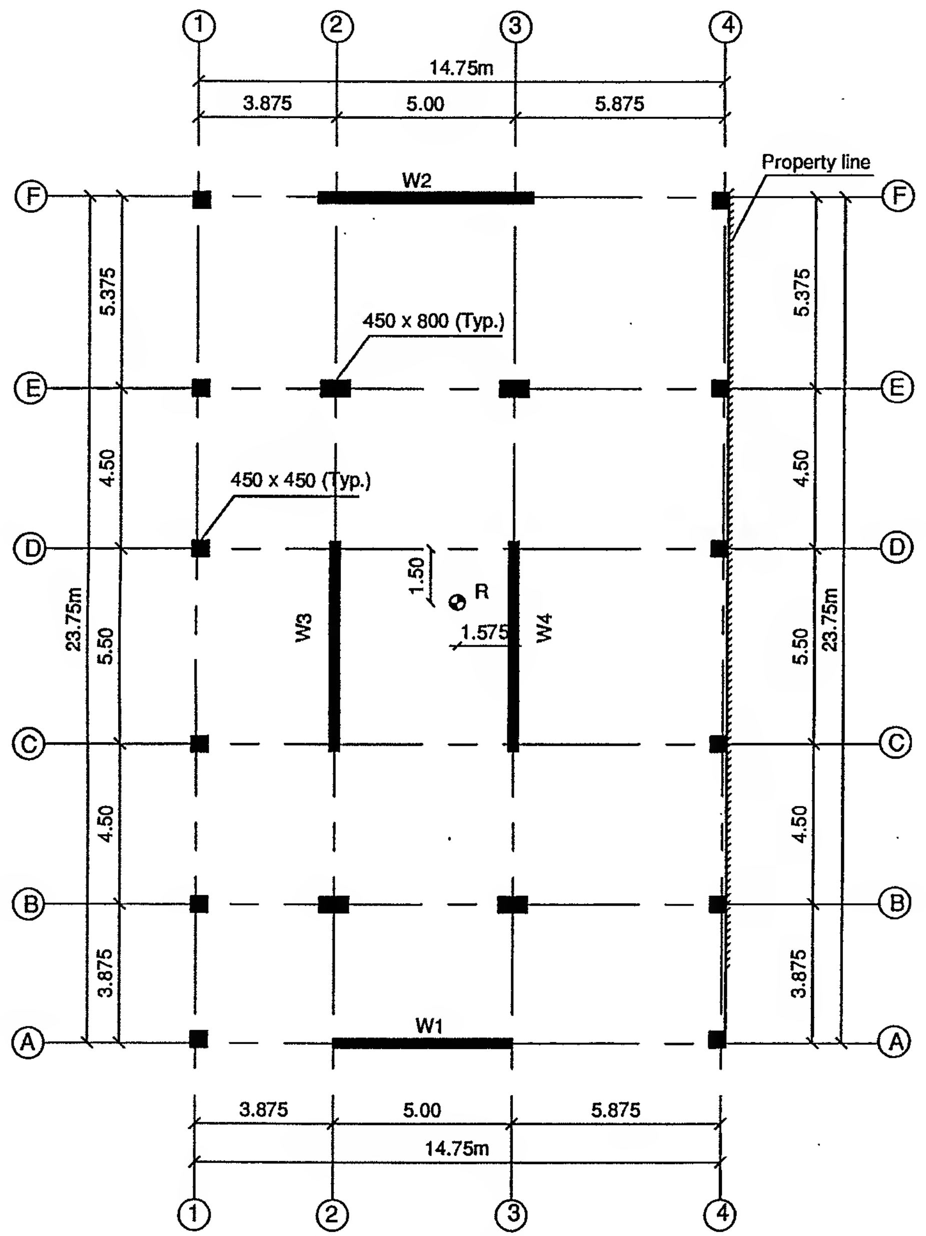


Fig. EX. 5.9a Axes and Columns

Solution

Step 1: Dimensions of the raft

Wherever possible, it is recommended to extend the raft beyond the edge columns by a distance that ranges from 0.5 m to 1.0 m. In this building, it is decided to extend the raft by 0.75 m from all the edge columns, except those located near the property line as shown in Fig. Ex 5.9b.

As *a role of thumb*, it is a common practice to assume the thickness of the raft for multistory buildings to be equal to the number of stories multiplied by (80 mm to 100 mm). Accordingly, the thickness of the raft is assumed 1100 mm.

Step 2: Check stresses on soil

In order to check the stresses on the soil, one has to calculate the area of the raft (A), the moment of inertia about the X-axis and the moment of inertia about the Y-axis.

$$A = 15.75 \times 25.5 = 401.6 \text{ m}^2$$

$$I_x = \frac{15.75 \times 25.5^3}{12} = 21763\ m^4 \qquad \& \qquad I_y = \frac{25.5 \times 15.75^3}{12} = 8302.3\ m^4$$

The resultant of the gravity loads does not coincide with the center of gravity of the raft. It can be easily proved that the eccentricities e_x and e_y, as shown in Fig. EX 5.9b, are given by:

$$e_x = 0.3 \text{ m} \qquad\qquad e_y = 0.5\text{m}$$

Since the moment due to the earthquake is reversible, one should consider the direction in which the moment due to the earthquake and that due to the eccentricity of the resultant of the gravity loads have the same sign.

$(M_y)_{total}$ = Moment due to earthquake + Moment due to eccentricity of the resultant of the gravity loads

$$M_y)_{total} = M_y + (e_x \times N)$$

$$(M_y)_{total} = 16000 + 0.3 \times 68000 = 36400 \text{ kN.m}$$

$(M_x)_{total}$ = Moment due to earthquake + Moment due to eccentricity of the resultant of the gravity loads

$$(M_x)_{total} = M_x + (e_y \times N)$$

$$(M_x)_{total} = 0 + 0.5 \times 68000 = 34000 \text{ kN.m}$$

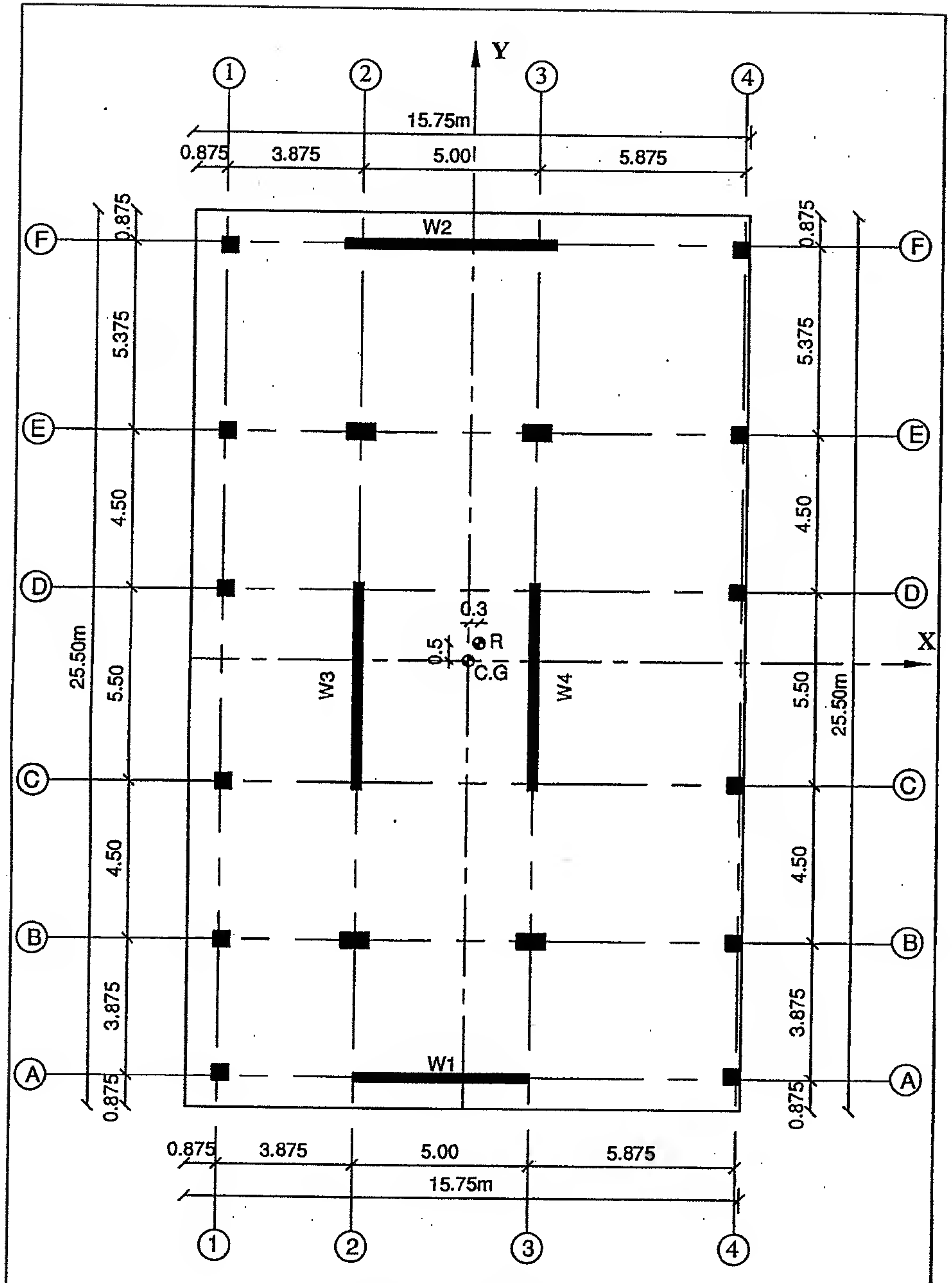


Fig. EX 5.9b Plan dimensions of the raft

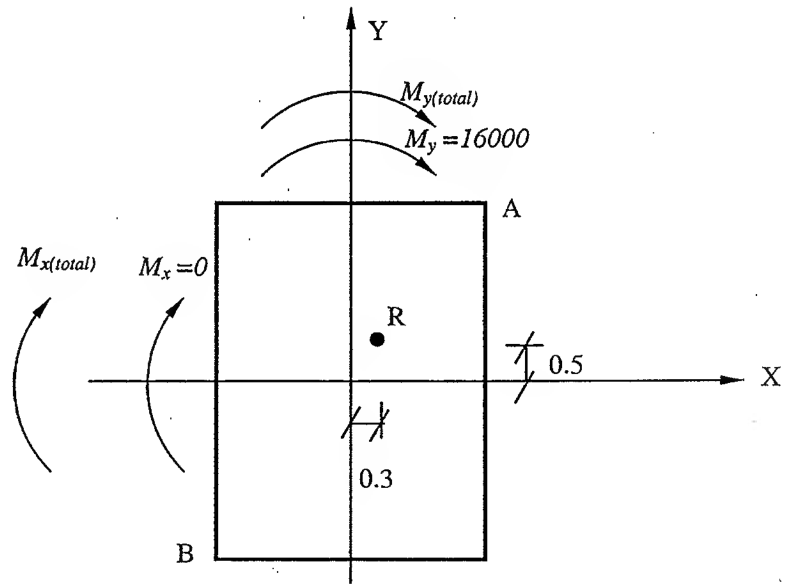
$$f = \frac{N}{A} + \frac{M_{x\,(total)}}{I_x} y + \frac{M_{y\,(total)}}{I_y} x$$

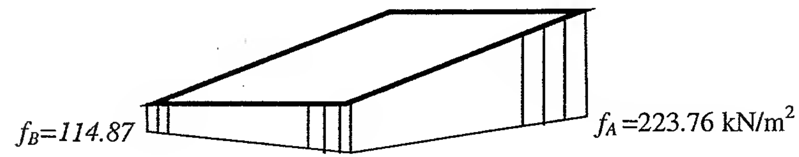
The coordinates of points A and B are (7.875,12.75) and (-7.875,-12.75), respectively.

$$f_A = \frac{68000}{401.6} + \frac{34000}{21763} 12.75 + \frac{36400}{8302} 7.875 = 223.76\ KN/m^2 < (1.3 \times 200)$$

$$f_B = \frac{68000}{401.6} + \frac{34000}{21763}(-12.75) + \frac{36400}{8302}(-7.875) = 114.87\ KN/m^2 < (1.3 \times 200)$$

According to the Egyptian Code for Foundations, the allowable bearing capacity of the soil can be increased to 1.3 of its recommended value whenever the earthquake load is considered. Hence, the stresses on soil are safe.





Step 3: Computer analysis of the raft

Step 3.1: Modeling the raft and the soil

The raft was modeled using shell elements and the soil was modeled using spring elements. Many commercial computer programs are well documented and can be used. In this example, the well-known structural analysis program SAP-2000 was used in the analysis. Figure 5.9d shows the finite element mesh used in the analysis.

The majority of the elements had dimensions 0.5m x 0.5m. At some locations, however, smaller element dimensions were used.

An approximate estimate of the coefficient of subgrade reaction is obtained as follows:

$k_s = 120 \times \textit{soil bearing capacity} = 120 \times 200 = 24000\ \text{N/m}^3$

The soil at each joint is modeled as a spring having a stiffness K. The stiffness of each spring is obtained by multiplying the coefficient of subgrade reaction by the area served of each node as follows:

Stiffness of a spring at a corner node $K = k_s \dfrac{Area}{4} = 24000 \times 0.0625 = 1500\, kN/m$

Stiffness of a spring at an exterior node $K = k_s \dfrac{Area}{2} = 24000 \times 0.125 = 3000\, kN/m$

Stiffness of a spring at an interior node $K = k_s \times Area = 24000 \times 0.25 = 6000\, kN/m$

Step 3.2: Modeling the acting forces and moments

A: Columns

Interior columns were represented by three nodes to take into account their relatively large dimensions. Forces and moments acting on each column were assumed to be acting on the three nodes. Exterior and corner columns were represented by two nodes.

B: Shear walls

In order to model the forces and moments acting on the shear walls, it was assumed that the gravity load could be divided among all the points representing the wall, whereas the moment could be represented by compression forces and tension forces acting at the nodes.

Distribution of loads on (W_1)

The wall is subjected to a normal force (-4200 kN) and a bending moment (8000 kN.m). The force at each node is evaluated by the superposition principle as follows:

A-Normal force

This wall is modeled using 22 nodes. Hence, the share of each node is equal to:

$$N_i = \frac{N}{No.of\ Nodes} = \frac{-4200}{22} = -190.1\ kN \downarrow$$

B-Bending moment

$$I = \frac{b \times t^3}{12} = \frac{0.25 \times 5^3}{12} = 2.604\ m^4$$

$$\sigma = \frac{M}{I} x = \frac{8000}{2.604} 2.5 = 7680\ kN/m^2$$

$$P_t = P_c = \frac{1}{2} \times 7680 \times 0.25 \times 2.5 = \pm\ 2400\ kN$$

Where P_t and P_c are the tension and compression forces, respectvely, resulting from the bending moment. Since we have two rows of nodes (2 x11), the share of each row is given as (refer to the figure):

$$P_c' = \frac{P_c}{No.of\ rows} = \frac{-2400}{2} = -1200\ kN$$

To distribute the forces along the nodes, a conservative approach shall be followed. It shall be assumed that the loads at the nodes are proportion to their distance from point of zero stress.

$$P_i = \frac{x_i}{\sum x_i} P_c$$

Where P_i is the force at node i and x_i is the distance from node i to the center of gravity of the wall.

$$\sum x_i = 2.5 + 2.0 + 1.5 + 1.0 + 0.5 = 7.5\, ms$$

$$P_1 = \frac{2.5}{7.5} \times -1200 = -400\ kN \downarrow$$

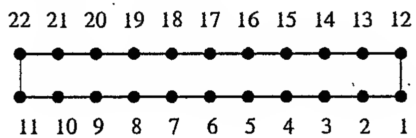
$$P_2 = \frac{2.0}{7.5} \times -1200 = -320\ kN \downarrow \qquad P_3 = \frac{1.5}{7.5} \times -1200 = -240\ kN \downarrow$$

$$P_4 = \frac{1.0}{7.5} \times -1200 = -160\ kN \downarrow \qquad P_5 = \frac{0.5}{7.5} \times -1200 = -80\ kN \downarrow$$

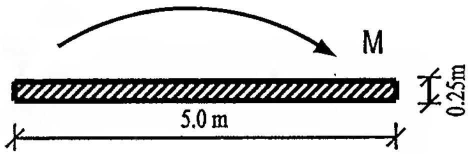
C-Total force

The total force at each node is given in Table EX 1.

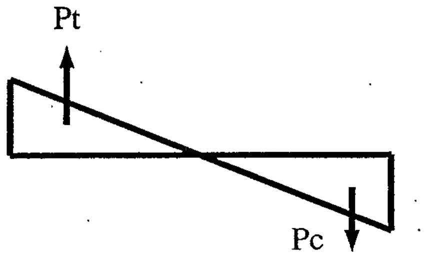
Nodes numbering



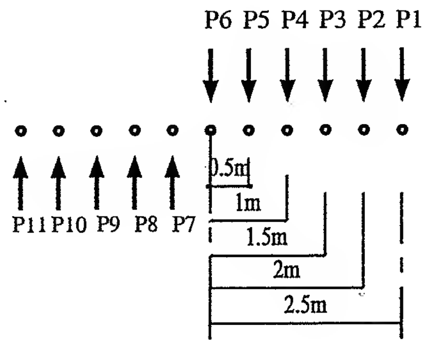
Wall W1



Stresses diagram and the resultant forces



Transformation of moment into nodal forces



Transformation of moment into nodal forces for wall W1

Distribution of loads on (W_2)

The wall is subjected to a normal force (-7000 kN) and a bending moment (12000 kN.m). The force at each node is evaluated by the superposition principle as follows :

A-Normal force

This wall is modeled using 26 nodes. Hence, the share of each node equals:

$$N_i = \frac{N}{No.of\ \ Nodes} = \frac{-7000}{26} = -269.2\ kN \downarrow$$

B-Bending moment

$$I = \frac{b \times t^3}{12} = \frac{0.30 \times 6^3}{12} = 5.4\ m^4$$

$$\sigma = \frac{M}{I} x = \frac{12000}{5.4} 3.0 = 6666.7\ kN / m^2$$

$$P_t = P_c = \frac{1}{2} \times 6666.7 \times 0.3 \times 3.0 = \pm 3000\ kN$$

Since we have two rows of nodes (2 x13), the share of each row is given by (refer to the figure):

$$P_c' = \frac{P_c}{No.of\ \ raws} = \frac{-3000}{2} = -1500\ kN$$

To distribute the forces along the nodes, it shall be assumed that the loads at the nodes are proportion to their distance from point of zero stress.

$$P_i = \frac{x_i}{\sum x_i} P_c$$

$$\sum x_i = 3.0 + 2.5 + 2.0 + 1.5 + 1.0 + 0.5 = 10.5 ms$$

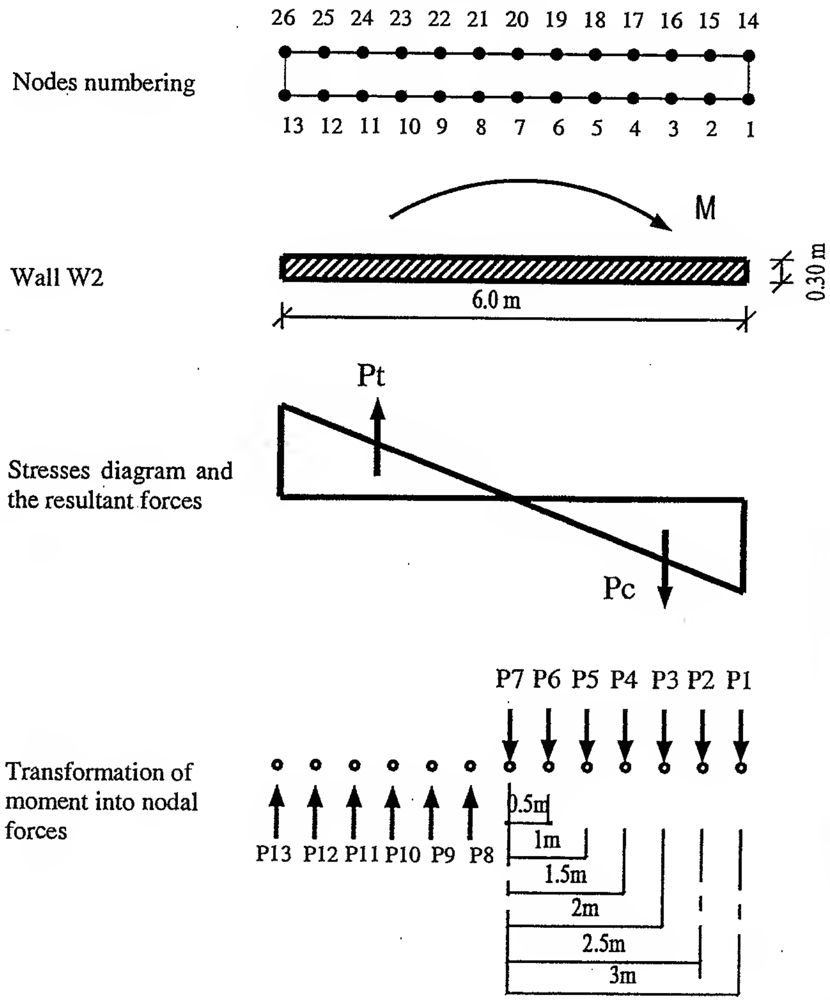
$$P_1 = \frac{3.0}{10.5} \times -1500 = -428.6\ kN \downarrow \qquad P_2 = \frac{2.5}{10.5} \times -1500 = -357.1\ kN \downarrow$$

$$P_3 = \frac{2.0}{10.5} \times -1500 = -285.7\ kN \downarrow \qquad P_4 = \frac{1.5}{10.5} \times -1500 = -214.3 kN \downarrow$$

$$P_5 = \frac{1.0}{10.5} \times -1500 = -142.9\ kN \downarrow \qquad P_6 = \frac{0.50}{10.5} \times -1500 = -71.4\ kN \downarrow$$

C-Total force

The total force at each node is given in Table EX 1.



Transformation of moment into nodal forces for wall W2

Table EX1 Total force for the shear walls at each node

Node	W1			W2		
	Force resulting form Normal force	Force resulting form bending moment	Resulting force at each node	Force resulting form Normal force	Force resulting form bending moment	Resulting force at each node
1	-190.9	-400.0	-590.9	-269.2	-428.6	-697.8
2	-190.9	-320.0	-510.9	-269.2	-357.1	-626.4
3	-190.9	-240.0	-430.9	-269.2	-285.7	-554.9
4	-190.9	-160.0	-350.9	-269.2	-214.3	-483.5
5	-190.9	-80.0	-270.9	-269.2	-142.9	-412.1
6	-190.9	0.0	-190.9	-269.2	-71.4	-340.7
7	-190.9	80.0	-110.9	-269.2	0.0	-269.2
8	-190.9	160.0	-30.9	-269.2	71.4	-197.8
9	-190.9	240.0	49.1	-269.2	142.9	-126.4
10	-190.9	320.0	129.1	-269.2	214.3	-54.9
11	-190.9	400.0	209.1	-269.2	285.7	16.5
12	-190.9	-400.0	-590.9	-269.2	357.1	87.9
13	-190.9	-320.0	-510.9	-269.2	428.6	159.3
14	-190.9	-240.0	-430.9	-269.2	-428.6	-697.8
15	-190.9	-160.0	-350.9	-269.2	-357.1	-626.4
16	-190.9	-80.0	-270.9	-269.2	-285.7	-554.9
17	-190.9	0.0	-190.9	-269.2	-214.3	-483.5
18	-190.9	80.0	-110.9	-269.2	-142.9	-412.1
19	-190.9	160.0	-30.9	-269.2	-71.4	-340.7
20	-190.9	240.0	49.1	-269.2	0.0	-269.2
21	-190.9	320.0	129.1	-269.2	71.4	-197.8
22	-190.9	400.0	209.1	-269.2	142.9	-126.4
23				-269.2	214.3	-54.9
24				-269.2	285.7	16.5
25				-269.2	357.1	87.9
26				-269.2	428.6	159.3
Total	-4200.0		-4200.0	-7000.0		-7000.0

In the computer model, the forces and moments acting on the columns were assigned to the joints. For shear walls, however, the resulting forces given in Table EX 1 that represents the applied moments and normal force were assigned to the nodes of the shear walls. The output of the program is shown in Fig. EX 9.5d

Step 4: Design of the raft
Step 4.1: Check of punching

$d = 1100 - 70 = 1030\ mm$

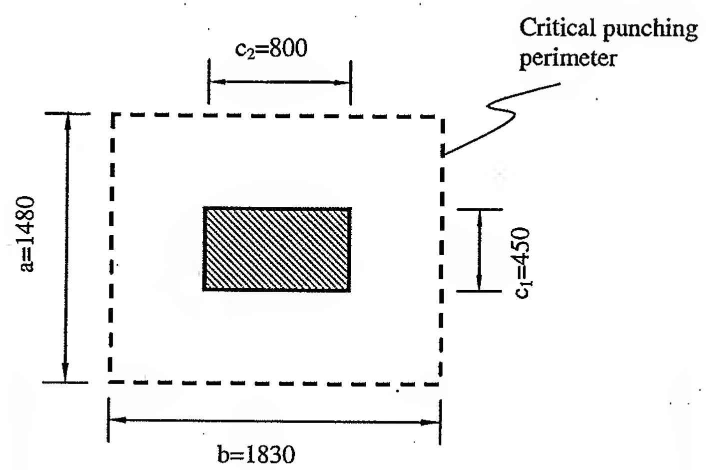
$Q_{up} = P_u = 5800$ kN $\qquad q_{up} = \dfrac{Q_{up}}{U \times d}$

$a = c_1 + d = 450 + 1030 = 1480\ mm$

$b = c_2 + d = 800 + 1030 = 1830\ mm$

$U = 2\,(a + b) = 2\,(1480 + 1830) = 6620\,mm$

$$q_{up} = \frac{5800 \times 10^3}{6620 \times 1030} = 0.85 N/mm^2$$



The concrete punching strength is the least of the three values:

1. $q_{cup} = 0.316\sqrt{\dfrac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\dfrac{35}{1.5}} = 1.52 N/mm^2 < 1.6.\ N/mm^2 ......o.k$

2. $q_{cup} = 0.316\,(0.50 + \dfrac{a}{b})\sqrt{\dfrac{f_{cu}}{\gamma_c}} = 0.316\,(0.50 + \dfrac{0.45}{0.80})\sqrt{\dfrac{35}{1.5}} = 1.62 N/mm^2$

3. $q_{cup} = 0.8\,(0.20 + \dfrac{\alpha\ d}{U})\sqrt{\dfrac{f_{cu}}{\gamma_c}} = 0.8\left(0.20 + \dfrac{4 \times 1.03}{6.62}\right)\sqrt{\dfrac{35}{1.5}} = 3.17\ N/mm^2$

$q_{cup} = 1.52$ N/mm^2 $\longrightarrow$ $q_{cup} > q_{up}$ .... ok

Step 4.2: Flexural Design

Step 4.2.1: Critical sections

The computer output of the raft foundation consists of bending moments acting in the two directions M_{11} (x-direction in this case) and M_{22} (y-direction in this case). The graphical representation is in the form of contour lines, in which each contour line represents a certain bending moment value as shown in Fig. EX. 9.5d . It should be mentioned that closely spaced contour lines indicate concentration of stresses. This usually occurs at the locations of the columns.

When designing the bottom reinforcement of the raft one should use the value of the bending moment at the face of the column.

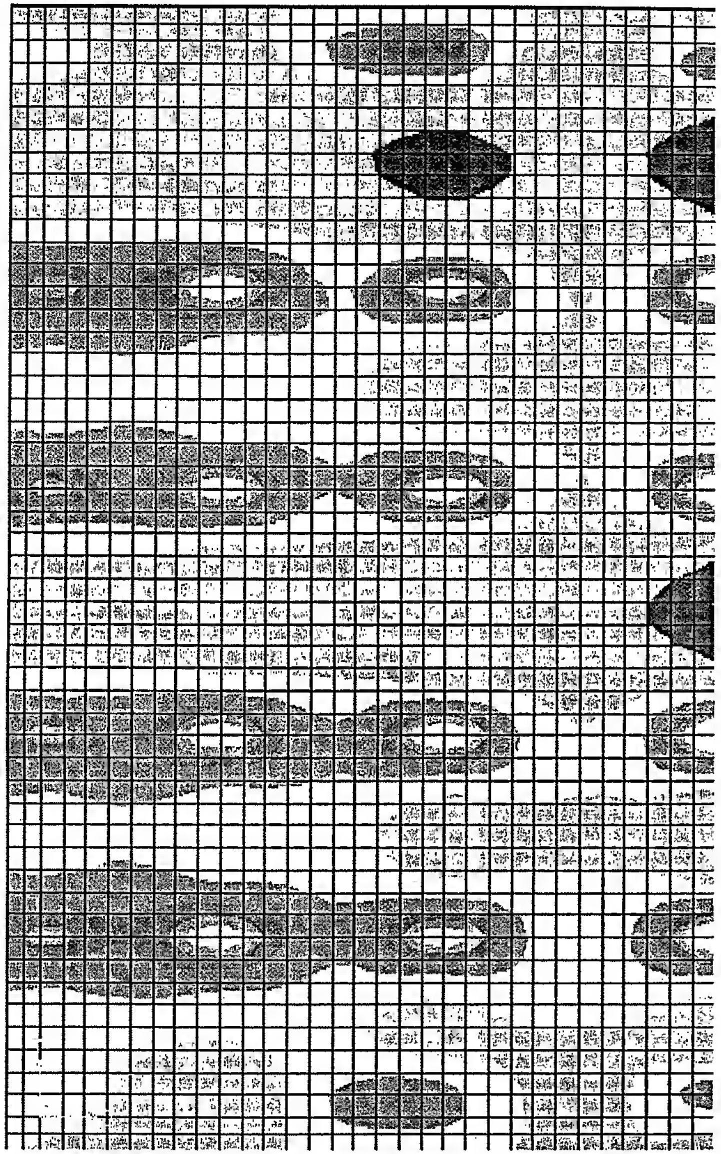


Fig. EX 9.5d Computer output

Step 4.2.2: Design of sections

The design for flexure for a critical section of 1.0 m width is carried out as follows :

Using the design aids (C1-J) curve

$$d = C_1 \sqrt{\frac{M_u}{f_{cu}\ B}}$$

$$1030 = C_1 \sqrt{\frac{M_u}{35 \times 1000}}$$

Get C_1 from (C1-J) curve and the corresponding value of J.

$$A_s = \frac{M_u}{f_y\ J\ d} = \frac{M_u}{360 \times J \times 1030} \geq A_{s\,min}$$

$$A_{s\,min} = \frac{0.6}{f_y} b\ d = \frac{0.60}{360} \times 1000 \times 1030 = 1716\ mm^2 \quad (5\Phi 22/m')$$

It is decided to use a mesh 5 Φ 22 /m′ top and bottom, and use addtioal reinforcement where needed.

It should be mentioned that the above procedure should be repeated for the case in which the earthquake load acting in the X-direction but in the reversed direction. In such a case, the moments acting on the shear walls will be reversed and the procedure described for transforming the moments and the normal forces acting on the shear walls into nodal forces will be followed.

Moreover, the raft should be analyzed for the case in which the earthquake load is actin in the Y-direction (straining actions are not given for that case).

The final reinfocement of the raft should cover all the cases.

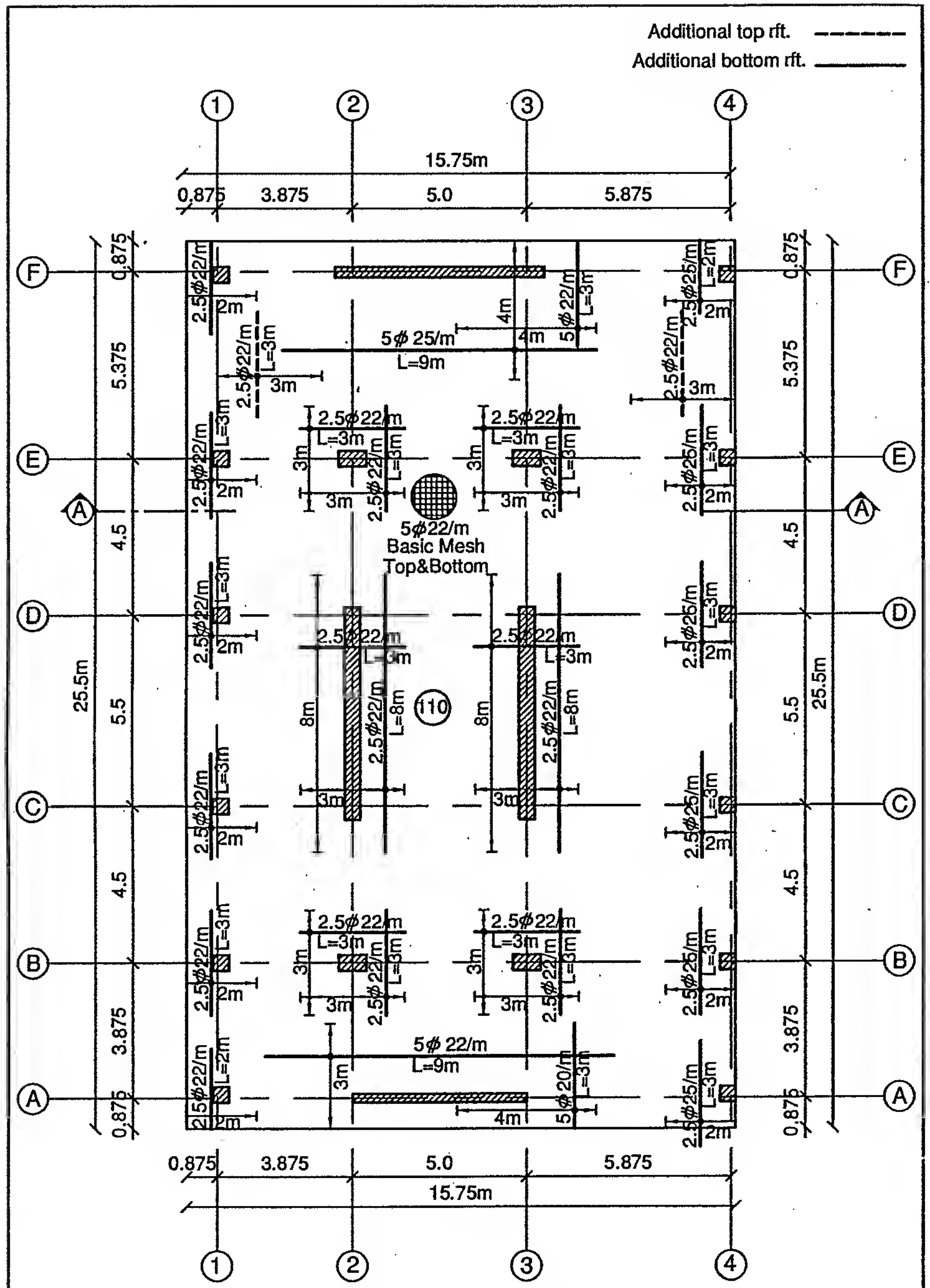


Fig. 5.9c Reinforcement of the raft

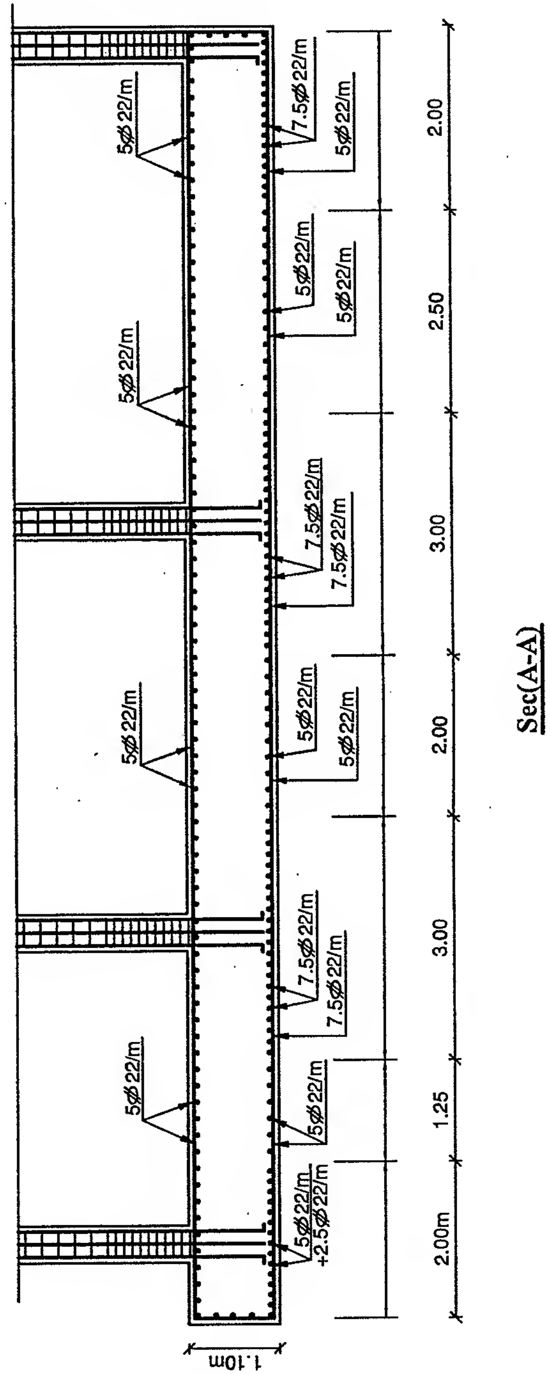
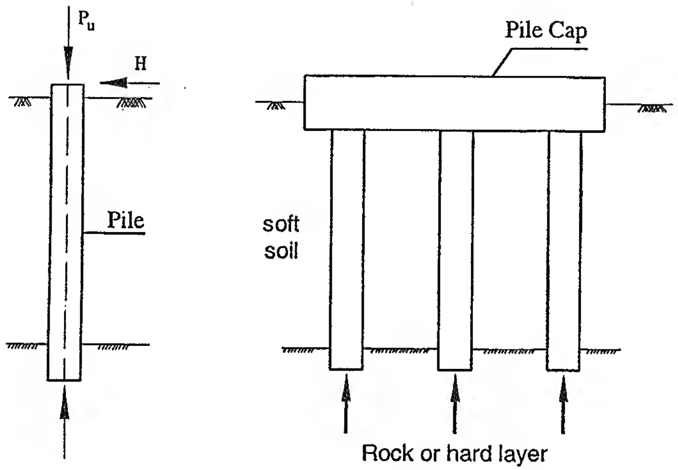


Fig. 5.9d Reinforcement of the raft(cont.)

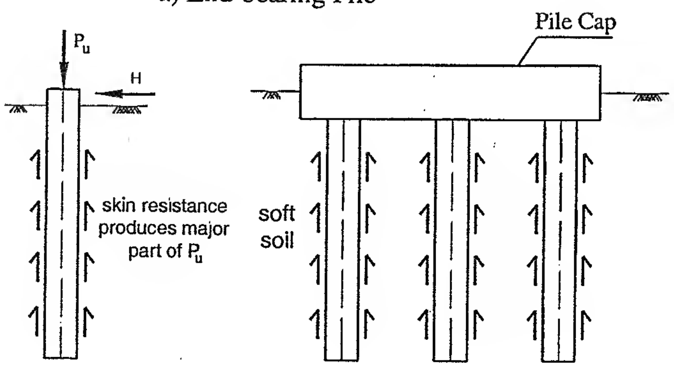
5.10 Design of Pile Caps

5.10.1 Introduction

Piles are structural members used to transmit surface loads to lower levels in the soil mass. This transfer could be made by a vertical distribution of the load along the pile shaft or by a direct application of load to a lower stratum through the pile base. A direct load application is made by an *end-bearing pile* as shown in Fig. 5.27a and a vertical distribution of the load is made using a *friction pile* as shown in Fig. 5.27b. In general, most piles carry loads as a combination of side resistance and point bearing except when the pile penetrates an extremely soft soil to a solid base.



a) End-bearing Pile



b) Friction Pile

Fig. 5.27 Friction and end bearing piles

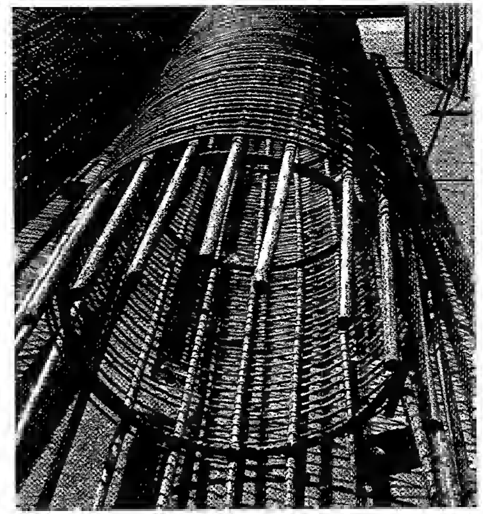


Photo 5.10 Reinforcement of a pile

Piles are commonly used for the following purposes:

- To carry the superstructure loads into or through a soil stratum.
- To resist uplift such as for basement rafts below the water-table.
- To resist overturning such as for tower legs subjected lateral loads.
- To control settlements in case the structure is underlain by a highly compressible stratum.

Photo 5.11 Construction of pile caps

5.10.2 Configurations of Pile Caps

Unless a single pile is used, a cap is necessary to spread the vertical and horizontal loads and any overturning moments to all the piles in the group. Pile caps take different shapes according to the number of piles used as shown in Fig. 5.28. The pile cap has a reaction that is a series of concentrated loads at the locations of the piles.

The acting loads on the pile cap includes the column loads and moments, any soil overlying the cap (if it is below the ground surface), and the weight of the cap.

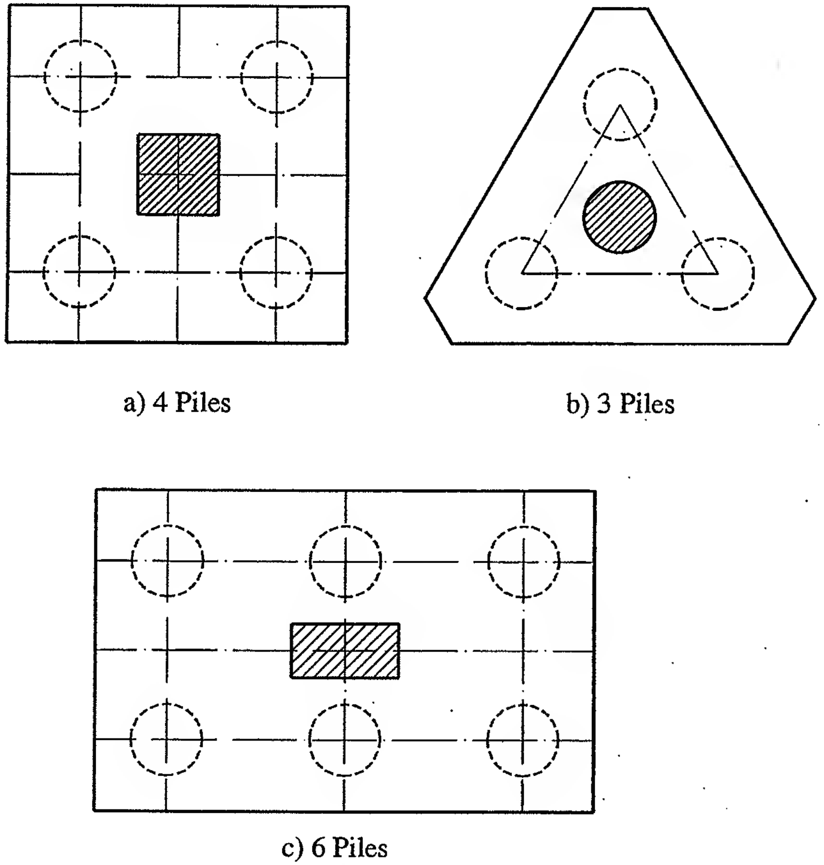


Fig. 5.28 Pile cap shapes according to the number of piles

5.10.3 Design of Pile Caps

Pile caps may be designed using one of the following methods:

1. Conventional design method.
2. Finite element method.
3. Strut and Tie method

5.10.3.1 Design using the conventional Method

Step 1: Determine the load of each pile

For a concentrically loaded pile cap, the load per pile is given by:

$$P_{pile} = \frac{1.05 \times N}{n} \leq P_{allowable} \quad \text{.................................} (5.43)$$

in which

P = Pile working load
1.05 = factor taking into account the pile cap self weight (5% of the load)
n = Number of piles
N = Working load of the column
$P_{allowable}$ = Allowable pile load

For eccentrically loaded pile caps, the load per pile is given by:

$$P_{pile} = \frac{N}{n} + \frac{M_y}{\sum x^2} x + \frac{M_x}{\sum y^2} y \leq P_{allowable} \quad \text{....................} (5.44)$$

where

M_x, M_y = moments about x and y axes, respectively
x, y = distances from y and x axes to any pile
$\sum x^2$, $\sum y^2$ = summation of the square distance from pile group center.

Step 2: One-way shear strength of pile-caps

The critical section is located at d/2 from the face of the column; where d is the depth of the pile cap.

With reference to Fig 5.29, the computation of the one-way shear on any section through a footing supported on piles shall be in accordance with the following:

- The entire reaction from any pile whose center is located $\Phi/2$ or more outside the critical section shall be considered (case (a)).

- The reaction of the pile is neglected if the pile center is located at $\Phi/2$ or more inside the critical section, case (b).
- For intermediate positions of pile center, the portion of the pile reaction to be considered as producing shear on the section shall be based on straight-line interpolation between full value at $\Phi/2$ outside the section and zero value at $\Phi/2$ inside the section, case (c).

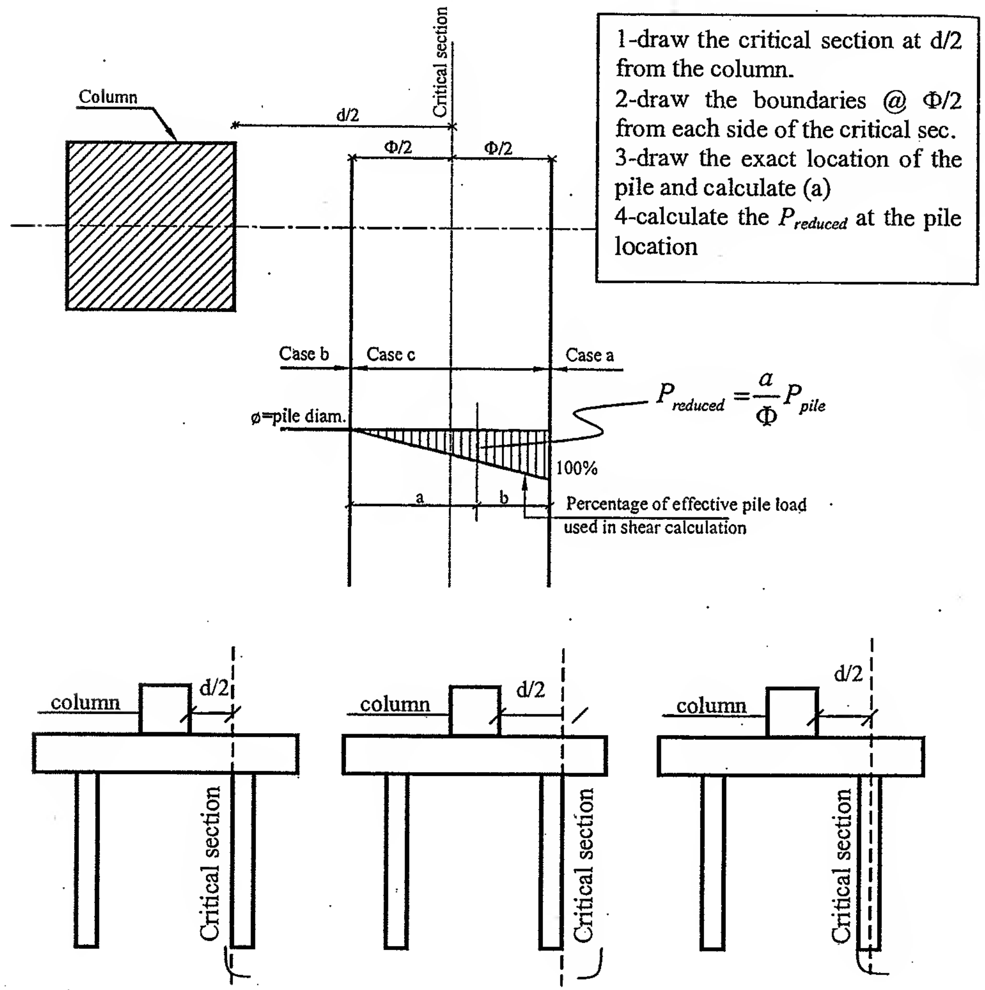


Fig. 5.29 Effective pile load for one-way shear

Step 3: Two-way (Punching) shear strength of pile caps

The calculation of the punching load for a pile cap is minimally addressed in the literature. The ECP 203 does not give explicit procedure for calculating the punching load for pile caps. In this text, an approximate procedure is proposed. It should be emphasized, however, that such an approximate procedure does not reflect the actual complicated behavior.

Consider the pile cap shown in Fig. 5.30. The critical section for punching is located at d/2 from the face of the column. It will be assumed that parts of the piles located inside the punching perimeter shall participate in reducing the punching load. Referring to Fig. 5.30, the punching load can be calculated as follows:

$$\lambda = \frac{\text{hatched area of the pile}}{\text{gross area of the pile}} \quad \text{.........................} (5.45)$$

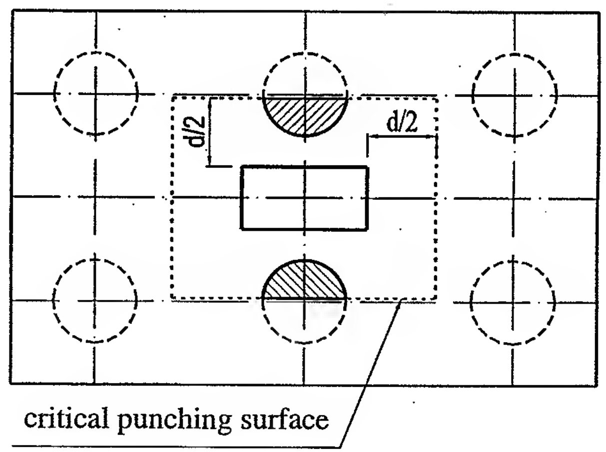


Fig. 5.30 Critical section for punching

$$Q_{up} = (\text{Column ultimate load} + \text{o.w. of pile cap within the punching perimeter}) - 2 \times \lambda \times \text{pile load}$$

$$\text{The punching stress } q_{up} = \frac{Q_{up}}{U \times d} \quad \text{...} (5.46)$$

The concrete punching strength is given as the least of the following values:

1. $q_{cup} = 0.316 \sqrt{\frac{f_{cu}}{\gamma_c}} \leq 1.6\ N/mm^2$ ....(5.47a)

2. $q_{cup} = 0.316\ (0.50 + \frac{a}{b})\sqrt{\frac{f_{cu}}{\gamma_c}}$ .... (5.47b)

3. $q_{cup} = 0.8\ (0.20 + \frac{\alpha\ d}{U})\sqrt{\frac{f_{cu}}{\gamma_c}}$ ....(5.47c)

where q_{cup} is the punching shear strength provided by concrete; (a/ b) is the ratio of long side to short side of column, α= 4, 3, and 2 for interior, edge, and corner columns, respectively, d is the effective shear depth of the pile cap (average flexural depth in the two directions), U is the perimeter of the critical section, and f_{cu} is the concrete compressive strength. Check of punching should be performed around the individual pile.

Step 4: Design for Flexure

The ECP 203 requires the critical section for flexure to be taken at the face of the column as shown in Fig. 5.31. Pile caps must be reinforced in two perpendicular directions. In most cases, an isolated centrally-loaded pile cap supporting a single column needs only bottom reinforcement. However, eccentrically loaded pile caps and pile caps supporting more than one column might need top reinforcement as well.

The minimum cover for the reinforcement is 70 mm (concrete cast against soil).

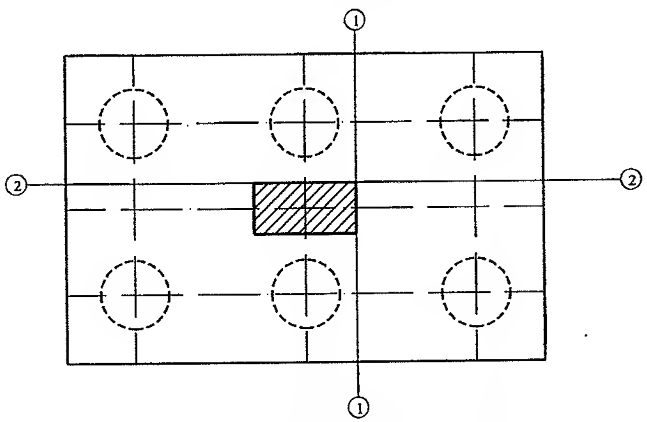


Fig. 5.31 Critical section for flexure

5.10.3.2 Finite Element Analysis of Pile Caps

The number of piles can be determined using the procedure mentioned in step 1 of the conventional method. Pile cap bending moments can be obtained using the FEM. Such a procedure can be carried out using commercially available computer programs. It should be mentioned, however, that one-way shear and punching shear can not be obtained from such an analysis. One-way shear punching shear calculations should be made according to the procedures mentioned in the conventional method.

The pile cap is modeled using shell elements while the piles are modeled using spring elements as shown in Fig. 5.32. Due to the fact that pile caps are usually thick, the use of shell elements that do not consider the effect of shear deformation is not advisable. Past experience, however, proved that the use of ordinary (thin) shell elements is adequate for design purposes. Deign moments should be calculated at the face of the column.

It is a common practice to calculate the spring constant of piles based on the permissible settlement of the pile during the pile load test. In other words, the pile spring constant K_{pile} is calculated as follows:

$$K_{pile} = \frac{\text{Pile working load}}{\text{Permissible settlement}} \quad \text{.....................} (5.48)$$

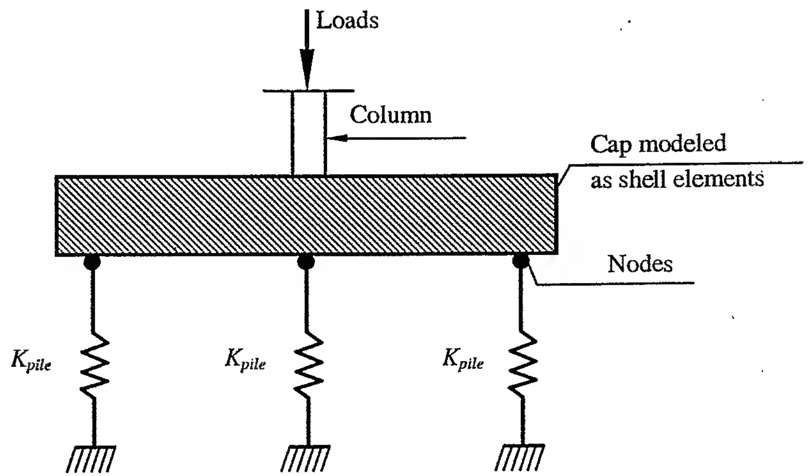
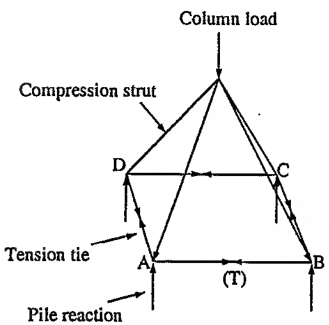


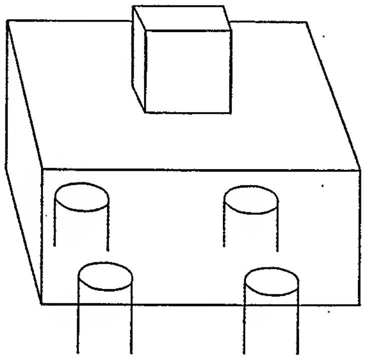
Fig. 5.32 Modeling of the pile cap

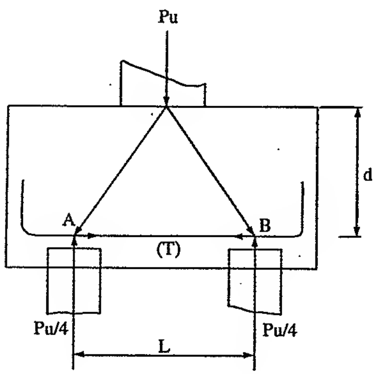
5.10.3.3 Design using The Strut and Tie Method

The Egyptian Code ECP 203 allows the use of *the Strut-and-Tie* method for designing pile caps. The structural action of a four-pile group is shown in Fig. 5.33. The pile cap is a special case of a deep beam and can be idealized as a space truss with four compression struts transferring load from the column to the piles, and four tension ties equilibrating the outward components of the compression struts. The tension ties have constant force in them and must be anchored for the full horizontal tie force outside the intersection of the pile and the compression strut. Hence, bars must either extend a distance equal to the anchorage length past the centerlines of the piles, or they must be hooked outside this point. For the pile cap shown in Fig. 5.33, the total tie force in one direction can be calculated from the force triangular shown.



(b) internal forces in pile cap


(a) Pile cap



(C) Force in tie A-B

Fig. 5.33 Strut and Tie method for a pile cap

Example 5.10

Design and give complete reinforcement detailing for a pile cap that constitutes a part of a deep foundations system of a high-rise building. The design data are as follows:

Column dimensions	= 900 mm x 900 mm
Unfactored column load	= 5000 kN
Factored column load	= 7500 kN
Pile diameter	= 800 mm
Pile working load	= 1400 kN
f_{cu}	= 40 N/mm^2
f_y	= 360 N/mm^2

Solution

Step 1: Dimensions of the pile cap

In order to determine the dimensions of the pile cap, one has to determine the number of piles.

$$\text{Number of plies} = \frac{\text{Unfactored load of column x 1.05}}{\text{Pile working load}} = \frac{5000x\ 1.05}{1400} = 3.75$$

Choose 4 piles. It should be noted that the multiplier 1.05 takes into consideration the own-weight of the pile cap.

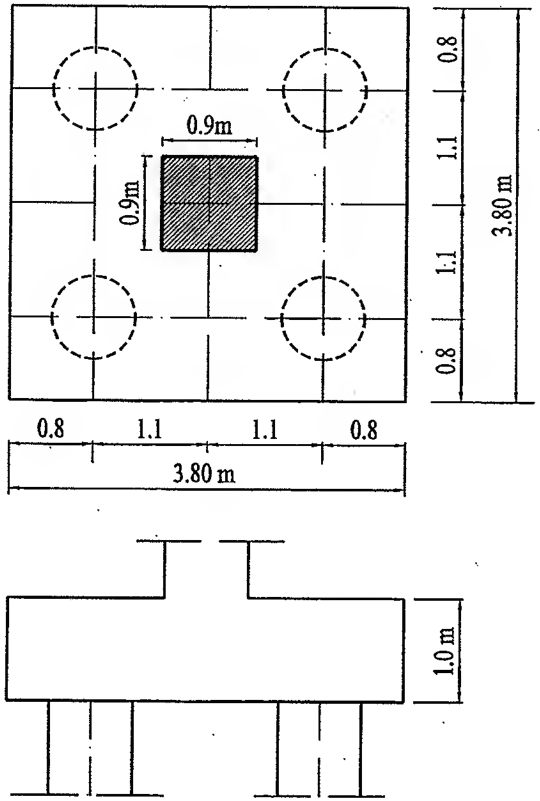
The spacing between the piles is usually taken (2.5 $\phi - 3$ ϕ). In this example, the spacing between piles is taken = 2.2m. The distance from the centerline of the pile to the edge is taken (0.8 ϕ -1 ϕ).

The dimensions of the pile cap are shown in the following figure. The thickness of the pile cap shall be assumed equal to 1.0 m.

Unfactored own-weight of the pile cap = 3.8 x 3.8 x 1.0 x 25 = 361 kN

$$\text{Exact pile load} = \frac{\text{unfactored column load} + \text{own weight of pile cap}}{\text{number of piles}}$$

$$= \frac{5000+361}{4} = 1340.25\,kN \ < 1400\,kN \ ....ok$$



Step 2: Design for shear

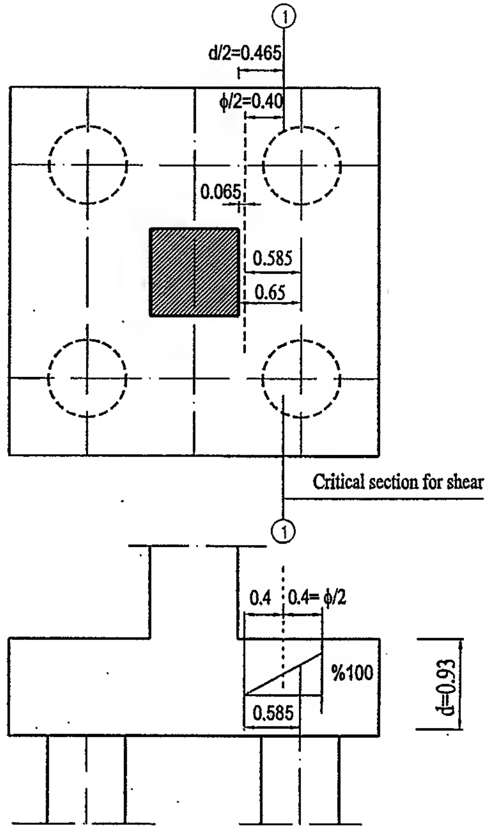
Step 2.1: One-way shear

$$\text{Ultimate load of pile} = \frac{\text{factored load of column} + \text{factored O.W. of pile cap}}{4}$$

$$\text{Ultimate load of pile} = \frac{7500 + 1.4 \times 361}{4} = 2001.35 \quad \text{kN.m}$$

$d = 1000 - 70 = 930\,mm$

The critical section for one-way shear is at d/2 from the face of the column as shown in the following figure.



SEC(1-1)

According to the ECP-203, the pile load that should be considered when checking the shear strength of pile caps can be reduced depending on the location of the center of the pile with respect to the critical section.

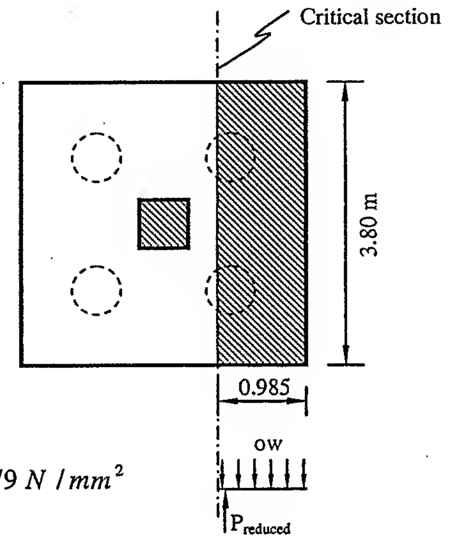
$P_{reduced}$ = reduced pile load for checking shear strength

$P_{reduced}$ = reduction factor × ultimate load of pile

$$= \frac{585}{800} \times 2001.35 = 1463.487 \text{ kN}$$

Q_u = 2×reduced pile load - O.W. of pile cap outside of the critical section (hatched area)

$$Q_u = 2\times1463.487 - 1.4\times25\times3.8\times0.985\times1.0 \cong 2796 \text{ kN}$$



$$q_u = \frac{Q_u}{b\times d} = \frac{2796\times10^3}{3800\times930} = 0.79\ N/mm^2$$

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.16\sqrt{\frac{40}{1.5}} = 0.826\ N/mm^2$$

Since $q_u < q_{cu}$, the thickness of the pile cap is considered adequate for one-way shear.

Step 2.2: Punching shear

The critical section is at d/2 from the column face as shown in the figure below.

$$a_1 = c_1 + d = 900 + 930 = 1830\,mm = 1.83\,m$$

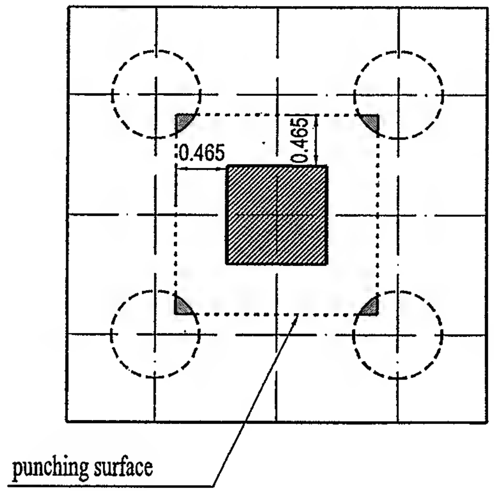
$$b_1 = c_2 + d = 900 + 930 = 1830\,mm = 1.83\,m$$

$$U = 2\times(a_1 + b_1) = 4\times1830 = 7320\,mm$$

The ECP 203 does not give explicit procedure for calculating punching load for pile caps. However, it can be assumed that the punching load equals to the column load minus the parts of the piles' loads located within d/2 from the face of the column. It will further be assumed that the load resisted by a certain area of a pile is equal to the total load resisted by the pile multiplied by the ratio of that area to the gross area of the pile.

Referring to figure, it can be noted that very small area of each pile is located inside the critical punching area (3.5%). According to the previous procedure, the punching load could be calculated as follows:

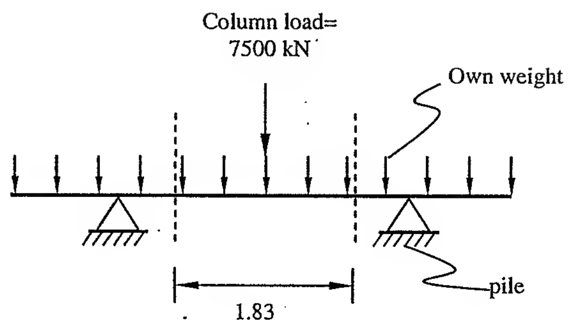
$$\lambda = \frac{\text{hatched area of the pile}}{\text{gross area of the pile}} = 0.035$$



Q_{up} = (Column load + O.W. of pile cap within the punching perimeter) - 4×λ×pile load

$$Q_{up} = (7500 + 1.4\times25\times1.83\times1.83\times1.0) - 4\times0.035\times2001.35 = 7337 \text{ kN}$$

$$q_{up} = \frac{Q_{up}}{U\times d} = \frac{7337\times1000}{7320\times930} = 1.07\,N/mm^2$$



The concrete strength for punching is the least of the three values:

1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{40}{1.5}} = 1.63\,\text{N/mm}^2 > 1.6\ \text{N/mm}^2 \rightarrow = 1.6\ \text{N/mm}^2$

2. $q_{cup} = 0.316\ (0.50 + \frac{a}{b})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\ (0.50 + \frac{0.90}{0.90})\sqrt{\frac{40}{1.5}} = 2.44\,N\ /mm^2$

3. $q_{cup} = 0.8\ (0.20 + \frac{\alpha\ d}{U})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\left(0.20 + \frac{4\times0.93}{7.32}\right)\sqrt{\frac{40}{1.5}} = 2.92\ N\ /mm^2$

$q_{cup} = 1.60\ \text{N/mm}^2$

Since $q_{up} \langle\ q_{cup}$, the thickness of the pile cap is adequate for punching shear.

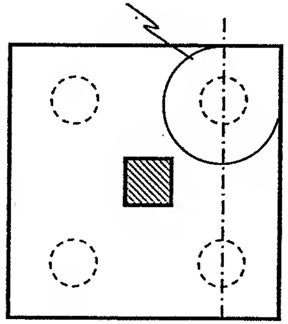
Note: The reader might notice that the reduction of the punching load due to the existence of the parts of the areas of the piles within the perimeter of punching complicates the calculations. Accordingly, the designer could conservatively neglect such a reduction in cases where it has trivial effect on the results.

Check of punching for individual piles

Pile load = 2001.35 kN

From the figure U=3.40 m

$$q_{up} = \frac{Q_{up}}{U \times d} = \frac{2001.35\times1000}{3400\times930} = 0.63\,N\ /mm^2 \rightarrow \text{safe}$$

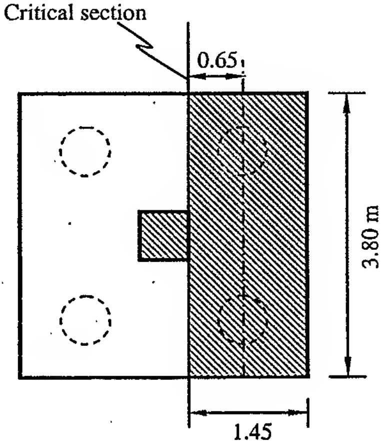


Step 3: Design for flexure

The critical section for flexure is at the face of the column.

M_u = 2 x factored load of pile x 0.65 - moment developed due to the O.W. of the hatched part of the pile cap

$$M_u = 2\times2001.35\times0.65 - 1.4\times25\times3.8\times\frac{1.45^2}{2} \cong 2462\ kN.m$$



$$d = C_1\sqrt{\frac{M_u}{f_{cu}\ B}} \qquad 930 = C_1\sqrt{\frac{2462\times10^6}{40\times3800}}$$

$C_1 = 7.3 \rightarrow$ Take c/d=0.125 $\rightarrow$ J = 0.825

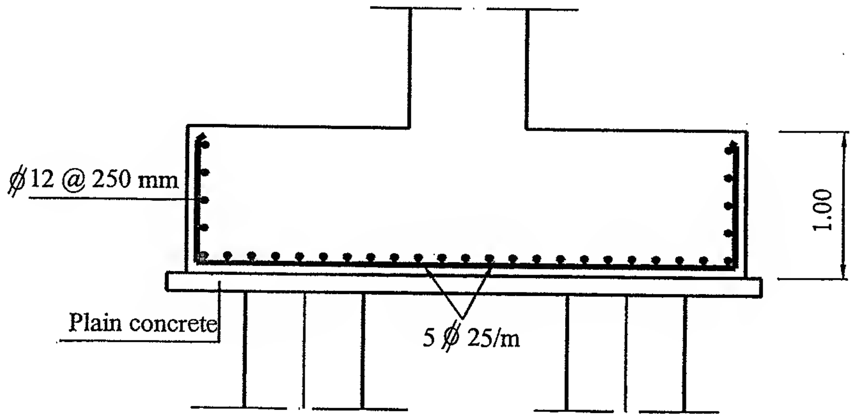
$$A_s = \frac{M_u}{f_y\ .J\ .d} = \frac{2462\times10^6}{360\times0.825\times930} = 8902\ mm^2$$

$$A_s/m = \frac{8902}{3.8} = 2342 \qquad mm^2/m'$$

Check the minimum steel requirement

$$A_{s,\min} = \frac{0.60}{f_y}\times b\times d = \frac{0.6}{360}\times1000\times930 = 1550\ mm^2 < A_s$$

Choose $5\phi25/m'$ (2454 mm²)



Reinforcement Details

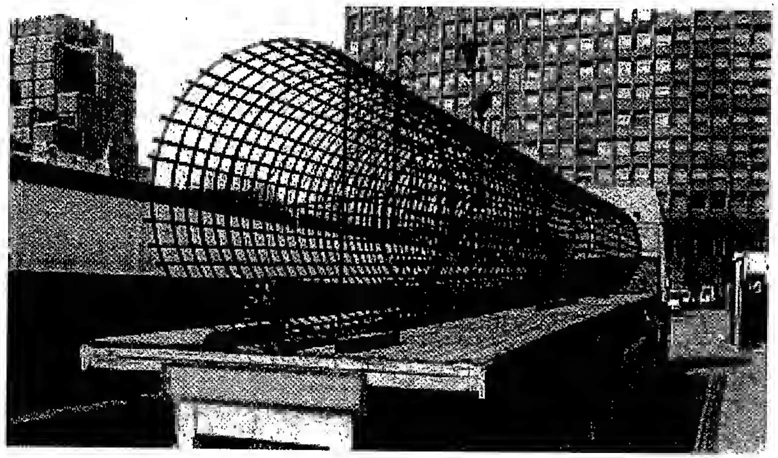


Photo 5.12 Reinforcement arrangement of a pile

Example 5.11

Design and give complete reinforcement detailing for a pile cap that constitutes a part of a deep foundations system of a factory. The design data are as follows:

Diameter of circular Column	= 850 mm
Unfactored column load	= 4500 kN
Factored column load	= 6750 kN
Pile diameter	= 800 mm
Pile working load	= 1750 kN
f_{cu}	= 35 N/mm^2
f_y	= 400 N/mm^2

Solution

Step 1: Dimensions of the pile cap

In order to determine the dimensions of the pile cap, one has to determine the required number of piles.

$$\text{Number of plies} = \frac{\text{Unfactored load of column x 1.05}}{\text{Pile working load}} = \frac{4500 \times 1.05}{1750} = 2.7$$

Choose 3 piles. The multiplier 1.05 takes into consideration the own weight of the pile cap.

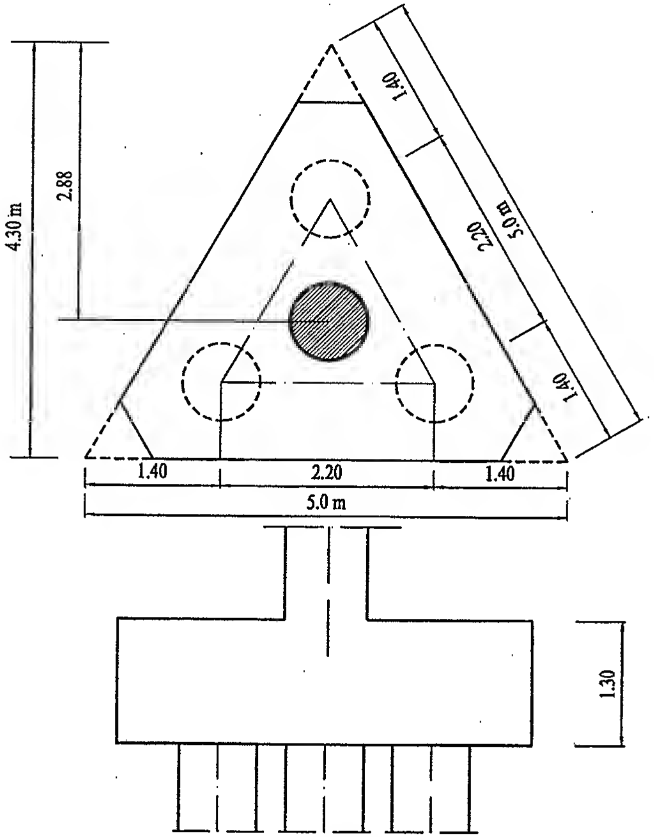
The spacing between the piles is usually taken (2.5 ϕ – 3 ϕ). In this example, the spacing between piles is taken = 2.2 m. The distance from the centerline of the pile to the edge is taken (0.8 ϕ -1 ϕ).

The plan dimensions of the pile cap are shown in the following figure. The thickness of the pile cap shall be assumed equal to 1.3 m.

Unfactored own weight of the pile cap = 5.0 x 4.33/2 x 1.3 x 25 = 351.8 kN

$$\text{Exact pile load} = \frac{\text{unfactored column load} + \text{own weight of pile cap}}{\text{number of piles}}$$

$$= \frac{4500 + 351.8}{3} = 1617.35\ kN .. < 1750\ kN ......ok$$



Pile cap arrangement

Step 2: Design for shear

Step 2.1: One-way shear

$$\text{Ultimate load of pile} = \frac{\text{factored load of column} + \text{factored O.W. of pile cap}}{3}$$

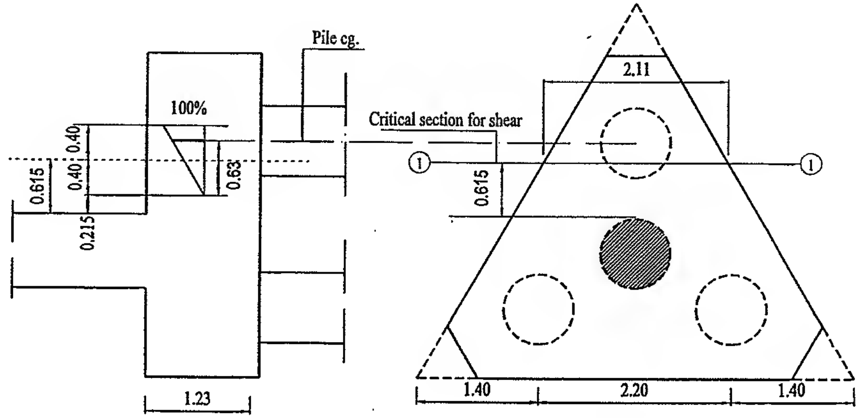
$$\text{Ultimate load of pile} = \frac{6750 + 1.4 \times 351.8}{3} = 2414.2 \quad \text{kN}$$

$$d = 1300 - 70 = 1230\,mm$$

There are two possible sections for one-way shear as follows:

Critical section 1

The critical section for one-way shear is at d/2 from the face of the column as shown in the figure below.



According to the ECP-203, the pile load considered when checking the shear strength of pile caps can be reduced depending on the location of the center of the pile with respect to the critical section.

From the figure, the distance (x) from the center of gravity of the column to the center of gravity of the pile equals = 1.1/cos 30° =1.27 m.

$P_{reduced}$ = reduction factor × ultimate load of pile

$$= \frac{630}{800} \times 2414.2 = 1901.2 \text{ kN}$$

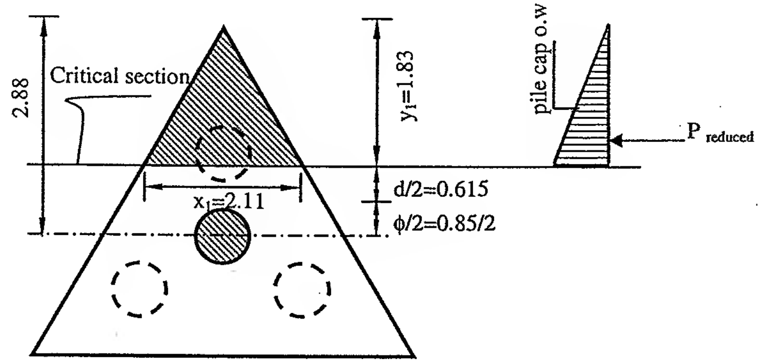
Q_u = reduced pile load - O.W. of pile cap outside of the critical section (hatched area)

$$y_1 = 2.88 - \frac{0.85}{2} - 0.615 = 1.83$$

$$x_1 = \frac{1.83}{\sin 60} = 2.11\ m$$

O.W. of the pile cap = O.W. of the hatched part in the following figure

$= 1.4 \times 2.11 \times 1.83/2 \times 1.3 \times 25 = 87.8$ kN



Q_u =1901.2 − 87.8=1813.4 kN

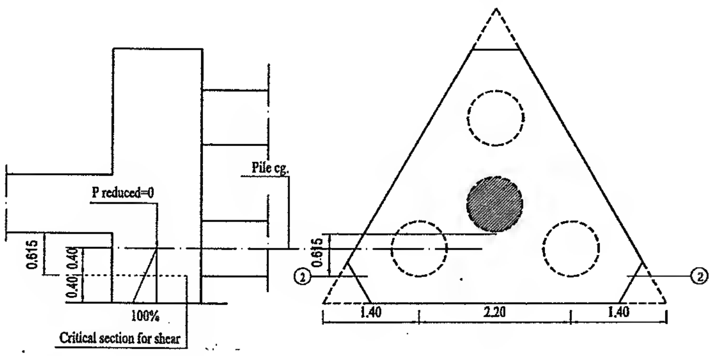
$$q_u = \frac{Q_u}{b \times d} = \frac{1813.4 \times 10^3}{2110 \times 1230} = 0.70\ N/mm^2$$

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.16\sqrt{\frac{35}{1.5}} = 0.77\ N/mm^2$$

Since $q_u \langle q_{cu}$, the thickness of the pile cap is adequate for one-way shear.

Critical section 2

From the figure below, it is clear that Q_u at the center of gravity of the pile is almost equal to zero. Therefore, the section is considered adequate.

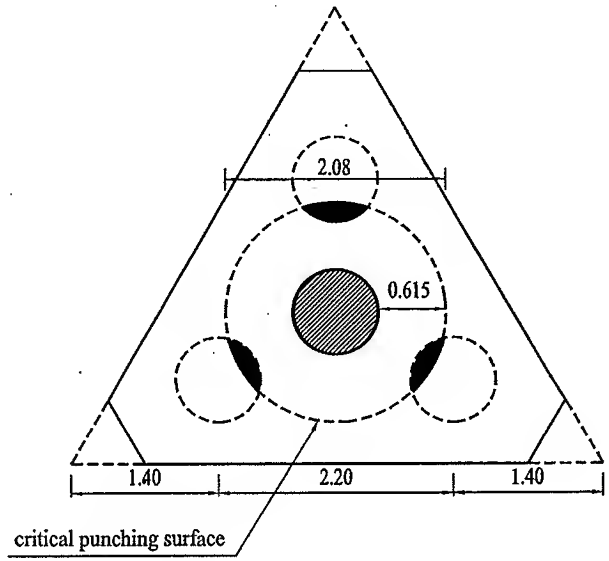


Step 2.2: Punching shear

The critical section for punching shear is at d/2 from the column face as shown in the figure below.

$D_1 = D_{column} + d = 850 + 1230 = 2080\,mm = 2.08\,m$

$U = \pi\ D_1 = \pi \times 2080 = 6535\,mm$



The punching load could be calculated as follows:

$\lambda = \dfrac{\text{hatched area of the pile}}{\text{gross area of the pile}} = 0.12$ (hatched area is calculated using AutoCAD)

Q_{up} =(Column load + O.W. of pile cap within the punching perimeter)- 3×λ×pile load

$$Q_{up} = (6750 + 1.4 \times 25 \times \frac{\pi}{4} \times (2.08)^2 \times 1.3) - 3 \times 0.12 \times 2414.2 = 6035.5 \text{ kN}$$

$$q_{up} = \frac{Q_{up}}{U \times d} = \frac{6035.5 \times 1000}{6535 \times 1230} = 0.75\,N/mm^2$$

The concrete strength for punching is the least of the three values

1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{35}{1.5}} = 1.53\,N/mm^2 < 1.6\,N/mm^2$

2. $q_{cup} = 0.316\,(0.50+\frac{a}{b})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\,(0.50+1)\sqrt{\frac{35}{1.5}} = 2.29\,N/mm^2$

3. $q_{cup} = 0.8\,(0.20+\frac{\alpha\,d}{U})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\left(0.20+\frac{4\times1.23}{6.535}\right)\sqrt{\frac{35}{1.5}} = 3.68\,N/mm^2$

q_{cup}= 1.53 N/mm^2

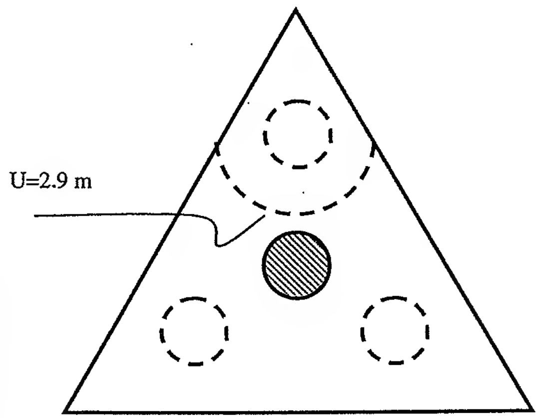
Since $q_{up} \langle q_{cup}$, the thickness of the pile cap is adequate for punching shear.

Check of punching for individual piles

Pile load = 2414.3 kN

From the figure U=2.90 m

$$q_{up} = \frac{Q_{up}}{U\times d} = \frac{2414.3\times1000}{2900\times1230} = 0.68\,N/mm^2 \rightarrow \text{safe}$$



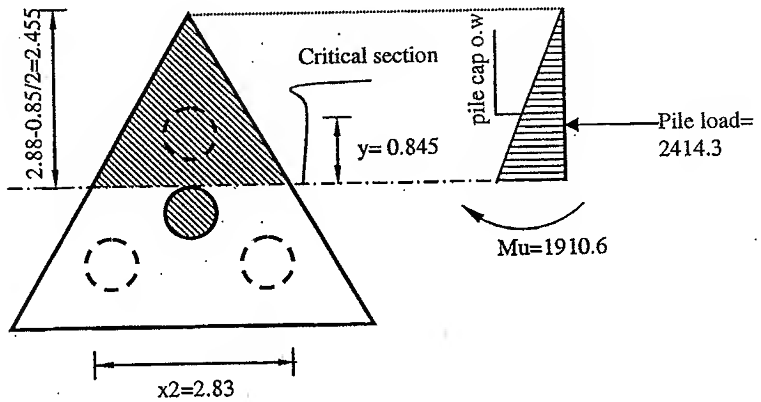
Step 3: Design for flexure

The critical section for flexure is at the face of the column

$$x_2 = \frac{2.455}{\sin 60} = 2.83\,m$$

M_u = factored load of pile • y - moment developed due to the O.W. of the hatched pile cap

$$M_u = 2414.2\times0.845 - 1.4\times25\times1.3\times\frac{2.455\times2.83}{2}\times\frac{2.455}{3} = 1910.6\,kN.m$$



$$d = C_1\sqrt{\frac{M_u}{f_{cu}\,B}} \qquad 1230 = C_1\sqrt{\frac{1910.5\times10^6}{35\times2830}}$$

$C_1 = 8.86 \rightarrow$ Take c/d=0.125 $\rightarrow$ J = 0.825

$$A_s = \frac{M_u}{f_y\,.J\,.d} = \frac{1910.5\times10^6}{400\times0.825\times1230} = 4701\,mm^2$$

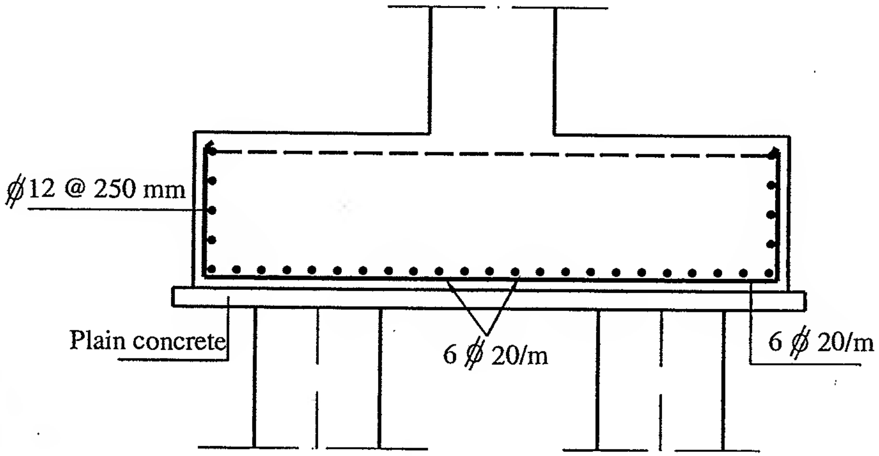
$$A_s/m = \frac{4701}{2.83} = 1658 \qquad mm^2/m'$$

Check the minimum steel requirement

$$A_{s,\min} = \frac{0.60}{f_y}\times b\times d = \frac{0.60}{400}\times1000\times1230 = 1845\,mm^2$$

Since As < A_{smin}, use A_{smin}

Choose $6\Phi20/m'$ (1884 mm^2)



Reinforcement details for the pile cap

Example 5.12

Design and give complete reinforcement detailing for a pile cap that constitute a part of a deep foundations system of an office building. Design data:

Column dimensions	= 600 mm x 1200 mm
Unfactored column load	= 7000 kN
Factored column load	= 10500 kN
Pile diameter	= 800 mm
Pile working load	= 1350 kN
f_{cu}	= 35 N/mm^2
f_y	= 360 N/mm^2

Solution

Step 1: Dimensions of the pile cap

In order to determine the dimensions of the pile cap, one has to determine the number of piles.

$$\text{Number of plies} = \frac{\text{Unfactored load of column x 1.05}}{\text{Pile working load}} = \frac{7000 \, x \, 1.05}{1350} = 5.44$$

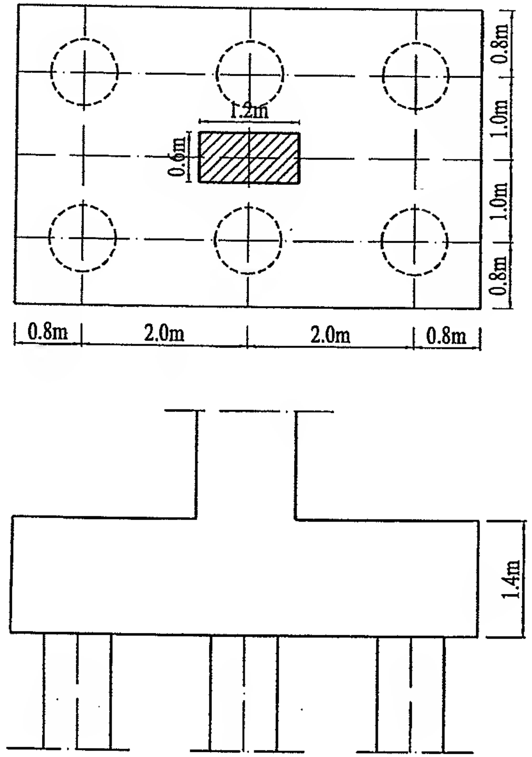
Choose 6 piles. The multiplier 1.05 takes into consideration the won weight of the pile cap.

The spacing between the piles is usually taken (2.5 $\phi - 3\ \phi$). In this example, the spacing between piles is taken = 2.0m. The distance from the centerline of the pile to the edge is taken 0.80 m. The plan dimensions of the pile cap are shown in the following figure. The thickness of the pile cap shall be assumed equal to 1.4 m.

Unfactored own weight of the pile cap = $25 \times 5.6 \times 3.6 \times 1.4$ = 705.6 kN

$$\text{Exact pile load} = \frac{\text{unfactored column load} + \text{own weight of pile cap}}{\text{number of piles}}$$

$$= \frac{7000 + 705.6}{6} = 1284.3\,kN < 1350 \ldots.ok$$



Pile cap arrangement

Step 2: Design for shear

Step 2.1: One-way shear

$$\text{Ultimate load of pile} = \frac{\text{factored load of column} + \text{factored O.W. of pile cap}}{6}$$

$$\text{Ultimate load of pile} = \frac{10500 + 1.4 \times 705.6}{6} = 1914.6 \quad \text{kN}$$

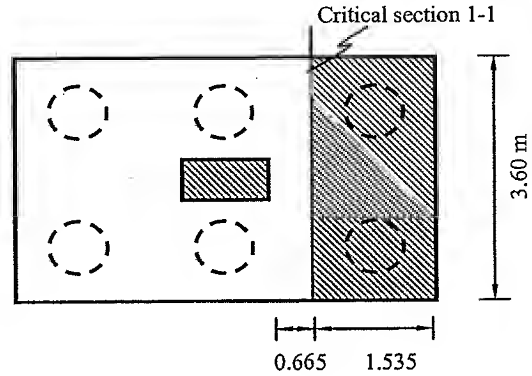
$$d = 1400 - 70 = 1330\, mm$$

The critical section for one-way shear is at d/2 from the face of the column as shown in the following figure.

Critical section 1-1

The distance between the C.G. of piles and the critical section for one-way shear is more than d/2 (0.665). Hence, no reduction in pile loads.

$$Q_u = 2 \times 1914.6 - 1.4 \times 25 \times 3.6 \times (1.535) \times 1.4 = 3559 \text{ kN}$$



$$q_u = \frac{Q_u}{b \times d} = \frac{3559 \times 10^3}{3600 \times 1330} = 0.74\ N/mm^2$$

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.16\sqrt{\frac{35}{1.5}} = 0.77\ N/mm^2$$

Since $q_u < q_{cu}$, the thickness of the pile cap is considered adequate for one-way shear

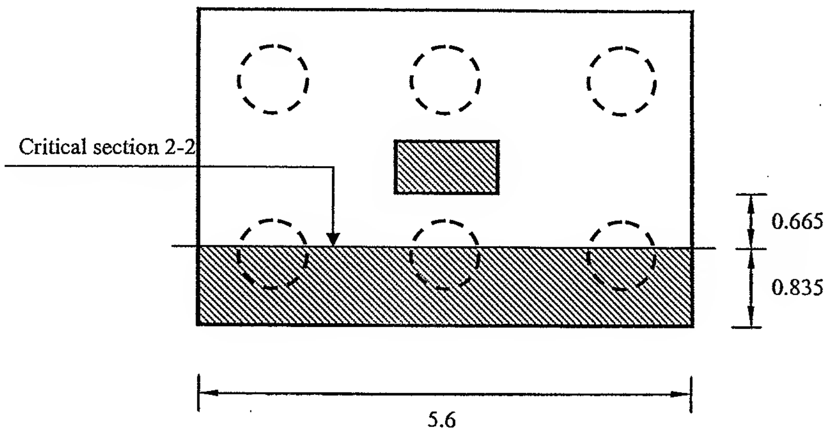
Critical section 2-2

$P_{reduced}$ = reduction factor x ultimate load of pile

$$= \frac{435}{800} \times 1914.6 = 1041.1 \text{ kN}$$

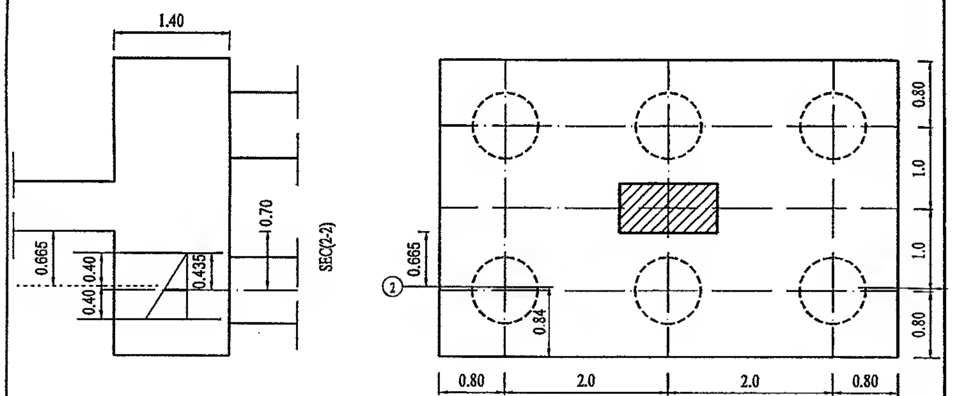
Q_u = 3 × reduced pile load - o.w. of pile cap outside of the critical section(hatched area)

$$Q_u = 3 \times 1041.1 - 1.4 \times 25 \times 5.6 \times 0.835 \times 1.4 = 2894.1 \text{ kN}$$



$$q_u = \frac{Q_u}{b \times d} = \frac{2894.1 \times 10^3}{5600 \times 1330} = 0.39\ N/mm^2$$

Since $q_u < q_{cu}$, the thickness of the pile cap is adequate for one-way shear

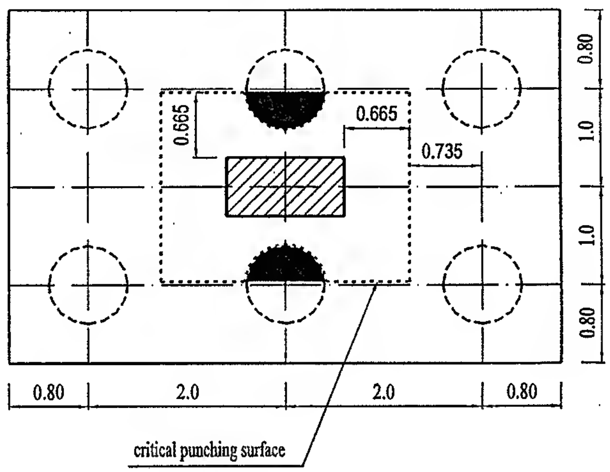


Step 2.2: Punching shear

The critical section for punching shear is at d/2 from the column face as shown in the figure below.

$$a_1 = c_1 + d = 600 + 1330 = 1930\,mm = 1.93\,m$$
$$b_1 = c_2 + d = 1200 + 1330 = 2530\,mm = 2.53\,m$$
$$U = 2 \times (a_1 + b_1) = 2\ (1930 + 2530) = 8920\,mm$$

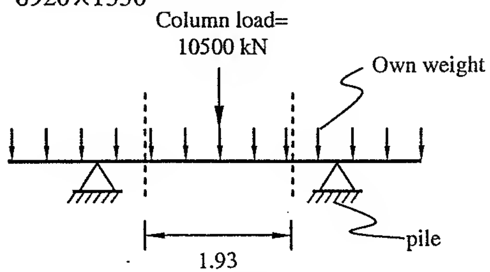
Referring to figure, it can be noted that the areas the piles that are located inside the critical punching area (λ=45%) of two piles. According to the previous procedure, the punching load could be calculated as follows:



Q_{up} =(Column load + O.W. of pile cap within the punching perimeter)- 2.λ.pile load

$$Q_{up} = (10500 + 1.4 \times 25 \times 1.93 \times 2.53 \times 1.4) - 2 \times 0.45 \times 1914.6 = 9016.1\ \text{kN}$$

$$q_{up} = \frac{Q_{up}}{U \times d} = \frac{9016.1 \times 1000}{8920 \times 1330} = 0.76\,N/mm^2$$



The concrete strength for punching is the least of the three values:

1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{35}{1.5}} = 1.53 N/mm^2 < 1.6 N/mm^2$

2. $q_{cup} = 0.316\,(0.50 + \frac{a}{b})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\,(0.50 + \frac{0.60}{1.20})\sqrt{\frac{35}{1.5}} = 2.23 N/mm^2$

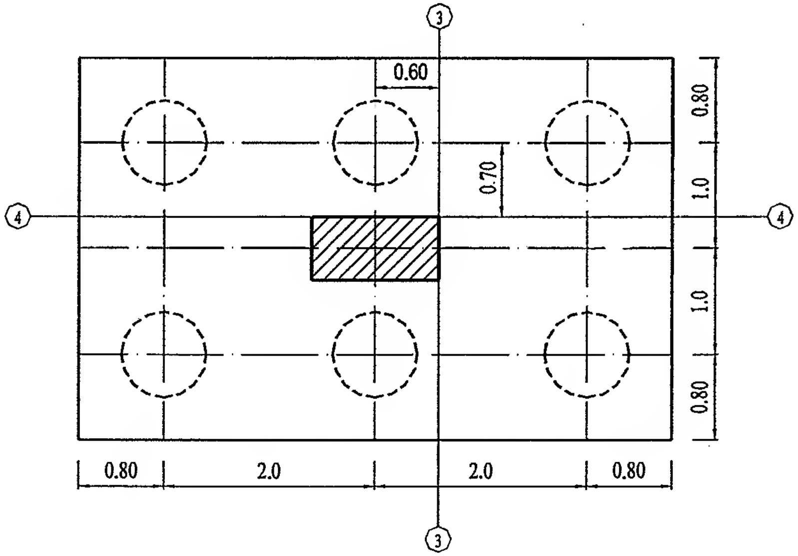
3. $q_{cup} = 0.8\,(0.20 + \frac{\alpha\, d}{U})\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\left(0.20 + \frac{4\times1.33}{8.92}\right)\sqrt{\frac{35}{1.5}} = 3.08 N/mm^2$

$q_{cup} = 1.53$ N/mm^2

Since $q_{up} < q_{cup}$, the thickness of the pile cap is adequate for punching shear.

Step 3: Design for flexure

The critical section for flexure is at the face of the column. There are two critical sections:



Section 1

$M_u = 2\cdot$factored load of pile $\cdot\ x_f$ - moment developed due to the O.W. of the hatched pile cap ($=1.4\times\gamma_c\times B\times t\times x_w^{\ 2}/2$)

$$x_f = 2.0 - \frac{1.2}{2} = 1.40 m$$

$$x_w = 2.8 - \frac{1.2}{2} = 2.2\ m$$

$$M_u = 2\times1914.6\times1.4 - 1.4\times25\times3.6\times1.4\times\frac{2.2^2}{2} \cong 4934\ kN.m$$

$$d = C_1\sqrt{\frac{M_u}{f_{cu}\,B}} \qquad 1330 = C_1\sqrt{\frac{4934\times10^6}{35\times3600}}$$

$C_1 = 6.72 \rightarrow$ Take c/d=0.125 $\rightarrow$ J = 0.825

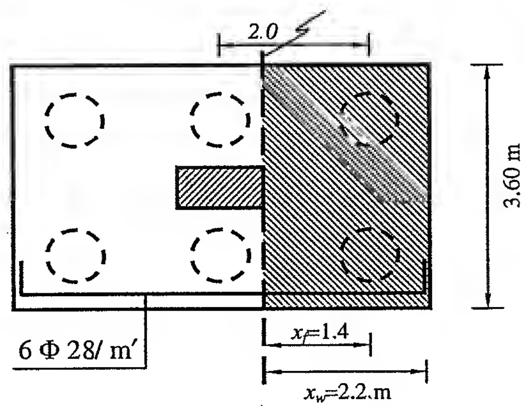
$$A_s = \frac{M_u}{f_y\,.J\,.d} = \frac{4934\times10^6}{360\times0.825\times1330} = 12476\ mm^2$$

$$A_s/m = \frac{9032.6}{3.6} = 3465 \qquad mm^2/m$$

Check the minimum steel requirement

$$A_{s,min} = \frac{0.60}{f_y}\times b\times d = \frac{0.60}{360}\times1000\times1330 = 2217\ mm^2$$

Choose $6\Phi28/m'$ (3695 mm^2)



Section 2

$M_u = 3 \cdot$ factored load of pile $\cdot\ x_f$ - moment developed due to the O.W. of the hatched pile cap ($=1.4\times\gamma_c\times B\times t\times x_w{}^2/2$)

$$x_f = 1.0 - \frac{0.6}{2} = 0.70\,m$$

$$x_w = 1.8 - \frac{0.60}{2} = 1.5\ m$$

$$M_u = 3\times1914.6\times0.70 - 1.4\times25\times5.6\times1.4\times1.5\times\frac{1.5}{2} \cong 3877\ kN.m$$

$$d = C_1\sqrt{\frac{M_u}{f_{cu}\,B}} \qquad 1330 = C_1\sqrt{\frac{3877\times10^6}{35\times5600}}$$

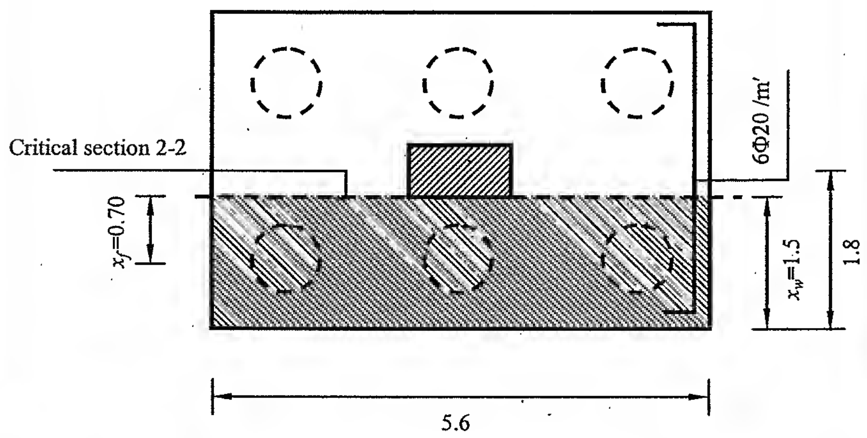
$C_1 = 9.46 \rightarrow$ Take c/d=0.125 $\rightarrow$ J = 0.825

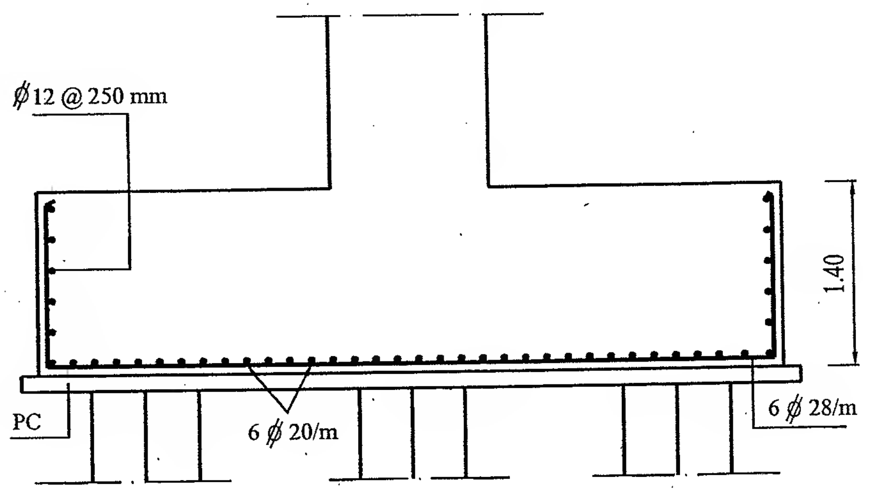
$$A_s = \frac{M_u}{f_y\,.J\,.d} = \frac{3877\times10^6}{360\times0.825\times1330} = 9802\ mm^2$$

$$A_s/m = \frac{9802}{5.6} = 1750 \qquad mm^2/m < A_{s,min}$$

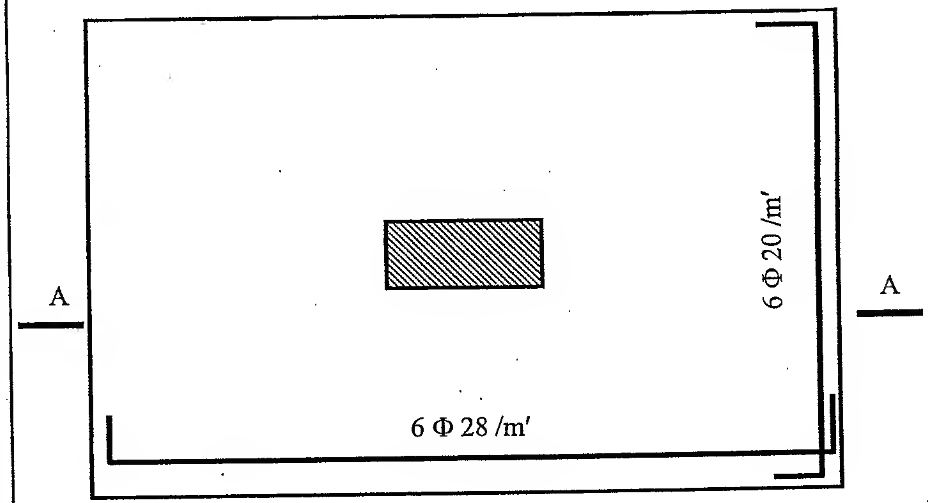
Use $A_s = A_{s,min} = 2217$ mm^2

Choose $6\Phi20/m'$





Section A-A



Plan

Reinforcement details of the pile cap

6

STRUT- AND -TIE MODEL

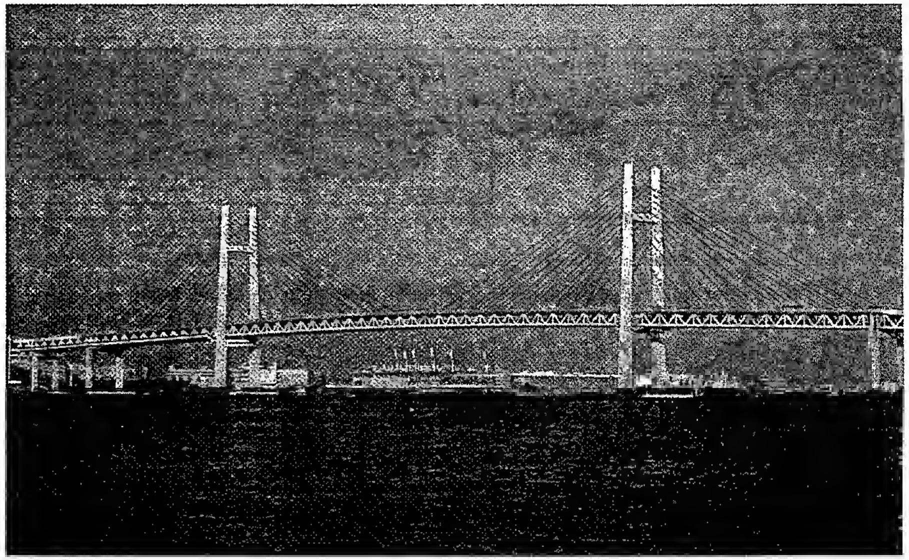


Photo 6.1 Yokohama Bay Bridge

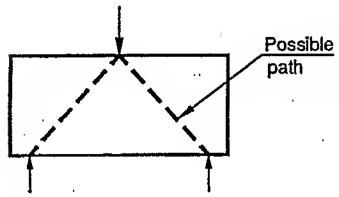
6.1 Introduction

In a structure, forces tend to follow the shortest possible path to transfer loads. In a beam subjected to concentrated loads, the shortest paths to transfer load are the straight lines connecting the points of loading and the supports.

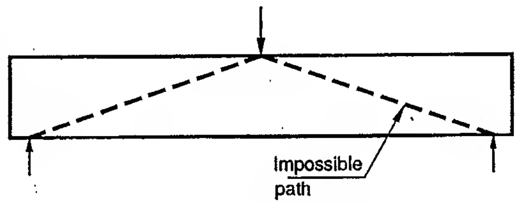
For deep beams, those shortest paths are possible paths, see Fig. 6.1a. The load is directly transferred to the supports through compression struts with reasonable inclination.

For slender beams, however, those shortest paths are not possible paths as shown in Fig. 6.1b. In such beams, the compression struts would be very flat. In order to develop a vertical component that is large enough to equilibrate the applied force, the actual force in the strut will be too large to cause concrete crushing. Vertical web reinforcement (ties) provides possible paths, as shown in Fig. 6.1d, since it increases the inclination of the struts.

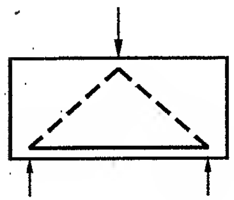
Comparison between Fig. 6.1c and Fig. 6.1d indicates that the strut-and-tie model is a special case of the truss model in which no vertical ties are statically needed.



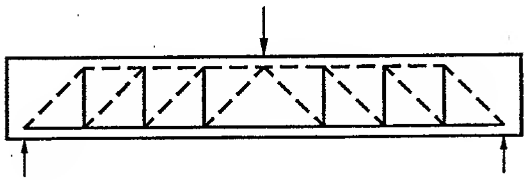
a)The shortest path in deep beam



b)The shortest path in slender beam



c) Strut and tie model for deep beam



d) Truss model for slender beam

Fig. 6.1 Possible load paths for beams

6.2 Principle of B and D Regions

B-region

A portion of a member in which the Bernoulli hypothesis of plane strain distribution is assumed to be valid.

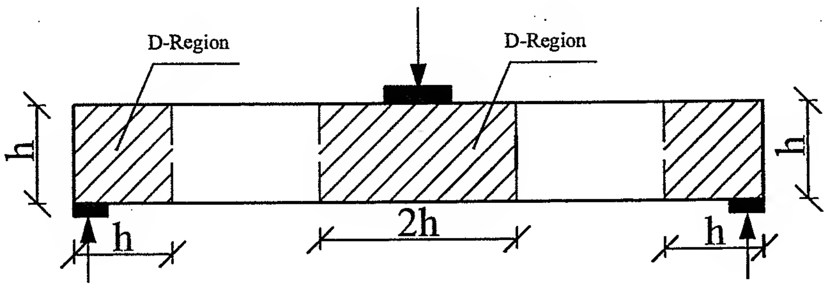
Discontinuity

A discontinuity of the stress distribution occurs at a change in the geometry of a structural element or at a concentrated load or reaction. St. Venant's principle indicates that the stresses due to axial load and bending moment approach a linear distribution at a distance approximately equal to the overall height of the member away from the discontinuity (See Fig. 6.2). For this reason, discontinuities are assumed to extend a distance h from the section where the load or change in geometry occurs.

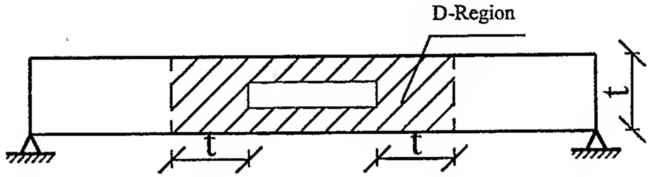
D-region

It is the portion of a member within a distance equal to the member height h from the face of discontinuity. The plane section assumption is not valid in such regions. Those disturbed portions are designated D-regions, where D denotes discontinuity or disturbance. Typical structures in which D-region behavior dominates are brackets and deep beams.

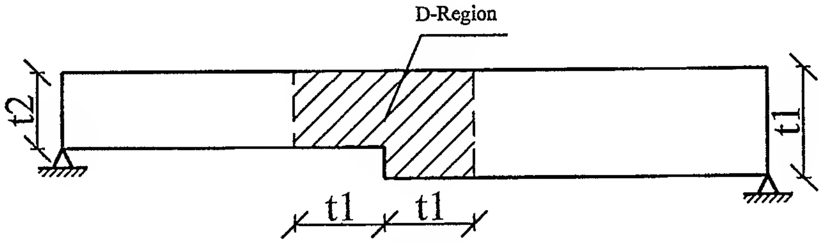
Figure 6.2 shows some typical D-regions. The regions between D-regions can be treated as B-regions. At disturbed or discontinuous regions of a structure such as corners, openings or concentrated loads and supports, plane sections do not remain plane, and the behavior is very different from that in B-regions. In such members the load carrying mechanism may be idealized as a truss made up of concrete compression struts and steel ties. Crushing of the concrete struts is one of the major failure modes for D-regions and the ultimate load is very dependent on the compressive strength of concrete. Because of transverse tension and cracking in the region of the strut, an effective concrete strength generally less than the cube strength, must be used in the design of the concrete strut.



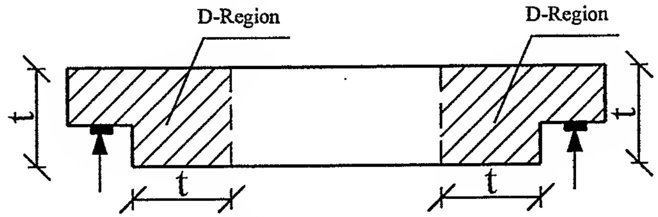
A- At locations of concentrated loads



B- Near openings

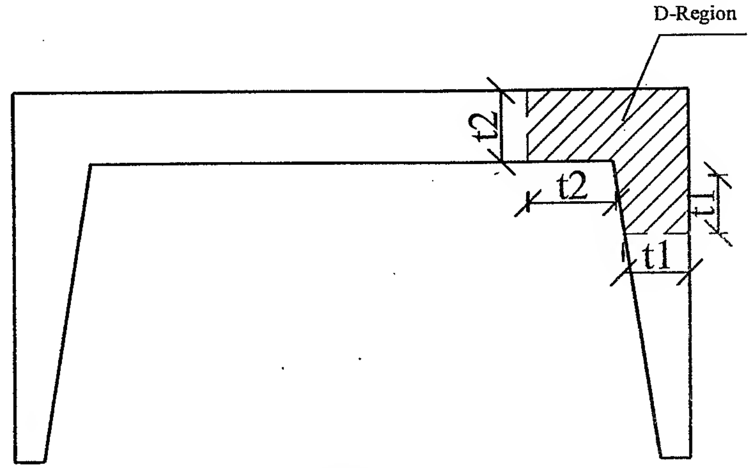


C- Sudden variation in beam thickness

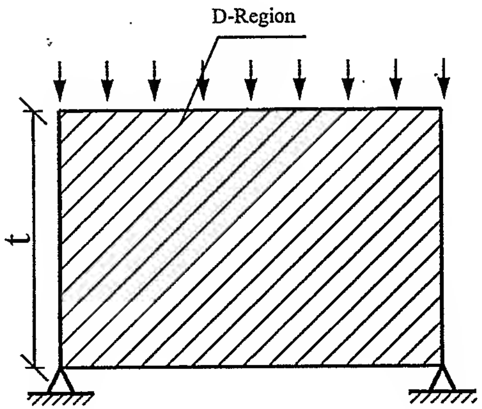


D- Tapered beam

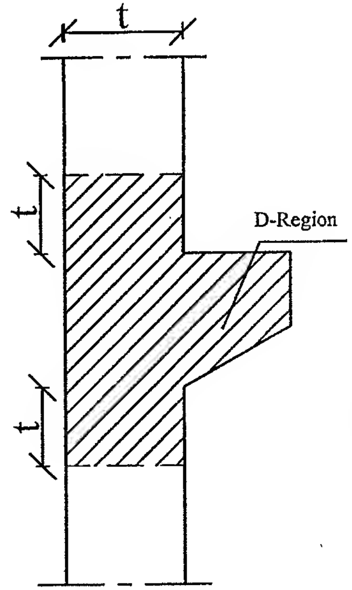
Fig. 6.2 Typical D-regions



E- Beam column joint



F- Deep beam



G- Short cantilever

Fig. 6.2 Typical D-regions (contd.)

6.3 Components of the Strut- and -Tie Model

The Strut-and-Tie model consists of:

- Major diagonal compression diagonals (struts)
- Tension ties (or Ties)
- Truss nodes.

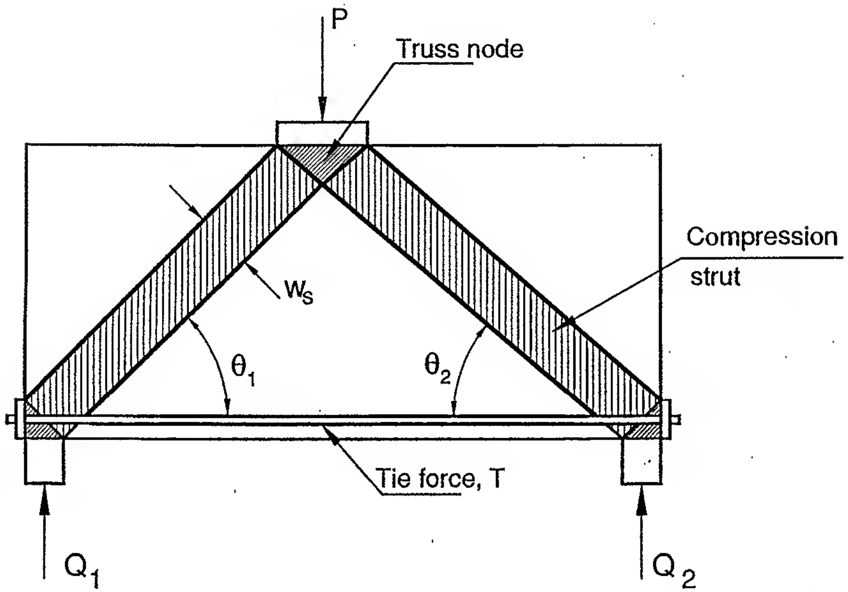


Fig. 6.3 Strut-and-Tie model for a deep beam

In Fig. 6.3, the concentrated load, P, is resisted by two major inclined diagonal *struts*, shown by the light shaded areas. The horizontal component of the force in the strut is equilibrated by a tension tie force, T. The three darker shaded areas represent *truss nodes*. These are wedges of concrete loaded on all sides except the side surfaces of the beam with equal compressive stress. The loads, reactions, struts, and ties in Fig. 6.3 are all laid out such that the centroid of each truss member and the line of action of all externally applied loads coincide at each joint.

The strut-and-tie shown in Fig. 6.3 can fail in one of three ways:

- The tension tie could yield.
- One of the struts could crush when the stress in the strut exceeds the effective compressive strength of concrete.
- A truss node could fail by being stressed greater than the effective compressive strength of concrete.

Since a tension failure of the steel will be more ductile than either a strut failure or a node failure, a deep beam should be proportioned so that the strength of the steel governs.

According to the Strut-and-Tie Model shown in Fig. 6.3, the shear strength can be calculated as:

$$Q = A_{strut} f_{ce} \sin\theta \quad \text{.......................................} (6.1)$$

where A_{strut} is the cross sectional area of the strut, f_{ce} is the effective compressive strength of concrete and θ is the angle of inclination of the strut.

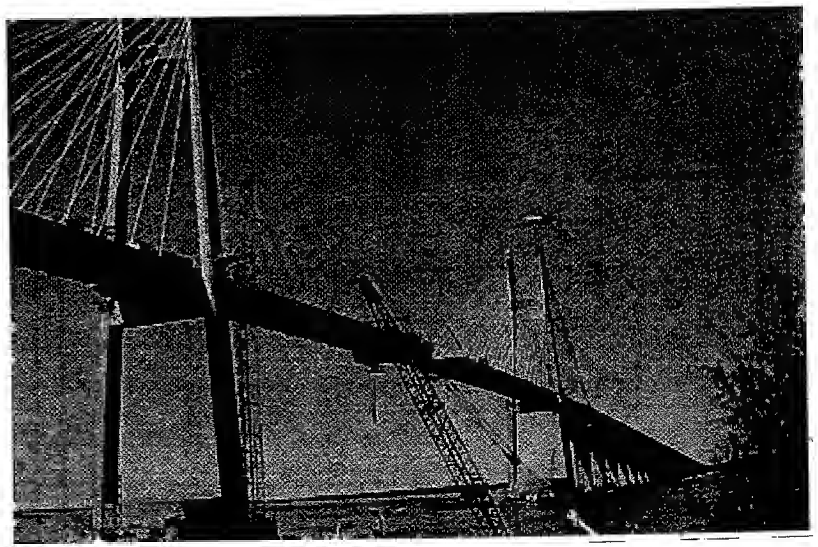


Photo 6.2 A Cable-stayed bridge during construction

The validity of a Strut-and-Tie model for a given member depends on whether the model represents the true situation. Concrete beams can undergo a limited amount of redistribution of internal forces. If the chosen Strut-and-Tie model requires excessive deformation to reach the fully plastic state, it may fail prematurely.

An example of an unsuitable model is given in Fig. 6.4 that shows a deep beam with two layers of longitudinal reinforcement. One layer is located at the bottom and the other at mid-depth. A possible Strut-and-Tie model for this beam consists of two trusses, one utilizing the lower reinforcing steel as its tension tie, the other using the upper reinforcing steel. For an ideally plastic material, the capacity would be the sum of the shears transmitted by the two trusses $Q_1 + Q_2$. It is clear, however, that the upper layer of reinforcing steel has little effect on strength. When this beam is loaded, the bottom tie yields first. Large deformations are required before the upper tie can yield. Before these can fully develop, the lower truss will normally fail.

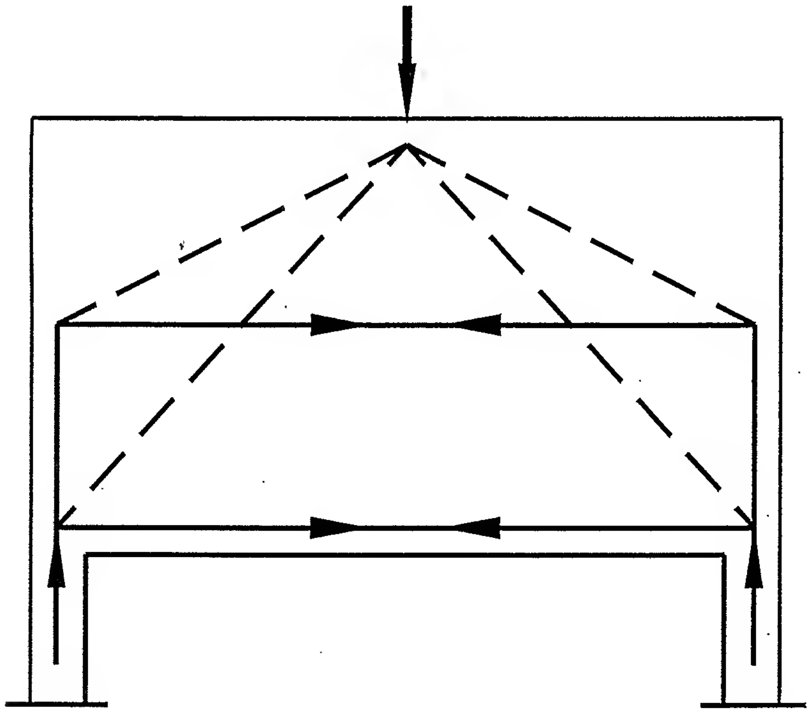


Fig. 6.4 Invalid Strut-and-Tie model

6.4 Design of the Struts

In the design using Strut-and-Tie models, it is necessary to check that crushing of the compressive struts does not occur. The cross-sectional area of the compressive struts is highly dependent on the details at their ends.

6.4.1 Idealization of the Strut

The most common types of struts utilized in design are:

PRISMATIC STRUT

Struts could be idealized as prismatic compression members ***(prismatic struts)*** as shown by the straight line outlines of the struts in Fig. 6.3.

TAPERED STRUT

If the effective compression strength at the two ends of the strut differs due to different bearing lengths (See Fig. 6.5), the strut is idealized as a uniformly tapered compression member ***(tapered strut)***.

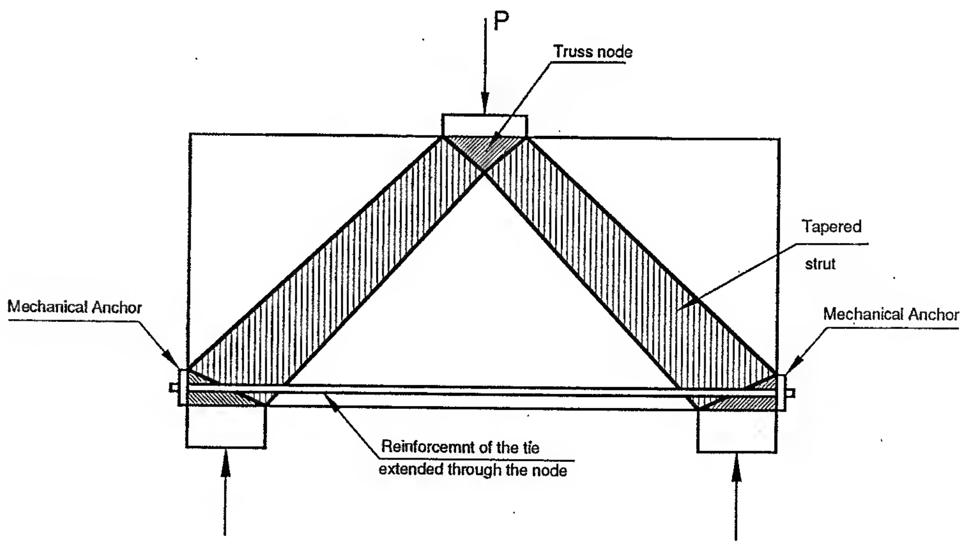


Fig. 6.5 Tapered strut.

BOTTLE-SHAPED STRUT

Is a strut located in a part of a member where the width of the compressed concrete at mid-length of the strut can spread laterally. The curved solid outlines in Fig. 6. 6 approximate the boundaries of the bottle-shaped struts. A split cylinder test is an example of a bottle-shaped strut. The spread of the applied compression force in such a test leads to a transverse tension that splits the specimen.
To simplify design, bottle-shaped struts are idealized either as prismatic or as uniformly tapered. Crack-control reinforcement is provided to resist the transverse tension. The amount of such reinforcement can be computed using the Strut-and-Tie model shown in Fig. 6.6 with the struts that represent the spread of the compression force acting at a slope of 1:2 to the axis of the applied compressive force.

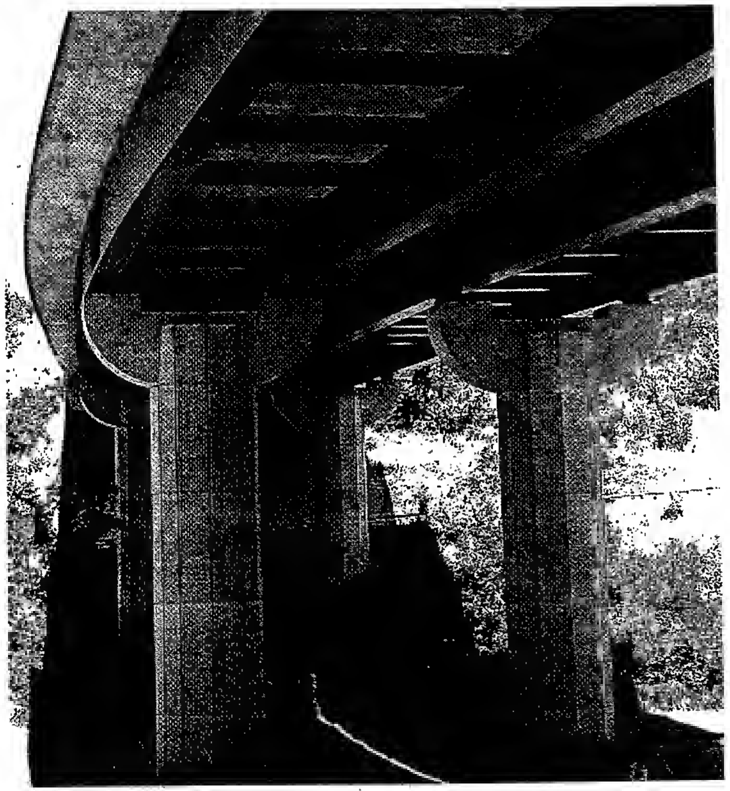


Photo 6.3 Double short cantilevers supporting a composite bridge

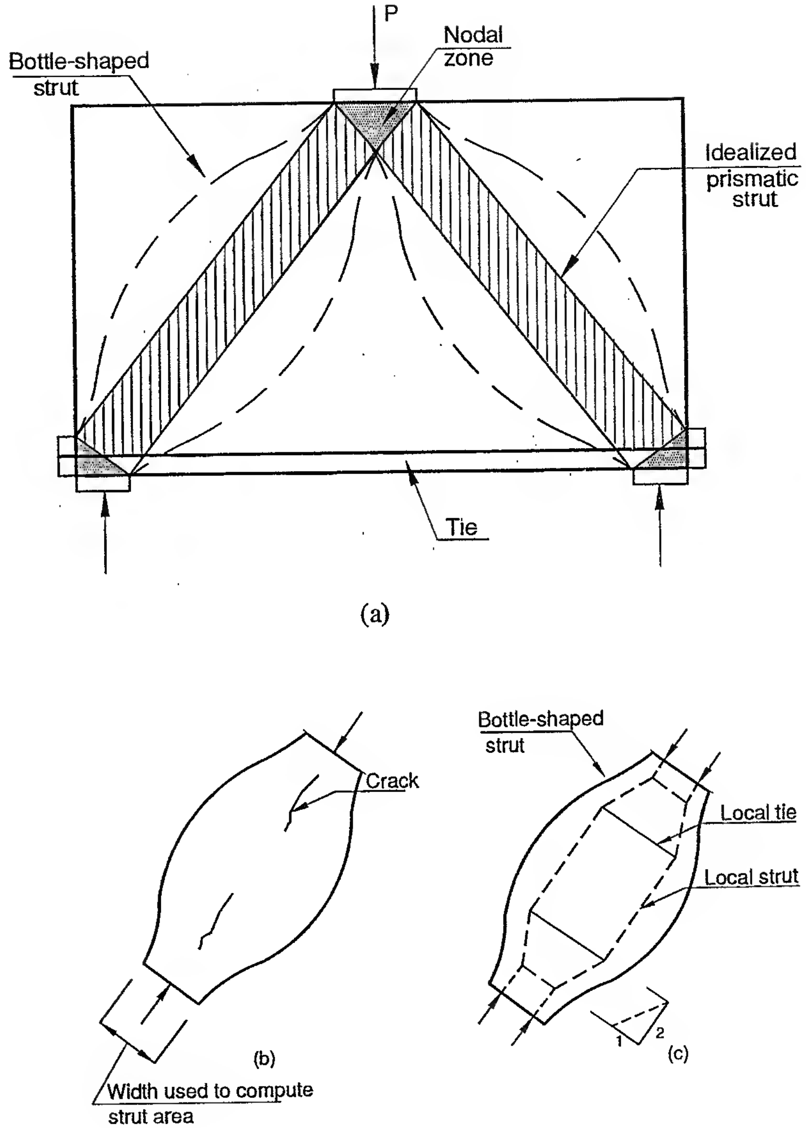


Fig. 6.6 Bottle shaped strut:(a) bottle-shaped strut in a deep beam, (b)cracking of a bottle-shaped strut; (c) strut-and-tie model of a bottle-shaped strut

6.4.2 Strength of Un-reinforced Struts

The compressive strength of an un-reinforced strut (F_c) shall be taken as the smaller value of the compressive strength at the two ends, given as:

$$F_c = f_{cd} \cdot A_c \quad \text{.......... (6.2)}$$

where

A_c = cross-sectional area of the strut at the strut end under consideration

f_{cd} = the smaller of (a) and (b):

(a) The effective compressive strength of the concrete in the strut;

(b) The effective compressive strength of concrete in the nodal zone.

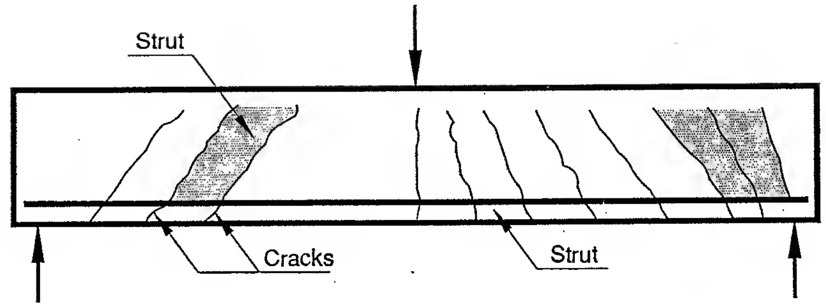
The compressive strength of concrete in the strut is given by:

$$f_{cd} = 0.67\, \beta_s\, f_{cu} / \gamma_c \quad \text{.......... (6.3)}$$

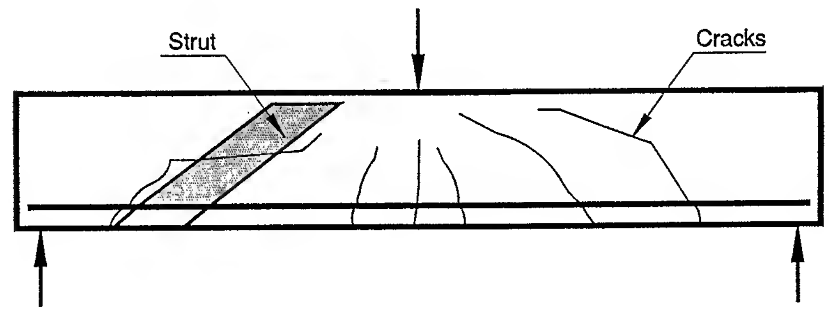
The strength coefficient, ($0.67\, f_{cu}/\gamma_c$), in Eq. 6.3 represents the cube concrete strength under sustained compression. The factor β_s is a factor that takes into account the stress conditions and the angle of cracking surrounding the strut. The value of β_s is given in Table 6.1. The strength coefficient ($0.67\, \beta_s f_{cu} / \gamma_c$) represents the effective concrete strength of the strut. The material strength reduction factor γ_c is taken 1.6.

Table 6.1 Values of the Coefficient β_s

Strut condition	β_s
A strut with constant cross-section along its length (for example a strut equivalent to the rectangular stress block in a compression zone in a beam)	1.0
Bottled-shape strut parallel to the direction of the cracks (Fig. 6.7a) provided that there are reinforcing bars normal to the center-line of the strut to resist the transversal tensile force. Such force can be assumed spreading with inclination of 26 degrees to the centerline of the strut.	0.70
Bottled-shape strut that is not parallel to the direction of the cracks (Fig. 6.7b).	0.60
Struts in tension members or the tension flanges of members.	0.40
All other cases	0.60



(a) Struts in a beam web with inclined cracks parallel to struts



(b) Struts crossed by skew cracks

Fig. 6.7 Types of struts

If the value of $\beta_s = 0.70$ specified in Table 6.1 is used, the axis of the strut shall be crossed by reinforcement proportioned to resist the transverse tensile force resulting from the compression force spreading in the strut as shown in Fig. 6.8. Otherwise, one has to use $\beta_s = 0.60$.

The designer may use a local strut-and-tie model to compute the amount of transverse reinforcement needed in a given strut. In the *American Concrete Institute Code (ACI 318)* ; for concrete strengths not exceeding 40 N/mm^2, the requirement is considered to be satisfied if the axis of the strut being crossed by layers of reinforcement satisfies the following equation:

$$\sum \frac{A_{si}}{b\ S_i} \sin\gamma_i \geq 0.003 \quad \text{.............................. (6.4)}$$

where A_{si} is the total area of reinforcement at spacing S_i in a layer of reinforcement with bars at an angle γ_i to the axis of the strut.

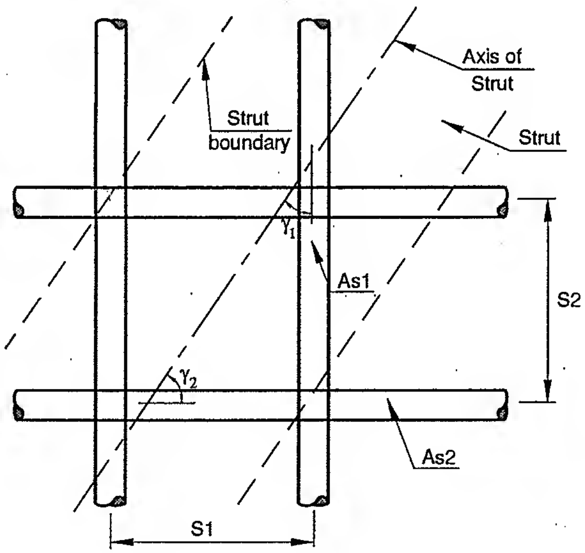


Fig. 6.8 Reinforcement crossing a strut.

6.4.3 Strength of Reinforced Struts

In order to increase the strength of the strut, it is permitted to reinforce it with compression reinforcement that satisfies the following requirements:

- The compression reinforcement should be placed within the strut and parallel to its axis.
- The reinforcement should be properly anchored.
- The reinforcement should be enclosed in ties or spirals satisfying the conditions applied to columns.

An example of a deep beam with reinforced struts is given in Fig. 6.9. The strength of a reinforced strut is given by:

$$F_c = A_c f_{cd} + A_s\ f_y / \gamma_s \quad \text{.............................. (6.5)}$$

Where $f_{cd} = \dfrac{0.67 f_{cu}}{\gamma_c} \beta_s$

$\gamma_s = 1.3$, $\gamma_c = 1.6$, β_s is obtained from table 6.1, A_s is the area of the reinforcing steel parallel to the direction of the strut and f_y is the yield strength.

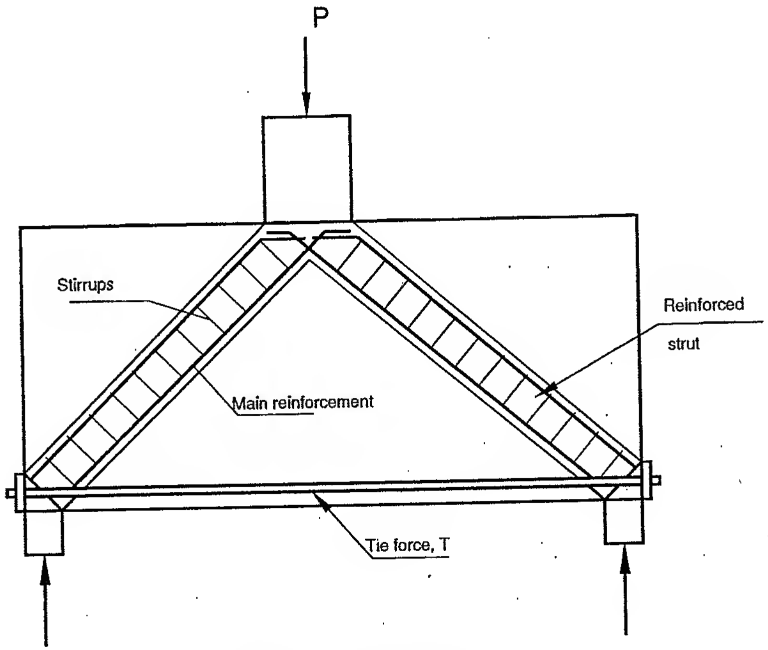


Fig. 6.9 Reinforced strut

6.5 Design of Ties

6.5.1 Strength of the Tie

The strength of the tie is calculated as

$$T_{ud} = A_s f_y / \gamma_s \quad \text{...} \quad (6.7)$$

where

T_{ud} = design tension force.
A_s = cross-sectional area of steel.
f_y = yield strength of steel.
γ_s =1.15

The width of the tie is determined to satisfy safety conditions for compressive stresses at nodal points for struts and ties meeting at that node. Such a width can approximately be taken not more than 70% of the width of the largest strut connected to the tie at the node.

6.5.2 Development Length of the Reinforcement

The reinforcement in a tie should be developed with a length equal to L_{bd} at the ends of the tie as shown in Fig. 6.10.

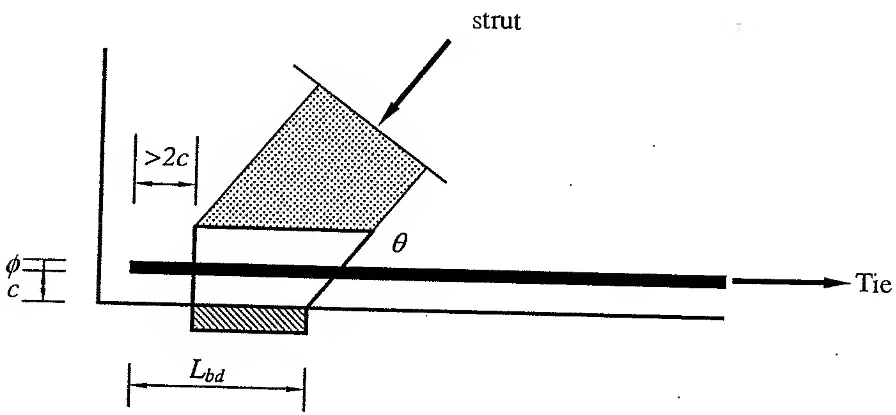


Fig. 6.10 Calculation of the development length at nodal zone

6.6 Design of Nodal Zones

6.6.1 Types of Nodal Zones

The locations of the intersection of the truss members are called the nodes of the truss. They represent regions of multi-directionally stressed concrete or nodal zones of the Strut-and-Tie model.

The compressive strength of concrete of the nodal zone depends of many factors, including the tensile straining from intersecting ties, confinement provided by compressive reactions and confinement provided by transverse reinforcement.

To distinguish between the different straining and confinement conditions for nodal zones, it is helpful to identify these zones as follows:

a) CCC – nodal zone bounded by compression struts only (hydrostatic node)
b) CCT – nodal zone bounded by compression struts and one tension tie
c) CTT – nodal zone bounded by a compression strut and tension ties
d) TTT - nodal zone bounded by tension ties only

Figure 6.11 illustrates the different types of nodal zones.

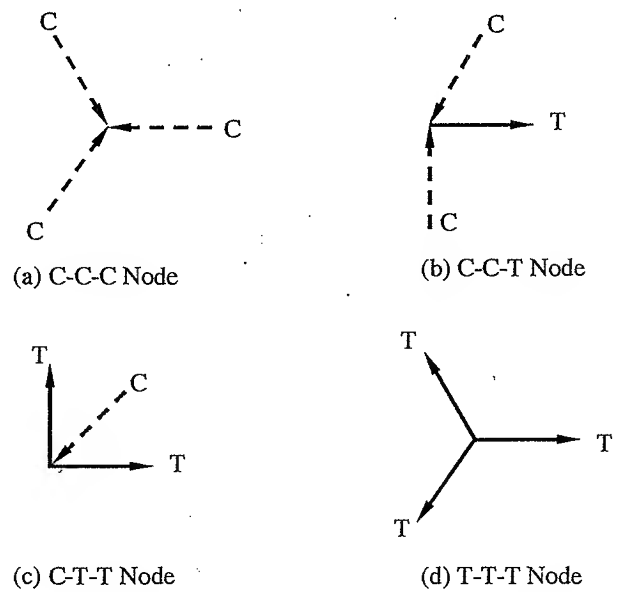


Fig. 6.11 Classification of nodes

6.6.2 Strength of the Nodal Zones

The compressive strength of a the nodal zone is given by

$$F_{cn} = A_{cn} \cdot \beta_n \left(0.67 f_{cu} / \gamma_c\right) \quad \text{.......... (6.6)}$$

in which

A_{cn} = the area of the face of the nodal zone taken perpendicular to the direction of the strut.

γ_c = material strength reduction factor for concrete = 1.6.

β_n = factor that takes into account the stress conditions at the nodal zone

Table 6.2 Values of β_n

Type of Node	β_n
C-C-C	1.0
C-C-T *	0.80
C-T-T or T-T-T	0.60

* In C-C-T nodes, the value β_n = 1.0 can be utilized if the tie is extended through the node and is mechanically anchored as shown in Fig. 6.12

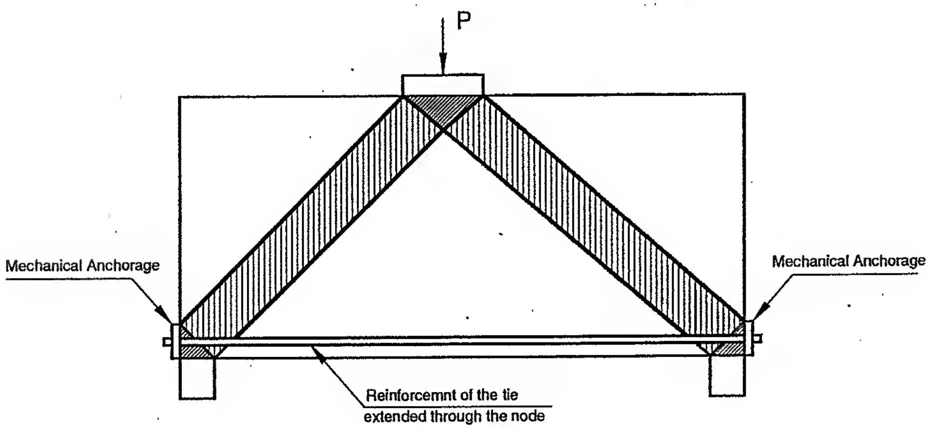


Fig. 6.12 Mechanical anchorage of tie reinforcement

In order to determine the dimensions of nodes subjected to tension and compression (CCT or CTT), the height U of the tension tie can be calculated as follows:

- In case of using one row of bars without providing sufficient development length beyond the nodal zones (Fig. 6.13a):

$$U = 0 \quad \text{.......... (6.7a)}$$

- In case of using one row of bars and providing sufficient development length beyond the nodal zones for a distance not less than 2c, where c is the concrete cover (Fig. 6.13b):

$$U = \varphi + 2c \quad \text{.......... (6.7b)}$$

Where ϕ is the bar diameter.

- In case of using more than one row of bars (Fig. 6.13c) and providing sufficient development length beyond the nodal zones for a distance not less than 2c, where c is the concrete cover:

$$U = \varphi + 2c + (n-1)\,s \quad \text{.......... (6.7c)}$$

Where n is number of bars and s the center line distance between bars.

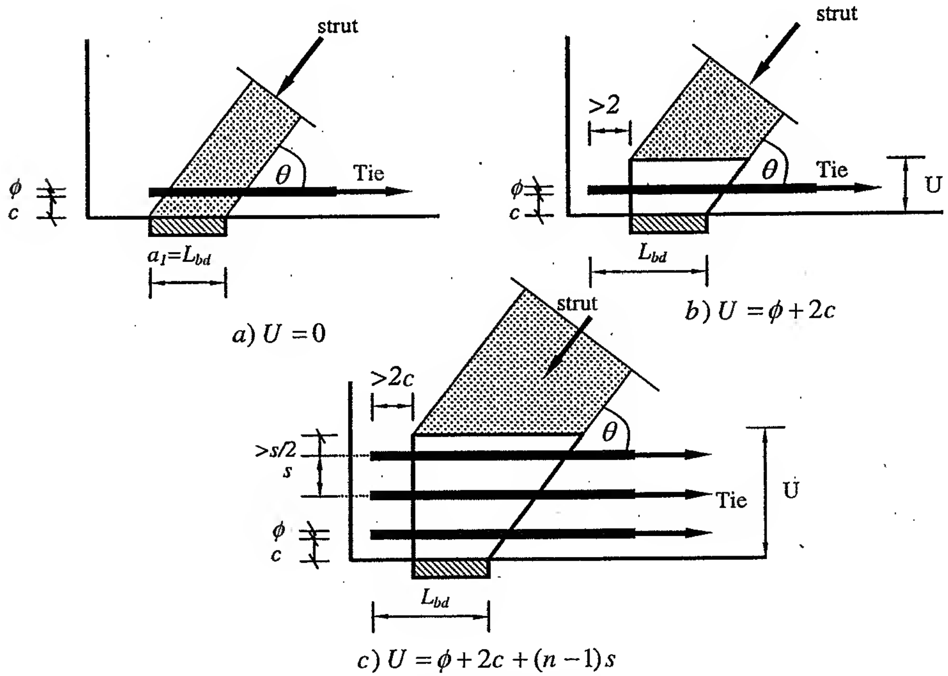


Fig. 6.13 The height (U) used to determine the dimensions of the node

6.7 Applications

The choice of a Strut-and-Tie model is a major issue and different engineers may propose various models for the same application. Figure 6.14 presents some applications of the Strut-and Tie model for designing deep beams. Struts are indicated as dashed lines; solid lines represent ties. The struts and ties are positioned by considering the likely paths of the loads to the supports and the orthogonal reinforcement pattern. The forces in the struts and ties are determined by equilibrium. As long as the resulting model satisfies equilibrium, and the struts, ties and nodal zones satisfy the provisions previously discussed in his chapter, the structure should develop the ultimate strength required.

Figure 6.14-a shows a simply supported deep beam that supports two planted columns and contains two web openings. The locations of the openings do not interfere with shortest load path between the loads and the supports. Hence, a simple Strut-and –Tie model, similar to that developed in a solid deep beam, can be proposed. It should be noted that the existence of the openings might limit the width of the strut.

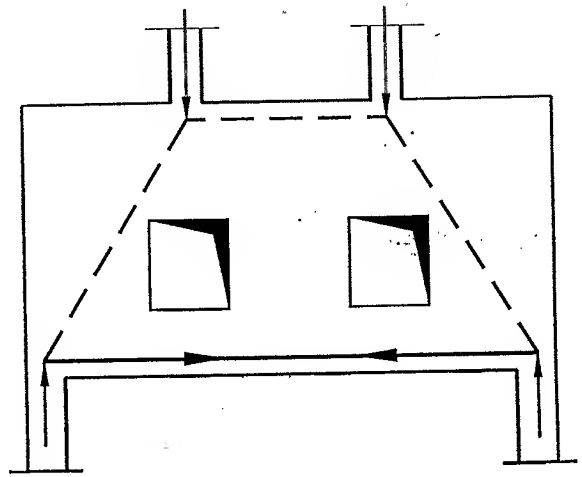
Figure 6.14-b shows a simply supported deep beam that supports two planted columns and contains two web openings. The locations of the openings interfere with shortest load path between the loads and the supports. Hence, the vertical load of each column has to travel around the opening. Accordingly, a tie has to be located above the openings, together with the traditional one located at the bottom part of the beam.

Figure 6.14-c shows a deep beam having an opening near the left-hand side support. The beam supports a planted column. The Strut-and-Tie model for such a beam is developed based on engineering judgment about how the force paths might flow around the opening. The model demonstrates that the main reinforcement of the beam shall follow the directions of the ties. The designer should provide light reinforcement mesh on both sides of the beam to control cracking and to enhance the performance of the struts.

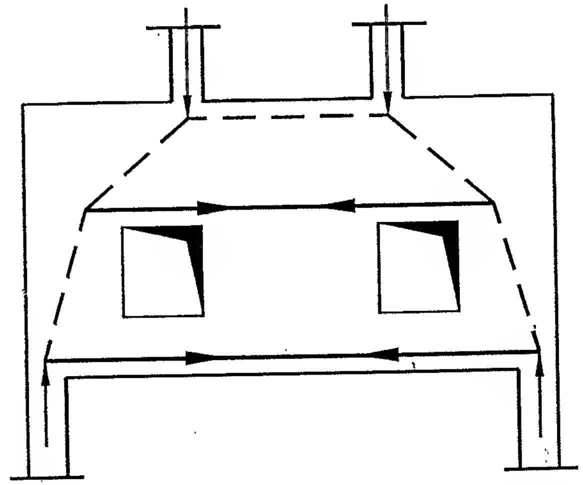
Figure 6.14-d shows a deep beam that is bottom loaded through two hangers. A simplified Strut-and-Tie model that is suitable for design is also shown. It is assumed that the bottom load is transferred to the beam by bond stresses between the vertical stirrups, that hang-up the load, and the concrete.

Figures 6.14-e-f-g show Strut-and-Tie models for cantilever and continuous beams subjected to different loading conditions.

Finally, Fig. 6.14-h shows a Strut-and-Tie model for a box girder bridge supported on a pier and a shallow foundation. In spite of the fact that such a bridge structure can be easily analyzed using simpler tools; the shown model gives some insight into the flow of forces.

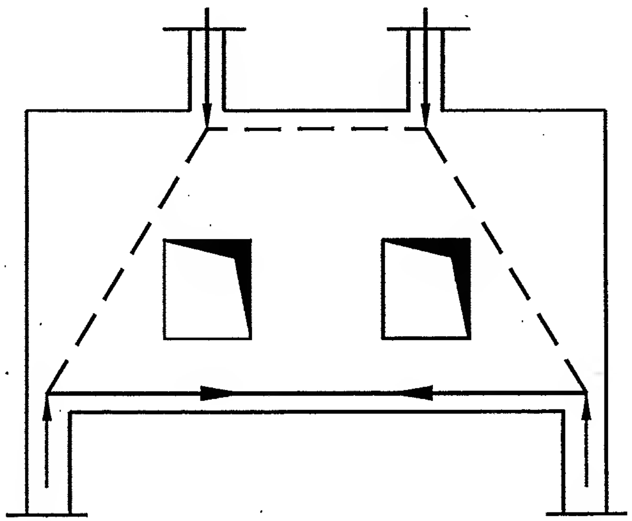


a) Deep beam with symmetrical openings

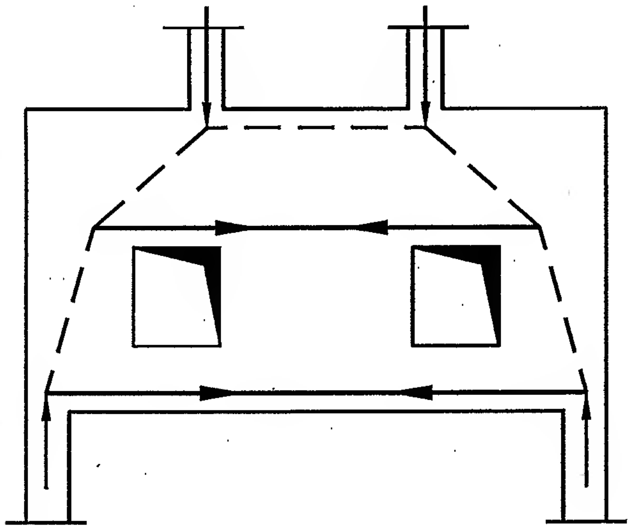


b) Deep beam with openings interfering with the shortest load path.

Fig. 6.14 Typical Strut-and-Tie applications.

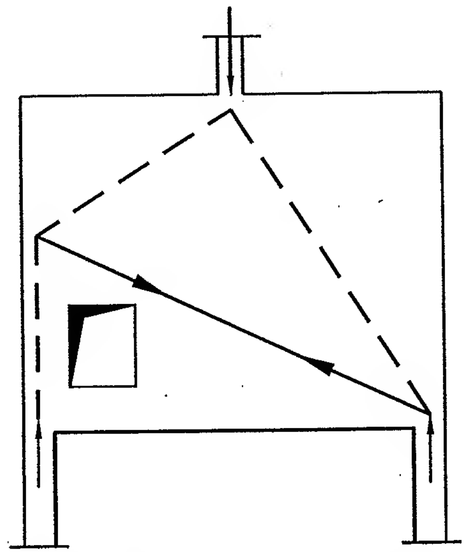


a) Deep beam with symmetrical openings

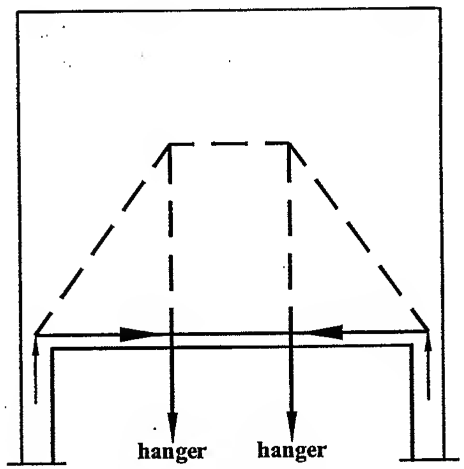


b) Deep beam with openings interfering with the shortest load path.

Fig. 6.14 Typical Strut-and-Tie applications.

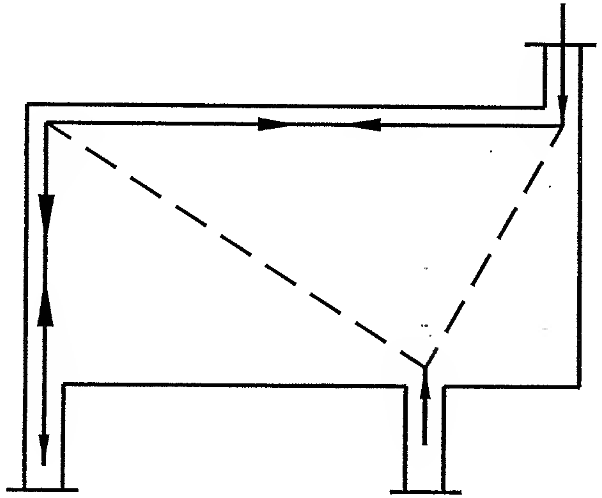


c) Deep beam with an eccentric opening

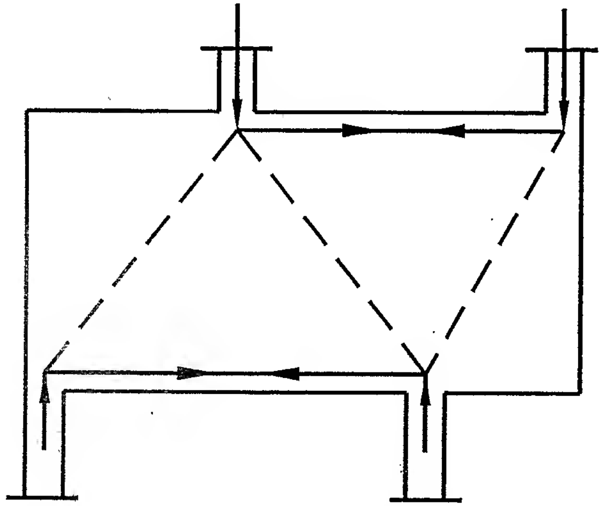


d) Bottom loaded deep beam

Fig. 6.14 Typical Strut-and-Tie applications (cont.)



e) Deep beam having a cantilever loaded at the edge

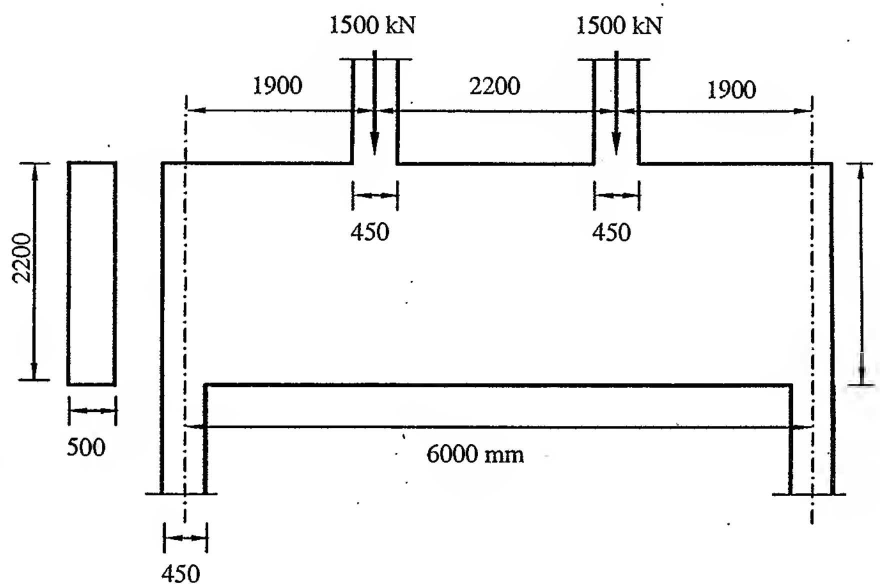


f) Deep beam having a cantilever with two concentrated loads

Fig. 6.14 Typical Strut-and-Tie applications (cont.)

Example 6.1

A transfer girder supports two planted columns, each having a factored load of 1500 kN as shown in the figure given below. The material properties are $f_{cu} = 30 N/mm^2$ and $f_y = 400 N/mm^2$. Design the beam using the Strut-and-Tie method presented in the ECP 203.



Step 1: Check bearing capacity at loading points and at supports

The area of the cross section of the column is (450 mm × 500 mm). The bearing stresses at points loading and at supports are:

$$f_b = \frac{P_u}{A_c} = \frac{1500 \times 1000}{450 \times 500} = 6.66\ N/mm^2$$

The nodal zones beneath the loading locations are (C-C-C) Nodes. The effective compressive strength of such type of nodes (β_n=1.0) is limited to:

$$f_{cd} = 0.67 \times \beta_n \times \frac{f_{cu}}{\gamma_c} = 0.67 \times 1.0 \times \frac{30}{1.6} = 12.56\ N/mm^2$$

The nodal zone over the support location is a compression-tension (C-C-T) Node. The effective compressive strength of this node (β_n=0.80) is limited to:

$$f_{cd} = 0.67 \times \beta_n \times \frac{f_{cu}}{\gamma_c} = 0.67 \times 0.80 \times \frac{30}{1.6} = 10.05\ N/mm^2$$

Since the applied bearing stresses (6.66 N/mm^2) are less than the limiting values at the loading locations and at the supports, the area of contact is considered adequate

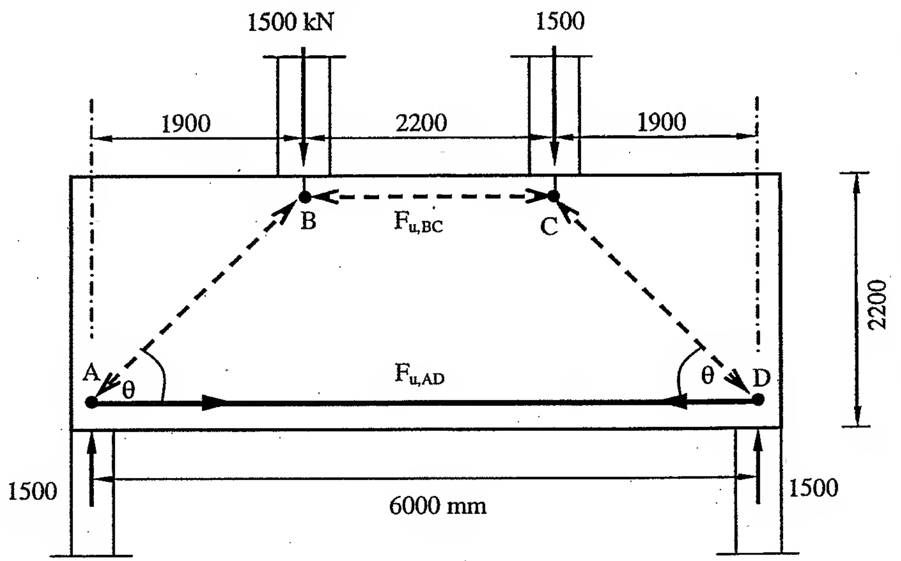
Step 2: Establish the Strut-and-Tie model

A simple Strut-and-Tie model is shown in the figure below. It consists of a direct strut AB (or CD) from the applied load to the support. To equilibrate the truss, tie AD and strut BC needs to be established as shown in figure. From equilibrium:

$$F_{u,BC} = F_{u,AD}$$

The horizontal position of nodes A, B is easy to determine, but the vertical position of these nodes must be calculated.

To fully utilize the beam, the positions of these nodes have to be as close to the top of the beam. In other words, the lever arm *(jd)* of the force couple must be set to the maximum and this means that the width of strut BC, w_s, and the width of the tie AD, w_t, must be minimum.



To minimize w_s, strut force BC, $F_{u,BC}$ must equal its capacity as follows:

$$F_{u,BC} = 0.67 \times \beta_s \times \frac{f_{cu}}{\gamma_c}\, b\, w_s \quad \text{where } \beta_s\text{=1.0 for prismatic struts}$$

To minimize w_t, the force in the tie $F_{u,AD}$ must reach the node capacity to anchor the tie as follows:

$$F_{u,AD} = 0.67 \times \beta_s \times \frac{f_{cu}}{\gamma_c}\, b\, w_t \quad \text{where } \beta_s\text{=0.80 for (C-C-T) node}$$

Since $F_{u,BC} = F_{u,AD}$, it can be easily proven that:

$$w_t = 1.25\, w_s$$

The distance between the compression and the tension force *jd* equals:

$$jd = t - \frac{w_s}{2} - \frac{w_t}{2} = t - \frac{w_s}{2} - \frac{1.25\, w_s}{2} = t - 1.125\, w_s$$

The moment of the forces about point A gives:

$$P_u \times L_x = F_{u,BC} \times jd$$

$$P_u \times L_x = 0.67 \times \beta_s \times \frac{f_{cu}}{\gamma_c}\, b\, w_s\, (t - 1.125\, w_s)$$

$$1500\,(1000) \times 1900 = 0.67 \times 1.0 \times \frac{30}{1.6}\, 500\, w_s\, (2200 - 1.125\, w_s)$$

The previous equation is quadratic in w_s. Solving for w_s gives:

$$w_s = 234.3\, mm$$

$$w_t = 1.25\, w_s = 1.25 \times 234.3 = 292.9\, mm$$

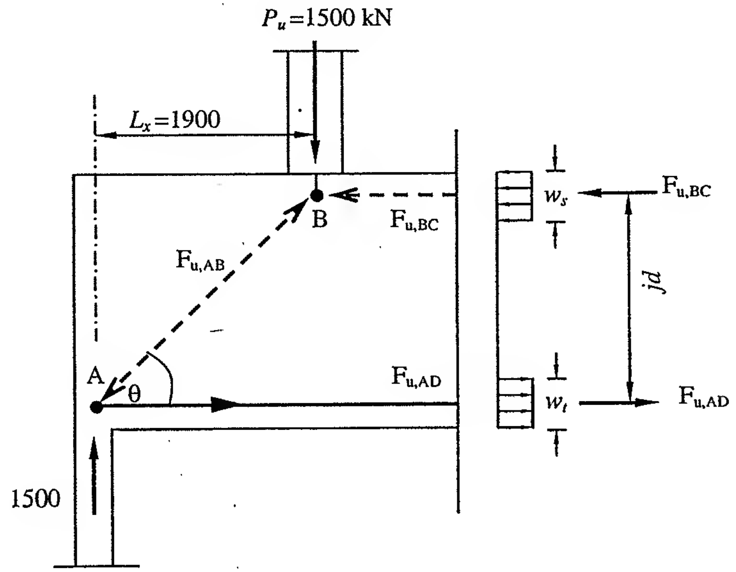
Choose $\quad w_s = 250\, mm$ and $\quad w_t = 300\ mm$

The depth of the deep beam equals:

$$d = t - \frac{w_t}{2} = 2200 - \frac{300}{2} = 2050\, mm$$

The distance *jd* equals:

$$jd = t - \frac{w_s}{2} - \frac{w_t}{2} = 2200 - \frac{250}{2} - \frac{300}{2} = 1925\, mm$$



The force in the strut and the tie equals:

$$F_{u,BC} = \frac{P_u \times L_x}{jd} = \frac{1500 \times 1900}{1925} = 1480.5\ kN$$

The angle of the strut AB equals:

$$\theta = \tan^{-1}\left(\frac{jd}{L_x}\right) = \tan^{-1}\left(\frac{1925}{1900}\right) = 45.37^o$$

$$F_{u,AB} = \frac{P_u}{\sin\theta} = \frac{1500 \times 1000}{\sin 45.37} = 2107.6\ kN$$

Step 3: Select the tie reinforcement

The tie reinforcement can be obtained as follows:

$$F_{u,AD} = A_s \times f_y / \gamma_s$$

$$F_{u,AD} = F_{u,BC} = 1480.5\ kN$$

$$1480.5 \times 1000 = A_s \times 400 / 1.15$$

$$A_s = 4256.5\ mm^2$$

Choose 9 Φ 25 mm (2 layers) $\rightarrow$ $A_s = 4417 mm^2$

Step 4: Design the nodal zones and check the anchorages

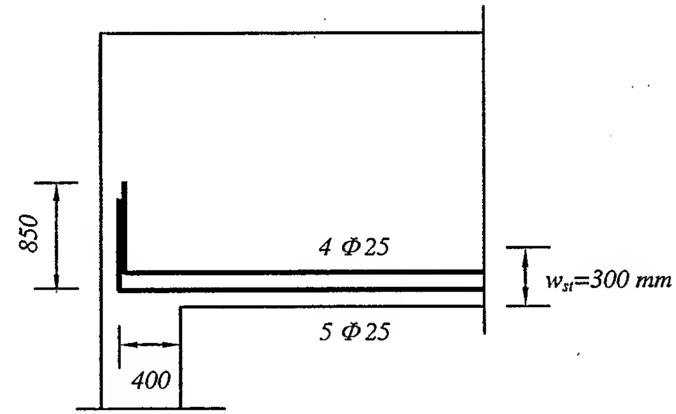
The 90° standard hook is used to anchor tie AD. The required anchorage is given by:

$$L_d = \frac{\alpha\ \beta\ \eta\ f_y / \gamma_s}{4\ f_{bu}}\ \varphi$$

L_d can also be directly obtained from the tables provided in the ECP 203.

$L_d = 50\ \Phi$ for f_{cu}=30 N/mm^2 $\rightarrow$ $L_d = 50\ \Phi = 50 \times 25 = 1250\ mm$

The Egyptian code requires measuring the development length from the point where the centroid of the tie reinforcement leaves the nodal zone and enters the span. However, for simplicity, it can be measured from the end of the column. The distance from the column to the end of the beam is about 400 mm as shown in figure below and the bent part is about 850 mm.



Step 5: Check the diagonal struts

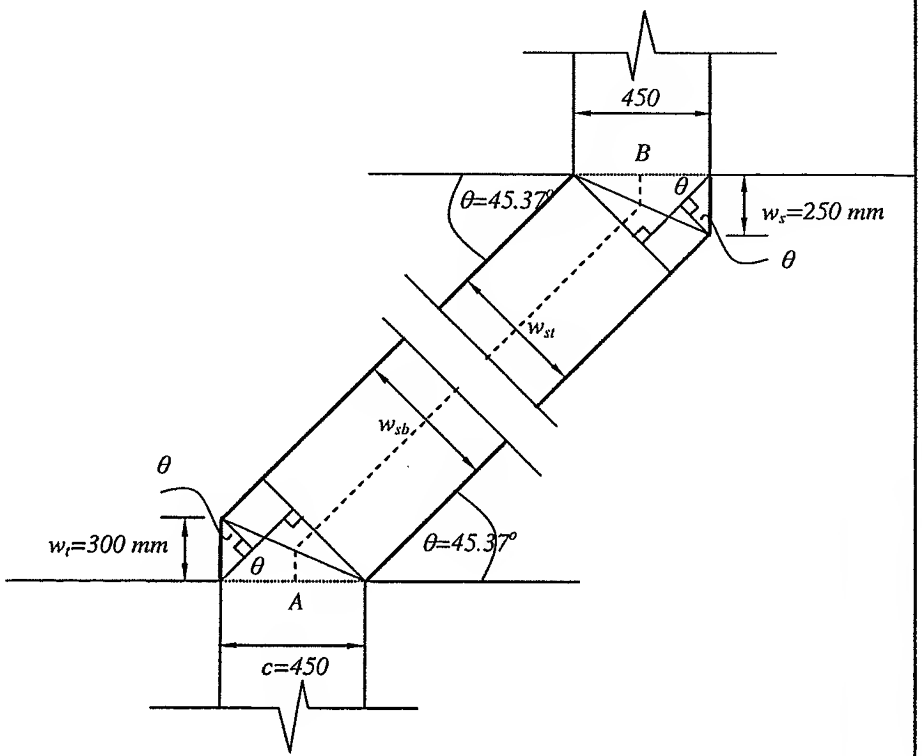
The force in the diagonal strut AB equals:

$$F_{u,AB} = \frac{P_u}{\sin\theta} = \frac{1500 \times 1000}{\sin 45.37} = 2107.6\ kN$$

Referring to the figure below, the width at the top of the strut is given by:

$$w_{st} = c \sin\theta + w_s \cos\theta$$

$$w_{st} = 450 \sin 45.37 + 250 \cos 45.37 = 495.89\ mm$$



The width at the bottom of the strut is given by:

$w_{sb} = c \sin\theta + w_t \cos\theta$

$w_{sb} = 450 \sin 45.37 + 300 \cos 45.37 = 531.01\ mm$

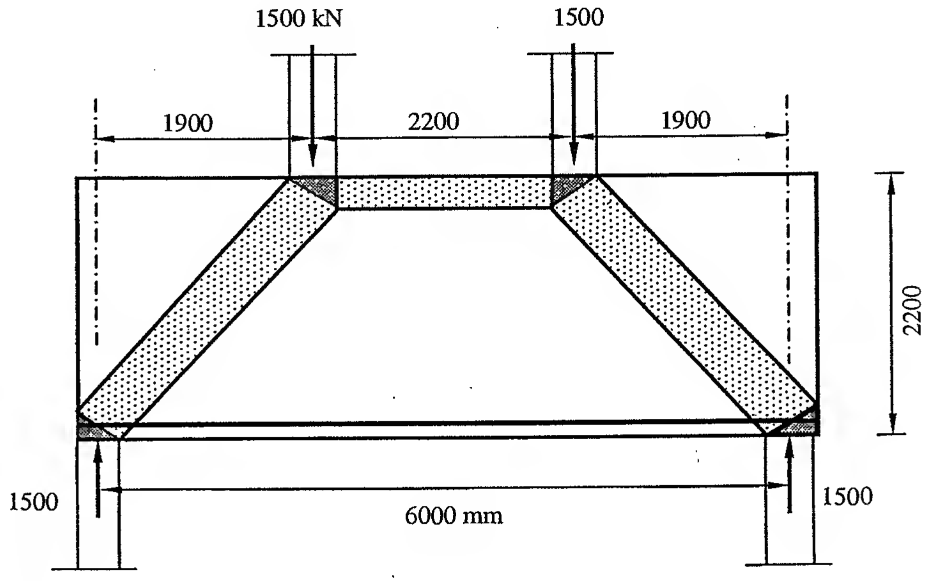
w_s is taken as the smaller of w_{sb} and w_{st}

The Strut AB is expected to be a bottle-shaped strut. By assuming that sufficient crack control reinforcement is used to resist the bursting force in the strut (β_s=0.7), the capacity of strut AB is limited to:

$$F_{u,AB} = 0.67 \times \beta_s \times \frac{f_{cu}}{\gamma_c}\ b\ w_s$$

$$F_{u,AB} = 0.67 \times 0.7 \times \frac{30}{1.6}\ 500 \times 495.89 = 2180.36\ kN$$

Because this is higher than the required force, strut AB (or CD) is considered adequate.



Step 6: Provide minimum web reinforcement

The minimum vertical web reinforcement required by the code is given by:

$A_{sv} = 0.0025\ b\ s$

The minimum horizontal shear reinforcement required by the code is given by:

$A_{vh} = 0.0015\ b\ s$

Choose s =200 mm

$A_{sv} = 0.0025\ 500\ (300) = 250\ mm^2$

For one leg, $A_{sv} = 125\ mm^2 \rightarrow A_s\ for\ \Phi 14 = 154\ mm^2$

Choose ϕ 14 @ 200 mm

$A_{sh} = 0.0015\ b\ (300) = \ mm^2$

For one leg, $A_{sh} = 150\ mm^2 \rightarrow A_s\ for\ \Phi 10 = 78.5\ mm^2$

Choose ϕ 10 @ 200 mm

The arrangement of the reinforcement is shown in the following figure.

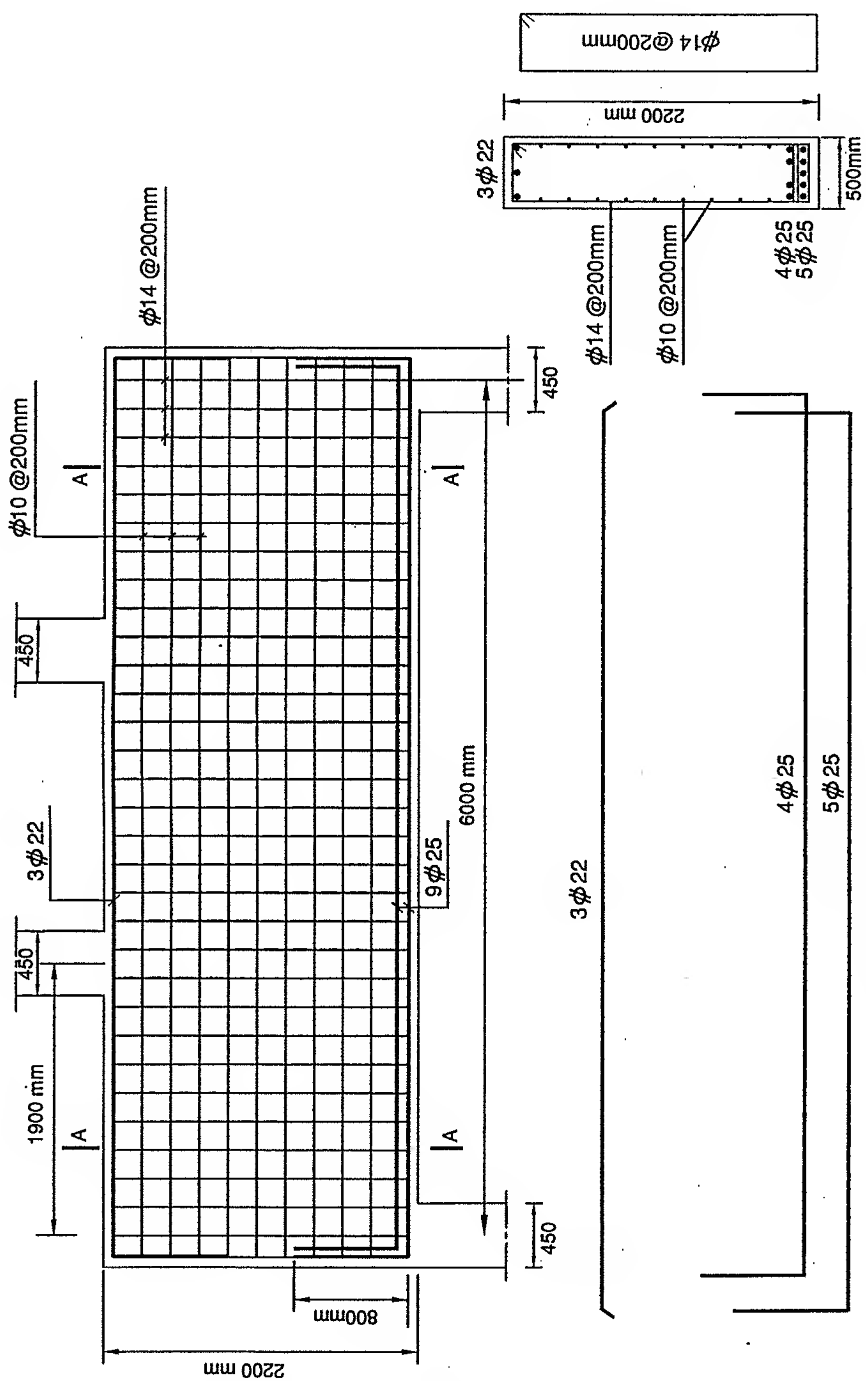


Fig. EX 6.1 Reinforcement details

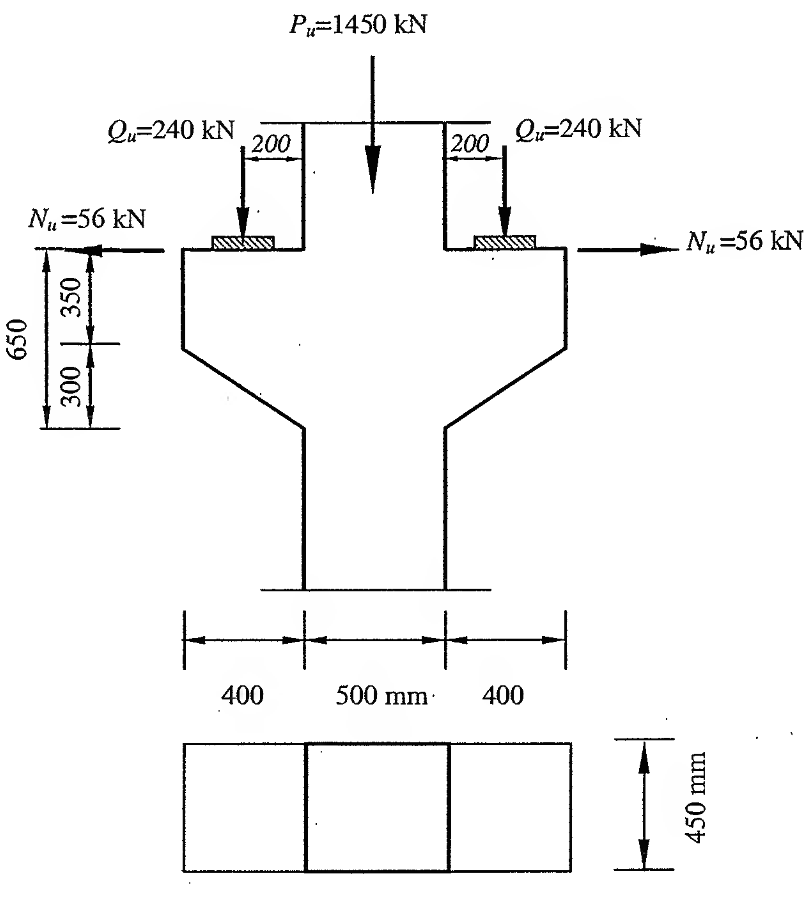
Example 6.2

Give a complete design for the bracket shown in figure using the Strut-and-Tie method presented in the ECP 203 according to the following data:

$f_{cu} = 30 N/mm^2$ and $f_y = 400 N/mm^2$

Factored vertical load $Q_u = 240$ kN

Factored horizontal load $N_u = 56$ kN



Column cross-section

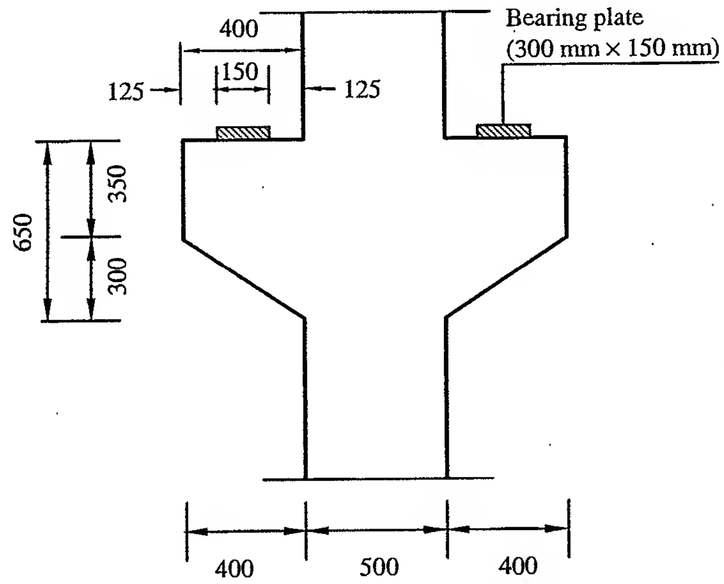
Step 1: Determine the bearing plate dimensions

The nodal zone underneath the bearing plate is a compression-tension (C-C-T) node (β_n=0.80). The effective compressive strength of this node is limited to:

$$f_{cd} = 0.67 \times \beta_n \times \frac{f_{cu}}{\gamma_c} = 0.67 \times 0.8 \times \frac{30}{1.6} = 10.05 N/mm^2$$

$$A_c = \frac{Q_u}{f_{cd}} = \frac{240 \times 1000}{10.05} = 23880\ mm^2$$

Choose a (300 mm × 150 mm) bearing plate (A_c=45000 mm²).



Step 2: Establish the Strut-and-Tie model

The figure below shows the geometry of the Strut-and-Tie model. The location of the tie AA′ is assumed to be 50 mm from the top of the corbel.

$$d = 650 - 50 = 600\,mm$$

As shown in the figure, the column axial load, P_u is resolved into two equal loads acting in line with strut CB. The location of the centerline of strut CB can be found by calculating its width, w_s. This width can be obtained from:

$$F_{u,BC} = f_{cd}\ b\ w_s$$

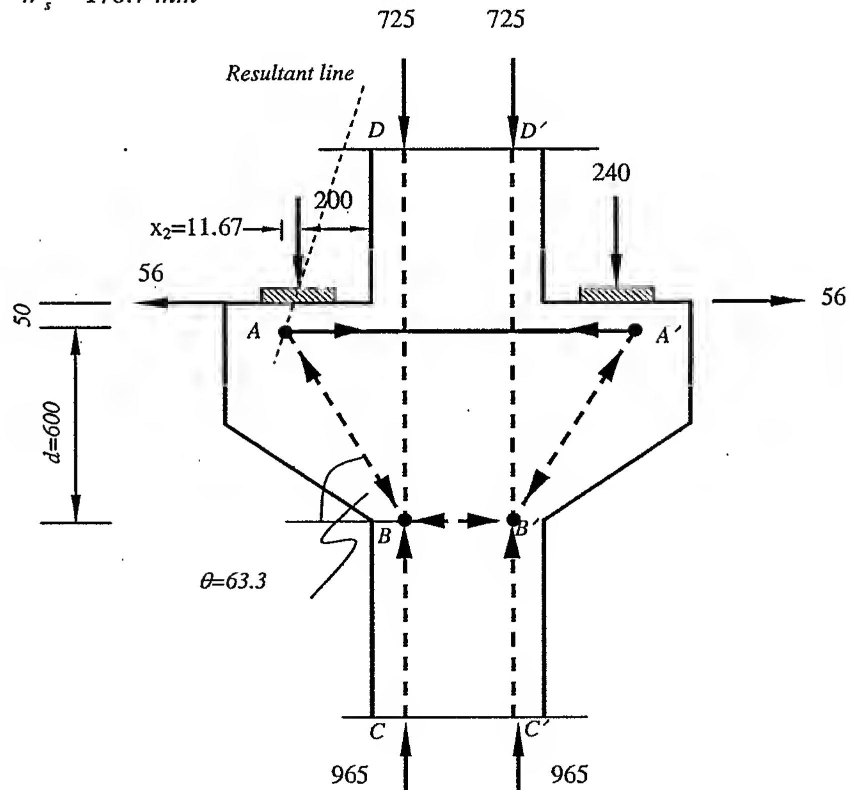
$$F_{u,BC} = 240 + \frac{1450}{2} = 965\,kN$$

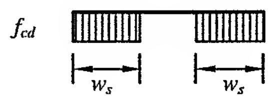
The nodal zone B is an all compression (C-C-C) node and strut *CB* is of prismatic type, the effective compressive strength f_{cd} is given by:

$$f_{cd} = 0.67 \times \beta_n \times \frac{f_{cu}}{\gamma_c} = 0.67 \times 1.0 \times \frac{30}{1.6} = 12.56 N/mm^2$$

$$965 \times 1000 = 12.56 \times 450 \times w_s$$

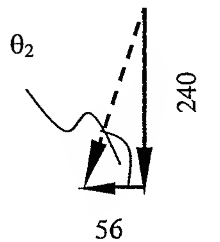
$$w_s = 170.7\ mm$$





The corbel is subjected to a vertical force of a value 240 kN and a horizontal force of a value 56 kN. The resultant of these forces shall be used in establishing the Strut-and-Tie model. The direction of the resultant can be obtained from the triangle of forces.

$$\theta_2 = \tan^{-1}\left(\frac{56}{240}\right) = 13.13^o$$



The distance x_2 form the concentrated load to the node A equals:

$$x_2 = 50\tan 13.13 = 11.66\,mm$$

This fixes the geometry of the Strut-and-Tie model.

Step 3: Determine the required truss forces

$$\theta = \tan^{-1}\left(\frac{600}{200+11.67+170.7/2}\right) = 63.66^o$$

$$F_{AB} = \frac{Q_u}{\sin\theta} = \frac{-240}{\sin 63.66} = -267.8\ kN \text{ .... (compression)}$$

$$F_{AA'} = F_{AB}\cos\theta + N_u = 267.8\ \cos 63.66 + 56 = 174.82\ kN \text{ .... (tension)}$$

$$F_{BB'} = F_{AB}\sin\theta = 267.8\cos 63.66 = -118.8\ kN \text{ .... (compression)}$$

The following table summarizes the forces in all members. Note that positive sign indicates tension and negative sign indicates compression.

Member	AA'	AB	BB'	CB	BD
Force (kN)	174.8	-267.8	-118.8	-965.0	-725.0

Step 4: Select the tie reinforcement

$$F_{u,AA'} = A_s \times f_y / \gamma_s$$

$$F_{u,AA'} = 174.8\ kN$$

$$174.8\times1000 = A_s \times 400/1.15$$

$$A_s = 502\ mm^2$$

$$A_{s\,\min} = 0.03\frac{f_{cu}}{f_y}b\,d = 0.03\times\frac{30}{400}\times450\times600 = 607.5\,mm^2$$

Choose 6 Φ 12 mm $\rightarrow$ $A_s = 678mm^2$

Step 5: Design the nodal zones and check the anchorages

The width w_s of the nodal zone B was determined to be 170.7 mm (refer to Step 2). Therefore, only nodal zone A is checked in this section.

To satisfy the stress limit of nodal zone A, the tie reinforcement must engage an effective depth of concrete w_t that can be obtained from:

$$F_{u,AA'} = 0.67\times\beta_n\times\frac{f_{cu}}{\gamma_c}b\,w_t \qquad \longrightarrow \beta_n = 0.80$$

$$174.8\times1000 = 0.67\times0.8\times\frac{30}{1.6}\,450\ w_t \qquad \rightarrow \qquad w_t = 38.65\ mm$$

As shown in figure, this limit is easily satisfied because the nodal zone available is 2x50=100 mm.

Step 6: Check the struts

Step 6.1: Strut AB

Strut AB shall be checked based on the sizes determined by nodal zones A and B. Other struts shall be checked by computing the strut widths and verifying whether they will fit within the space available

$$F_{ns} = 0.67\times\beta_s\times\frac{f_{cu}}{\gamma_c}\ b\ w_{st}$$

w_{st} is taken as the smaller width at the two ends of the strut as shown in figure.

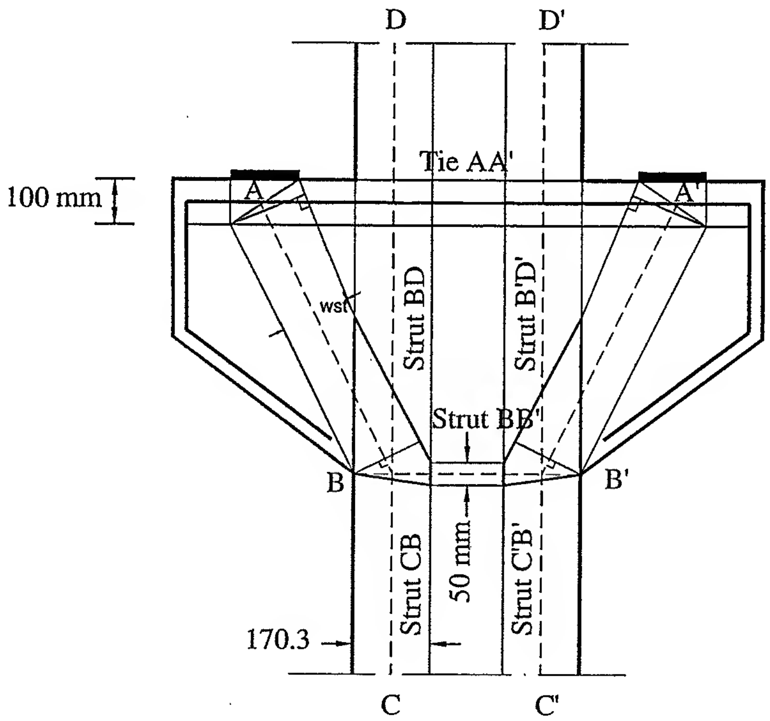
The width at the bottom of the strut can be accurately computed using AutoCAD program as shown in figure below or approximately as follows:

$w_{st} = w_s \sin\theta \quad \rightarrow \quad w_{st} = 170 \sin 63.33 \quad \rightarrow \quad w_{st} = 152.98\,mm$

Strut AB is expected to be a bottle-shaped strut. By assuming that sufficient crack control reinforcement is used to resist bursting force in the strut (β_s=0.7), the capacity of strut AB is limited to

$$F_{ns} = 0.67 \times 0.7 \times \frac{30}{1.6}\; 450 \times 152.98 = 605.38\ kN$$

Since the node strength F_{ns} is higher than the required force $F_{u,AB}$ (=267.8, refer to the table in step 3), strut AB is considered adequate.



Dimensions of the Strut-and-Tie model components

Step 6.2: Strut BD

$$F_{u,BD} = 0.67 \times \beta_s \times \frac{f_{cu}}{\gamma_c}\; b\, w_{s,BD}$$

β_s=1.0 (inside the column zone)

$$F_{u,BD} = 725.0 \times 1000 = 0.67 \times 1.0 \times \frac{30}{1.6}\; 450\, w_{s,BD}$$

$$w_{s,BD} = 128.24\,mm$$

Since the required width for strut CB is less than the available for the node (170.3 mm), the design is considered adequate.

Step 6.3: Strut BB′

$$F_{u,BB} = 0.67 \times \beta_s \times \frac{f_{cu}}{\gamma_c}\; b\, w_{s,BB} \rightarrow\rightarrow\rightarrow \beta_s = 1.0$$

$$F_{u,BD} = 118.8 \times 1000 = 0.67 \times 1.0 \times \frac{30}{1.6}\; 450\, w_{s,BB'}$$

$w_{s,BB'} = 21\ mm \rightarrow\rightarrow$ Choose 50 mm width for strut BB′

Step 7: Calculate the minimum reinforcement

Step 7.1: Vertical reinforcement

Assume that the spacing of the vertical stirrups is 200 mm.

$$A_{st} = \mu_{min} b\ s = \frac{0.4}{f_y} \times b \times s = \frac{0.4}{240} \times 300 \times 200 = 100\,mm^2$$

Choose vertical stirrups with diameter =8 mm (two branches) spaced at 200 mm

The available area = 50 x 2 = 100 mm^2 (O.K.)

Step 7.2: Horizontal reinforcement

$A_h = (A_s - A_n)$

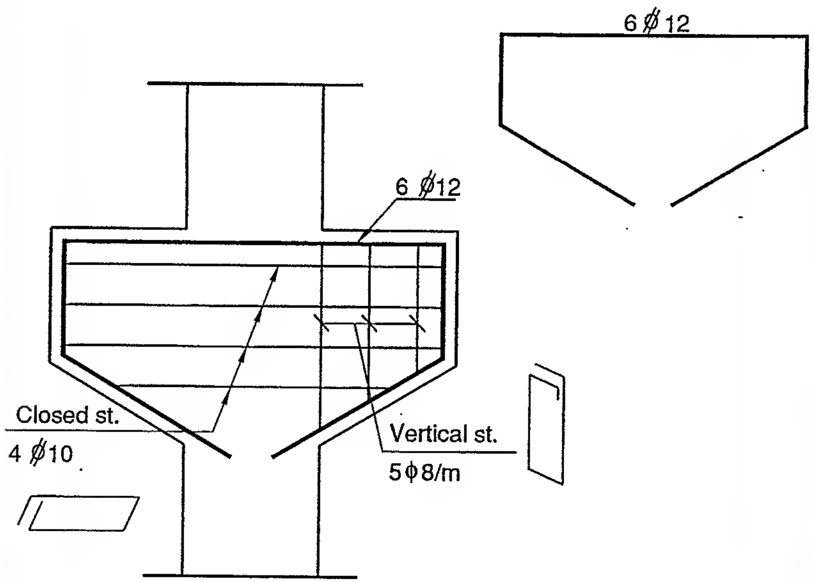
$$A_n = \frac{N_u}{f_y / \gamma_s} = \frac{56\times10^3}{400/1.15} = 161.0 \text{ mm}^2$$

$A_h = 0.50(A_s - A_n) = 0.50(502-161) = 170.5\ mm^2$

Choose 4 closed stirrups with diameter = 10 mm (two branches) @ 200mm

A_h= 78.5×2×4 = 628 mm^2 > 170.5 mm^2 (O.K.)

$$Avg.spacing = \frac{d}{n-1} = \frac{600}{4-1} = 200\,mm \rightarrow o.k$$



Reinforcement details of the corbel

Example 6.3

Design and give complete reinforcement detailing for a pile cap that constitutes a part of a deep foundations system using the Strut-and-Tie method. Design data:

Column dimensions	= 750 mm× 750 mm
Unfactored dead column load	= 2200 kN
Unfactored dead column load	= 1100 kN
Pile diameter	= 500 mm
Pile working load	= 750 kN
f_{cu}	= 30 N/mm^2
f_y	= 360 N/mm^2

Solution

Step 1: Arrangement of piles

The total loads acing on the pile cap are given by:

$P = P_{DL} + P_{LL} = 2200 + 1100 = 3300\,kN$

The required number of piles is given by:

$$n = \frac{1.05\times P}{P_{pile}} = \frac{1.05\times3300}{750} = 4.62 \quad \text{piles}$$

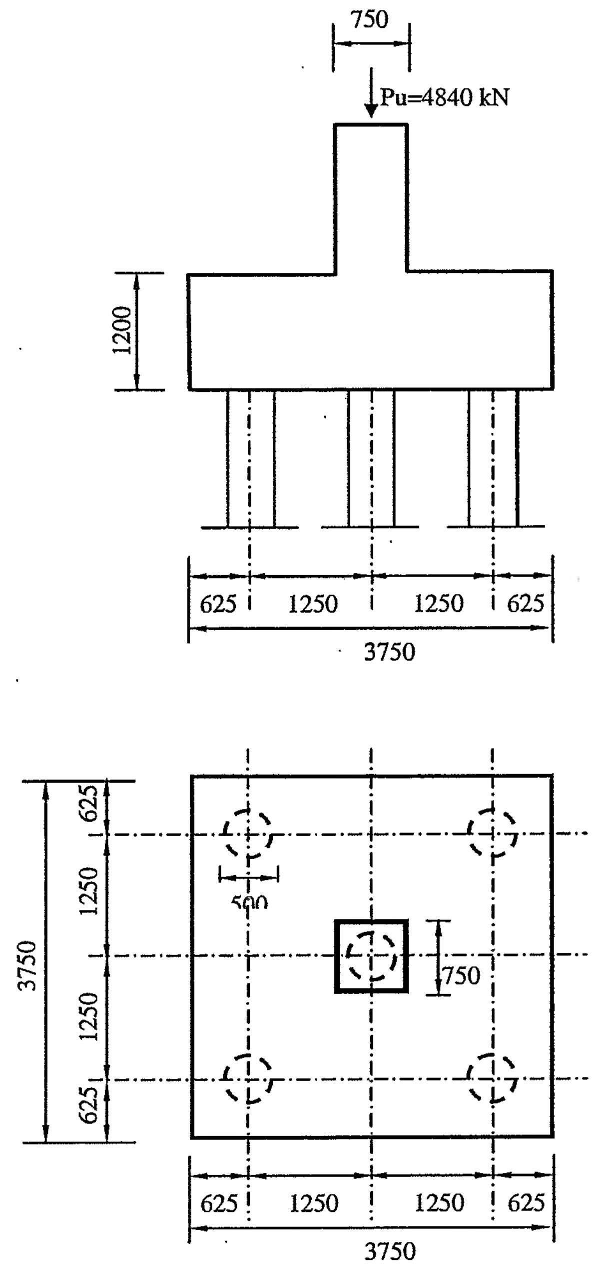
Choose 5 piles. The arrangement is shown in the following figure.

Assume that the thickness of the pile cap is 1200 mm.

The ultimate applied load is given by:

$P_u = 1.4\ P_{DL} + 1.6\ P_{LL} = 1.4\times2200 + 1.6\times1100 = 4840\ kN$

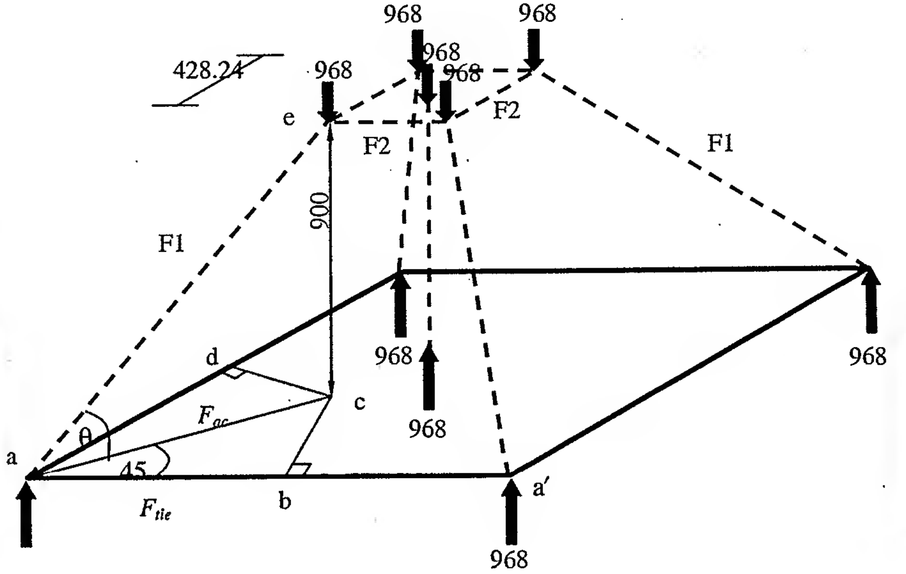
$$P_{u/pile} = \frac{4840}{5} = 968\ kN$$



Pile cap arrangement

Step 2: Establish the Strut-and-Tie model

The column load is divided into 5 equal loads of 968 kN each. Each of these loads is connected to the center of one pile through an inclined strut as shown in the figure.



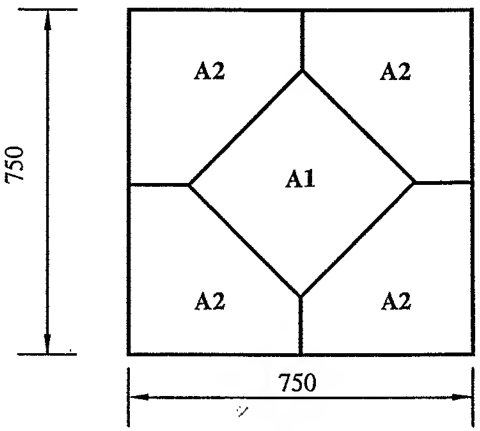
The location of each load at the column cross-section should be determined in order to establish the Strut-and-Tie model. Such locations are determined based on satisfying the stress limits of the struts.

The figure given below shows the cross-section of the column divided into 5 areas, each of them is connected to one strut. The area A_1 of the vertical strut (Bottle –shaped strut) at the top node can be found from:

$$A_1 = \frac{F_u}{0.67\,\beta_s f_c/\gamma_c} = \frac{968\times 10^3}{0.67\times0.7\times30/1.6} = 110078\ mm^2 \approx (332\text{ mm x }332\text{ mm})$$

Accordingly, the area A_2 of the intersection of the inclined strut with the top node (with the cross-section of the column) is found from:

$$A_2 = \frac{\left(700^2 - 332^2\right)}{4} = 113069\ mm^2$$

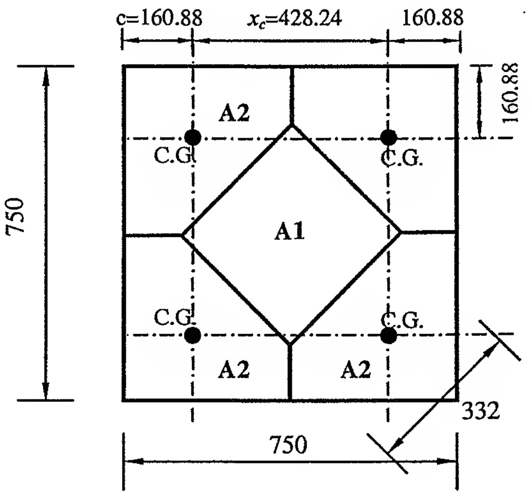


The center of gravity (C.G.) of the area A_2 can be easily obtained as shown in figure (*calculations not shown*).

$c = 160.88\,mm$

$x_c = 700 - 2 \times 160.88 = 428.24\ mm$

It is assumed that the center-line of the inclined strut connects the C.G. of the area A2 and the C.G. of the pile.



Step 3: Calculate the forces

Assume a clear concrete cover of 70 mm, and that the distance from the center-line of the bottom tie to the bottom concrete fibers is 150 mm.

Assume that the thickness of the top horizontal struts is 300 mm. Hence, the distance between the bottom tie and top horizontal strut equals:

$ec = 1200 - 150 - 300/2 = 900\ mm$

$ab = ad = 1250 - 428.24/2 = 1035.88\ mm$

$ac = \sqrt{1035.88^2 + 1035.88^2} = 1465\,mm$

$$\theta = \tan^{-1}\left(\frac{ec}{ac}\right) = \tan^{-1}\left(\frac{900}{1465}\right) = 31.56^o$$

The force in the strut $F_1 = \dfrac{968}{\sin 31.56} = 1849\ kN$

The force in the tie is obtained as follows:

$F_{ac} = F_1 \cos\theta = 1849 \times \cos 31.56 = 1575.6\ kN$

$F_{tie} = F_{ac}\ \cos 45 = 1575.6\ \cos\ 45 = 1114\,kN$

Similarly, F_2=1114 kN (compression)

Step 4: Select the tie reinforcement

The tie reinforcement can be obtained from the following equation:

$F_{tie} = A_s \times f_y / \gamma_s$

$1114 \times 1000 = A_s \times 360/1.15$

$A_s = 3559\ mm^2$ Choose 8 Φ 25 mm in 2 layers $\rightarrow$ $A_s = 3927\ mm^2$

Provide A_{smin} at locations other than the ties.

$$A_{s,\min} = \frac{0.6}{f_y}\,b\ d \qquad (d = 1200 - 100 = 1100\ mm\,)$$

$$A_{s,\min} = \frac{0.6}{360} \times 1000 \times 1100 = 1833\ mm^2/m' \ \rightarrow\ \text{Use } 8\ \Phi\ 18/\text{m}'$$

Step 5: Check the struts

Step 5.1: Strut F1

At bottom node

$w_t = 2c + \phi$

Since it has been assumed earlier that the distance from C.G. of the tie to bottom outermost concrete fibers is 150 mm → w_t = 300 mm.

$w_{sb} = c \sin\theta + w_t \cos\theta$

$w_{sb} = 500 \sin 31.56 + 300 \cos 31.56 = 517.3\ mm$

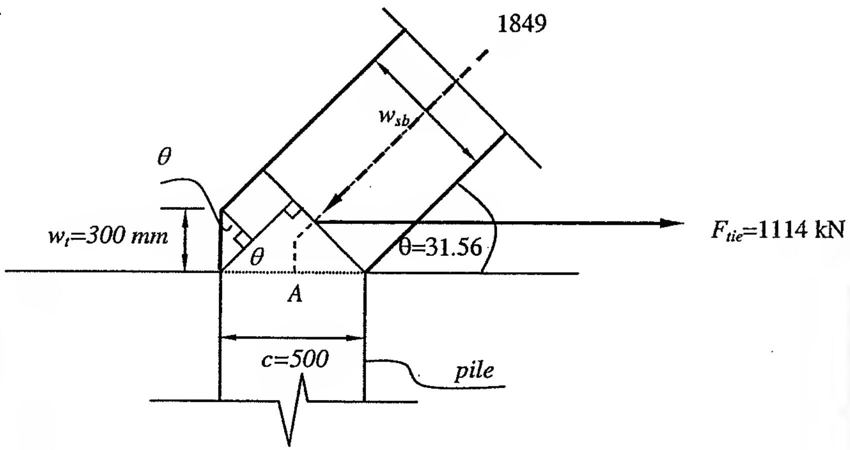
The capacity of the inclined strut is given by:

$F_{cs} = 0.67 \times \beta_s \times \dfrac{f_{cu}}{\gamma_c} w_{sb} \times b$ (b is the pile diameter)

$\beta_s = 0.60$ for bottle-shaped strut inclined with the cracks without special reinforcement

$F_{cs} = 0.67 \times 0.60 \times \dfrac{30}{1.6} \times 500 \times 517.3 = 1949.6\ kN$

Because this is higher than the required force (1849 kN), strut F1 is considered adequate.



At top node

Required area of the strut at the top node

$F_1 = 0.67 \times \beta_s \times \dfrac{f_{cu}}{\gamma_c} A_{CsT}$

$1849.2 \times 1000 = 0.67 \times 0.60 \times \dfrac{30}{1.6} A_{CsT}$

$A_{CST} = 245337.9\ \text{mm}^2$

Assume that the diagonal struts are square in shape. The side dimensions of the struts (h_2) can be obtained as follows:

$h_2 = \sqrt{245337.9} = 495.32\,mm$

Step 5.2: Strut F2

The top node is the node where the inclined struts, the horizontal top struts and the top load meet. At this node, the horizontal area A_2 and the inclined area of the inclined strut form parts of the nodes.

To calculate the thickness h_1, an average width (h_3) for the area A_2 is used. Thus,

$h_3 = \dfrac{A_2}{h_2} = \dfrac{113069}{495.32} = 228.2\ mm$

$h_2 = h_3 \sin\theta + h_1 \cos\theta$

$495.3 = 228.2 \sin 31.56 + h_1 \cos 31.56$

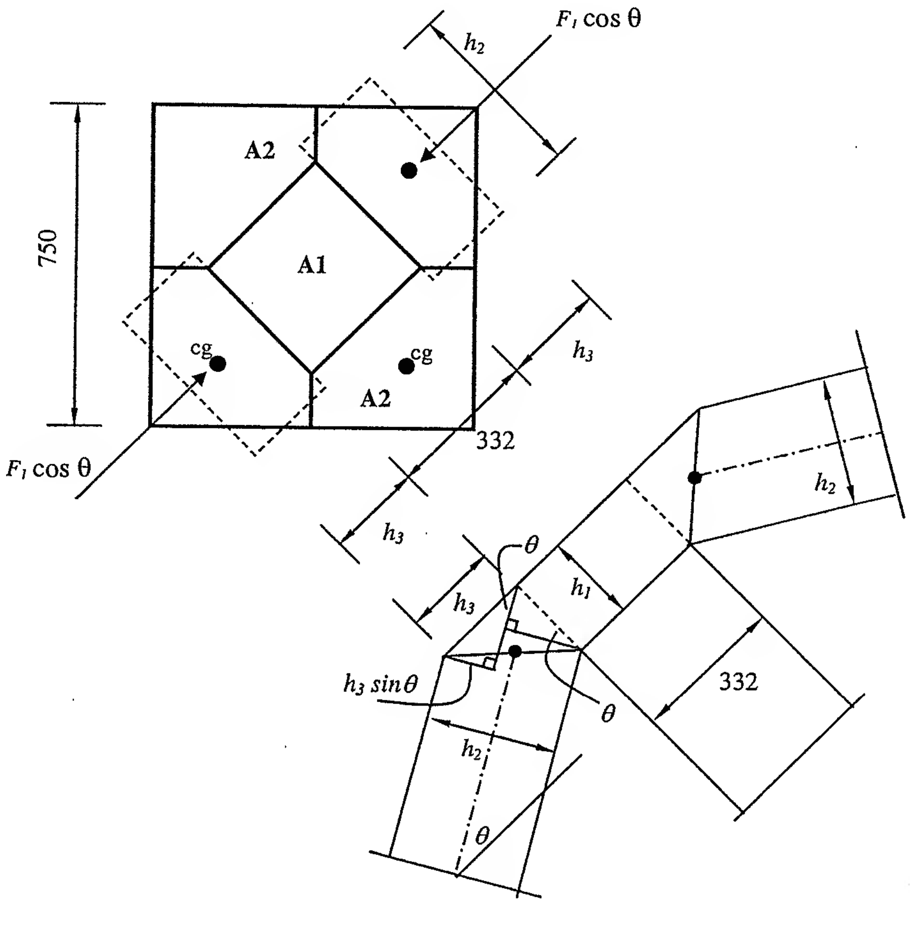
$h_1 = 441\,mm$

$f_c = \dfrac{F_1 \cos\theta}{h_1 \times h_2} = \dfrac{1849 \times 1000 \times \cos 31.56}{441 \times 495.32} = 7.2\,N/mm^2$

$f_{cn} = 0.67 \times \beta_n \times \dfrac{f_{cu}}{\gamma_c} = 0.67 \times 1.0 \times \dfrac{30}{1.6} = 12.56\ N/mm^2$

(β_n=1.0 for (C-C-C) node)

Since f_c is less than f_{cn}, the design is consider adequate.

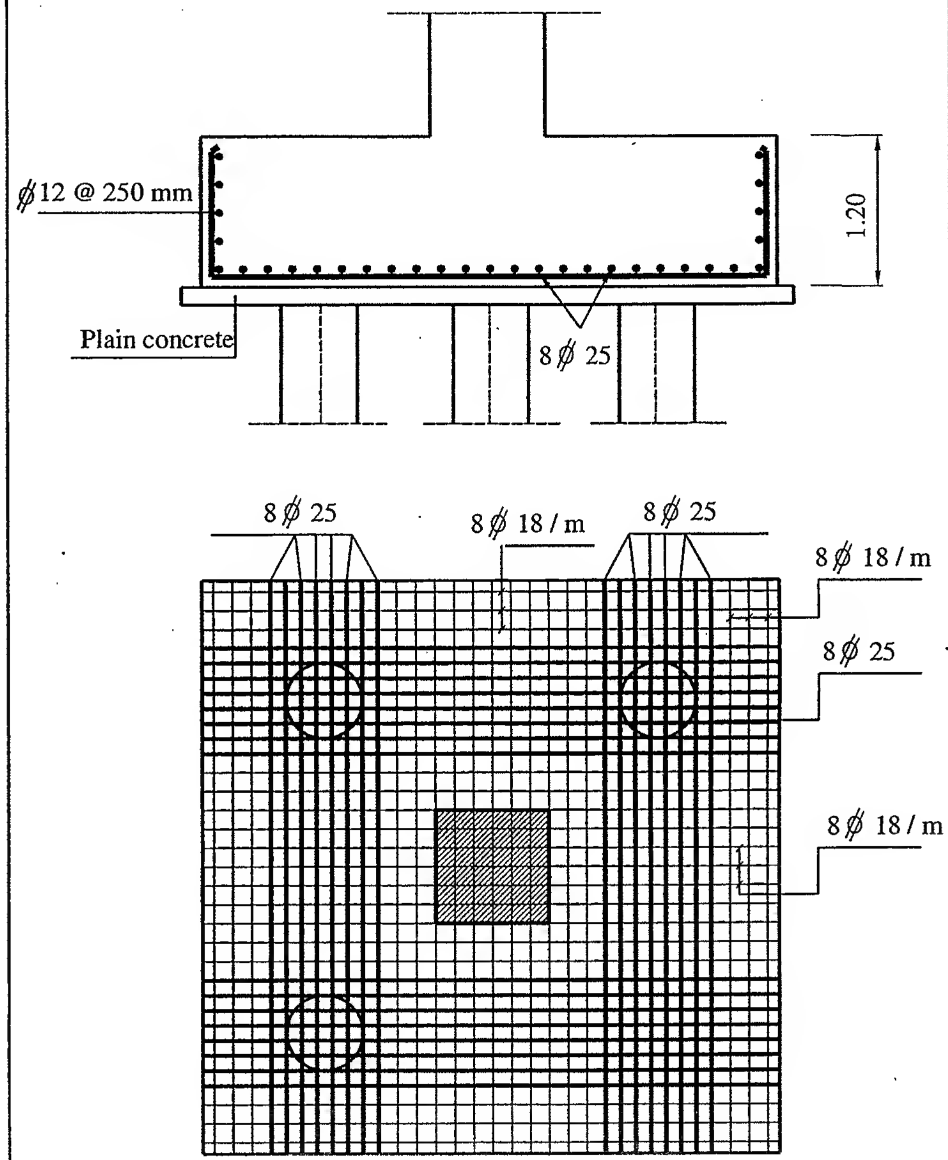


Step 6: Reinforcement arrangement

Section 6-7-1-4 of the ECP 203 states that when the Strut-and-Tie model is used for designing pile caps, the tie reinforcement must be distributed in a distance greater than three times the pile diameter if the distance between the centerlines of the piles is more than 3D where D is the pile diameter.

In this example, the distance between the centerlines of the piles is less than 3D. The reinforcement of the tie is arranged such that the distance between the bars is 100 mm (the minimum accepted distance).

Other locations should be reinforced with the minimum reinforcement calculated in Step 5.



Reinforcement Details

7

INTRODUCTION TO PRESTRESSED CONCRETE

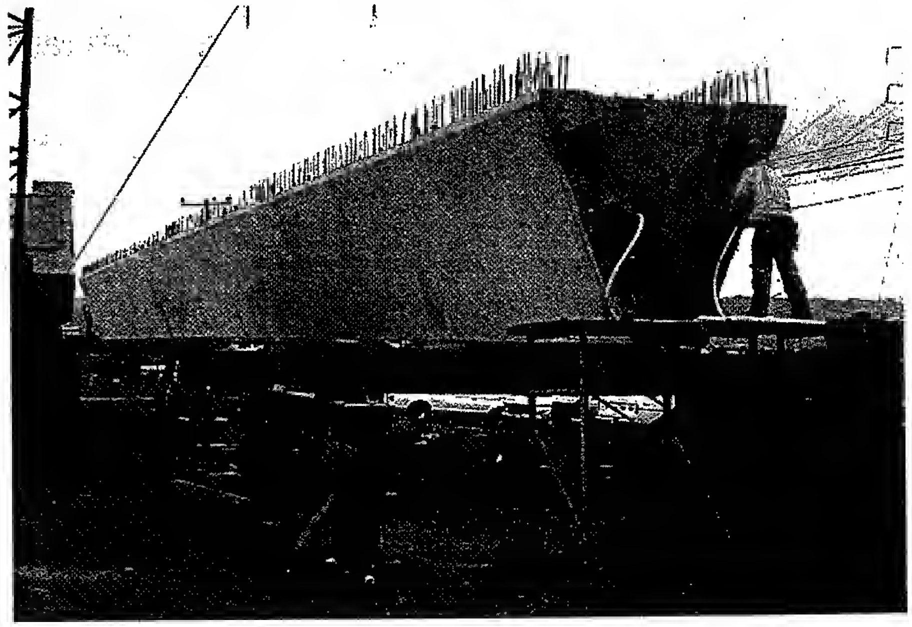


Photo 7.1 Prestressed concrete girder during grouting

7.1 Introduction

The idea of prestressing was introduced to overcome the main disadvantage of concrete which is the low tensile strength. Introducing compressive longitudinal force, called prestressing, prevents the cracks from developing by reducing or even eliminating the tensile stresses at critical sections. Thus, prestressing is a technique of introducing compressive stresses of a pre-determined magnitude

into a structural member to improve its behavior. Therefore, all sections can reach the full capacity of concrete in compression.

Although prestressed concrete has many benefits, it requires more attention to specific design considerations that are not usually considered in construction of ordinary reinforced concrete.

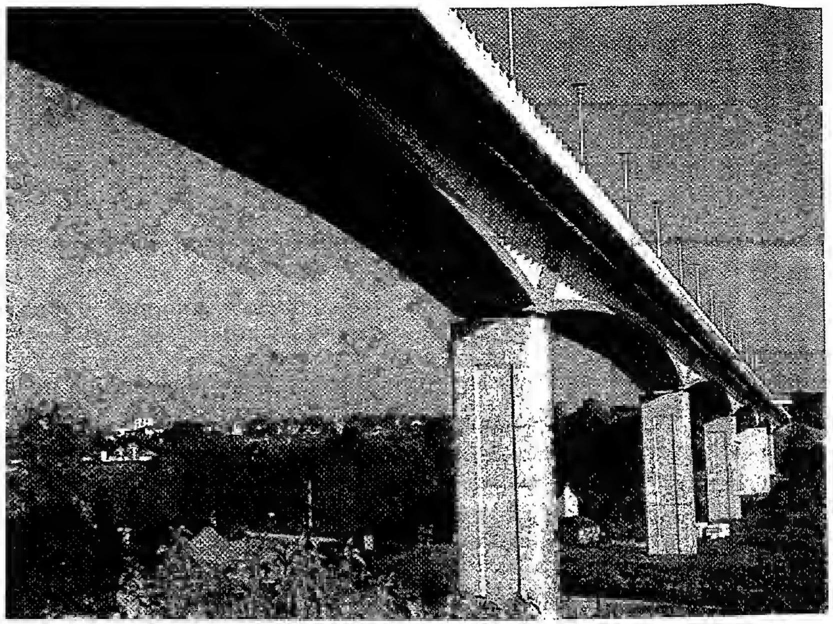
Prestressed concrete is used in buildings, towers, tanks, underground structures, and bridges. The wide spread use of prestressing is mainly due to the new technology of developing high strength steel or fiber reinforced plastics (FRP) and the accumulated knowledge of computing the short and long-term losses. Prestressing significantly reduce the dead weight of flexural members. The small span-to-depth ratio accompanied by short construction time makes prestressed concrete very attractive solution as a construction material.

The idea of prestressed concrete can be traced back to 1872, when P.H Jackson, an engineer from California, USA, developed a prestressing system that used a tie rod to build beams and arches from individual blocks. Early attempts of prestressing were not successful because of prestressing losses over time. In 1920s, the concept of circular prestressing was introduced but with a little progress because of the unavailability of high strength material that can compensate the long-term losses.

Linear prestressing continued to develop in Europe especially in France through the work of Eugene Freyssinet. In 1928 he proposed the use of high strength steel to overcome the losses. P.W. Abeles of England introduced the concept of partial prestressing. The work of T. Y. Lin of developing the load-balancing technique simplifies the design process particularly in indeterminate structures. Since 1950s, the number of buildings and bridges constructed of prestressed concrete has grown enormously.

7.2 Systems of Prestressing

Prestressed members are classified into two main groups; *pre-tensioned* and *post-tensioned*. The member is called pre-tensioned, if the steel is stressed before casting the concrete. The member is called post-tensioned, if the steel is stressed after hardening of the concrete.



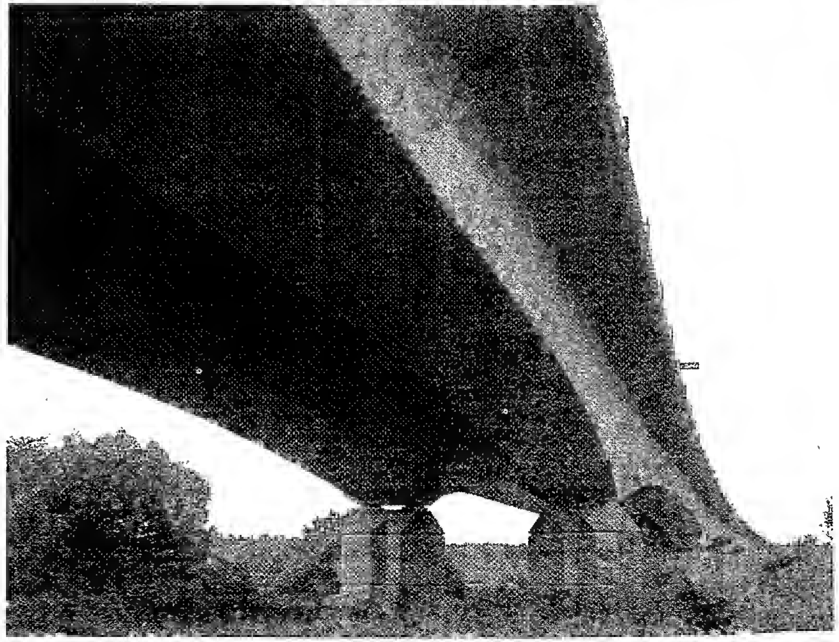


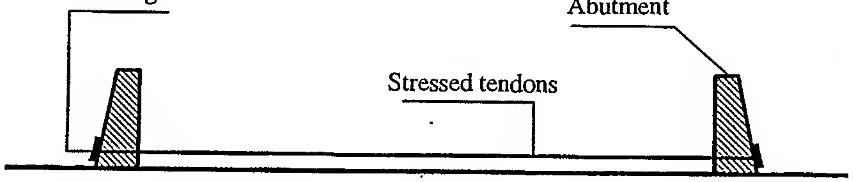
Photo 7.2 Sample of prestressed concrete bridges

7.2.1 Pretensioned Concrete

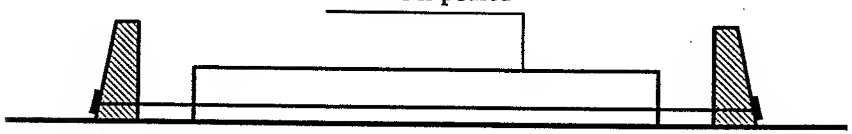
Figure 7.1 illustrates the procedure for pre-tensioning a concrete member. Such a procedure can be summarized as follows:

1- The prestressing tendons are initially tensioned between fixed rigid walls and anchored.
2- With the formwork in place, the concrete is cast around the stressed steel tendons and cured.
3- When the concrete has reached its required strength, the wires are cut (or released from the rigid walls).
4- As the tendons attempt to contract, the concrete is compressed. Prestress is transmitted through bond between the steel and the concrete.

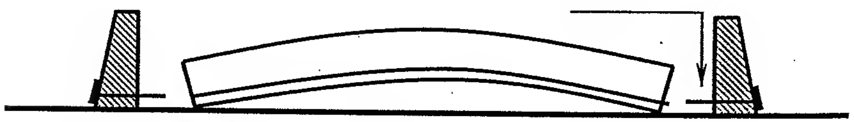
Pretensioned concrete members are often precast in pre-tensioning yards that are usually long enough to accommodate many identical units simultaneously.



a) Stressing the tendons before pouring the concrete



b) The concrete is poured over the stressed tendons



c) The wires are cut and the prestressing force is transferred to concrete

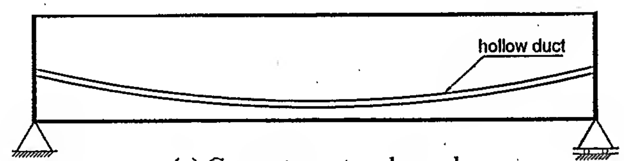
Fig. 7.1 A Pre-tensioned beam during manufacturing

7.2.2 Post-tensioned Concrete

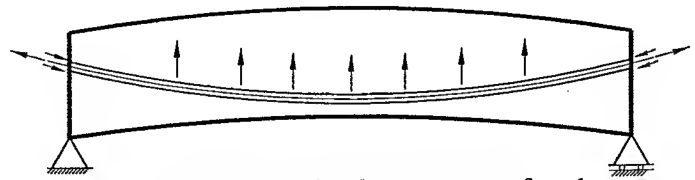
The procedure for post-tensioning a concrete member is shown in Fig. 7.2.

1. With the formwork in position, the concrete is cast around the hollow ducts, which are fixed to any desired profile.
2. The steel tendons are usually in place, unstressed in the ducts during the concrete pour, or alternatively may be threaded through the ducts later.
3. When the concrete has reached its required strength, the tendons are tensioned. Tendons may be stressed from one end with the other end anchored or may be stressed from both ends.
4. The tendons are anchored at each stressing end. In post-tensioning, the concrete is compressed during the stressing operation and the prestressing is maintained after the tendons are anchored by bearing of the end plates onto concrete. The ducts containing the tendons may be filled with grout under pressure. In this way, tendons are bonded to the concrete and are more efficient in controlling the cracks and providing ultimate strength. Bonded tendons are also less likely to corrode.

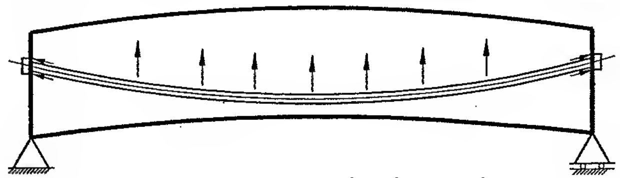
It should be mentioned that most in-situ prestressed concrete is post-tensioned. Post-tensioning is also used for segmental construction of large-span bridge girders.



(a) Concrete cast and cured



(b) Tendons stressed and prestress transferred



(c) Tendons anchored and grouted

Fig. 7.2 A post-tensioned beam

7.3 General Design Principle

Flexural stresses in prestressed members are the result of internal prestressing force P, the internal moment due to eccentric cable configuration (P. e), and the external applied moments (M). The prestressing force results in stresses that are opposite to those resulting from the external loads. The entire section is mainly subjected to compression stresses and is free from cracks. Fig. 7.3 illustrates the distribution of the stresses at mid-span. It is clear that the tensile stresses that result from the applied loads are eliminated by the compressive stresses due to prestressing

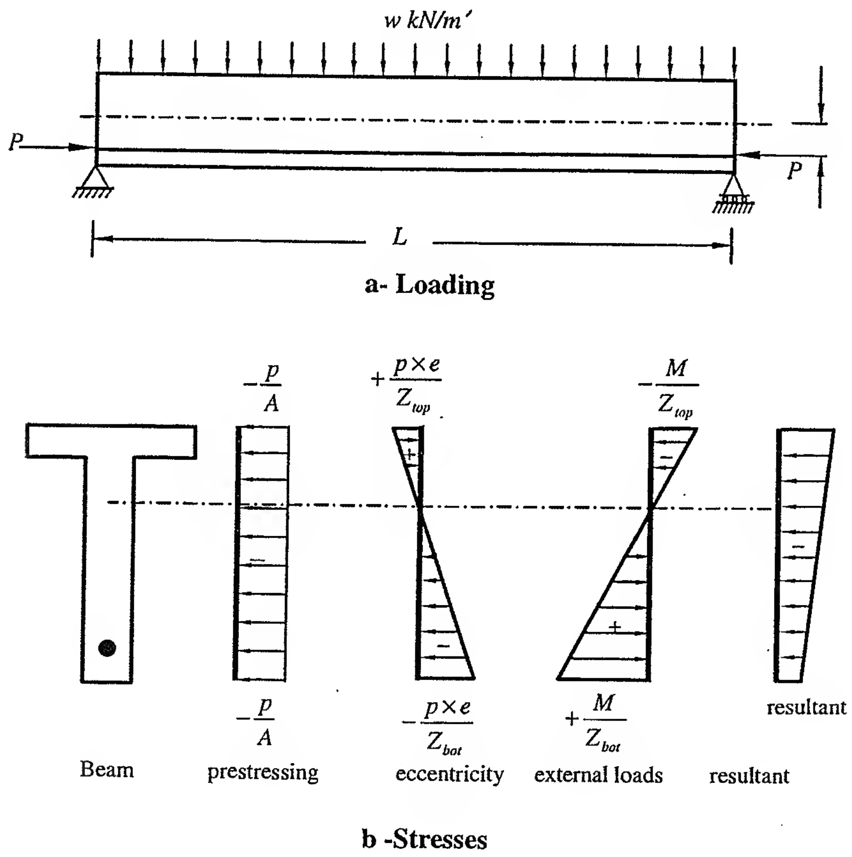


Fig 7.3 Distribution of stresses in a prestressed beam

7.4 Materials

7.4.1 Concrete

The Egyptian Code ECP 203 presents an idealization for the stress-strain curve of concrete in compression. The initial part of the curve is a parabolic curve up to a strain of 0.002 and the second part is a straight horizontal line up to a strain of 0.003, as shown in Fig. 7.4. Referring to Fig. 7.4, the equation of the concrete stress (f_c) in terms of the concrete strain (ε_c) can be expressed as:

$$f_c = \begin{cases} f_c^* \left[\dfrac{2\,\varepsilon_c}{0.002} - \left(\dfrac{\varepsilon_c}{0.002} \right)^2 \right] & for \quad \varepsilon_c < 0.002 \\ f_c^* & for \quad 0.002 \le \varepsilon_c \le 0.003 \end{cases} \qquad \text{.....................} (7.1)$$

where $f_c^* = \dfrac{0.67\, f_{cu}}{\gamma_c}$

To take the full advantages of prestressed concrete, concrete with high compressive strength is usually used. The ECP 203 specifies the concrete grade that should be used in prestressed concrete as shown in Table 7.1

Table 7.1 Concrete compressive strength used in prestressed concrete (N/mm^2)

Concrete grade	30	35	40	45	50	55	60

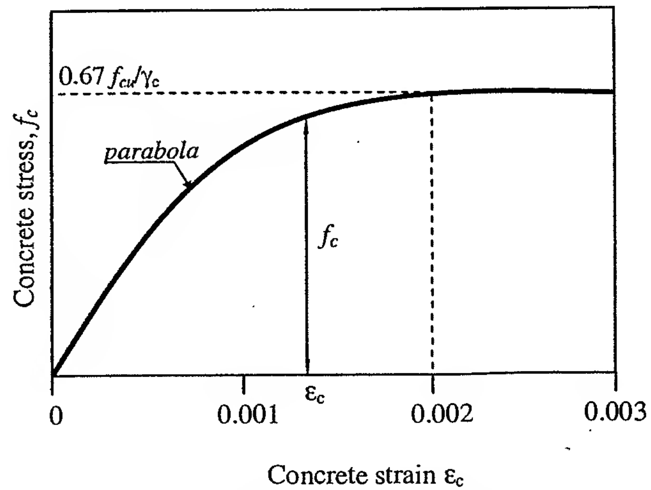


Fig 7.4 ECP 203 idealized stress-strain curve for concrete

The advantages of using high-strength concrete in prestressed concrete construction can be summarized in the following points:

1. Due to its speed in gaining strength, the shattering can be removed faster reducing time and cost.
2. It minimizes losses in prestressing force by reducing creep, elastic shortening and shrinkage.
3. It reduces the size and weight of the member.
4. It reduces the required area for shear reinforcement.
5. It produces the high bond strength required to anchor the strands used in pre-tensioned construction.

7.4.2 Non-prestressing Reinforcement

The behavior of the non-prestressing steel reinforcement is idealized by the Egyptian code as an elasto-plastic material as shown in Fig 7.5. The reinforcing steel stress can be calculated using Eq. 7.2.

$$\begin{aligned} f_s &= \varepsilon_s \times E_s \quad when \;\; \varepsilon_s < \varepsilon_y / \gamma_s \\ f_s &= f_y / \gamma_s \quad when \;\; \varepsilon_s \geq \varepsilon_y / \gamma_s \end{aligned} \qquad \text{.................................. (7.2)}$$

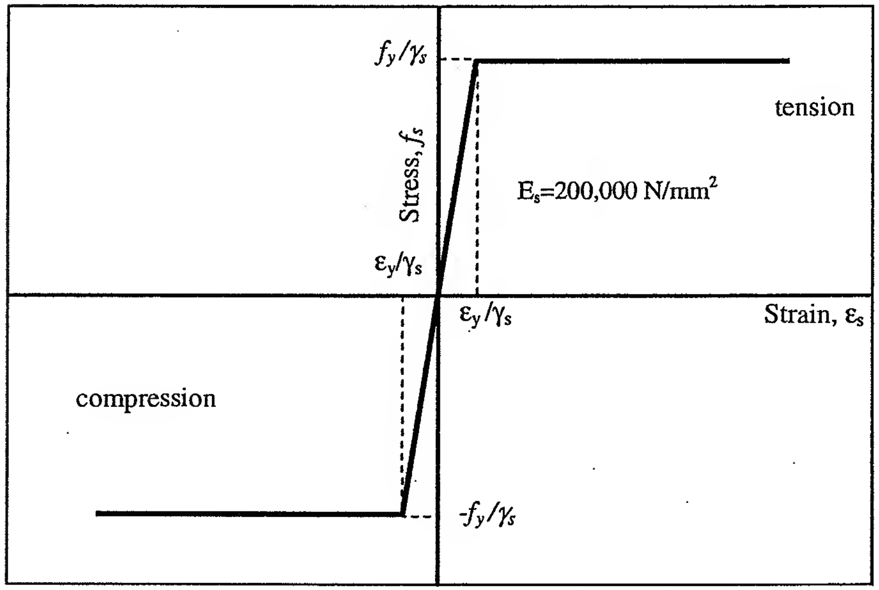


Fig 7.5 Idealized stress-strain curve for non-prestressed steel

7.4.3 Prestressing Reinforcement

Prestressing reinforcement is available in different forms such as cold drawn wires, cables, and alloyed steel bars. The most common type of prestressing reinforcement is the seven wire strands cable as shown in Fig. 7.6. The ultimate tensile strength of these cables is several times that of non-prestressing reinforcement. For example, the ultimate strength of wires and cables ranges from 1700-1900 N/mm^2 (about 4-5 times that of high grade steel). This high strength is attributed to adding alloying elements (manganese and carbon), and by the use of heat treating and tempering. The favorable high tensile strength is accompanied by a loss of ductility and toughness. Therefore steel reinforcement with yield point above 1900 N/mm^2 is not commonly used because of their extremely brittle nature.

A typical stress-strain relation for prestressing reinforcement is shown in Fig. 7.6. It is clear that the prestressing steels lack a sharply defined yield point. Therefore, most codes including the ECP 203, specifies the yielding point as the stress associated with a 1% strain. For high strength bars, the yield strength is frequently specified as the stress associated with the intersection of the curve and a line parallel to the initial slope starting at strain of 0.002. The yield stress f_{py} for stress relieved steel equals to approximately 85% of f_{pu} and equals to 90% of f_{pu} for low relaxation steel.

The modulus of elasticity E_{ps} can be taken as 200 GPa (200,000 N/mm^2) for bars and wires and 180 GPa (180,000 N/mm^2) for strands.

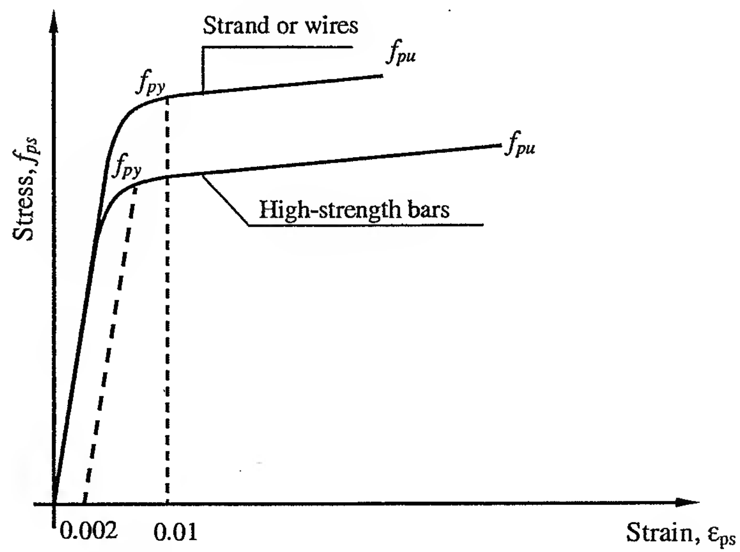
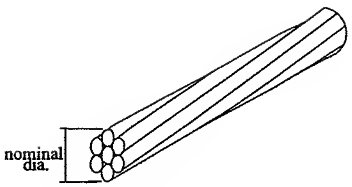
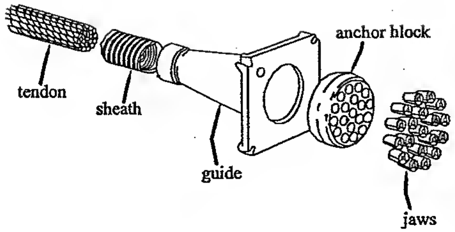


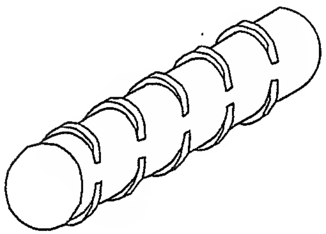
Fig. 7.6 Stress-Strain curve for prestressing reinforcement



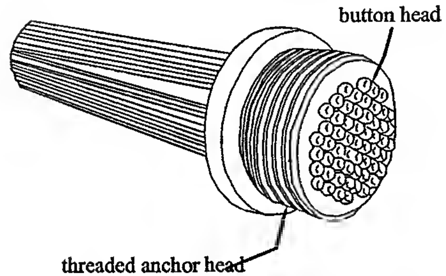
(a) 7-wire monostrand tendon



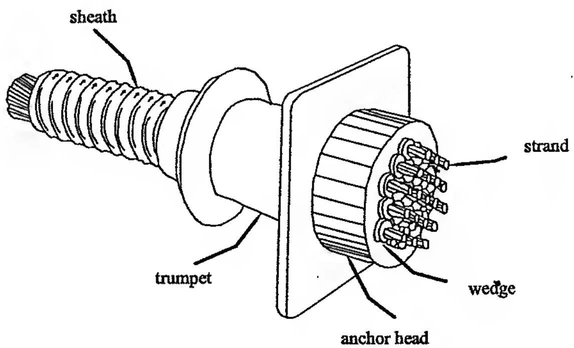
(b) Multi-strand tendon tendon



(c) single bar tender



(d) Multi-wire tendon



(e) Multi-wire tendon

Fig.7.6 Prestressing tendons (cont.)

7.5 Losses in Prestressed Members

7.5.1 Introduction

The applied prestressing force after jacking undergoes a number of reductions. Some of these reductions occur immediately and others occur over a period of time. Therefore it is important to establish the level of prestressing at each loading stage as shown in the diagram below. Following the transfer of force from the jack to the member, a loss in tendon stress ranging from 10-15% of the initial force occurs.

Prestressed losses can be categorized in two groups.

Immediate losses: These are the losses that occur during fabrication, including elastic shortening Δf_{pe}, anchorage loss Δf_{pA} and frictional losses Δf_{pf}.

Time-deponent losses: These are the losses that increase over time, including creep Δf_{pcr}, shrinkage Δf_{psh}, and steel relaxation Δf_{pR}.

$$P_j \xrightarrow{\text{Immediate losses}} P_i \xrightarrow{\text{Time-dependent losses}} P_e$$

Jacking force — Prestressing force immediately after transfer — Final or effective prestressing force

Some types of these losses occur only in post-tensioned members. An example of such losses is the friction losses that develop between the tendon and the concrete at the time of jacking. The following is a summary for the losses that need to be considered for each type.

Pretensioned members

$$\Delta f_{pT} = \Delta f_{pe} + \Delta f_{psh} + \Delta f_{pcr} + \Delta f_{pR} \quad \text{.............................} (7.3)$$

Post-tensioned members

$$\Delta f_{pT} = \Delta f_{pA} + \Delta f_{pe} + \Delta f_{pw} + \Delta f_{pf} + \Delta f_{psh} + \Delta f_{pcr} + \Delta f_{pR} \quad \text{.........} (7.4)$$

where

Δf_{pA} = anchorage slip losses

Δf_{pe} = Elastic shortening losses

Δf_{pw} = wobble friction losses

Δf_{pf} = curvature friction losses

Δf_{psh} = shrinkage losses

Δf_{pcr} = creep losses

Δf_{pR} = steel relaxation losses

In the following sections, each type of losses is briefly discussed, and step by step examples for losses calculations are given.

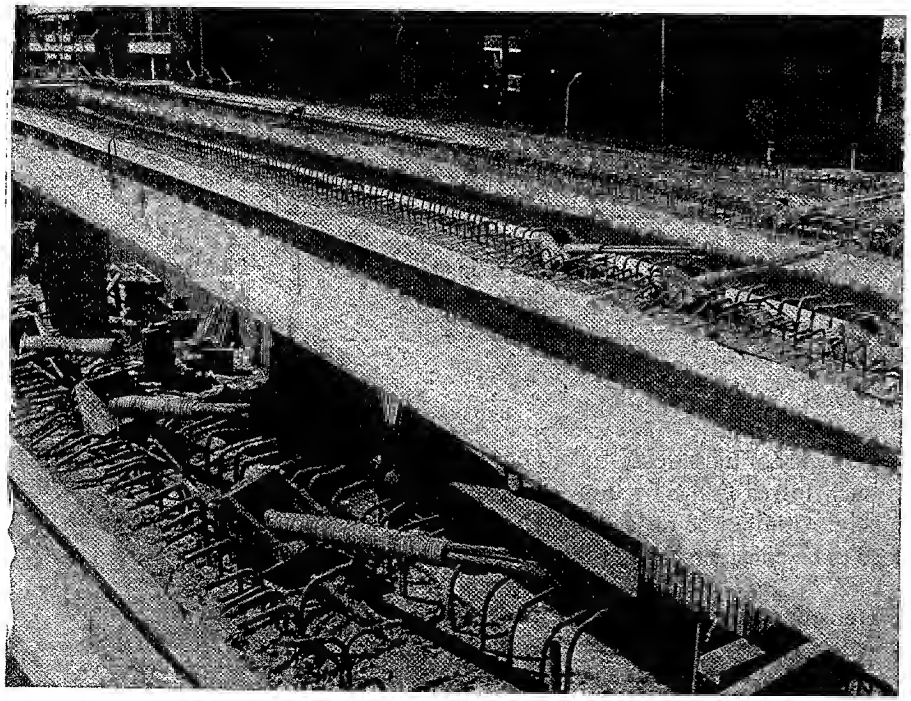


Photo 7.3 Prestressed concrete girders during construction

7.5.2 Anchorage Slip Losses (A)

At transfer in post-tension construction, when the jack is released, a small amount of tendon shortening occurs because of the anchorage fitting and movement of the wedges. The magnitude of this slip is function of the anchorage system and specified by the manufacturer. It usually varies from 2-6 mm. From Hook's law, the loss of stress in the cable Δf_{pA} due to slippage is given by:

$$\Delta f_{pA} = \frac{\Delta_A}{L} E_p \qquad (7.5)$$

where

Δ_A = magnitude of slip

L = tendon length (the horizontal distance can be used)

E_p = modulus of elasticity of the prestressing steel

The loss in prestressing steel stress due anchorage slip is inversely proportional to the length of the cable. Hence, the loss of stress due to slippage decreases as the length of the cable increases. At, transfer, if the tendon can be stressed by additional increment of length equal to the predicted anchorage slip without overstressing the cable, the loss in stress due to slippage can be eliminated.

Photo 7.4 Prestressing tendons at the anchorage plate

7.5.3 Elastic Shortening Losses(e)

When the prestressing force is transferred to the concrete, the concrete shorten and part of the prestressing is lost. To establish the loss of tendon stress due to elastic shortening, we shall consider the deformations of pre-tensioned member stressed by a tendon at the centroid of the beam (Fig. 7.8). Since the concrete and the tendon are fully bonded, the strain experienced by concrete must equal to that in the prestressing steel. This compatibility of strain can be expressed as:

$$\varepsilon_{ci} = \Delta\varepsilon_s \quad \text{.. (7.6)}$$

where ε_{ci} is the concrete strain and $\Delta\varepsilon_s$ is the reduction in steel strain due to elastic shortening. Applying Hook's law to the previous equation gives:

$$\frac{f_{ci}}{E_{ci}} = \frac{\Delta f_{pe}}{E_p} \quad \text{.. (7.7)}$$

where f_{ci} is the concrete strain at the centroid of the tendons, E_{ci} is the concrete modulus of elasticity at the time of transfer, and Δf_{pe} is the loss in prestressing force as a result of elastic shortening of the beam.

Rearranging, Eq. 7.7 gives:

$$\Delta f_{pe} = \frac{E_p}{E_{ci}} f_{pci} \quad \text{.. (7.8)}$$

If eccentric tendon is used, the eccentricity of the tendon and beam self-weight should be taken into account. The stress in concrete at the level of prestressing steel is given by:

$$f_{ci} = -\frac{P_i}{A} - \frac{P \times e \times e}{I} + \frac{M_{ow} \times e}{I} \quad \text{.................................. (7.9)}$$

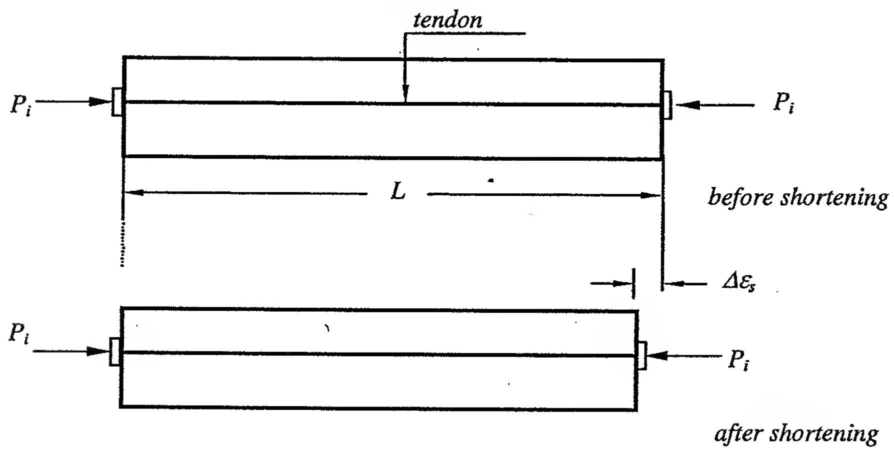


Fig. 7.8 Elastic shortening of a concrete member

For post-tensioned members, the calculations of the elastic shortening losses is more complicated because the losses vary with the greatest losses occurring in the first strand stressed and the least losses occurring in the last strand stressed. For this reason the ECP 203 requires that for post-tensioned members to use only half the value calculated for pre-tensioned members as follows:

$$\Delta f_{pe} = \frac{1}{2}\frac{E_p}{E_{ci}} f_{pci} \quad \text{(for post-tensioned members) .... (7.10)}$$

This type of losses equals to zero if all tendons are jacked simultaneously because the jack that elongates the tendon simultaneously compress the concrete and the elastic shortening takes place before the tendon is anchored.

7.5.4 Wobble Friction Losses (W)

This type of losses exists only in post-tensioned members due to the friction between the tendons and the surrounding ducts. When the ducts are positioned in forms, some degree of misalignment is unavoidable because of workmanship. Actually, it is impossible to have a perfectly straight duct in post-tensioned construction, and the result is friction. Fig. 7.9 shows the misalignment in a duct for a straight tendon. These deviations occur both in elevation and in plan. At each point of contact a normal force, which is proportional to the tendon force, develop between the tendon and the surrounding material. Because of the normal force, frictional forces develop at the point of contact. This type of friction is called the *length effect.*

If the variation of the tension in the cable is neglected and the cable force is taken equal to the tension at the ends of the cable, the loss in force due to friction can be expressed as:

$$P_x = P_o \cdot e^{-kx} \quad \text{.. (7.11)}$$

Where, P_x is the force at a distance x, P_o is the required force to produce P_x at any point x along the tendon profile, x is the distance from the end. k is coefficient of friction between the tendon and the surrounding due to wobble effect. It equals 0.0033 for ordinary cable and equals 0.0017 for fixed ducts.

The wobble losses equal:

$$\Delta f_{pw} = P_o - P_x \quad \text{.. (7.12)}$$

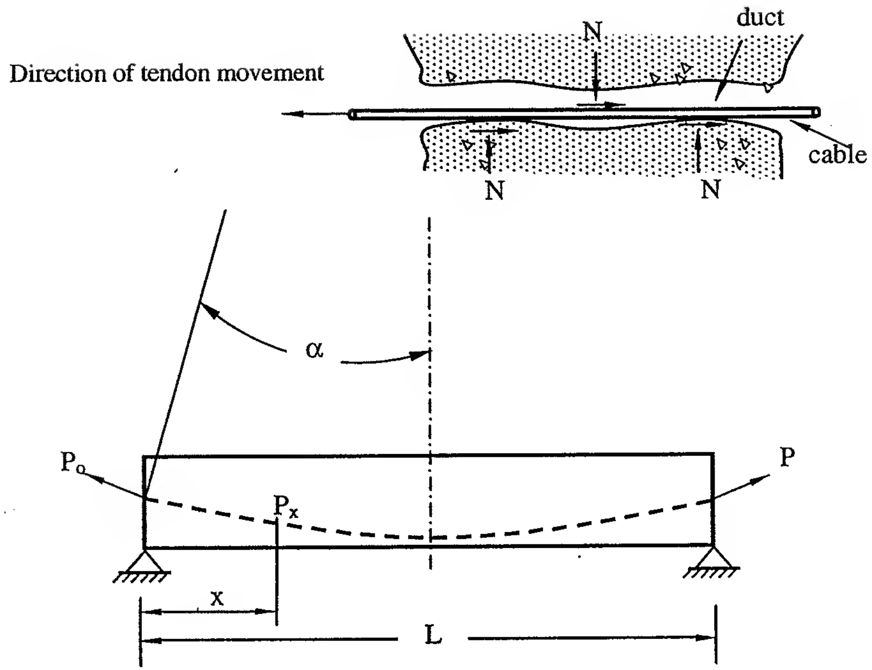


Fig. 7.9 Wobble friction losses

7.5.5 Curvature Friction Losses (F)

This type of losses is also limited to post-tensioned members. The resulting loss is due to the friction between the cables and the duct. These friction losses are a function of the curvature of the tendon axis and the roughness of the surrounding material. As a result, the force in the tendon decreases with the distance from the jack. If a certain force is required at any desired section, the friction force between that section and the jack must be estimated and added to the required force to establish the jacking force. It should be noted that the maximum frictional losses occur at the far end of the beam. Although, friction losses vary along the beam span, such calculation is not usually performed and the maximum value is used.

The ECP 203 gives the following formula to estimate the force at distance x produced by jacking force P_o as follows:

$$P_x = P_o \cdot e^{\left(\frac{-\mu \cdot x}{r_{ps}}\right)} \qquad (7.13)$$

where r_{ps} is the radius of ducts that contain the tendons as shown in Fig. 7.10 and μ is the friction coefficient and can be assumed as:

$\mu = 0.55$	case of friction between steel and concrete
$\mu = 0.30$	case of friction between steel and steel
$\mu = 0.25$	case of friction between steel and lead

It is worth noting that the quantity $(\mu \cdot x / r_{ps})$ represents the losses due to curvature. The code permits the use of a simplified expression for calculating that type of losses if $\left(\mu \cdot x / r_{ps}\right) \le 0.2$. Such an expression is given by:

$$P_x = P_o \left(1 - \frac{\mu \cdot x}{r_{ps}}\right) \qquad (7.14)$$

Thus, the curvature losses equal to:

$$\Delta f_{pf} = P_o - P_x \qquad (7.15)$$

Furthermore, the code permits combining the wobble and the curvature losses in one formula by approximating the logarithmic relation by straight line given by:

$$P_x = P_o \left(1 - \left\{k \cdot x + \frac{\mu \cdot x}{r_{ps}}\right\}\right) \qquad (7.16)$$

With the condition that the looses is less than 20%:

$$\left(k \cdot x + \frac{\mu \cdot x}{r_{ps}}\right) \le 0.20 \qquad (7.17)$$

Thus the total frictional losses in this case equal:

$$\Delta f_{p(w+f)} = P_o \left(k \cdot x + \frac{\mu \cdot x}{r_{ps}}\right) \qquad (7.18)$$

or

$$\Delta f_{p(w+f)} = P_o - P_x \qquad (7.19)$$

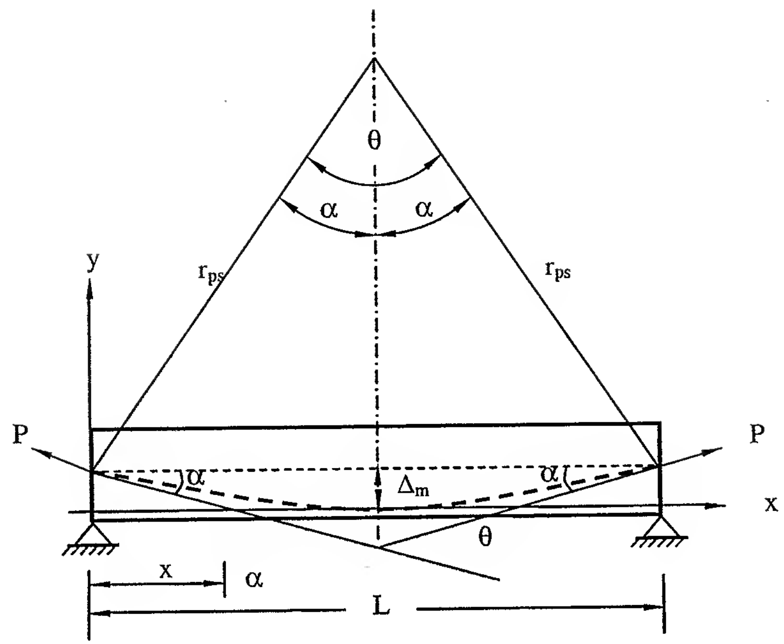


Fig. 7.10 Evaluation of the tendon profile curvature

To evaluate the radius of curvature for a parabolic tendon, the following formula is assumed:

$$y = a\left(x - \frac{L}{2}\right)^2 \quad \text{.......... (7.20)}$$

where a is a constant to be evaluated by applying the boundary conditions at $x = 0 \rightarrow y = \Delta_m$

$$\Delta_m = a\frac{L^2}{4} \quad \text{.......... (7.21)}$$

$$a = \frac{4\,\Delta_m}{L^2} \quad \text{.......... (7.22)}$$

$$y = \frac{4\,\Delta_m}{L^2}\left(x - \frac{L}{2}\right)^2 \quad \text{.......... (7.23)}$$

Differentiating the previous equation gives the angle α at any point.

$$y' = \tan\alpha = \frac{8\,\Delta_m}{L^2}\left(x - \frac{L}{2}\right) \quad \text{.......... (7.24)}$$

Since the ratio of the depth of the beam to its span is small, it is sufficiently accurate to assume that α = tan α and L= length of the arc.

At $x = 0,\ \alpha = -\frac{4\,\Delta_m}{L}$ & at $x = L,\ \alpha = \frac{4\,\Delta_m}{L}$

$$\theta = 2\alpha = \frac{8\,\Delta_m}{L} \quad \text{.......... (7.25)}$$

Assuming the curvature of the tendon is based on that of circular arc, then
$L \approx r_{ps} \cdot \theta$

$$\boxed{r_{ps} \approx \frac{L}{\theta} = \frac{L^2}{8\,\Delta_m}} \quad \text{.......... (7.26)}$$

7.5.6 Shrinkage Losses (sh)

The losses in tendon stress due to shrinkage in a prestressed member depend on many factors. They include the amount of mixing water, the relative humidity, the curing period, and size and shape of the cross section. The shrinkage losses are approximately 7% in pre-tensioned members and 5% in post-tensioned ones.

Approximately 80% of the shrinkage takes place in the first year. The code specifies an average value of the ultimate shrinkage in Code Table 2.8.A according to the size of the member and the relative humidity. In cases where the environmental factors are not known, the following table (7.2) is used

Table 7.2 Values of the shrinkage strain ε_{sh}

Prestressing system	Shrinkage strain
Pre-tensioned members (3-5 days after casting)	300×10^{-6}
Post-tensioned members (7-14 days after casting)	200×10^{-6}

In case of stage construction, the code permits assuming that half the amount of shrinkage occurs in the first month and 75% during the first six months. The shrinkage losses are given by the following equation:

$$\Delta f_{psh} = \varepsilon_{sh} \times E_p \quad (7.27)$$

For post-tension members, the loss in prestressing force due to shrinkage is less than that for pre-tensioned members. The code states that the only amount of shrinkage that needs to be considered is that occurred after transferring the force to the member.

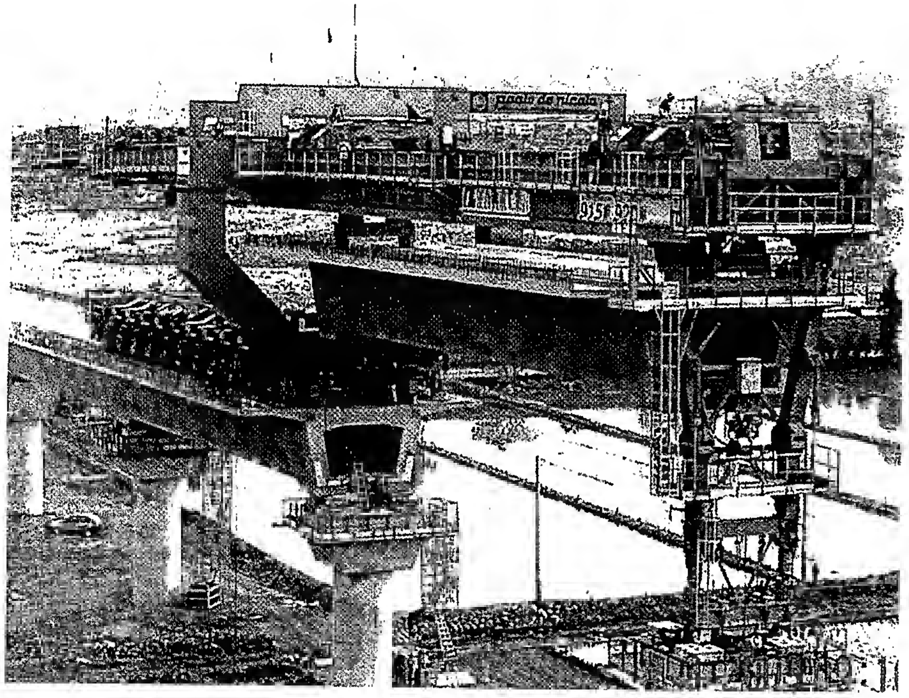


Photo 7.5 Prestressed concrete bridge during construction

7.5.7 Creep Loss (CR)

Experimental research over the years indicates that deformations continue to increase over time. This deformation under constant longitudinal force is termed *creep*. The amount of creep depends on the applied load, duration, properties of concrete, curing conditions, the age of element at first loading, and environmental conditions. It should be emphasized that losses due to creep result only from sustained loads during the loading history of the structural element.

Since the relationship due to creep is linear, it is possible to relate the creep strain ε_{cr} to the elastic strain ε_{el} such that a creep coefficient ϕ can be defined as

$$\phi = \frac{\varepsilon_{cr}}{\varepsilon_{el}} \quad (7.28)$$

The creep strain ε_{cr} can be taken from Table 2-8-b in the code or from Table 7.3 in the absence of environmental factors. The value of the creep coefficient ϕ ranges from 1.5 to 3. The Egyptian code permits the use of ϕ=2.0 for pre-tensioned members and ϕ=1.6 for post-tensioned members.

Table 7.3 Values of creep strain ε_{cr}

Prestressing system	ε_{cr} for every N/mm^2 of the working stress	
	Concrete stress at the time of prestressing f_{ci}	
	$f_{ci} > 40$	$f_{ci} \leq 40$
Pre-tensioned beams (3-5 days after casting)	48 x 10^{-6}	48 x $(40/f_{ci})$ x10^{-6}
Post-tensioned beams (7-14 days after casting)	36 x 10^{-6}	36 x $(40/f_{ci})$ x10^{-6}

If the working concrete stresses at service loads is greater than 1/3 the concrete strength f_{cu}, the creep strain should be increased by the factor α determined from Fig. 7.11. This increased strain value (ε^*_{cr}) is given by the following formula:

$$\varepsilon^*_{cr} = \varepsilon_{cr} \cdot \alpha \quad (7.29)$$

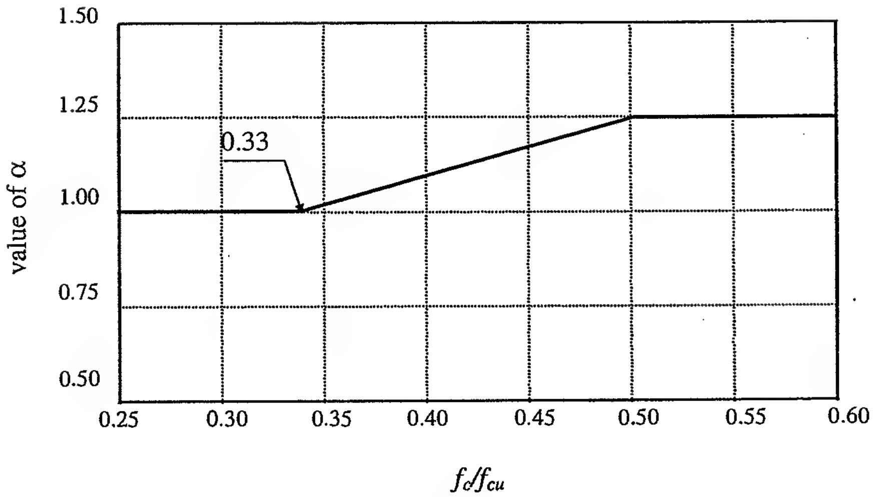


Fig. 7.11 Values of a with respect to concrete stress

Another formula for determining creep losses for bonded prestressed members is given by:

$$\Delta f_{pcr} = \frac{\phi\, E_p}{E_c}\, f_{cs} \quad \text{.. (7.30)}$$

where E_p = the prestressing steel modulus of elasticity.
E_c = the concrete modulus of elasticity.
f_{cs} = the stress in concrete at the level of centroid of the prestressing tendons.

In general, this loss is a function of the stress in the concrete at the section being analyzed. The ECP 203 expression for f_{cs} is;

$$f_{cs} = f^*_{cs} - f^*_{csd} \quad \text{.. (7.31)}$$

where
f^*_{cs} = stress in concrete at level of steel cg immediately after transfer.
f^*_{csd} = stress in concrete at level of steel cg due to all sustained loads applied after prestressing is completed.

Equation 7.30 can be expressed as:

$$\Delta f_{pcr} = \phi\, \frac{E_p}{E_c}\left(f^*_{cs} - f^*_{csd}\right) \quad \text{.............................. (7.32)}$$

7.5.8 Steel Relaxation Losses (R)

Relaxation is defined as the loss of stress under constant strain, while creep is defined as the change in strain under constant stress. This type of losses occurs under constant loading due to the elongation of the tendons with time. A typical relaxation curve showing relaxation losses as a function of time for a specimen that is initially loaded to 70% of its ultimate strength and held at a constant strain, is shown in Fig. 7.12. The loss in stresses due to relaxation depends on the duration and the ratio of initial prestressed f_{pi} to the yield strength f_{py}.

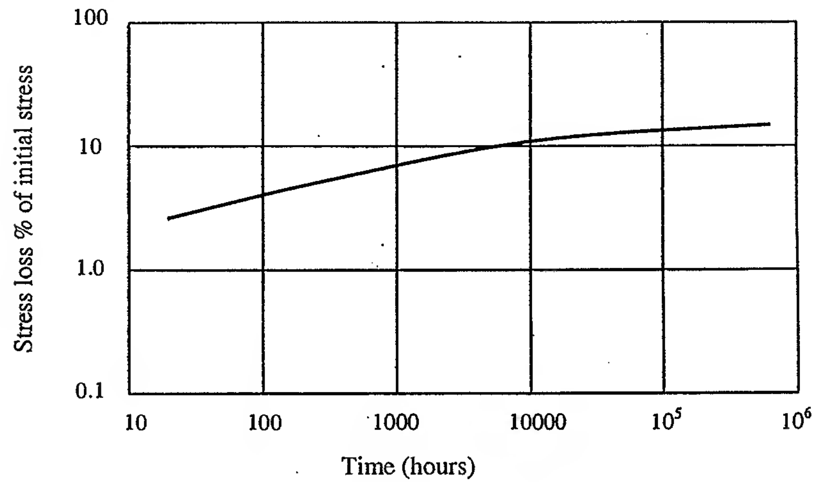


Fig. 7.12 Typical stress relaxation losses .

The ECP 203 gives the following equation to estimate the relaxation losses:

$$\Delta f_{pR} = \frac{f_{pi} \times \log t}{k_1}\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \quad \text{..................... (7.33)}$$

where
Δf_{pr} = steel relaxation losses due to relaxation
f_{pi} = initial prestressing stress before time dependent losses
t = time elapsed since jacking (max 1000 hrs.)
k_1 = coefficient depends on prestressing steel type and is taken:

- 10 for normal relaxation stress relieved strands.
- 45 for low relaxation stress relieved.

This relationship is applicable only when the ratio f_{pi}/f_{py} is greater than 0.55. If a step-by-step loss analysis is necessary, the loss increment at any particular loading stage can be determined from :

$$\Delta f_{pR} = \frac{f_{pi} \times (\log t_2 - \log t_1)}{k_1} \left(\frac{f_{pi}}{f_{py}} - 0.55 \right) \quad \text{.................} \quad (7.34)$$

where t_1 is the time at the beginning, and t_2 is the time at the end of interval from jacking to the time being considered.

Examples 7.1 and 7.2 illustrate the procedure for calculating the losses in pretensioned and post-tensioned beams respectively.

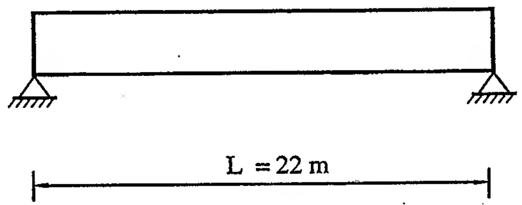
Example 7.1: Calculations of losses for a pre-tensioned beam

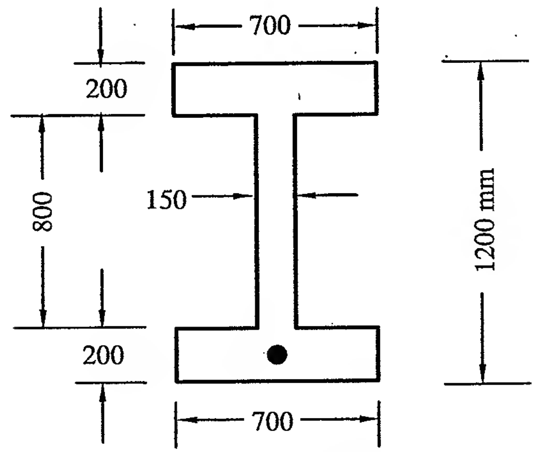
The prestressed beam shown in the figure below is pre-tensioned. Calculate the prestressing losses knowing that the beam is prestressed with normal relaxation stress relieved tendons.

(Note: calculate relaxation losses after 200 days)

Data

f_p = 1360 N/mm^2
A_{ps} = 2097 mm^2
f_{cu} = 40 N/mm^2
f_{cui} = 30 N/mm^2
E_p = 190000 N/mm^2
Cover =100 mm
Unfactored super-imposed load = 4 kN/m′





Beam cross section

Solution

Step 1: Calculation of elastic shortening losses

The cross sectional area (A) equals:

$$A = 2\times700\times200+800\times150 = 400000 \ mm^2$$

Since the section is symmetrical; $y_{top} = y_{bottom}$= 600 mm

$$I = 2\times\left(\frac{700\times200^3}{12}+700\times200\times(600-100)^2\right)+\frac{150\times800^3}{12} = 7.733\times10^{10} \ mm^4$$

The modulus of elasticity at the time of prestressing E_{ci} equals:

$$E_{ci} = 4400\sqrt{f_{cui}} = 4400\sqrt{30} = 24099.8 \ N/mm^2$$

$$P = f_p \times A_{ps} = 1360\times2097/1000 = 2852 \ kN$$

The concrete stress at the level of the prestressing tendons is given by:

$$f_{pci} = -\frac{P}{A}-\frac{P\times e\times e}{I}+\frac{M_{ow}\times e}{I}$$

$$f_{pci} = -\frac{2852\times1000}{400000}-\frac{2852\times1000\times500\times500}{7.73\times10^{10}}+\frac{605\times10^6\times500}{7.73\times10^{10}} = -12.44 \ \text{N/mm}^2$$

The loss of prestressing force due to elastic shortening is given by:

$$\Delta f_{pe} = \frac{E_p}{E_{ci}} f_{pci} = \frac{190000}{24099.8}12.44 = 98.08 \ N/mm^2$$

Step 2: Calculation of creep losses

The modulus of elasticity of concrete at full strength equals:

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{40} = 27828 \ N/mm^2$$

The moment due to the superimposed dead load equals:

$$M_{sd} = \frac{w_{sd}L^2}{8} = \frac{4\times22^2}{8} = 242 \ kN.m$$

The additional stress due to the superimposed dead load is given by:

$$f^*_{csd} = \frac{M_{sd}\times e}{I} = \frac{242\times10^6\times500}{7.73\times10^{10}} = 1.56 \ \text{N/mm}^2$$

The initial prestressing stress f_{pi} equals to the prestressing stress after the occurrence of immediate losses.

$$f_{pi} = f_p - \Delta f_{pe} = 1360-98.06 = 1261.94 \ N/mm^2$$

$$P_i = f_{pi}\times A_{ps} = \frac{1261.9\times2097}{1000} = 2646.4 \ KN$$

$$f_{pci} = -\frac{2646.4\times1000}{400000}-\frac{2646.4\times1000\times500\times500}{7.73\times10^{10}}+\frac{605\times10^6\times500}{7.73\times10^{10}}$$

$$f_{pci} = -11.26 \ \text{N/mm}^2$$

For pre-tensioned beams, ϕ=2.0. Hence, the loss of the prestressing force due to creep is given by:

$$\Delta f_{pcr} = \phi\frac{E_p}{E_c}\left(f^*_{cs}-f^*_{csd}\right)$$

$$= 2.0\frac{190000}{27828}(11.26-1.56) = 132.46 \ \text{N/mm}^2$$

Step 3: Calculation of shrinkage losses

For pre-tensioned beams, the shrinkage strain ε_{sh} is equal to $300\text{x}10^{-6}$.

$$\Delta f_{psh} = \varepsilon_{sh}\times E_p = 300\times10^{-6}\times190000 = 57 \ \text{N/mm}^2$$

Step 4: Calculation of steel relaxation losses

K_1=10 $\rightarrow$ stress relieved tendons

Time (t) = 200 day = 200 x 24 = 4800 hours

The Egyptian Code ECP 203 requires calculating the relaxation losses at a time of not more than 1000 hours. Hence, assume t = 1000 hour.

$$\Delta f_{pr} = \frac{f_{pi}\times\log(t)}{K_1}\left(\frac{f_{pi}}{f_{py}}-0.55\right) = \frac{1261.9\times\log(1000)}{10}\left(\frac{1261.9}{1700}-0.55\right)$$

$$\Delta f_{pr} = 72.8 \ N/mm^2$$

The total losses can be summarized in the following table

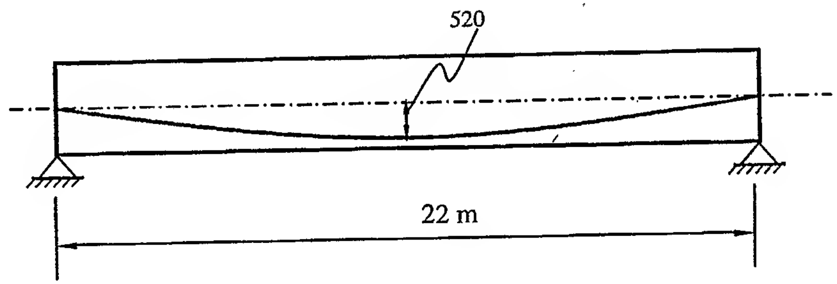
Type of loss	**Stress (N/mm^2)**	**Percent of total losses**
Elastic shortening losses	98.08	27.22%
Shrinkage losses	132.46	36.76%
Creep losses	57.00	15.82%
Relaxation losses	72.80	20.20%
Total losses	360.34	100.00%
Stress before losses	1360.00	
Stress after losses	999.66	
Losses (%)	26.5%	

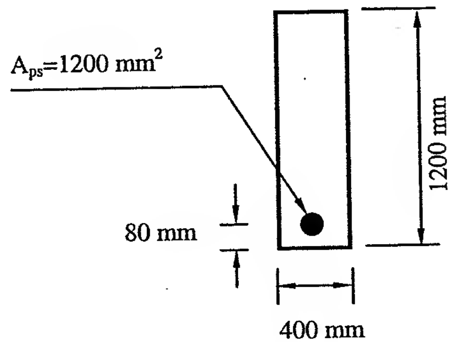
Example 7.2: Step by step computation of losses in post-tensioned beam

A simply supported post-tensioned beam is shown in the figure below. The area of the prestressing tendons is 1200 mm^2 and f_{pu}=1900 N/mm^2. Compute the prestressing losses at the critical section of the beam at 100 days knowing that normal stress relieved strands are used inside steel ducts.

Data

w_{DL} = 3.5 kN/m′
f_{cu} = 35 N/mm^2
f_{cui} = 26.25 N/mm^2
E_p = 193000 N/mm^2
Anchorage slip = 4 mm





Solution

Step 1: Calculate section properties

$A = 400 \times 1200 = 480000\ mm^2$

$$I = \frac{b \times t^3}{12} = \frac{400 \times 1200^3}{12} = 5.76 \times 10^{10}\ mm^4$$

$$Z_{bot} = Z_{top} = \frac{I}{y_{bot}} = \frac{5.76 \times 10^{10}}{600} = 96 \times 10^6\ mm^3$$

$e = 600 - 80 = 520\ mm$

$$w_{o.w} = \gamma_c \times A = 25 \times \frac{480000}{1000000} = 12\,kN/m'$$

For normal relaxation stress relieved stands, the ECP 203 specifies the yield stress as:

$f_{py} = 0.85 \times f_{pu} = 0.85 \times 1900 = 1615$ N/mm^2

The initial prestressing force at the time of jacking equals:

$$f_{pi} = smaller\ of \begin{cases} 0.70\ f_{pu} = 0.7 \times 1900 = 1330\ \text{N/mm}^2 \\ 0.80\ f_{py} = 0.8 \times 1615 = 1292\ \text{N/mm}^2 \end{cases}$$

f_{pi}=1292 N/mm^2

Step 2: Anchorage slip Losses

The strain due to anchorage slip is $\varepsilon_A = \frac{\Delta_A}{L} = \frac{4}{22 \times 1000} = 1.818 \times 10^{-4}$

The loss in stresses due to anchorage slip is given by:

$\Delta f_{pA} = \varepsilon_A \times E_{ps} = 1.818 \times 10^{-4} \times 193000 = 35.09\ N/mm^2$

Step 3: Wobble friction losses

The code specifies k=0.0033 for normal conditions, thus the stress at the end of the beam equals:

$P_{x1} = P_o \cdot e^{-kx} = 1292.e^{(-0.0033 \times 22)} = 1201.5\ N/mm^2$

Losses = 1292-1201.525 = 90.47 N/mm^2

Step 4: Curvature friction losses

The radius of curvature r_{ps} can be approximated by:

$$r_{ps} \approx \frac{L^2}{8 \times e} = \frac{22^2}{8 \times (520/1000)} = 116.35\ m$$

The stress after frictional losses equals:

$$P_{x2} = P_o \cdot e^{\left(-\frac{\mu x}{r_{ps}}\right)} = 1292 e^{\left(-\frac{0.3 \times 22}{116.35}\right)} = 1220.75\ N/mm^2$$

Losses = 1292-1220.75 =71.25 N/mm^2

Total frictional losses = Δf_{pf} = 90.47+71.25 =161.73 N/mm^2

Net force = P_x= 1292-161.73 =1095.18 N/mm^2

Alternative method

The quantity $\alpha = \left(k \cdot x + \frac{\mu \cdot x}{r_{ps}}\right) = \left(0.0033 \times 22 + \frac{0.30 \times 22}{116.35}\right) = 0.1293$

Since this quantity α is less than 0.2, the net force after the frictional losses is

$P_x = P_o(1 - \alpha) = 1292(1 - 0.1293) = 1124.9$ N/mm^2

Step 5: Elastic shortening losses

The modulus of elasticity at the time of prestressing E_{ci} equals:

$E_{ci} = 4400\sqrt{f_{cui}} = 4400\ \sqrt{26.25} = 22543.3 N/mm^2$

At transfer, the beam self weight is the only acting moment that equals:

$$M_{ow} = \frac{w_{ow} \times L^2}{8} = \frac{12 \times 22^2}{8} = 726\ kN.m$$

The tTotal losses including (anchorage + Wobble + friction losses) equals:

=35.09+161.73=196.82 N/mm^2

Net prestressing stress =1292-196.82 =1095.18 N/mm^2

$$P_i = f_{pi} \times A_{ps} = \frac{1095.18 \times 1200}{1000} = 1314.22\ kN$$

The concrete stress at the level of the prestressing tendons is given by:

$$f_{pci} = -\frac{P_i}{A} - \frac{P_i \times e \times e}{I} + \frac{M_{ow} \times e}{I}$$

$$f_{pci} = -\frac{1314.22 \times 1000}{480000} - \frac{1314.22 \times 1000 \times 520 \times 520}{5.76 \times 10^{10}} + \frac{726 \times 10^6 \times 520}{5.76 \times 10^{10}}$$

$$f_{pci} = -2.35 \ \text{N/mm}^2$$

The loss of prestressing force due to elastic shortening is given by:

$$\Delta f_{pe} = \frac{E_p}{E_{ci}} f_{pci} = \frac{193000}{22543.29} 2.35 = 20.15 \ N/mm^2$$

$$f_{pi} = 1095.18 - 20.15 = 1075.03 \ N/mm^2$$

Step 6: Calculation of creep losses

$$P_i = f_{pi} \times A_{ps} = \frac{1075.03 \times 1200}{1000} = 1290.04 \ kN$$

The modulus of elasticity of concrete at full strength:

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{35} = 26030.7 \ N/mm^2$$

The moment due to the superimposed dead loads equals:

$$M_{sd} = \frac{w_{sd} L^2}{8} = \frac{3.5 \times 22^2}{8} = 211.75 \ kN.m$$

The additional stress due to this load is given by:

$$f^*_{csd} = \frac{M_{sd} \times e}{I} = \frac{211.75 \times 10^6 \times 520}{5.76 \times 10^{10}} = 1.91 \ \text{N/mm}^2$$

$$f^*_{cs} = -\frac{1290.04 \times 1000}{480000} - \frac{1290.04 \times 1000 \times 520^2}{5.76 \times 10^{10}} + \frac{726 \times 10^6 \times 520}{5.76 \times 10^{10}} = -2.19 \ \text{N/mm}^2$$

For post-tensioned beams, ϕ=1.6. The creep loss equals:

$$\Delta f_{pcr} = \phi \frac{E_p}{E_c}\left(f^*_{cs} - f^*_{csd}\right) = 1.6 \frac{193000}{26030.7}(2.19 - 1.91) = 3.30 \ \text{N/mm}^2$$

Step 7: Calculation of shrinkage losses

For post-tensioned beams, the shrinkage strain ε_{sh} is 200x10^{-6}.

$$\Delta f_{psh} = \varepsilon_{sh} \times E_p = 200 \times 10^{-6} \times 193000 = 38.6 \ \text{N/mm}^2$$

Step 4: Calculation of steel relaxation losses

K_1=10 for normal stress relieved tendons & t= 100 day = 100 x 24 =2400 hours.

The code requires calculating the relaxation losses at a time not exceeding 1000 hours. Thus, t = 1000 hour.

$$\Delta f_{pr} = \frac{f_{pi} \times \log(t)}{K_1}\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

$$\Delta f_{pr} = \frac{1075.03 \times \log(1000)}{10}\left(\frac{1075.03}{1615} - 0.55\right) = 37.30 \ N/mm^2$$

The itemized losses can be summarized in the following table

Type	**Stress (N/mm^2)**	**% of total losses**
Anchorage slip losses	35.09	11.8%
Wobble friction losses	90.48	30.5%
Curvature friction losses	71.25	24.1%
Elastic shortening	20.15	6.8%
Shrinkage losses	38.60	13.0%
Creep losses	3.30	1.1%
Relaxation losses	37.30	12.6%
Total immediate losses	216.97	73.3%
Total time dependent losses	79.20	26.7%
Total losses	296.16	100.0%
Stress before losses	1292	
Stress after losses	995.84	
Losses (%)	22.92%	

7.6 Anchorage Zones

7.6.1 Introduction

In prestressed concrete structural members, the prestressing force is usually transferred from the prestressing steel to the concrete in one of two different ways. *In post-tensioned construction*, relatively small anchorage plates transfer the force from the tendon to the concrete immediately behind the anchorage by bearing. *In pre-tensioned members*, the force is transferred by bond between the steel and the concrete. In either case, the transfer of the prestressing force occurs at the end of the member and involves high local pressures and forces.

The length of the member over which the concentrated prestressing force changes into a uniformly distributed over the cross section is called the transfer length (in the case of pre-tensioned members) and the anchorage length (for post-tensioned members). The stress concentrations within the anchorage zone in a pre-tensioned member are not usually as sever as in a post-tensioned anchorage zone. In pre-tensioned beams, there is a more gradual transfer of prestressing. The prestressing force is transmitted by bond over a significant length of the tendon and there are usually a number of tendons that are well distributed throughout the anchorage zone. In addition, the high concrete bearing stresses behind the anchorage plates in post-tensioned members do not occur in pre-tensioned construction. Only post-tensioned concrete anchorage zone are given attention in design and will be treated in details in this text.

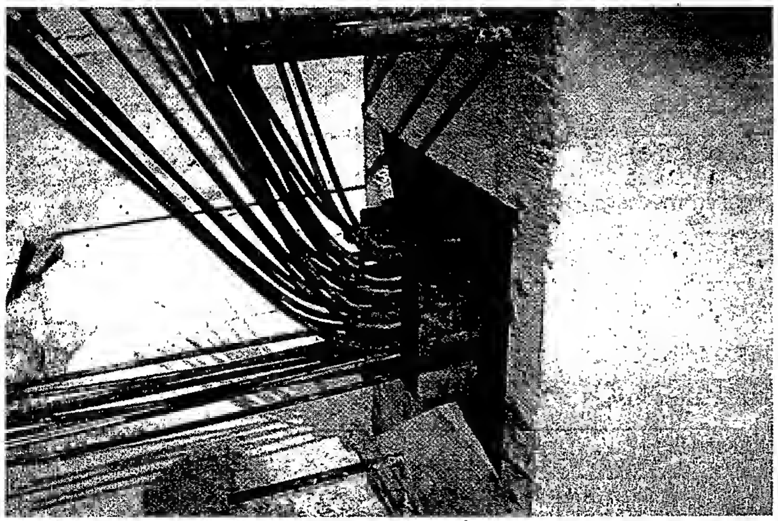


Photo 7.6 Anchorage zone of a prestressed concrete beam

7.6.2 Stress Distribution

In post-tensioned concrete structures, failure of the anchorage zone is perhaps the most common cause of problems arising during construction. Such failures are difficult and expensive to repair and might necessitate replacement of the entire member.

Anchorage zones may fail due to uncontrolled cracking or splitting of the concrete from insufficient transverse reinforcement. Bearing failures immediately behind the anchorage plates are also common and may be caused because of the inadequate dimensions of the bearing plates or poor quality of concrete.

Consider the case shown in Fig. 7.13 of a single square plate centrally positioned at the end of a member of depth t and width b. In the region of length L_a immediately behind the anchorage plate (i.e. the anchorage zone), plane sections do not remain plane and beam theory does not apply. High bearing stresses at the anchorage plate disappear throughout the anchorage zone, creating high transverse stresses. The spreading of stress that occurs within the anchorage zone is illustrated in Fig. 7.13. The stress trajectories are closely spaced directly behind the bearing plate where the compressive stresses are high, and become more widely spaced as the distance from the anchorage plate increases. In order to enhance the compressive strength of concrete, spiral reinforcement is usually provided as shown in Fig. 7.14. The confinement of concrete due to the spiral reinforcement enhances its strength and ductility.

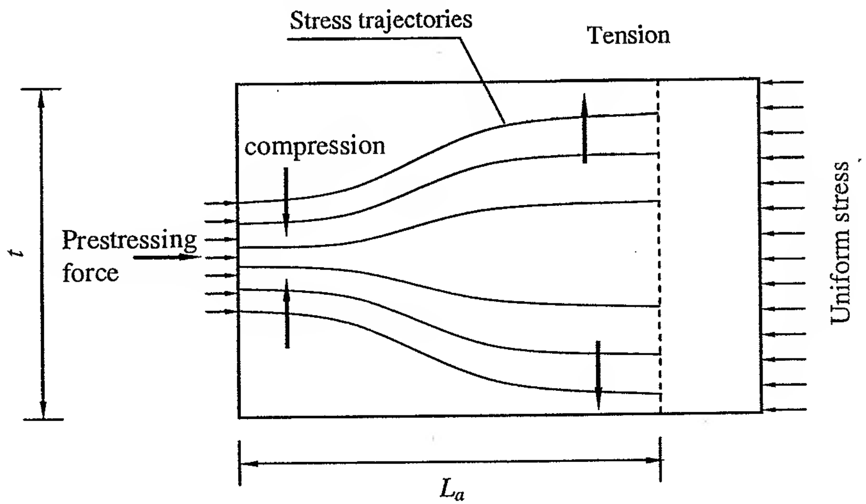


Fig. 7.13 Idealized stress paths in end block with single load

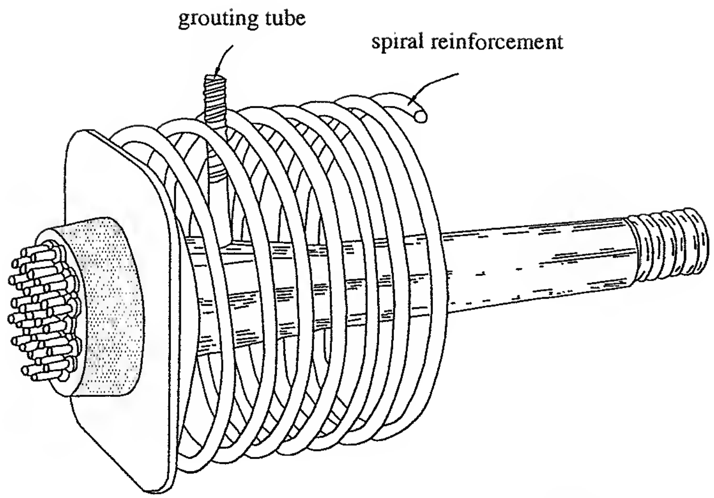


Fig. 7.14 Spiral reinforcement at the anchorage zone

Steel duct protecting tendons

Before casting the concrete

After casting the concrete and removing the duct

Photo 7.7 Anchorage zone in a bridge deck before and after casting the concrete

St Venant's principle suggests the length of the disturbed region for the single centrally located anchorage is approximately equal to the thickness of the member *t*. The high compressive stresses vanish after a short distance and tensile stresses form as shown in Fig. 7.15. The transverse tensile forces (often called bursting or splitting forces) need to be estimated accurately so that transverse reinforcement within the anchorage zone can be designed to resist them.

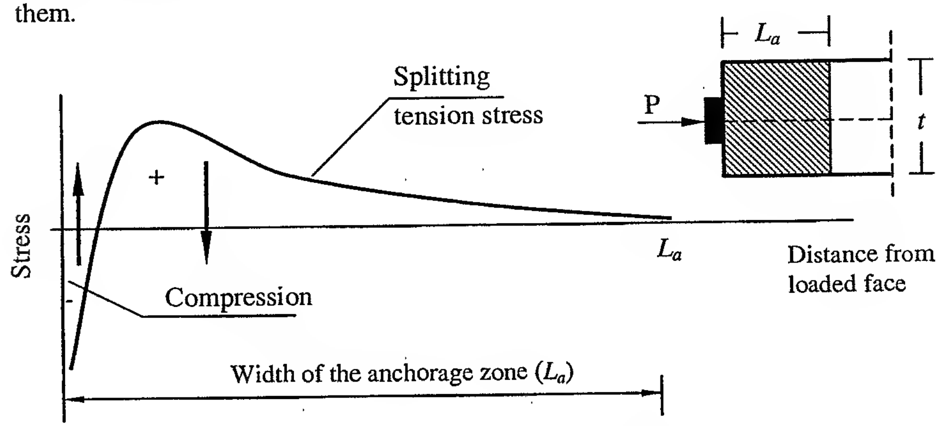


Fig. 7.15 Stress distribution at the center line of an anchorage zone

On the other hand, the stress trajectories for an eccentrically loaded member are not equally spaced as shown in Fig. 7.16. The length of the disturbed zone L_a is approximately equal to twice the distance of the prestressing force to the edge. High bursting tensile stresses developed along the axis of the plate. Moreover, end tensile stresses develop at the edge above the bearing plate. These tensile stresses called the spalling stresses and are usually exist in eccentrically loaded end zone.

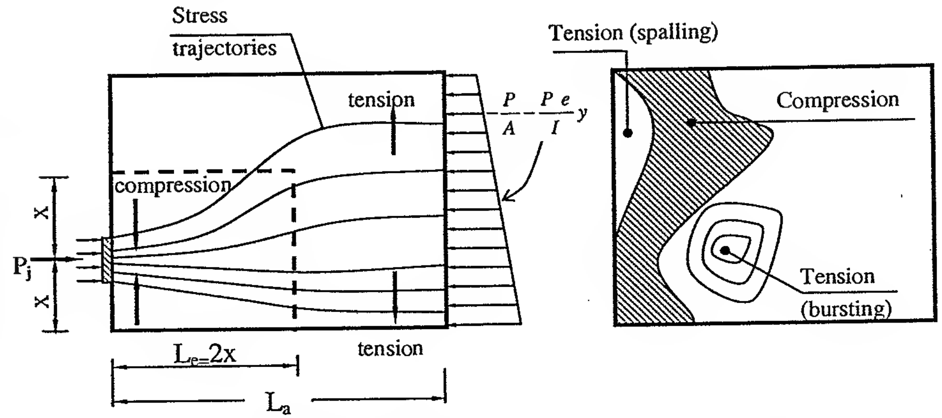


Fig. 7.16 Stress contours for an eccentric loading

7.6.3 Methods of Analysis

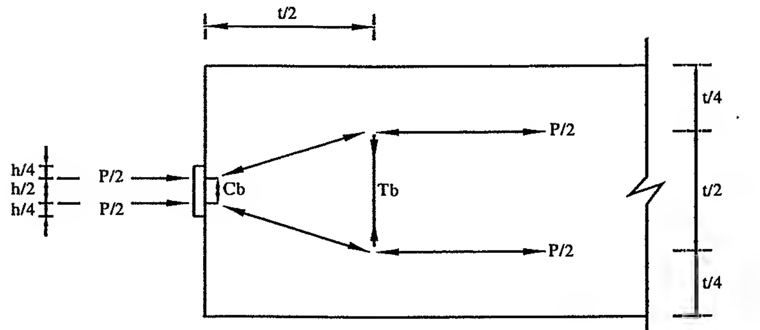
The design of the anchorage zone for a post-tensioned member involves both the arrangement of the anchorage plates, to minimize transverse stresses, and the determination of the amount and distribution of reinforcement to carry the transverse tension after cracking of the concrete. The ECP 203 states that the anchorage zone should be designed to withstand a force equals to 1.2 the jacking force.

The spreading of the prestressing forces occurs through both the depth and the width of the anchorage zone and therefore transverse reinforcement must be provided within the end zone in two orthogonal directions. The reinforcement quantities required in each direction are obtained from separate two-dimensional analyses, i.e., the vertical transverse tension is calculated by considering the vertical spreading of forces and the horizontal tension is obtained by considering the horizontal spreading of forces. The methods of analysis are:

1. Strut-and-Tie method
2. Beam analogy
3. Finite element method

7.6.3.1 Strut-and-Tie Method

The internal flow of forces in each direction can be visualized in several ways. A simple model is to consider truss action within the anchorage zone. For the anchorage zone of the rectangular beam shown in Fig. 7.17, the truss analogy shows that transverse compression exists directly behind the bearing plate, with transverse tension, often called the bursting force at some distance along the member. The truss analogy can be used in T-beams for calculating both the vertical tension in the web and the horizontal tension across the flange.



$$T_b = 0.25 \times P\left(1-\frac{h}{t}\right) \cong 0.25P$$

Fig. 7.17 Strut and Tie model for an anchorage zone

7.6.3.2 Beam Analogy

An alternative model for estimating the internal tensile forces in the anchorage zone is to consider it as a deep beam loaded from one side by the bearing stresses immediately under the anchorage plate and resisted on the other side by the statically equivalent, linearly distributed stresses in the beam. The depth of the deep beam is taken as the anchorage length L_a.

The beam analogy model is illustrated in Fig. 7.18 for a single central anchorage, together with the bending moment diagram for the idealized beam. Since the maximum moment tends to cause bursting along the axis of the anchorage, it is usually denoted by M_b and called the bursting moment.
By considering the free-body diagram of one-half of the end block, the bursting moment M_b required for the rotational equilibrium is obtained from statics. Referring to Fig. 7.18 and taking moment about any point on the member axis, one gets:

$$M_b = \frac{P}{2}\left(\frac{t}{4}-\frac{h}{4}\right) = \frac{P}{8}(t-h) \qquad (7.35)$$

The lever arm between C_b and T_b is approximately equal to t/2. Hence,

$$T_b \approx \frac{M_b}{t/2} = \frac{P}{4}\left(1-\frac{h}{t}\right) \qquad (7.36)$$

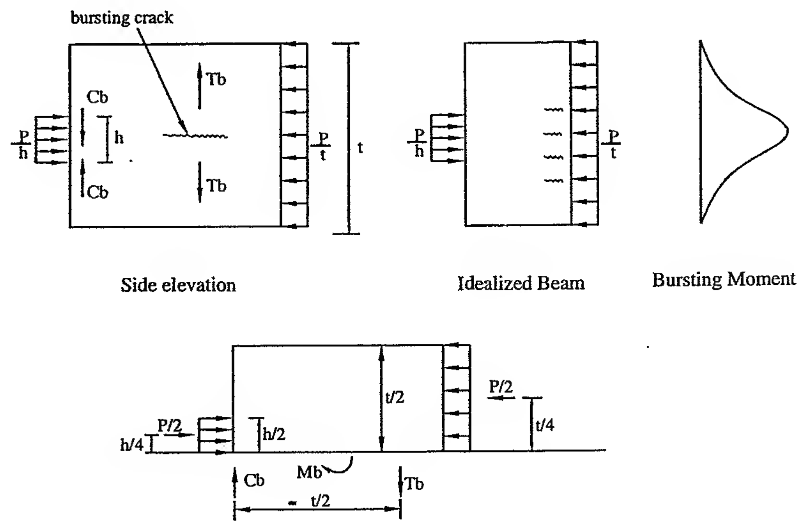


Fig. 7.18 Bursting moment at the end zone

Expressions for the bursting moment and the horizontal transverse tension resulting from the lateral dispersion of the bearing stresses across the width b are obtained by replacing the thickness t in Eqs. 7.35 and 7.36 with the width b.

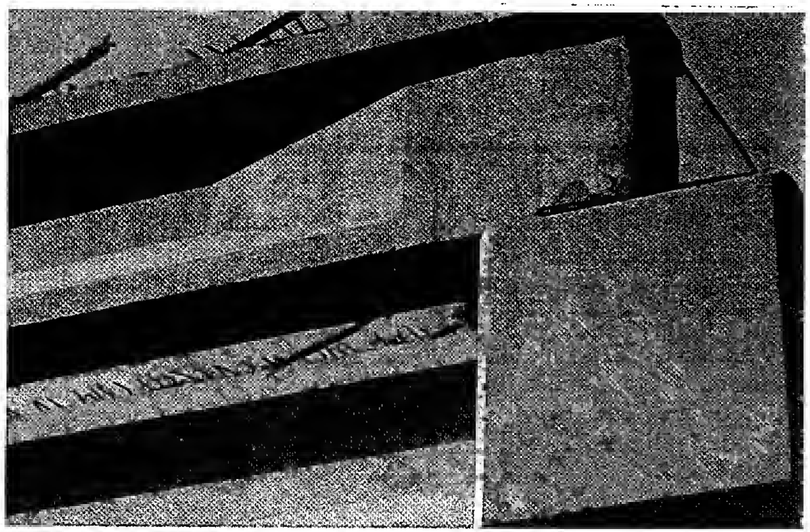


Photo 7.8 Increasing beam width at the anchorage zone area

7.6.3.3 Finite Element Method

Computer programs are commonly used to analyze the anchorage zone. These programs are based on the finite element method. The anchorage zone is modeled using the shell element and the external prestressing is applied through a series of concentrated loads. Typical output is shown in Fig. 7.19.

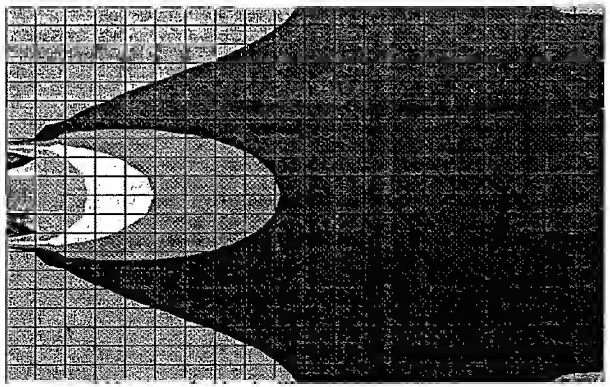


Fig. 7.19 Stress distribution in the anchorage zone

Example 7.3

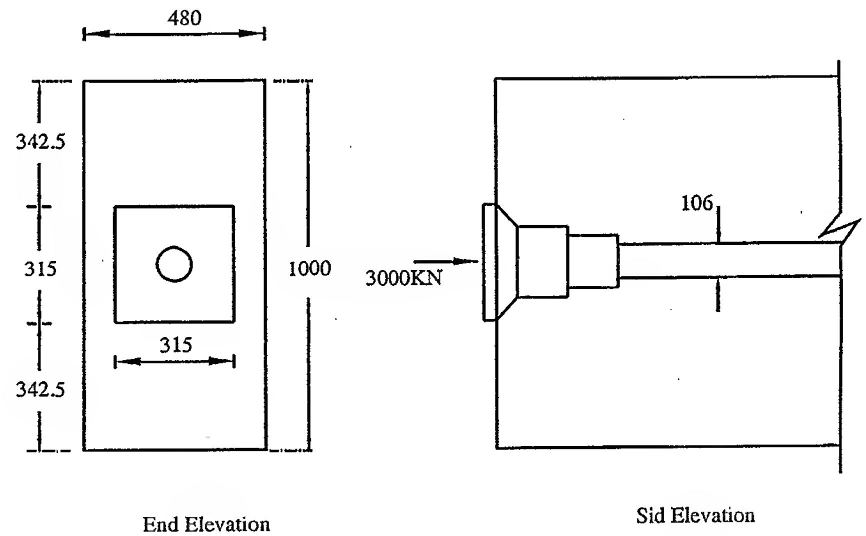
The figure given below shows the anchorage zone of a flexural member. The square bearing plate is 315 mm x 315 mm with a duct diameter of 106 mm. It is to design such an anchorage zone according to the beam theory

Data

P_j = 3000 kN

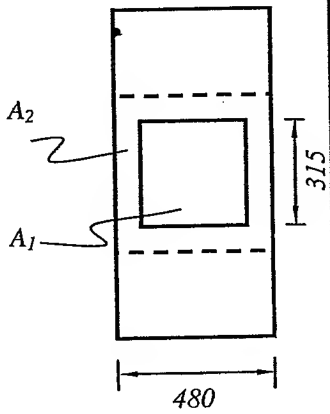
f_{cu} = 60 N/mm^2

f_y = 280 N/mm^2



Solution

Step 1: Check of bearing



$$A_1 = 315\times315-\frac{\pi}{4}\times106^2 = 90400\ mm^2$$

$$A_2 = 480\times480 = 230400\ mm^2$$

The bearing stresses equal:

$$f_b = \frac{1.2\times P_j}{A_1} = \frac{1.2\times3000\times1000}{90400} = 39.82\ N/mm^2$$

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{230400}{90400}} = 1.59 < 2.0 \rightarrow ok$$

$$f_{bcu} = 0.67\ \frac{f_{cu}}{1.5}\sqrt{\frac{A_2}{A_1}} = 0.67\ \frac{60}{1.5}\times1.59 = 42.8\ N/mm^2 < 39.8\ .....ok$$

Step 2: Design of transverse reinforcement

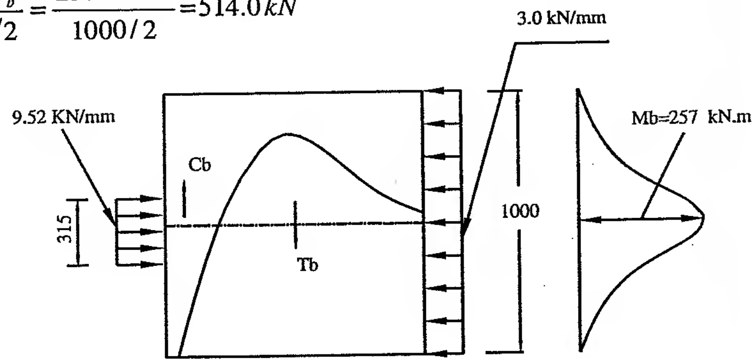
Step 2.1: Vertical plane

Consider moments in the vertical plane (vertical bursting tension).

The forces and bursting moments in the vertical plane are given by:

$$M_b = \frac{P}{8}(t-h) = \frac{3000}{8}(1000-315)\times10^{-3} = 257.0\ kN.m$$

$$T_b = \frac{M_b}{t/2} = \frac{257.0\times10^3}{1000/2} = 514.0\,kN$$



Bursting in the vertical plane

The allowable stress for f_y =280 N/mm^2 is 160 N/mm^2. The amount of vertical transverse reinforcement equals:

$$A_{sb} = \frac{T_b}{f_s} = \frac{514.0\times10^3}{160} = 3211\ mm^2$$

This area of transverse steel must be provided within the length of the beam located from (0.2 t) to (1.0 t), i.e. (=800 mm) from the loaded end face. Two 12 mm diameter stirrups (four vertical legs) are to be provided.

$$A_{sb} = 4\times112 = 448\ mm^2$$

The number of stirrups $n = \dfrac{3211}{448} = 7.16$

The spacing between stirrups $s = \dfrac{800}{7.16} = 111\ mm \rightarrow use\ 100\,mm$

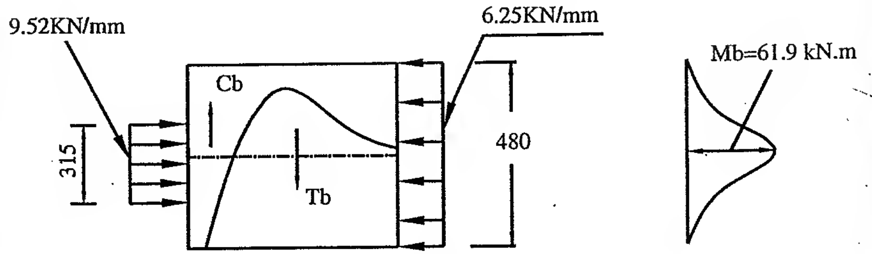
Use Two 12 mm @100 mm

Step 2.2: Horizontal plane

Now, consider the moments in the horizontal plane (horizontal bursting tension). The forces and bursting moments in the horizontal plane are obtained by replacing t with b = 480 mm. The bursting moment and horizontal tension are:

$$M_b = \frac{P}{8}(b-h) = \frac{3000}{8}(480-315)\times10^{-3} = 61.9\ kN.m$$

$$T_b = \frac{M_b}{b/2} = \frac{61.9\times10^3}{480/2} = 258.0\ kN$$



Bursting in the horizontal plane

The amount of horizontal transverse reinforcement equals:

$$A_{sb} = \frac{T_b}{f_s} = \frac{258.0\times10^3}{160} = 1611\ mm^2$$

Such an amount is required within the length of the beam located between 96 mm (0.2 t) and 480 mm (1.0 t) from the loaded face. Try 2-12 mm stirrups.

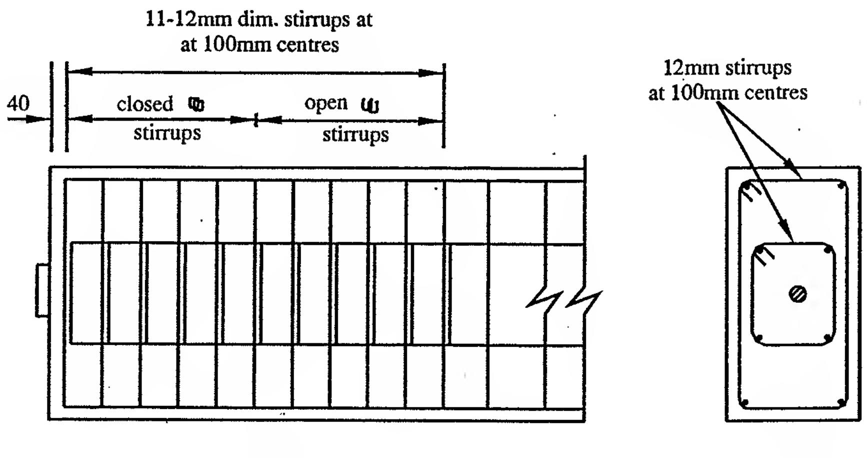
$A_{sb} = 4\times112 = 448\ mm^2$

The number of stirrups $n = \dfrac{1611}{448} = 3.59$

The spacing between stirrups $s = \dfrac{0.8\times480}{3.59} = 106\ mm \rightarrow use\ 100\,mm$

Use Two 12 mm @100 mm

Four pairs of closed 12 mm stirrups (i.e. four horizontal legs per pair of stirrups) at 100 mm centers ($A_{sb} = 1760\,mm^2$) are provided. To satisfy horizontal bursting requirements, this size and spacing of stirrups should be provided from the loaded face for a length of at least 480 mm.



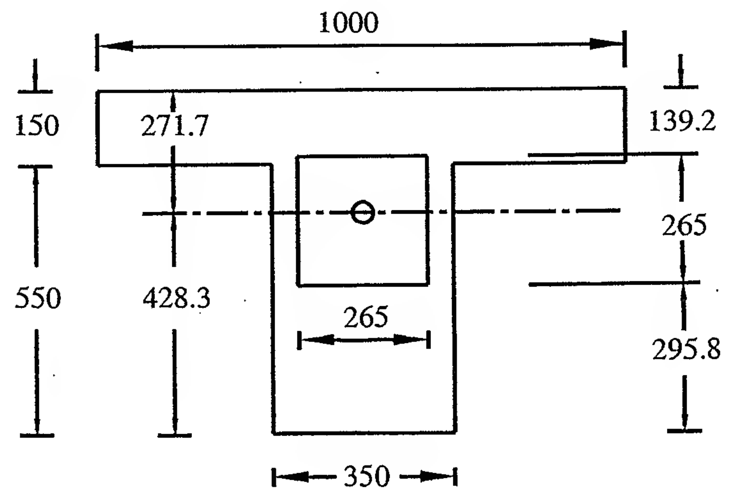
Elevation

Section

Example 7.4

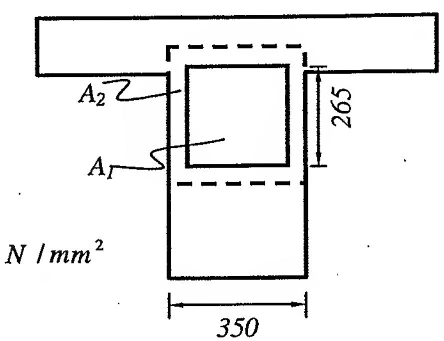
The figure given below shows the anchorage zone of a T-beam. The jacking force equals 2000 kN, f_y =280 N/mm^2, and the concrete strength is 60 N/mm^2. Design the anchorage zone using:

- Beam analogy
- Strut-and-Tie method



Solution

Step 1: Check of bearing



$A_1 = 265\times265 = 70225\ mm^2$

$A_2 = 350\times350 = 122500\ mm^2$

The design load is 1.2 P_j.

$$f_b = \frac{1.2\ P_j}{A_1} = \frac{1.2\times2000\times1000}{70225} = 34.17\ N/mm^2$$

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{122500}{70225}} = 1.32 < 2.0 \rightarrow ok$$

$$f_{bcu} = 0.67 \frac{f_{cu}}{1.5}\sqrt{\frac{A_2}{A_1}} = 0.67 \frac{60}{1.5} \times 1.32 = 35.4\ N/mm^2 < 34.17\ ....ok$$

Step 2: Design of transverse reinforcement (using beam method)

Step 2.1: Vertical plane

$$f_1 = \frac{2000}{265} = 7.547\ kN/mm$$

$$A = 1000 \times 150 + 350 \times 550 = 342500\ mm^2$$

$$f_{avg} = \frac{P_j}{A} = \frac{2000}{342500} = 0.005839\ kN/mm^2$$

$$f_{avg} = 5.839 N/mm^2 < \frac{0.56 f_{ci}}{\gamma_c} < \frac{0.56(0.75 \times 60)}{1.5} < 16.8\ N/mm^2 ....o.k$$

$f_{flange} = f_{avg} \times B = 0.005839 \times 1000 = 5.84 kN/mm$ (Refer to Fig. EX. 7.4.1b.)

$$f_{web} = f_{avg} \times b = 0.005839 \times 350 = 2.044 kN/mm$$

The distributed region distance L_e equals twice the distance of the prestressing zone to the top edge. $L_e = 2 \times 271.7 = 543 mm$

Referring to Fig. EX 7.4.1d, the maximum moment occurs at point of zero shear (x). Such a point is obtained as follows:

$$2.044x = 7.547(x - 295.8) \rightarrow x = 405.7\ mm$$

Taking moment about point (o) gives the maximum moment M_b:

$$M_b = \left(2.044 \times 405.7^2/2 - 7.547 \times (405.7 - 295.8)^2/2\right)/1000 = 122.6\ kN.m$$

$$T_b = \frac{M_b}{L_e/2} = \frac{122.6 \times 1000}{543/2} = 451.7\ kN$$

The allowable stress for f_y =280 N/mm^2 is 160 N/mm^2. The amount of vertical transverse reinforcement in the web equals:

$$A_{sb} = \frac{T_b}{f_s} = \frac{451.7 \times 1000}{160} = 2823\ mm^2$$

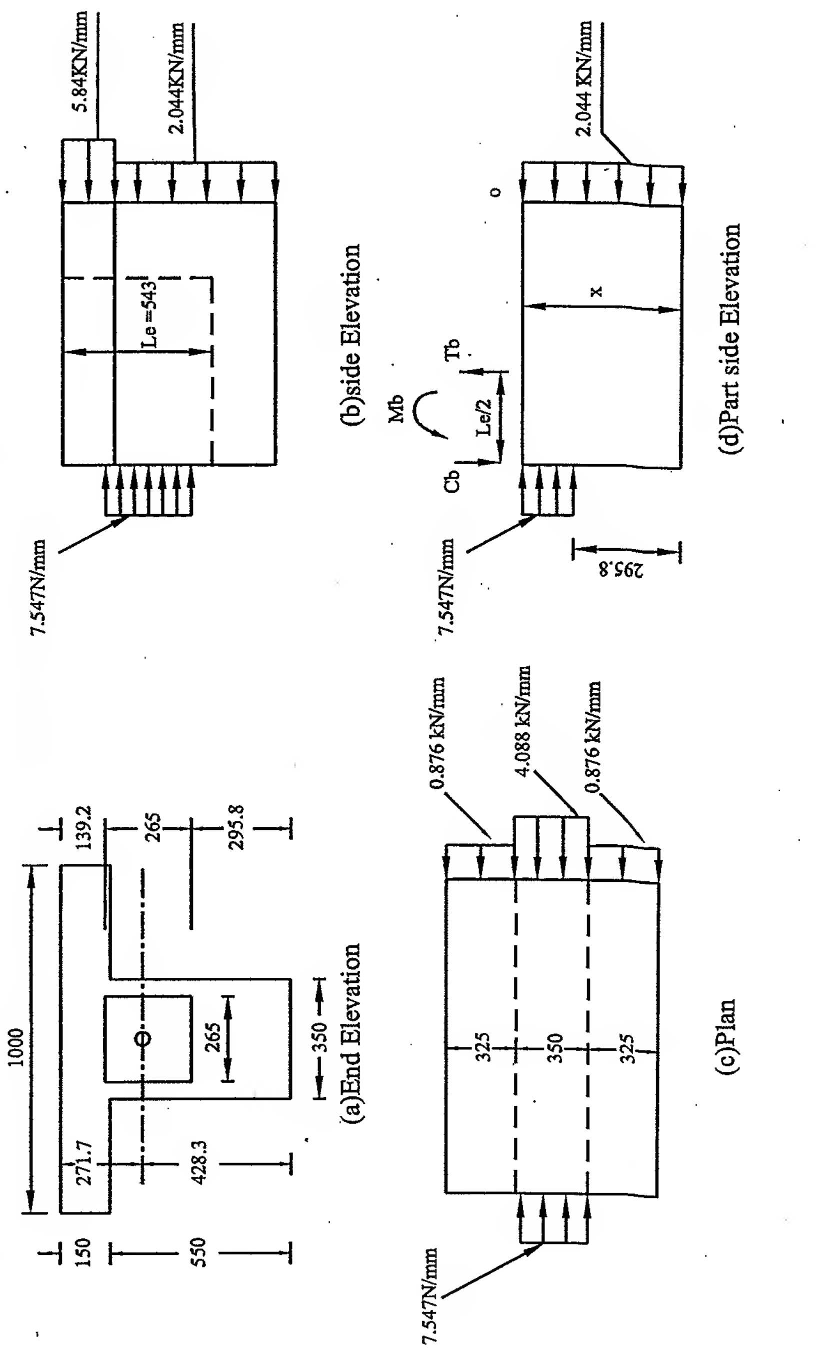


Fig. EX. 7.4.1 Stress distribution in the web and the flanges

This area of steel must be distributed within the length of the beam between 0.2 (L_e =109 mm) and (L_e =453 mm) from the loaded surface. Use stirrups with diameter of 16 mm over the full depth of the web and 12 mm stirrups immediately behind the anchorage as shown in Fig. EX. 7.4.3.

$$A_{sb,provided} = 2\times 201 + 2\times 113 = 628\,mm$$

The number of stirrups equals $\dfrac{2821}{628} = 4.5$

The required spacing $= \dfrac{453-109}{4.5} = 96\,mm \rightarrow use\ 90mm\ spacing$

Use Φ 12 mm + Φ 16 mm @90 mm

Step 2.1: Horizontal plane

Referring to Fig. EX. 7.4.1c, the stresses in the flanges equal:

$$f_2 = f_{avg}\times t_s = 0.00584\times 150 = 0.876\,kN/mm$$

$$f_3 = f_{avg}\times t = 0.00584\times 700 = 4.088\,kN/mm$$

Due to symmetry, the point of zero shear is located at the middle. Thus the bursting moment in the horizontal direction equals;

$$M_b = \left(\frac{4.088\times 175^2}{2} + 0.876\times 325\times(175+325/2) - 7.547\times\frac{(265/2)^2}{2}\right)/1000$$

$$M_b = 92.4\,kN.m$$

The bursting moment is resisted by horizontal tension and compression in the flange.

$$T_b = \frac{M_b}{B/2} = \frac{92.4\times 1000}{500} = 185\,kN$$

The area of the horizontal reinforcement required in the flange equals:

$$A_{sb,flange} = \frac{T_b}{f_s} = \frac{185\times 1000}{160} = 1156\ mm^2$$

Using top bars of 16 mm, the total number required is:

$$n = \frac{1156}{201} = 5.75$$

This area of steel must be distributed within the length of the beam between (0.2 B=200 mm) and (B=1000 mm) from the loaded surface.

The required spacing $= \dfrac{1000-200}{5.75} = 139\,mm \rightarrow 130mm$

Use Φ 16 mm @ 130 mm

Step 3: Design of transverse reinforcement (using strut-and-tie)

Step 3.1: Distribution of forces

Another approach for solving the same previous problem is the strut-and-tie approach. In this approach, the prestressing force is distributed to the flange and to the web through a series of struts and ties as shown in Fig. EX. 7.4.2.
The area of the cross section equals

$$A = 1000\times 150 + 350\times 550 = 342500\ mm^2$$

The average stress equals

$$f_{avg} = \frac{P_j}{A} = \frac{2000}{342500} = 0.005839\ kN/mm^2$$

$$F_{flange} = f_{avg}\times A_{flange} = 0.005839\times 1000\times 150 = 876\,kN$$

The force in the flange is located at the c.g of the flange (75 mm from the top)

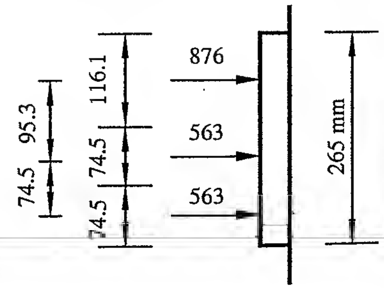
$$F_{web} = P - F_{flange} = 2000 - 876 = 1126\,kN$$

The force in the web is divided into two forces, each equals 563 kN.
Each force is located at quarter points of the web depth $t_{web}/4$ = 550/4=137.5 mm
The truss extends from the bearing plate into the beam for a length about the distance of the prestressing zone to the edge=271.7≈272 mm
The forces at the bearing plate must equal those in the flange and the web but with different spacing as shown in figure.

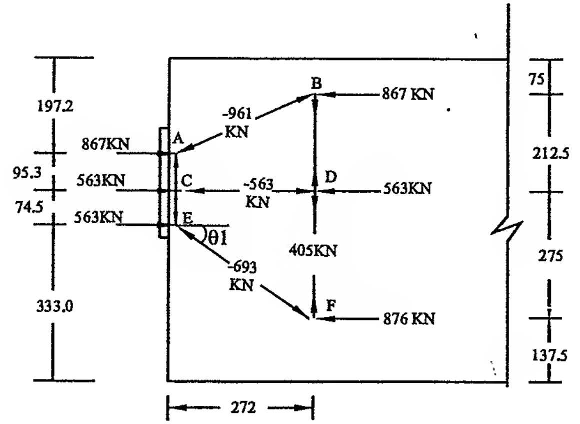
$$f_1 = \frac{2000}{265} = 7.547\ kN/mm$$

$$y_1 = \frac{F_{flange}}{f_1} = \frac{876}{7.547} = 116.1mm$$

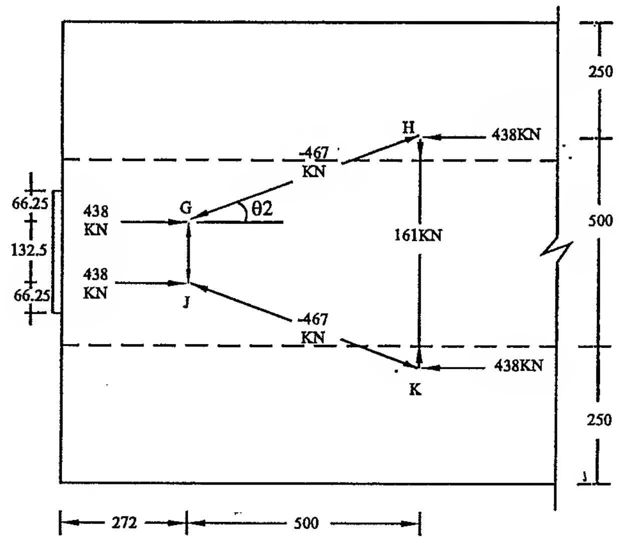
$$y_2 = \frac{F_{web}}{f_1} = \frac{563}{7.547} = 74.5\,mm$$



The distance between the force in the flange and the web=(y_1+y_2)/2=95.3 mm



(a)Vertical dispersion of prestress (elevation)



(b)Horizontal dispersion of prestress (plan)

Fig. EX. 7.4.2 Stress dispersion of the prestressing force

Step 3.2: Vertical direction

The inclination of the bottom strut equals

$$\theta_1 = \tan^{-1}\frac{333-137.5}{272} = 35.706^o$$

The force in the strut equals$=\dfrac{563}{\cos 35.706} = 693\,kN$

The tension in the tie equals$=693\times\sin 35.7 = 405\,kN$

The area of steel required to carry the force in the tie

$$A_{sb} = \frac{T_b}{f_s} = \frac{405\times1000}{160} = 2531\ mm^2$$

This area of steel must be distributed within the length of the beam between 0.2 L_e =109 mm and L_e =453 mm from the loaded surface. Use stirrups with diameter of 16 mm over the full depth of the web and 12 mm stirrups immediately behind the anchorage as shown in Fig. EX. 7.4.3

$$A_{sb\,,provided} = 2\times201+2\times113 = 628\,mm$$

The number of stirrups equals$\dfrac{2531}{628} = 4.03$

The required spacing $=\dfrac{453-109}{4.03} = 107\,mm \rightarrow use\ 90mm\ spacing$

Step 3.3: Horizontal direction

The horizontal spreading of the forces into the flange is shown in Fig EX. 7.4.2. The total force is applied at the quarter point of the flange (250 mm for the edge). The location of the horizontal force is at 272 mm from the edge. The distance at which the truss is formed equals B/2=500 mm.

At the bearing plate the force is located at the quarter points =(265/4=66.25 mm)

The inclination of the strut equals

$$\theta = \tan^{-1}\frac{250-66.25}{500} = 20.18^o$$

The force in the strut equals$=\dfrac{438}{\cos 20.18} = 467\,kN$

The tension in the tie equals$=467\times\sin 20.18 = 161\,kN$

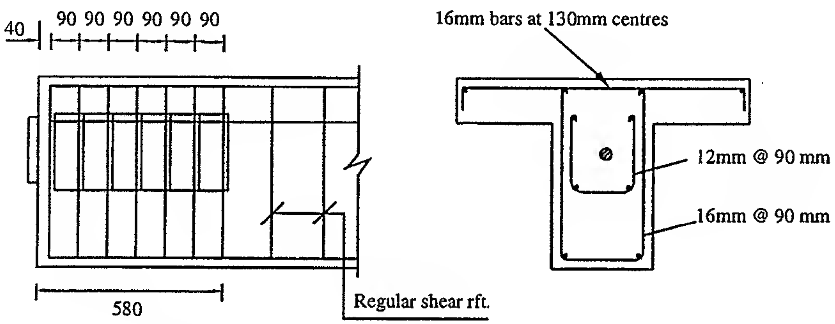
The area of steel required to carry the force in the tie

$$A_{sb} = \frac{T_b}{f_s} = \frac{161 \times 1000}{160} = 1006\, mm^2$$

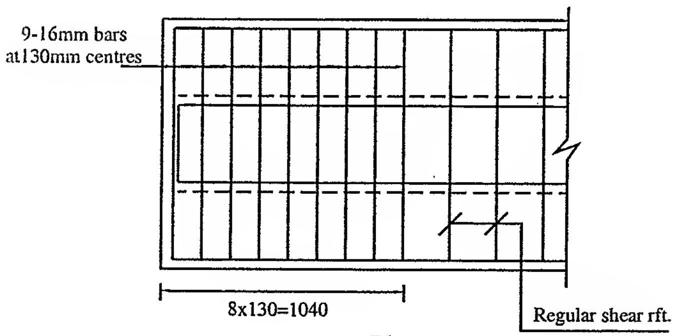
Using top bars of 16 mm, the total number required is $n = \dfrac{1006}{201} = 5$

This area of steel must be distributed within the length of the beam between 0.2 B=200 mm and B=1000 mm from the loaded surface.

$$s = \frac{1000 - 200}{5} = 160 \rightarrow take\ 130\, mm$$



Elevation Cross Section



Plan

Fig. EX. 7.4.3 Reinforcement details for the anchorage zone

8

FLEXURE IN PRESTRESSED CONCRETE BEAMS

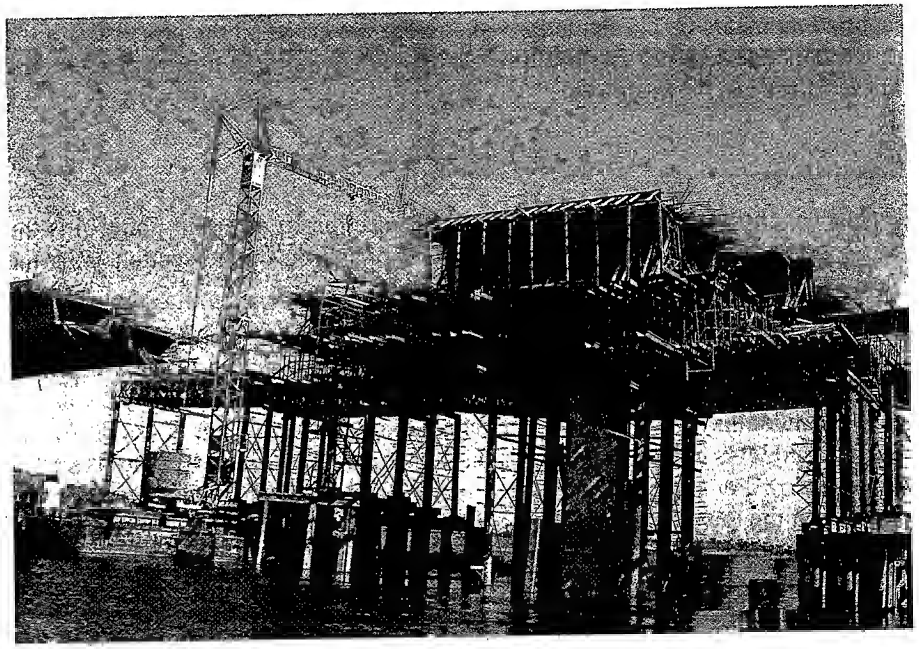


Photo 8.1 Prestressed box –girder bridge during construction

8.1 Introduction

The basic principles used in the flexural design and analysis of prestressed concrete beams are presented in this chapter. Two steps are considered in the analysis and design of prestressed beams in flexure namely;

- The analysis under service loads.
- The analysis at the ultimate state.

The fundamental relationships used in the service load analysis are based upon the basic assumptions of elastic design. However, at the ultimate stage, stresses and strains are not proportional and the ultimate limit analysis should be carried out. Deflections under service loads together with the prestressing forces should be calculated to confirm the compliance with the applicable design criteria.

8.2 Analysis of Prestressed Concrete Members under Service Loads

8.2.1 General

Flexural stresses in prestressed members are the result of the prestressing force *P*, the internal moment due to eccentric cable configuration, and the external applied moments (*M*). Calculations of stresses are based on the properties of the gross concrete section. The resulting stresses at any point in the beam caused by these forces can be written as:

$$f = -\frac{P}{A} \pm \frac{P \times e}{I} y \mp \frac{M \times y}{I} \quad \text{.............................} (8.1)$$

where y is the distance from the C.G. of the section to a certain point, A is the cross sectional area, I is the gross moment of inertia of the section, P is the prestressing force and M is the external applied moment.

The resulting stresses should be checked against the allowable values specified by the code and given in Tables 8.1 and 8.2. For calculation of stresses at the extreme fibers, it is usually more convenient to express the quantity I/y as the section modulus (Z). For non-symmetrical members such as T-sections, the section modulus at the top Z_{top} is different from the section modulus at the bottom Z_{bot}. For a simply supported beam, as the one shown in Fig. 8.1, the stresses at the top and bottom fibers of the beam can be calculated from:

$$f_{bot} = -\frac{P}{A} - \frac{P \times e}{Z_{bot}} + \frac{M}{Z_{bot}} \quad \text{...} (8.2)$$

$$f_{top} = -\frac{P}{A} + \frac{P \times e}{Z_{top}} - \frac{M}{Z_{top}} \quad \text{...} (8.3)$$

In which,

$$Z_{bot} = \frac{I}{y_{bot}} \qquad Z_{top} = \frac{I}{y_{top}}$$

It should be clear that the stresses induced due to (*P.e*) are opposite to those induced due to the external applied moment M.

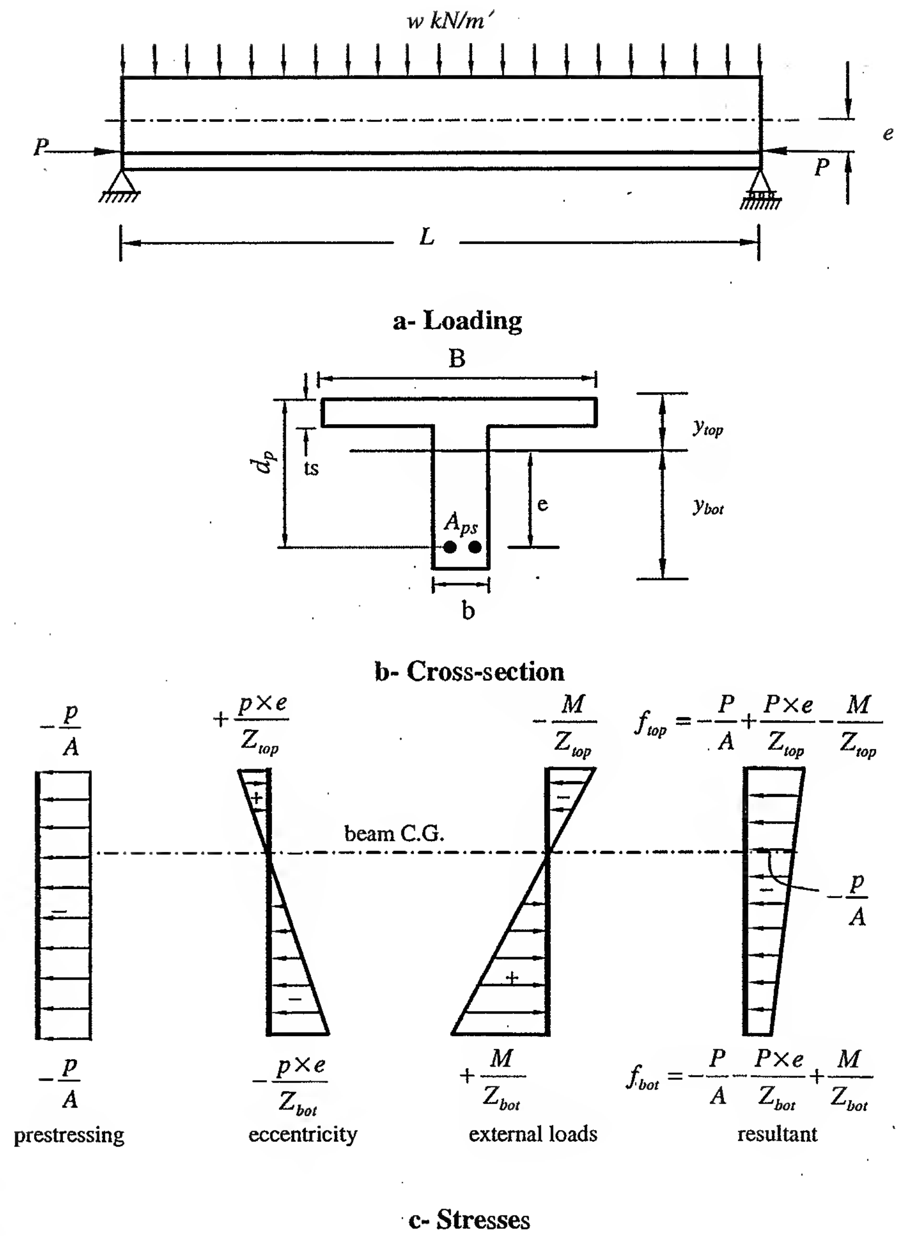


Fig. 8.1 Stress distributions in a concrete section due to The prestressing and the applied loads

8.2.2 Allowable Concrete and Steel Stresses

8.2.2.1 Allowable Steel Stresses

Prestressing steel is most commonly used in the form of wires or strands. The code specifies the ratio of the yield strength f_{py} of the prestressing steel to its ultimate strength f_{pu} as follows:

$f_{py}/f_{pu} = 0.80$ for deformed bars.

$f_{py}/f_{pu} = 0.85$ for normal relaxation stress-relieved strands, wires and bars.

$f_{py}/f_{pu} = 0.90$ for low relaxation stress-relieved strands and wires.

The strength reduction factor γ_{ps} for prestressing steel is taken the same as non-prestressed steel. Hence, the strength reduction factor for flexure γ_{ps} is taken as 1.15.

The tensile stresses allowed by the ECP 203 for prestressing wires, prestressing strands, or prestressing bars are dependent upon the stage of loading. At jacking, the maximum allowable stress is the lesser of 0.75 f_{pu} or 0.90 f_{py}. When the jacking force is first applied, a stress of 0.70 f_{pu} is allowed. Immediately after transfer of the prestressed force to the concrete, the permissible stress is 0.70 f_{pu} or 0.80 f_{py} whichever is smaller. The previous values are applied also in case of post-tensioned members. The ECP 203 allowable stresses in prestressing steel are summarized in Table 8.1

Table 8.1 Allowable tensile stresses for prestressing steel

Maximum stress produced by jacking (before transfer)*	$0.90 f_{py}$ or $0.75 f_{pu}$
Maximum tendon stress at tensioning process	$0.70 f_{pu}$
Maximum tendon stress immediately after transfer not to exceed the smaller of	$0.80 f_{py}$ or $0.70 f_{pu}$
Maximum stress in post-tensioned tendons at anchorages and couplers immediately after anchorage of the tendons not to exceed the smaller of	$0.80 f_{py}$ or $0.70 f_{pu}$

* Not to exceed the stress recommended by the manufacturer of the prestressing system

8.2.2.2 Allowable Concrete Stresses

Concrete with high compressive strength is normally used in prestressed members. This is because of its high modulus of elasticity (less creep and elastic shortening losses). In addition, its high bearing capacity permits the use of small anchorage zone. The allowable stresses in concrete are dependent on the stage of loading. The ECP 203 gives these stresses in two stages namely; at transfer and at service loads. The allowable stresses at transfer are given in Table 8.2.

At the service load stage, the ECP-203 classifies prestressed concrete elements into 4 cases depending on the tensile stresses developed in the section. These cases are as follows:

Case A: (Full prestressing)

These are elements in which there is no tensile stresses are allowed (developed tensile stress equals to zero). These elements are:

- Structural elements subjected to cyclic or dynamic loads.
- Structures with tension side severely exposed to corrosive environment of strong chemical attack which cause rusting of steel (category four according to ECP 203).

Case B: Uncracked sections

These are elements in which the tensile stresses due to all loads are less than:

$$0.44\sqrt{f_{cu}} \quad \text{...(8.4a)}$$

Examples of structural elements deigned according to case B are:

- Solid slabs and flat slabs.
- Prestressed concrete elements with unbonded tendons.
- Structures with severely exposed tension side (category three according to ECP 203).

Case C: An intermediate case between full and partial prestressing

Structural elements are subjected to tensile stresses larger than case B but less than the cracking strength of concrete given by:

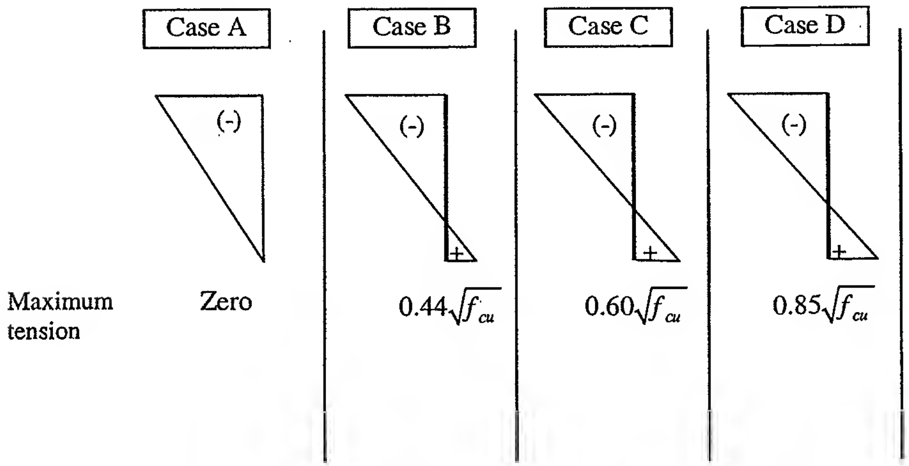
$$f_{ctr} = 0.6\sqrt{f_{cu}} \leq 4.0\ N/mm^2 \quad \text{............................ (8.4b)}$$

Case D: Cracked sections (Partial prestressing)

These are elements in which the tensile stresses due to *all loads* (using uncracked sections properties, I_g), are less than $0.85\sqrt{f_{cu}}$.

In addition, the tensile stresses developed in the section due to *permanent loads*, which might include permanent live loads, should be less than $0.6\sqrt{f_{cu}}$.

For cases C and D, ordinary reinforcing steel or non-prestressed strands are provided to resist the tension force developed in the section at the working stage.



f_{cu} (N/mm^2)	Full prestressing	Uncracked Section	Transition	Partial Prestressing
40	0	2.78	3.79	5.38
45	0	2.95	4.02	5.70
50	0	3.11	4.24	6.01
55	0	3.26	4.45	6.30
60	0	3.41	4.65	6.58

Fig. 8.2 Maximum allowable tension at full service loads

Table 8.2 Allowable concrete stresses (N/mm^2)

At the time of initial tensioning before time dependent losses produced by creep, shrinkage, or relaxation have occurred (At Transfer)	
1. Maximum compressive stress	$0.45 f_{cui}$
2. Maximum tensile stress except as permitted in item 3	$0.22\sqrt{f_{cui}}$
3. Maximum tensile stress at the ends of simply supported members	$0.44\sqrt{f_{cui}}$
Service load flexural stresses, assuming all prestressed losses have occurred (At Service Loads)	
1. Maximum compressive stress due to prestressed plus sustained loads	$0.35 f_{cu}$
2. Maximum compressive stress due to prestressed plus total loads	$0.40 f_{cu}$
3. Maximum tensile stress in pre-compressed zone tensile zone	Case A- zero Case B- $0.44\sqrt{f_{cu}}$ Case C- $0.60\sqrt{f_{cu}}$ $\leq 4\ N/mm^2$ Case D- $0.85\sqrt{f_{cu}}$
Axial compression	
1. Maximum compressive stress	$0.25 f_{cu}$

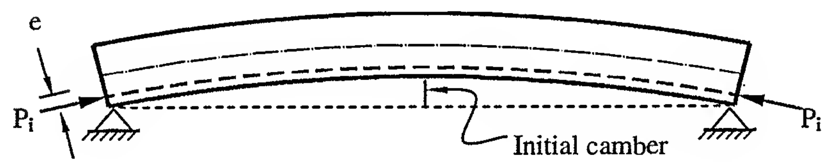
where
f_{cui} is the concrete characteristic strength at the time of transfer (N/mm^2)
f_{cu} is the concrete characteristic strength at service load (N/mm^2).

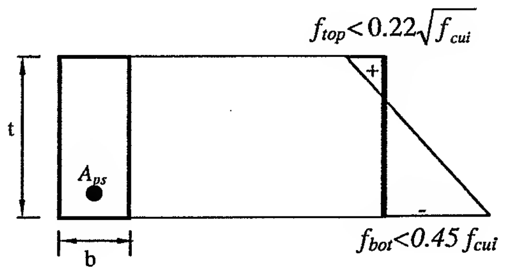
8.2.3 Calculations of Stresses at Transfer

After applying the prestressing force to the beam, the beam deflects upward (camber) and the only external applied moment is due to the self-weight of the beam M_{ow} as shown in Fig. 8.3. The bottom fibers are subjected to high compression stresses while, the top fibers are subjected to tension stresses. The concrete strength f_{cui} used in the calculation of the permissible stresses should be at the time of transferring the force to the concrete. Furthermore, the prestressing force (P_i) used in stress expressions is the initial prestressing force before losses.

$$f_{bot} = -\frac{P_i}{A} - \frac{P_i \times e}{Z_{bot}} + \frac{M_{ow}}{Z_{bot}} \le 0.45\ f_{cui} \quad \text{.......................} (8.5)$$

$$f_{top} = -\frac{P_i}{A} + \frac{P_i \times e}{Z_{top}} - \frac{M_{ow}}{Z_{top}} \le 0.22\sqrt{f_{cui}} \quad \text{....................} (8.6)$$





Stresses at mid section

Fig. 8.3 Stress distribution and allowable stresses at transfer

8.2.4 Calculations of Stresses at Full Service Loads

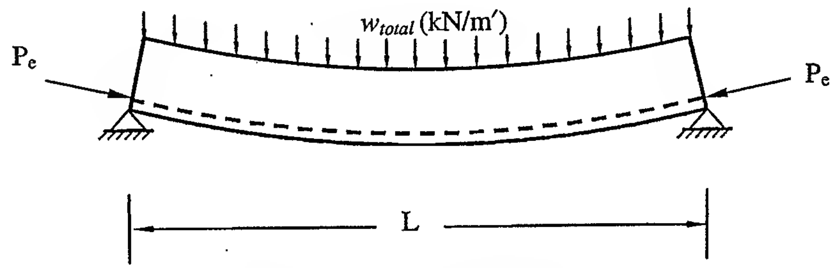
After the application of the superimposed dead loads and the live loads, the prestressed member shall be subjected to the total service moment M_{total}. The full intensity of such loads usually occurs after the building is completed and some time-dependent losses have taken place. Therefore, the prestressing force used in the computation of stresses is *the effective prestressing force* (P_e) as shown in Fig. 8.4. The total unfactored load on the beam is given as:

$$w_{total} = w_{ow} + w_{DL} + w_{LL} \quad \text{...} (8.7a)$$

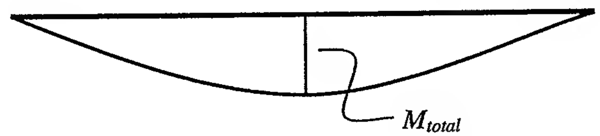
$$P_e = P_i \times [1 - losses\,(\%)] \quad \text{..} (8.7b)$$

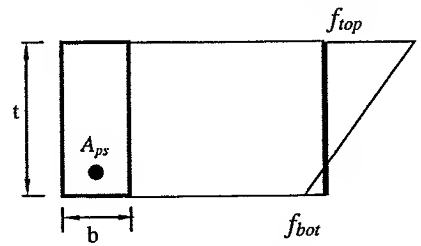
The maximum bending at mid span for simple beam equals:

$$M_{total} = \frac{w_{total} \times L^2}{8}$$



Loads at the service load stage





Stresses at mid section

Fig. 8.4 Stress distribution and allowable stresses at full service loads

The stresses are calculated using Eq. 8.8 and Eq. 8.9. The stresses at mid-span section should not exceed the limits allowed by the code and given in Table 8.2.

$$f_{top} = -\frac{P_e}{A} + \frac{P_e \times e}{Z_{top}} - \frac{M_{total}}{Z_{top}} \quad \text{.........} \quad (8.8)$$

$$f_{bottom} = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{bot}} + \frac{M_{total}}{Z_{bot}} \quad \text{.........} \quad (8.9)$$

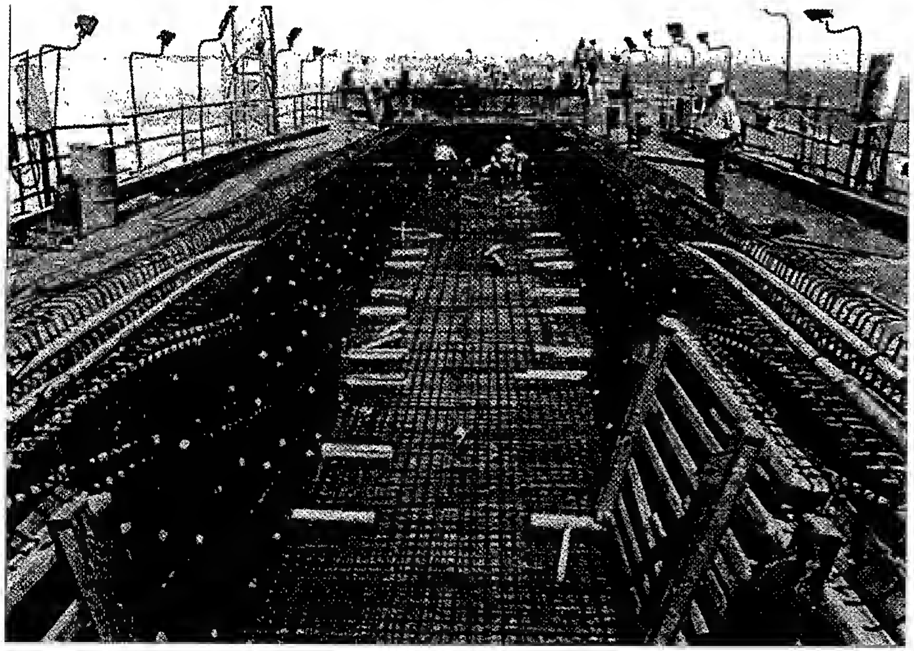
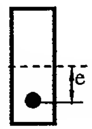
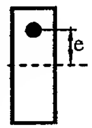


Photo 8.2 Tendon placement in a bridge box-section

8.2.5 Summary

Equations for stress computation are used to determine the concrete stress at the extreme fibers for positive and negative bending moments as summarized in Table 8.3. It is important to verify that the stresses for both load cases (at transfer and at full service load) are within the allowable limits.

Table 8.3 Stress calculations at top and bottom of sections subjected to either negative or positive bending moments

Item			At transfer	At full service load
positive bending (+ve)		f_{top}	$-\frac{P_i}{A}+\frac{P_i \times e}{Z_{top}}-\frac{M_{ow}}{Z_{top}} \le 0.22\sqrt{f_{cui}}$	$-\frac{P_e}{A}+\frac{P_e \times e}{Z_{top}}-\frac{M_{total}}{Z_{top}} \le 0.40\ f_{cu}$
		f_{bot}	$-\frac{P_i}{A}-\frac{P_i \times e}{Z_{bot}}+\frac{M_{ow}}{Z_{bot}} \le 0.45\ f_{cui}$	$-\frac{P_e}{A}-\frac{P_e \times e}{Z_{bot}}+\frac{M_{total}}{Z_{bot}} \le$ see Table 8.2
negative bending(-ve)		f_{top}	$-\frac{P_i}{A}-\frac{P_i \times e}{Z_{top}}+\frac{M_{ow}}{Z_{top}} \le 0.45\ f_{cui}$	$-\frac{P_e}{A}-\frac{P_e \times e}{Z_{top}}+\frac{M_{total}}{Z_{top}} \le$ see Table 8.2
		f_{bot}	$-\frac{P_i}{A}+\frac{P_i \times e}{Z_{bot}}-\frac{M_{ow}}{Z_{bot}} \le 0.22\sqrt{f_{cui}}$	$-\frac{P_e}{A}+\frac{P_e \times e}{Z_{bot}}-\frac{M_{total}}{Z_{bot}} \le 0.40\ f_{cu}$

In the following example problems, the allowable stresses are usually available and the determination of one unknown (the prestressing force, the span, or the loads) is required. For a certain load case (transfer or service load), two values can be obtained by solving the stress equations at the top and the bottom for the required unknown. Care should be given to the appropriate choice as given in Table 8.4.

Table 8.4 Analysis of prestressed sections

At Transfer	At Service load
• Smaller P_I	• Larger P_e
• Smaller A_{SP}	• Larger A_{SP}
• Longer span	• Smaller span
• Smaller eccentricity e	• Larger eccentricity e
• Larger self-weight loads	• Smaller live-loads
• Larger self-weight moment	• Smaller moment

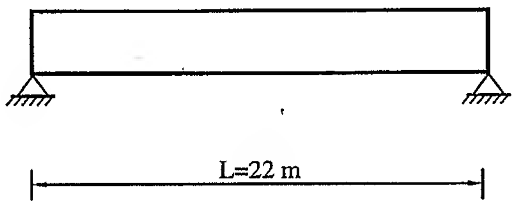
Example 8.1

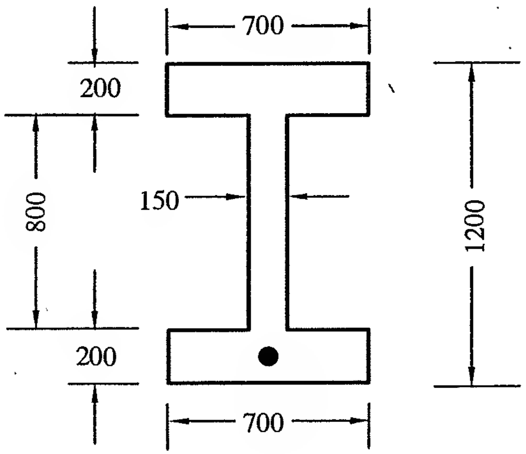
For the prestressed beam shown in figure and knowing that the beam is fully prestressed determine,

1. The required force at transfer.
2. The amount of prestressing steel.
3. The stresses at final stage.

Data

f_{cu} = 40 N/mm^2
f_{cui} = 30 N/mm^2
f_{py} = 1700 N/mm^2
f_{pu} = 2000 N/mm^2
LL = 18 kN/m′ (unfactored)
Flooring weight = 4 kN/m′ (unfactored)
cover =100 mm
Losses = 15%





Beam cross section

Solution

Step 1: Calculate Section Properties

The cross sectional area (A) equals:

$$A = 2\times700\times200+800\times150 = 400000\ mm^2$$

Since the section is symmetrical; $y_{top} = y_{bottom} = 600$ mm

$$I = 2\times\left(\frac{700\times200^3}{12}+700\times200\times(600-100)^2\right)+\frac{150\times800^3}{12} = 7.733\times10^{10}\ mm^4$$

$$Z_{bot} = \frac{I}{y_{bottom}} = \frac{7.733\times10^{10}}{600} = 1.289\times10^8\ mm^3$$

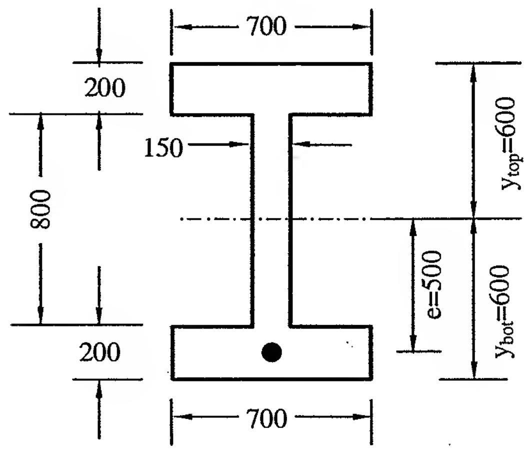
$Z_{top} = Z_{bot} = 1.289\text{x}10^8\ mm^3$

$$w_{ow} = \gamma_c \times A = 25\times\frac{400000}{10^6} = 10\,kN/m'$$

The allowable compression stress f_{ci} and allowable tension stresses f_{ti} can be obtained using the compressive strength of concrete at the time of pre-stressing f_{cui} as follows:

$f_{ci} = 0.45\text{ x }f_{cui} = 0.45\text{ x }30 = -13.5\ N/mm^2$

$$f_{ti} = 0.22\sqrt{f_{cui}} = 0.22\sqrt{30} = 1.2\ N/mm^2$$



Step 2: Calculate the initial prestressing force at transfer

At transfer, the self-weight of the beam is the only applied load, thus:

$$M_{ow} = \frac{w_{ow}\,L^2}{8} = \frac{10 \times 22^2}{8} = 605\ kN.m$$

The eccentricity of the cable from the C.G. of the section equals:

$$e = y_{bot} - cover = 600 - 100 = 500\ mm$$

The initial prestressing force should stratify the allowable stresses at transfer at the bottom and top fibers.

$$f_{bottom} = -\frac{P_i}{A} - \frac{P_i \times e}{Z_{bot}} + \frac{M_{ow}}{Z_{bot}}$$

Assuming that P_i is in kN, and applying in the previous equation gives P_{i1}

$$-13.5 = -\frac{P_{i1} \times 1000}{400000} - \frac{P_{i1} \times 1000 \times 500}{1.289 \times 10^8} + \frac{605 \times 10^6}{1.289 \times 10^8}$$

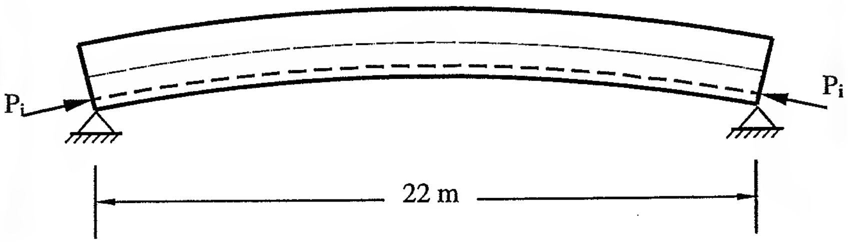
$P_{i1} = 2852$ kN

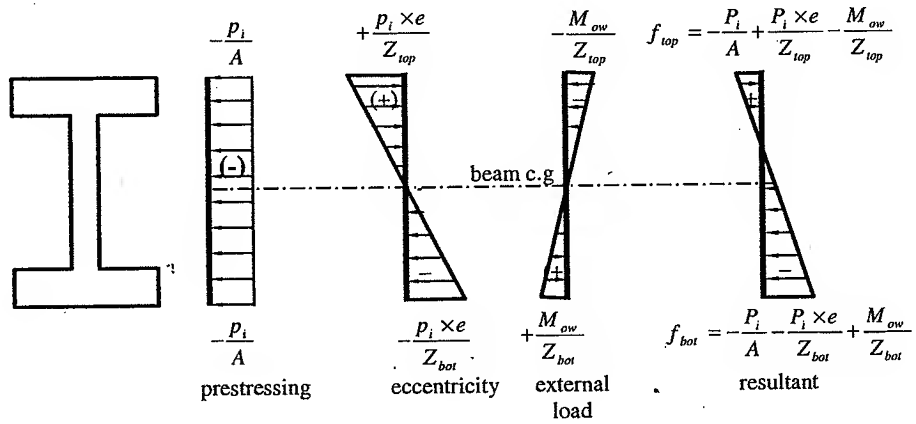
A second value P_{i2} is obtained from the analysis of the top fibers as follows:

$$f_{top} = -\frac{P_i}{A} + \frac{P_i \times e}{Z_{top}} - \frac{M_{ow}}{Z_{top}}$$

$$+1.2 = -\frac{P_{i2} \times 1000}{400000} + \frac{P_{i2} \times 1000 \times 500}{1.289 \times 10^8} - \frac{605 \times 10^6}{1.289 \times 10^8}$$

$P_{i2} = 4281$ kN





The chosen initial prestressing force is the smaller of the two values, because it will give stress at the opposite fiber that is less than the allowable. In this case, the critical load is 2852 kN giving -13.5 N/mm^2 at the bottom fiber. Applying this load at transfer will give stress at the top fiber of -7.6 N/mm^2 *(calculation is not shown)*, which is less than the allowable stress at transfer.

Final design P_i=P_{i1}=2852 kN

Step 3: Calculate the required prestressing steel

The allowable prestressing stress at time of transfer is given by:

$$f_{pi} = smaller\ of \begin{cases} 0.70 f_{pu} = 0.7 \times 2000 = 1400\ N/mm^2 \\ 0.8 f_{py} = 0.8 \times 1700 = 1360\ N/mm^2 \end{cases}$$

f_{pi}= 1360 N/mm^2

$$A_{ps} = \frac{P_i}{f_{pi}} = \frac{2852 \times 1000}{1360} = 2097\ mm^2$$

Step 4: Calculate the stresses at the service load stage

The total load is the summation of dead and live loads.

$$w_{total} = w_{ow} + w_{flooring} + w_{LL} = 10 + 4 + 18 = 32\,kN/m'$$

The total moment at the mid-span equals:

$$M_{tot} = \frac{w_{tot}\ L^2}{8} = \frac{32 \times 22^2}{8} = 1936\ \ kN.m$$

The effective prestressing force at the time of applying all service loads equals:

$P_e = (1\text{-losses})\ P_i$ $\rightarrow$ $P_e = (1\text{-}0.15) \times 2852 = 2424.2$ kN

The allowable stresses at service load stage equals

f_{te} = zero (no tension is allowed in fully prestressed beams)

$$f_{ce} = 0.40 f_{cu} = 0.40 \times 40 = -16\ N/mm^2$$

$$f_{bottom} = -\frac{P_e}{A} + \frac{P_e \times e}{Z_{top}} - \frac{M_{total}}{Z_{top}}$$

$$f_{top} = -\frac{2424.2 \times 1000}{400000} + \frac{2424.2 \times 1000 \times 500}{1.289 \times 10^8} - \frac{1936 \times 10^6}{1.289 \times 10^8}$$

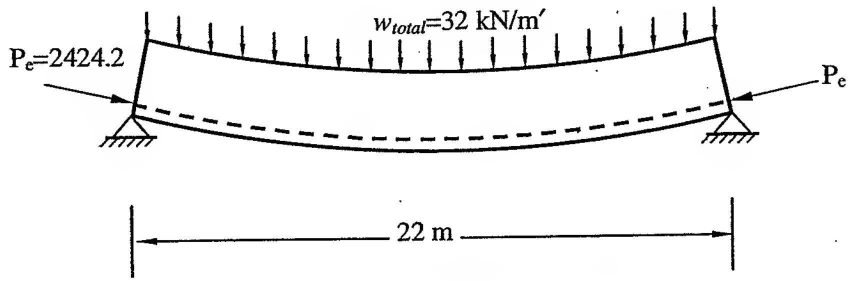
$$f_{top} = -11.67\ N/mm^2\ \ldots < -16 (safe)$$

$$f_{bottom} = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{bot}} + \frac{M_{total}}{Z_{bot}}$$

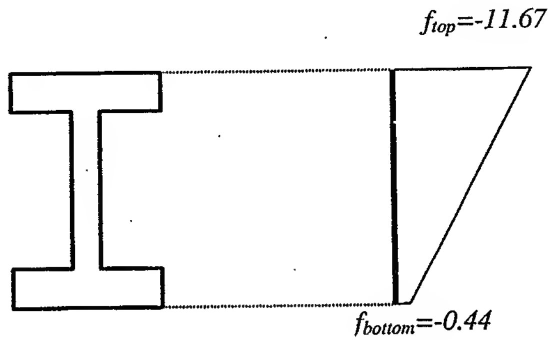
$$f_{bottom} = -\frac{2424.2 \times 1000}{400000} - \frac{2424.2 \times 1000 \times 500}{1.289 \times 10^8} + \frac{1936 \times 10^6}{1.289 \times 10^8}$$

$$f_{bottom} = -0.44\ N/mm^2\ \ldots < zero\ \ (safe)$$

The beam is considered safe because non of the extreme fiber stresses exceed the allowable stresses.



Loads at the service load stage

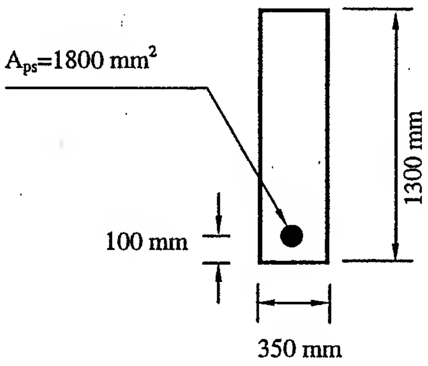


Stresses at mid section

Example 8.2

Figure EX 8.2 shows the cross-section of a simply supported post-tensioned beam. Determine the maximum span of the beam based on the stresses at the transfer. Assume that the beam is made of normal strength concrete with f_{cu}=40 N/mm^2, and the concrete strength at transfer (f_{cui}) is 75% of the cube strength. Assume also that the time dependent losses are 12% of the initial prestressing and that the yield strength and the ultimate strength of the tendons are 1700 and 2000 N/mm^2, receptively.

Knowing that the beam is categorized as case D, check stresses at full service stage if the beam is subjected to an unfactored live load of 12.5 kN/m′ and unfactored superimposed dead load of 4 kN/m′. Calculate the required non prestressed steel (f_y=400 N/mm^2).



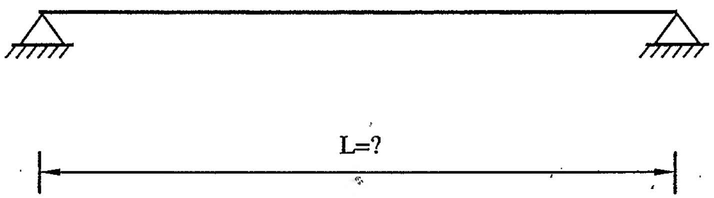


Fig. EX 8.2 Beam elevation and cross section

Solution

Step 1: Calculate Section Properties

The cross sectional area (A) equals:

$$A = 350\times1300 = 455000 \ \ mm^2$$

Since the section is symmetrical; $y_{top} = y_{bottom}$= 650 *mm*

$$I = \frac{350\times1300^3}{12} = 6.41\times10^{10} \ \ mm^4$$

$$Z_{bot} = \frac{I}{y_{bottom}} = \frac{6.41\times10^{10}}{650} = 98.583\times10^6 \ \ mm^3$$

$$Z_{top} = Z_{bot} = 98.583 \text{ x} 10^6 \ mm^3$$

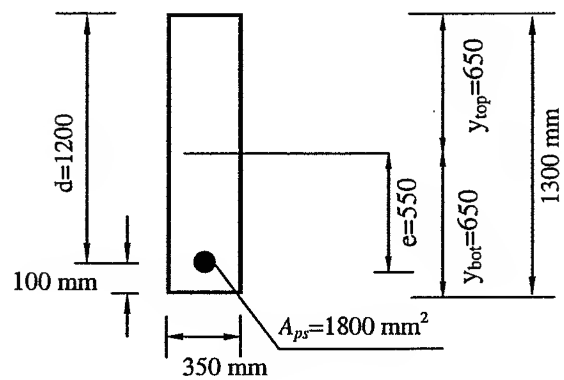
$$w_{o.w} = \gamma_c \times A = 25\times\frac{455000}{10^6} = 11.375 kN \ / m'$$

The allowable compression stress f_{ci} and the allowable tension stress f_{ti} can be obtained using the compressive strength of concrete at the time of pre-stressing f_{cui} as follows:

$$f_{cui} = 0.75\times f_{cu} = 0.75\times30 = 22.5 \ N / mm^2$$

$$f_{ci} = 0.45 \text{ x } f_{cui} = 0.45 \text{ x } 22.5 = -10.125 \ N/mm^2$$

$$f_{ti} = 0.22\sqrt{f_{cui}} = 0.22\sqrt{22.5} = 1.04 \ N / mm^2$$



Step 2: Calculate the prestressing force

The allowable prestressing stress at time of transfer is given by:

$$f_{pi} = smaller\ of \begin{cases} 0.70\, f_{pu} = 0.7\times 2000 = 1400\ N/mm^2 \\ 0.8\, f_{py} = 0.8\times 1700 = 1360\ N/mm^2 \end{cases}$$

f_{pi}=1360 N/mm^2

$$P_i = A_{ps} \times f_{pi} = \frac{1800\times 1360}{1000} = 2448\ kN$$

Step 3: Calculate beam span

At transfer, the self weight of the beam is the only applied load. Since the span is unknown the bending due to own weight equals:

$$M_{ow} = \frac{w_{ow}\, L^2}{8} = \frac{11.375\times L^2}{8} = 1.42\times L^2\ kN.m$$

The eccentricity of the cable from the c.g of the section equals:

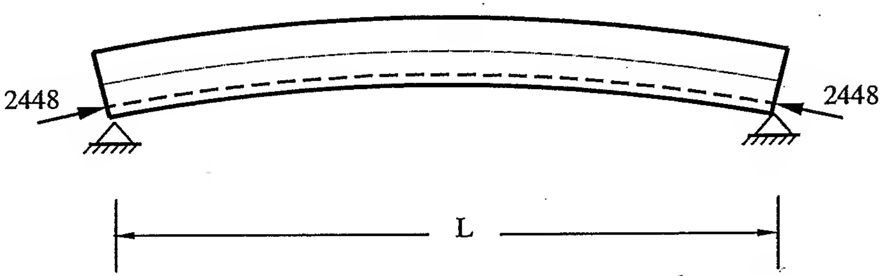
$$e = y_{bot} - \text{cover} = 650 - 100 = 550\ mm$$

Analysis of the stresses at the bottom fiber, which is compression, gives:

$$f_{bottom} = -\frac{P_i}{A} - \frac{P_i \times e}{Z_{bot}} + \frac{M_{ow}}{Z_{bot}}$$

$$-10.125 = -\frac{2448\times 1000}{455000} - \frac{2448\times 1000\times 550}{98.583\times 10^6} + \frac{M_{ow}\times 10^6}{98.583\times 10^6}$$

M_{ow}= 878.64 kN.m



Analysis of the stresses at the top fiber, which is tension, gives:

$$f_{top} = -\frac{P_i}{A} + \frac{P_i \times e}{Z_{top}} - \frac{M_{ow}}{Z_{top}}$$

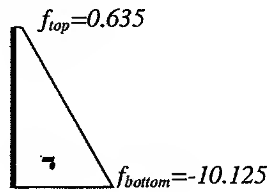
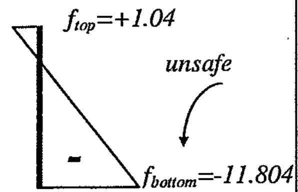
$$+1.04 = -\frac{2448\times 1000}{455000} + \frac{2448\times 1000\times 550}{98.583\times 10^6} - \frac{M_{ow}\times 10^6}{98.583\times 10^6}$$

M_{ow}= 713.12 kN.m

The allowable stresses at the bottom and top fibers of the cross-section are –10.125 N/mm^2 and + 1.04 N/mm^2, respectively. The calculation of the span of the beam should be based on the larger of the two values of the bending moments obtained from the analyses of the stresses at the bottom and top fibers (M_{ow}= 878.64 kN.m). This is attributed to the fact that at the case of transfer the self-weight moment produces stresses that are of opposite sign compared to the allowable stresses, both at the bottom and top fibers as shown in the following table.

$$M_{ow} = 878.64 = 1.42\times L^2$$

L=24.858 m

Item	bottom condition	Top condition
Allowable stress N/mm^2	-10.125	+1.04
M_{ow} (kN.m)	878.64	713.124
Span(m)	24.858	22.395
stress at opposite fiber N/mm^2	-0.635(top)<+1.04	-11.804(bottom)>-10.125
condition	safe	unsafe
		

Step 4: Calculate stresses at the service load stage

The total load is the summation of dead and live loads.

$$w_{total} = w_{ow} + w_{DL} + w_{LL} = 11.375 + 4 + 12.5 = 27.875\,kN/m'$$

The total moment at the midspan equals:

$$M_{tot} = \frac{w_{total}\,L^2}{8} = \frac{27.875 \times 24.858^2}{8} = 2153.2\ \ kN.m$$

The effective prestressing force at the time of applying all service loads is

P_e = (1-losses) P_i $\rightarrow$ P_e = (1-0.12) x 2448 = 2154.24 kN

For case D, the allowable stresses at service load stage equals:

$$f_{te} = 0.85\sqrt{f_{cu}} = 0.85\sqrt{40} = 5.4\ N/mm^2$$

$$f_{ce} = 0.40\,f_{cu} = 0.40 \times 40 = -16\ N/mm^2$$

$$f_{top} = -\frac{P_e}{A} + \frac{P_e \times e}{Z_{top}} - \frac{M_{total}}{Z_{top}}$$

$$f_{top} = -\frac{2154.24 \times 1000}{455000} + \frac{2154.24 \times 1000 \times 550}{98.583 \times 10^6} - \frac{2153.2 \times 10^6}{98.583 \times 10^6}$$

$$f_{top} = -14.56\ N/mm^2\ \ldots < -16\ (safe)$$

$$f_{bottom} = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{bot}} + \frac{M_{total}}{Z_{bot}}$$

$$f_{bottom} = -\frac{2154.24 \times 1000}{455000} - \frac{2154.24 \times 1000 \times 550}{98.583 \times 10^6} + \frac{2153.2 \times 10^6}{98.583 \times 10^6}$$

$$f_{bottom} = +5.09\,N/mm^2\ \ldots < 5.4\ \ (safe)$$

The beam is considered safe because both extreme fiber stresses (at top and *bottom*) are less than the allowable stress.

Step 5: Calculation of non-prestressed steel (As)

Since the stresses at the bottom (+5.09 N/mm^2) exceeds the tensile strength of concrete ($0.6\sqrt{40} = 3.79\,N/mm^2$), the beam must be reinforced with non-prestressed steel. The depth of the tension zone y_{ten} equals:

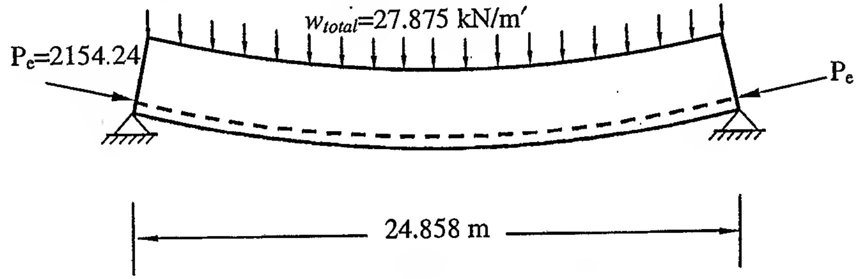
$$y_{ten} = \frac{5.09}{5.09 + 14.56} \times 1300 = 336.8\ \text{mm}$$

The tension force equals:

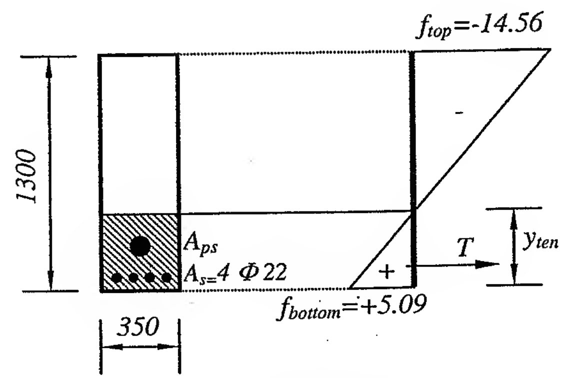
$$T = \frac{1}{2} f_{bottom} \times y_{ten} \times b = \frac{1}{2} \times 5.09 \times 336.8 \times 350 / 1000 = 300\,kN$$

The allowable working stress f_s for the non-prestressed steel (f_y=400 N/mm^2) is obtained from Table 5.1 of the ECP 203. $\rightarrow$ f_s=200 N/mm^2

$$A_s = \frac{T}{f_s} = \frac{300 \times 1000}{200} = 1500\ mm^2\ \rightarrow \text{Use } 4\ \Phi\ 22$$



Loads at the service load stage

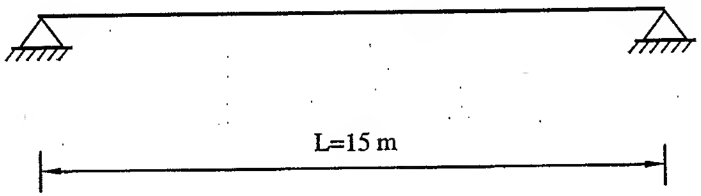


Stresses at mid section

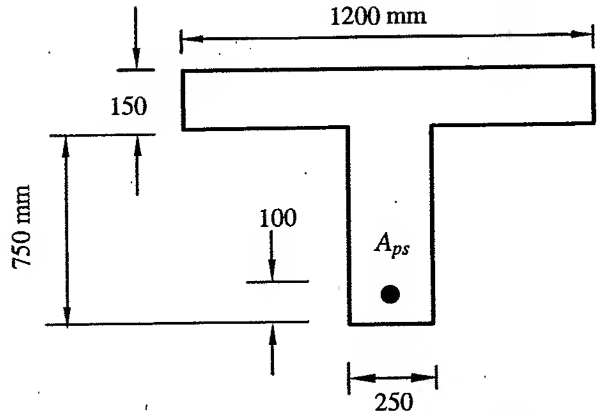
Example 8.3

Calculate the required prestressing force at the service load stage if the beam shown in the figure is subjected to dead loads of 8 kN/m′(not including own weight) and live loads of 20 kN/m′. The beam is prestressed with unbonded tendons and the losses may be assumed 16 %. It is also required to check the stresses at transfer.

Assume that $f_{cu} = 45$ N/mm^2, $f_{cui} = 34$ N/mm^2



Beam elevation



Beam cross-section

Solution

Step 1: Calculate Section Properties

The cross sectional area (A) equals:

$$A = 1200 \times 150 + 250 \times 750 = 367500 \ mm^2$$

Since the section is not symmetrical, calculate the location of the center of gravity. $\rightarrow$ $y = \dfrac{1200 \times 150 \times 75 + 250 \times 750 \times 525}{367500} = 304.6 \ mm$

$$I = \frac{1200 \times 150^3}{12} + 1200 \times 150 \times (304.6 - 75)^2 + \frac{250 \times 750^3}{12} + 250 \times 750 \times (525 - 304.6)^2$$

$I = 2.77 \times 10^{10} \ mm^4$

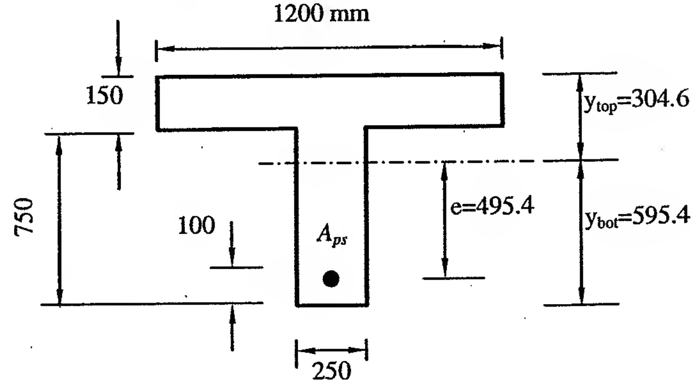
$$y_{top} = y = 304.6 \ mm$$

$$y_{bot} = t - y_{top} = (150 + 750) - 304.6 = 595.4 \ mm$$

$$Z_{top} = \frac{I}{y_{top}} = \frac{2.77 \times 10^{10}}{304.6} = 91.02 \times 10^6 \ mm^3$$

$$Z_{bot} = \frac{I}{y_{bot}} = \frac{2.77 \times 10^{10}}{595.4} = 46.6 \times 10^6 \ mm^3$$

$$w_{o.w} = \gamma_c \times A = 25 \times \frac{367500}{1000000} = 9.1875 \ kN/m'$$



Since the beam is prestressed with unbonded tendons, the beam is categorized as case B. The allowable compression stress f_{ce} and the allowable tension stress f_{te} at full service load (case B) can be obtained using the compressive strength of concrete f_{cu} as follows:

$$f_{ce} = 0.40 \times f_{cu} = 0.40 \times 45 = -18.0 \text{ N/mm}^2$$

$$f_{te} = 0.44\sqrt{f_{cu}} = 0.44\sqrt{45} = 2.95\ N/mm^2$$

Step 2: Calculate prestressing force at service load stage

$$w_{total} = w_{ow} + w_{DL} + w_{LL} = 9.1875 + 8 + 20 = 37.19\, kN/m'$$

The total moment at the mid-span equals:

$$M_{tot} = \frac{w_{total}\, L^2}{8} = \frac{37.19 \times 15^2}{8} = 1045.9\ \ kN.m$$

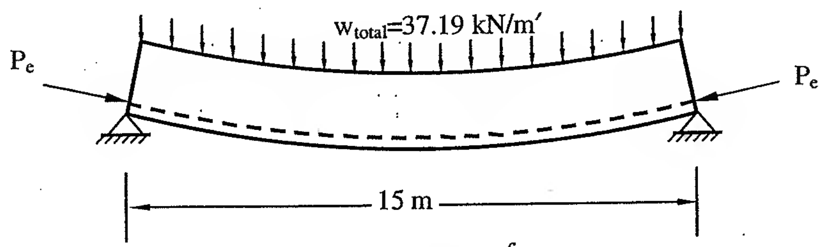
$$e = y_{bot} - \text{cover} = 595.4 - 100 = 495.4\ mm$$

The prestressing force should satisfy the allowable stresses at the top and bottom fibers. The first value for the prestressing force is obtained from the condition at the bottom fibers, which is given by:

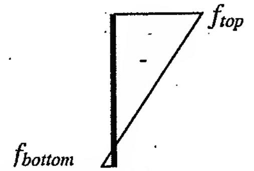
$$f_{bottom} = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{bot}} + \frac{M_{total}}{Z_{bot}}$$

$$+2.952 = -\frac{P_e \times 1000}{367500} - \frac{P_e \times 1000 \times 495.4}{46.6 \times 10^6} + \frac{1045.9 \times 10^6}{46.6 \times 10^6}$$

P_{e1}=1460.3 kN



Stresses at mid section at service load



The second value for the prestressing force is obtained from the critical condition at the top fibers, which is given by:

$$f_{top} = -\frac{P_e}{A} + \frac{P_e \times e}{Z_{top}} - \frac{M_{total}}{Z_{top}}$$

$$-18.0 = -\frac{P_e \times 1000}{367500} + \frac{P_e \times 1000 \times 495.4}{91.02 \times 10^6} - \frac{1045.9 \times 10^6}{91.02 \times 10^6}$$

P_{e2} = -2391 kN (negative)

The previous load value is negative and means that the prestressing force is tension and therefore it is rejected. Thus, the prestressing force is taken equal to 1460.3 kN (still the principle of choosing the bigger is valid).

Step 3: Check stress at transfer

At transfer, the self-weight of the beam is the only

$$M_{ow} = \frac{w_{ow}\, L^2}{8} = \frac{9.1875 \times 15^2}{8} = 258.4\ kN.m$$

$$P_e = P_i\,(1 - losses)$$

$$1460.3 = P_i\ (1 - 0.16)$$

P_i= 1738 kN

The allowable stress at transfer is given by:

$$f_{ci} = 0.45 \times f_{cui} = 0.45 \times 34 = -15.3 \text{ N/mm}^2$$

$$f_{ti} = 0.22\sqrt{f_{cui}} = 0.22\sqrt{34} = 1.283\ N/mm^2$$

$$f_{bottom} = -\frac{P_i}{A} - \frac{P_i \times e}{Z_{bot}} + \frac{M_{ow}}{Z_{bot}}$$

$$f_{bottom} = -\frac{1738 \times 1000}{367500} - \frac{1738 \times 1000 \times 495.4}{46.6 \times 10^6} + \frac{258.4 \times 10^6}{46.6 \times 10^6}$$

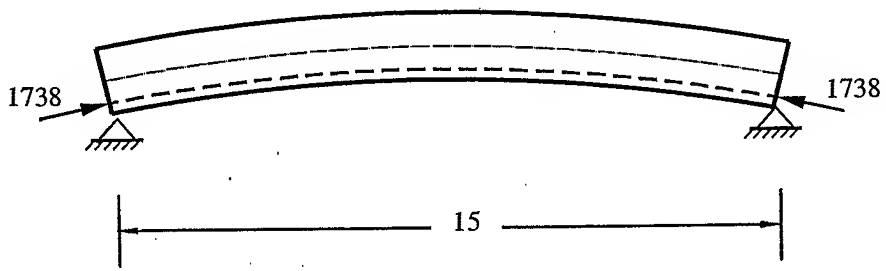
$$f_{bottom} = \left|-17.67\right| \text{ N/mm}^2 > \left|-15.3\right| \text{ N/mm}^2\ (unsafe)$$

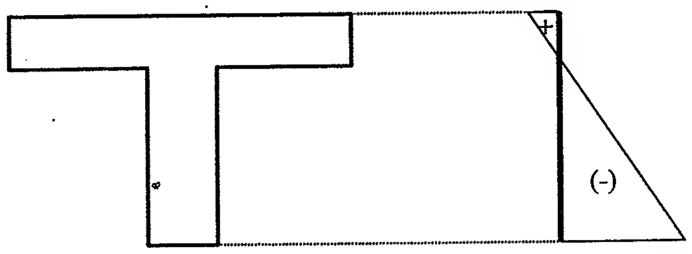
$$f_{top} = -\frac{P_i}{A} + \frac{P_i \times e}{Z_{top}} - \frac{M_{ow}}{Z_{top}}$$

$$f_{top} = -\frac{1738 \times 1000}{367500} + \frac{1738 \times 1000 \times 495.4}{91.02 \times 10^6} - \frac{258.4 \times 10^6}{91.02 \times 10^6}$$

$$f_{top} = +1.89 \ \text{N/mm}^2 > +1.283 \ \text{N/mm}^2 \ (unsafe)$$

The beam is considered **unsafe** because both the extreme fiber stresses exceed the allowable stresses.



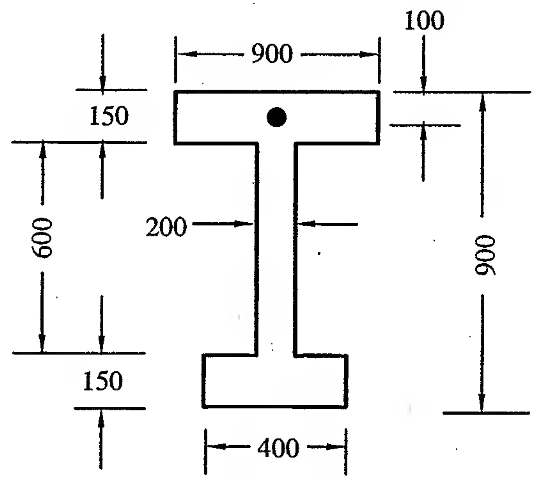


Stresses at transfer

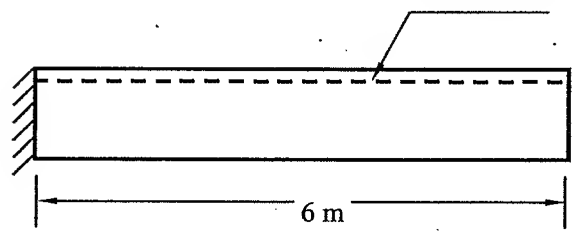
Example 8.4

The cantilever beam shown in figure supports a balcony in a stadium. It is required to determine the cross sectional area for low relaxation stress relieved strands based on the allowable stresses at transfer. Assume that f_{pu}= 1900 N/mm^2, f_{cu}=50 N/mm^2, f_{cui}= 38 N/mm^2.

Calculate the service load stress if the unfactored superimposed dead load is 8.0 k N/m′ and the unfactored live load is 14 kN/m′.Assume losses of 15% and that the beam is considered as case B.



Beam cross section



Beam elevation

Solution

Step 1: Calculate section properties

$$A = 900\times150 + 600\times200 + 400\times150 = 315000 \ mm^2$$

Since the section is not symmetrical, one should calculate the C.G.

$$y = \frac{900\times150\times75 + 600\times200\times450 + 400\times150\times825}{315000} = 360.7 \ mm$$

$$I = \frac{900\times150^3}{12} + 900\times150\times(360.7-75)^2 + \frac{200\times600^3}{12} + 200\times600\times(450-360.7)^2$$
$$+\frac{400\times150^3}{12} + 400\times150\times(825-360.7)^2 = 2.887\times10^{10} \ mm^4$$

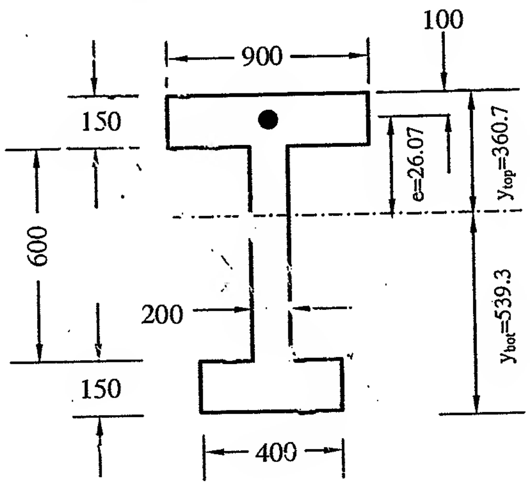
$y_{top} = y = 360.7$ mm

$$y_{bot} = t - y = (150+600+150) - 360.7 = 539.3 \ mm$$

$$Z_{top} = \frac{I}{y_{top}} = \frac{2.887\times10^{10}}{360.7} = 80.053\times10^6 \ mm^3$$

$$Z_{bot} = \frac{I}{y_{bot}} = \frac{2.887\times10^{10}}{539.3} = 53.546\times10^6 \ mm^3$$

$$w_{o.w} = \gamma_c \times A = 25 \times \frac{315000}{1000000} = 7.875 \ kN/m'$$



The allowable compression stress f_{ci} and the allowable tension stress f_{ti} can be obtained using the compressive strength of concrete at the time of pre-stressing f_{cui} as follows:

$f_{ci} = 0.45 \text{ x } f_{cui} = 0.45 \text{ x } 38 = -17.1 \text{ N/mm}^2$

$$f_{ti} = 0.22\sqrt{f_{cui}} = 0.22\sqrt{38} = 1.365 \ N/mm^2$$

Step 2: Calculate the initial prestressing force at transfer

At transfer, the self-weight is the only load that acts on the beam at transfer.

$$M_{ow} = \frac{w_{ow}\,L^2}{2} = \frac{7.875\times6^2}{2} = 141.75 \ kN.m$$

The eccentricity of the cable from the C.G, of the section equals:

$$e = y_{top} - \text{cover} = 360.7 - 100 = 260.7 \ mm$$

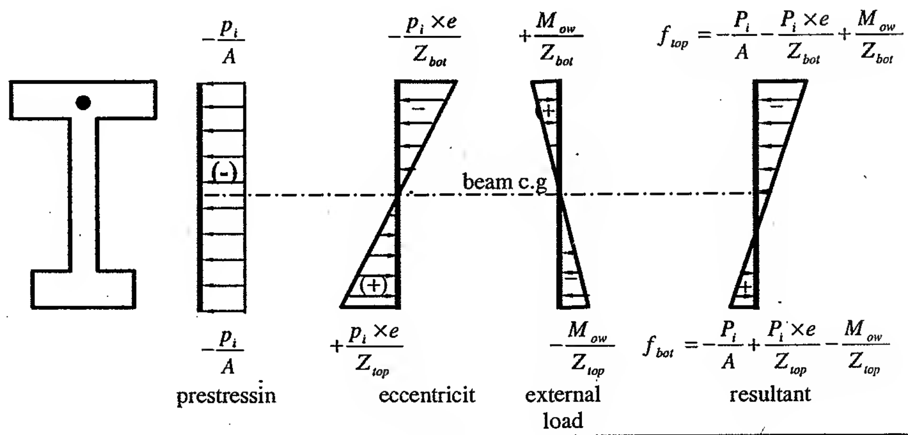
The first prestressing force is obtained from the critical condition at the bottom fibers, which is tension as shown in figure and is given by:

$$f_{bottom} = -\frac{P_i}{A} + \frac{P_i\times e}{Z_{bot}} - \frac{M_{ow}}{Z_{bot}}$$

Assuming that P_i is in kN, and applying in the previous equation, one gets:

$$+1.365 = -\frac{P_{i1}\times1000}{315000} + \frac{P_{i1}\times1000\times260.7}{53.546\times10^6} - \frac{141.75\times10^6}{53.546\times10^6}$$

P_{i1}=2367.7 kN



The second prestressing force is obtained from the critical condition at the top fibers, which is compression as shown in figure and is given by:

$$f_{top} = -\frac{P_i}{A} - \frac{P_i \times e}{Z_{top}} + \frac{M_{ow}}{Z_{top}}$$

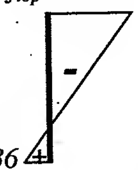
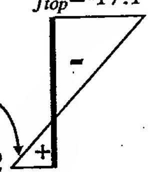
$$-17.1 = -\frac{P_{i2} \times 1000}{315000} - \frac{P_{i2} \times 1000 \times 260.7}{80.053 \times 10^6} + \frac{141.75 \times 10^6}{80.053 \times 10^6}$$

$P_{i2} = 2934.2$ kN

The allowable stresses at the bottom and top fibers of the section are -17.1 N/mm^2 and + 1.356 N/mm^2, respectively. The calculation of the prestressing force should be based on the smaller of the two values obtained from the analyses of stresses at the bottom and top fibers (P_{i1}=2367.7 kN.m). This is attributed to the fact that at the case of transfer the self-weight moment produces stresses that are of opposite sign compared to the allowable stresses.

Final design **$P_i = P_{i1}$=*2367.7 kN***

A summary of the calculations is given in the following table.

Item	bottom condition	Top condition
Allowable stress N/mm^2	+1.36	-17.1
Actual stress	+1.36	-17.10
Force (kN) N/mm^2	2367.7	2934.2
Stress at opposite fiber N/mm^2	-13.46	2.32
Condition	safe	unsafe
	f_{top}=-13.46  f_{bottom}=+1.36	f_{top}=-17.1 unsafe  f_{bottom}=-2.32

Step 3: Calculate the required prestressing steel

For low relaxation stress relieved strands, the code specify the yield stress as:

$$f_{py} = 0.9 \times f_{pu} = 0.9 \times 1900 = 1710 \text{ N/mm}^2$$

The allowable prestressing stress at time of transfer is given by:

$$f_p = smaller\ of \begin{cases} 0.70\ f_{pu} = 0.7 \times 1900 = 1330\ N/mm^2 \\ 0.8\ f_{py} = 0.8 \times 1710 = 1368\ N/mm^2 \end{cases}$$

f_{pi}=1330 N/mm^2

$$A_{ps} = \frac{P_i}{f_{pi}} = \frac{2367.7 \times 1000}{1330} = 1780\ mm^2$$

Step 4: Calculate stresses at the service load stage

The total load is the summation of dead and live loads:

$$w_{total} = w_{ow} + w_{DL} + w_{LL} = 7.875 + 8 + 14 = 29.875 kN\ /m'$$

The total moment at the support equals:

$$M_{tot} = \frac{w_{tot}\ L^2}{2} = \frac{29.875 \times 6^2}{2} = 537.8\ \ kN.m$$

The effective prestressing force at the time of applying all service loads is:

P_e = (1-losses) P_i

P_e = (1-0.15) x 2367.7 = 2012.5 kN

The allowable stresses at service load stage for stage B equals;

$$f_{te} = 0.44\sqrt{f_{cu}} = 0.44\ \sqrt{50} = 3.13\ N\ /mm^2$$

$$f_{ce} = 0.40 f_{cu} = 0.40 \times 50 = -20\ N\ /mm^2$$

$$f_{top} = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{top}} + \frac{M_{total}}{Z_{top}}$$

$$f_{top} = -\frac{2012.5\ \times 1000}{315000} - \frac{2012.5\ \times 1000 \times 260.7}{80.053 \times 10^6} + \frac{537.8 \times 10^6}{80.053 \times 10^6}$$

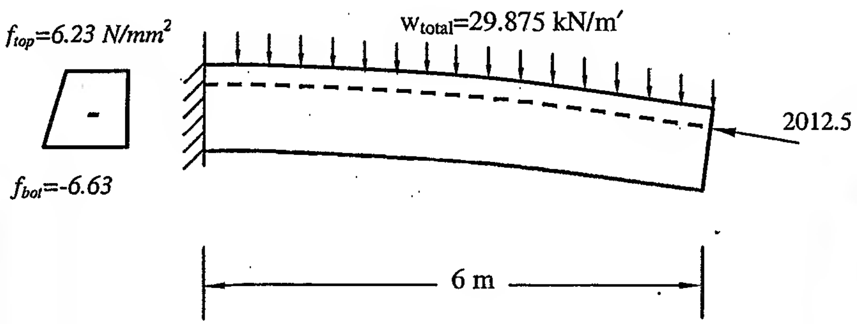
$$f_{top} = |-6.23|\ N\ /mm^2\ \ldots < |-20|\ (safe)$$

$$f_{bottom} = -\frac{P_e}{A} + \frac{P_e \times e}{Z_{bot}} - \frac{M_{total}}{Z_{bot}}$$

$$f_{bottom} = -\frac{2012.5 \times 1000}{315000} + \frac{2012.5 \times 1000 \times 260.7}{53.546 \times 10^6} - \frac{537.8 \times 10^6}{53.546 \times 10^6}$$

$$f_{bottom} = -6.63\ N/mm^2 \ \ldots < -20 (safe)$$

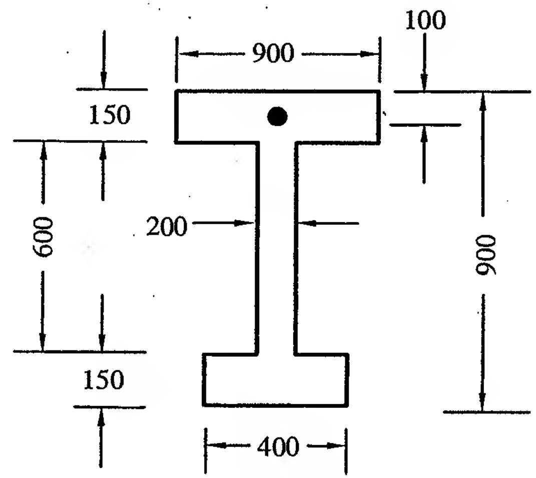
The beam is considered safe because non of the extreme fiber stresses exceed the allowable stresses.



Example 8.5

Assuming that the cantilever beam shown in Example 8.4 may be categorized as case D and based on the service load stage, calculate the maximum value of the live load that can be added to the beam.

Data:
f_{pu} =19 N/mm²
f_{cu} =50 N/mm²
$f_{cu\,i}$= 38 N/mm²
Losses 15%
L= 6.0 m



Beam cross section

Solution

Step 1: Calculate the maximum moment at service load stage

$$w_{tot} = w_{ow} + w_{DL} + w_{LL} + w_{add} = 7.875 + 8 + 14 + w_{add} = 29.875 + w_{add}$$

The total moment at the support equals:

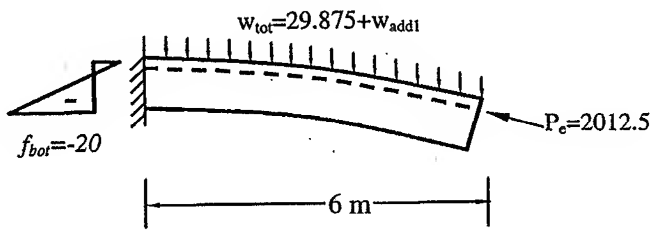
$$M_{total} = \frac{w_{tot}\ L^2}{2} = \frac{w_{tot} \times 6^2}{2} = 18\ w_{tot}$$

The allowable stresses at service load stage for case D equals:

$$f_{te} = 0.85\sqrt{f_{cu}} = 0.85\ \sqrt{50} = 6.01\,N\ /mm^2$$

$$f_{ce} = 0.40\,f_{cu} = 0.40 \times 50 = -20\ N\ /mm^2$$

The first critical moment is obtained from the critical stress at the bottom fibers.

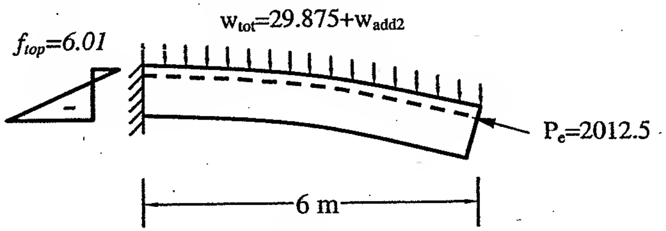


$$f_{bottom} = -\frac{P_e}{A} + \frac{P_e \times e}{Z_{bot}} - \frac{M_{total}}{Z_{bot}}$$

$$-20 = -\frac{2012.5\ \times 1000}{315000} + \frac{2012.5\ \times 1000 \times 260.7}{53.546 \times 10^6} - \frac{M_{total1} \times 10^6}{53.546 \times 10^6}$$

$M_{total,1}$=1253.5 kN.m

The second critical moment is obtained from the critical stress at the top fibers.



$$f_{top} = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{top}} + \frac{M_{total}}{Z_{top}}$$

$$+6.01 = -\frac{2012.5\ \times 1000}{315000} - \frac{2012.5\ \times 1000 \times 260.7}{80.053 \times 10^6} + \frac{M_{total,2} \times 10^6}{80.053 \times 10^6}$$

$M_{total,2}$=1517.2 kN.m

The chosen moment is the smaller of the two moments, because it will result in opposite fiber stress that is less than the maximum allowable stress (see the comparison table below).

M_{total} =$M_{total,1}$=1253.3 kN.m

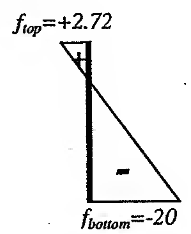
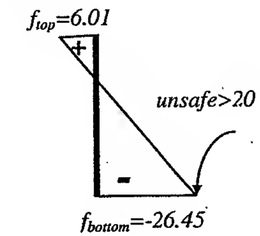
$$M_{total} = 1253.3 = \frac{w_{tot} \times 6^2}{2}$$

$$w_{tot} = 69.6\ kN\,/\,m'$$

$$w_{tot} = 69.6 = 29.875 + w_{add}$$

Thus the load that can be added to the beam equals:

w_{add} = 39.8 kN/m′

Item	bottom condition	Top condition
Actual stress N/mm^2	-20.00	6.01
w $_{tot}$ (KN/m′)	69.64	84.3
w_{add} (KN/m′)	39.76	54.4
Stress at opposite fiber* N/mm^2	2.72	-26.45
Condition	safe	unsafe
		

* Calculations are not shown.

8.3 Flexural Strength of Prestressed Beams

8.3.1 Introduction

The main objective of the prestressing procedure is to produce a member that is almost free of cracks at service loads. However, the satisfaction of concrete and steel stress limits at service loads does not necessarily ensure adequate strength and does not provide a reliable indication of either the actual strength or the safety of a structural member. It is important to consider the non-linear behavior of the member in the ultimate stage to ensure that it has an adequate structural capacity.

If external dead and live loads are applied to the prestressed concrete member shown in Fig. 8.5, various loading stages are noted. A typical loading history along with stress distribution is given in Fig. 8.5 and Fig 8.6. The following is a summary of loading stages:

1. The initial prestressing force P_i is applied to the beam and is transmitted to the concrete together with the beam self-weight. This is usually called the *transfer stage.*
2. At the *service load stage,* the full superimposed dead load is applied. In most cases, this will produce compression stresses all over the cross section as shown in Fig. 8.6. Most of the long-term losses including creep and shrinkage have occurred leading to net prestressing force of P_e.
3. If the load is further increased due to the introduction of the live loads, the upward deflection due prestressing is canceled (or balanced) by the applied external loads, and the resultant deflection is equal to zero. The stress over the cross section is uniform and equals P/A. This is called *the balanced stage.*
4. A further increase in loading will produce tension at the bottom fibers and zero stress at the prestressing steel level. This is called *the decompression stage.*
5. At this stage the beam probably is at *the full service load stage.* If the loads are furthered increased the developed tension stresses at the bottom fiber reach the concrete tensile strength f_{ctr}. At this stage the beam stats to crack and the inertia of the beam drops dramatically. This called *the cracking stage.*
6. Finally, overloading of the member occurs leading to the ultimate condition of the beam and the final collaps of the member.

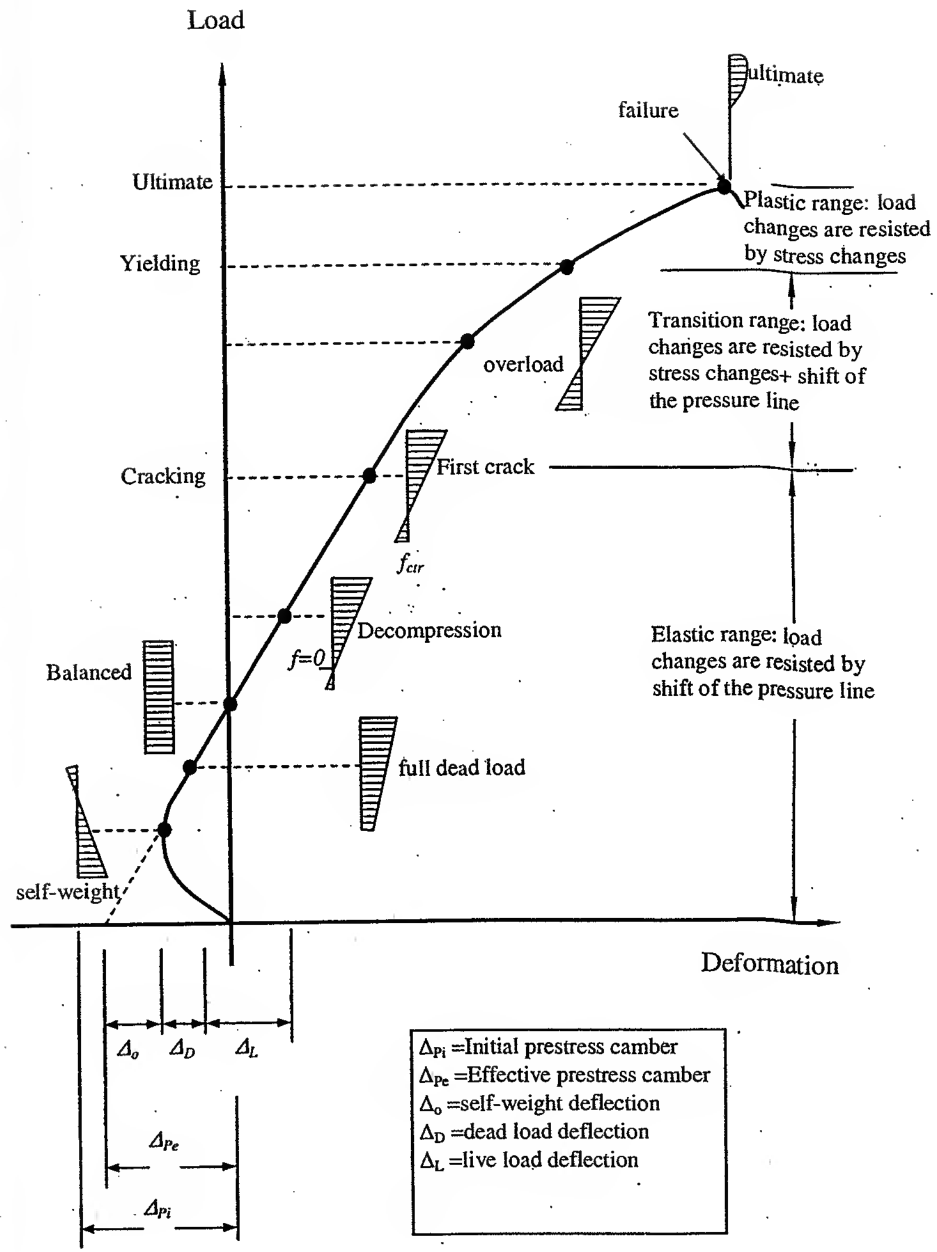


Fig. 8.5 Load-deformation curve at different loading stages for prestressed beams

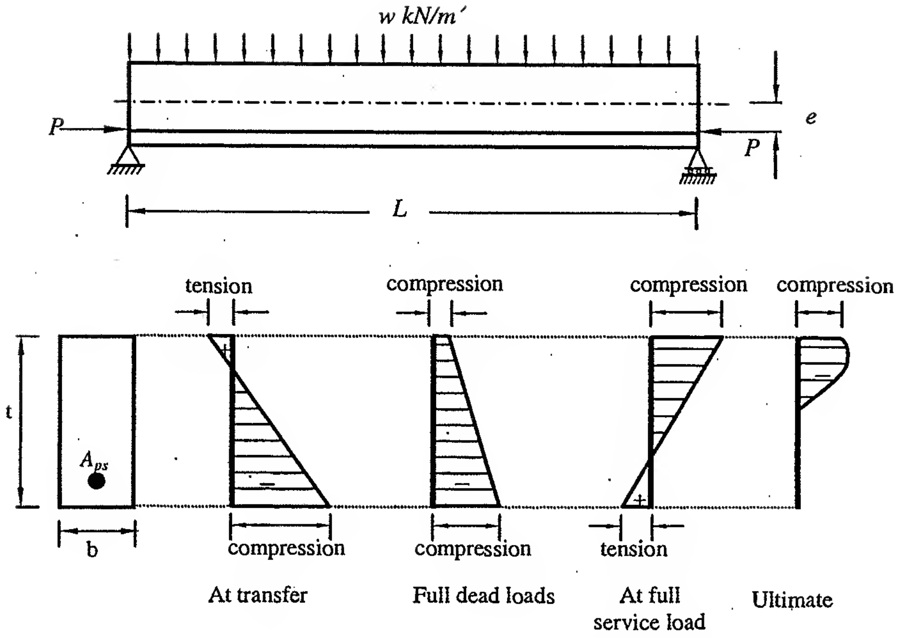


Fig. 8.6 Stress distributions at various stages

During the loading of a prestressed concrete beam, the neutral axis starts to rise at a relatively uniform rate as the external loads increase. This behavior continues until the beam cracks. After the cracking load has been exceeded, the neutral axis rate of movement decreases as additional loads are applied, and a significant increase in the stress in the prestressing tendon begins to take place. This change in action continues until the applied loads are entirely resisted by proportional changes in the internal forces, just as the ordinary reinforced concrete. At the ultimate stage, additional moment capacity is created by an increase in the magnitude of the components of the internal couple rather than by increase in the arm of the internal couple as shown in Fig. 8.5.

The fact that the load is carried at the ultimate by different actions that are significantly different than those in the elastic range makes the ultimate strength calculations essential for all prestressed members to ensure that adequate safety exists.

8.3.2 Calculations of the Ultimate Moment Capacity

The ECP 203 requires that the moment due to all factored loads not to exceed the ultimate capacity of the section. The ultimate moment capacity M_u is calculated in a manner similar to that adopted for ordinary non-prestressed beams. If a prestressed beam is loaded up to failure, the distribution of the stresses becomes nonlinear and the strain in the steel continues to increase with noticeable large deformations.

The code assumes that the final failure occurs when the concrete strain reaches 0.003. Since the stress-strain curve of the prestressing steel does not contain a well defined yield plateau, the stress in the tendons continues to increase beyond the yield point at a reduced slope. The final steel stress at ultimate f_{ps} must be predicted in order to compute the ultimate capacity of the beam. Referring to Fig. 8.7, equilibrium of the internal forces gives

$$C_c + Cs = T_p + T \quad \text{.......... (8.10)}$$

$$\frac{0.67\ f_{cu}\ b\ a}{1.5} + \frac{A'_s \times f_y}{1.15} = \frac{A_{ps} \times f_{ps}}{1.15} + \frac{A_s \times f_y}{1.15} \quad \text{.......... (8.11)}$$

in which

C_c is the compression force in the concrete.
C_s is the compression force in the non-prestressing steel
T_p is the tension force in the prestressing cables
T is the tension force in the non-prestressing steel

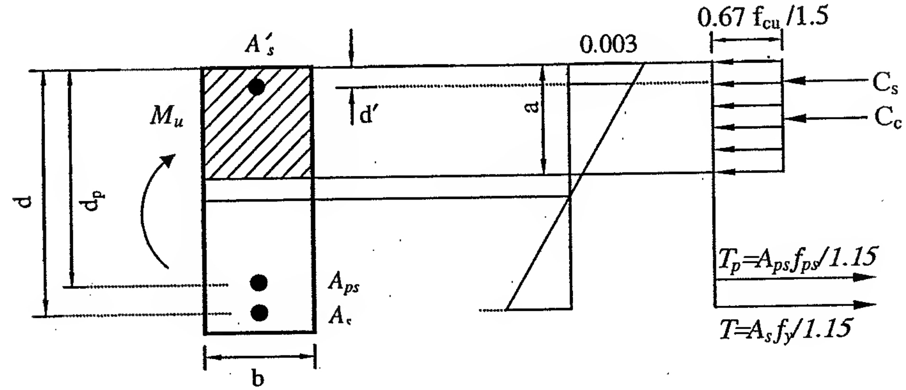


Fig. 8.7 Analysis of a prestressed section at ultimate

The depth of the compression stress block (a) is determined from Eq. 8.11. For rectangular sections with prestressing steel in the tension side only, the ultimate moment is given by:

$$M_u = \frac{A_{ps} \times f_{ps}}{1.15}\left(d_p - \frac{a}{2}\right) \quad \text{(8.12)}$$

where d_p is the distance from the prestressed reinforcement to the compression fiber.

For sections reinforced with both non-prestressing steel (tension and compression) and prestressing steel as the one shown in Fig. 8.7, the ultimate moment M_u is given by:

$$M_u = \frac{A_{ps} \times f_{ps}}{1.15}\left(d_p - \frac{a}{2}\right) + \frac{A_s \times f_y}{1.15}\left(d - \frac{a}{2}\right) + \frac{A_s' \times f_y}{1.15}\left(\frac{a}{2} - d'\right) \quad \text{(8.13)}$$

Total amount of prestressing steel and non-prestressing steel reinforcement shall be adequate to develop at least 1.2 the cracking moment M_{cr} given by the ECP 203 (explained in details in Chapter 9). Exception is made for flexural members with shear and flexural strength that exceed twice the ultimate design moment and for unbonded post-tensioned slabs.

In all of the above equations, prestressing steel stress at ultimate f_{ps} is unknown because the stress-strain curve is non-linear as shown in Fig. 8.8a.

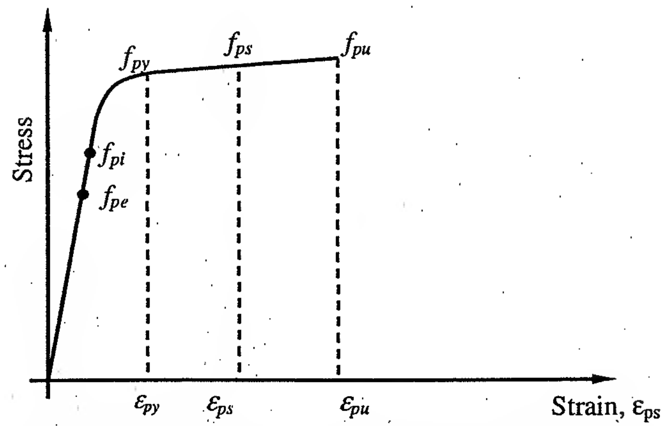


Fig. 8.8a Stress-Strain curve for prestressing reinforcement

8.3.3 Calculation of Prestressing Steel Stress at Ultimate f_{ps}

The calculation of f_{ps} depends on whether the tendons are bonded or unbonded.

8.3.3.1 Calculation of f_{ps} in Bonded Tendons

The ECP 203 provides two different methods for calculating the steel stress at ultimate for bonded tendons. These two methods are:

- The strain compatibility method.
- The simplified method.

A: Calculation of f_{ps} Using the Strain Compatibility

At transfer, tendon stress is equal to the initial prestressing stress f_{pi} and the external moment equals the self weight moment. As creep and shrinkage occur, the concrete shortens and the stress in the tendon drops to a value less than f_{pi} as shown in Fig. 8.8b. When service loads are applied, the beam deflects downward and the cable elongate. Since the developed bending strains are small compared to the strains in the prestressing steel, only a slight increase in the tendon force occurs. The increased internal moment (from M_{ow} to M_s) required to resist the moment produced by the live loads, is created by an increase in the arm of the internal couple (the neutral axis moves upwards) without any significant increase in tendon stress. When the tensile stress in the concrete equals the tensile strength (occurs at M_{cr}), cracking occurs and the tendon stress increases by the amount equal to the tension formerly carried by the concrete. If the load is further increased, the strain in the prestressing steel increases at an accelerated rate and the beam undergoes large deflections and reaches f_{ps} at the ultimate condition (M_u). For under-reinforced beams, the final failure occurs when the maximum compressive strain in the concrete reaches a value of 0.003.

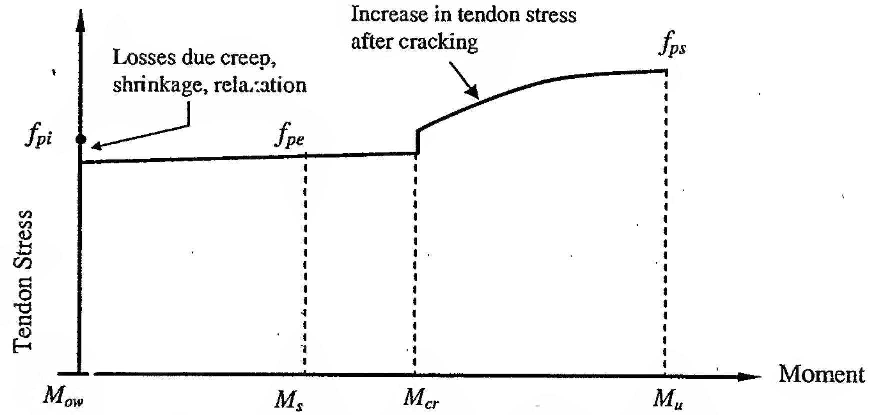


Fig. 8.8b Tendon stress as a function of the applied load

The ECP 203 states that the strain in the prestressing steel at ultimate ε_{ps} can be computed as the sum of three components as follows (refer to Fig. 8.9).

$$\varepsilon_{ps} = \varepsilon_{pe} + \varepsilon_{ce} + \varepsilon_{pc} \quad \text{.............................} (8.14)$$

Where

ε_{ps} = the strain in the prestressing steel due to prestressing after computing all losses.

ε_{ce} = the strain in concrete at the level of the prestressing steel due to prestressing after computing all losses (*decompression strain*).

ε_{pc} = the strain in the prestressing steel determined using equilibrium of forces.

Calculation of ε_{ps}

The first component ε_{pe} is the strain due to prestressing, and can be computed simply by applying Hook's law as follows:

$$\varepsilon_{pe} = \frac{f_{pe}}{E_p} = \frac{P_e / A_{ps}}{E_p} \quad \text{.............................} (8.15)$$

where E_p is the steel modulus of elasticity, P_e is the effective prestressing forceand A_{ps} is the cross sectional area of the prestressing steel.

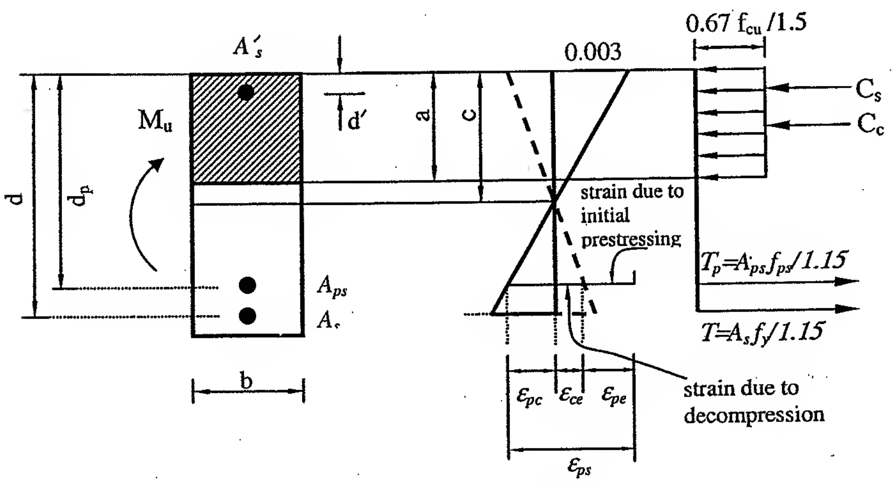


Fig. 8.9 Strain and stress distributions at ultimate for a prestressed beam

Calculation of ε_{ce}

At transfer, the bottom fibers are subjected to compressive stresses due to the existence of the prestressing steel. After applying the service load, the stresses at the bottom fibers decrease gradually until reaching zero. The strain required for the concrete stress to reach zero at the level of prestressing steel is called the *decompression strain* ε_{ce} as shown in Fig. 8.10. From compatibility of the strains, the strain in the concrete must be equal that of the prestressing steel.

$$\varepsilon_{ce} = \frac{f_{ce}}{E_c} \quad \text{.......................................} (8.16)$$

where

E_c is the concrete modulus of elasticity at full strength and f_{ce} is the stress in the concrete due to prestressing force after considering all losses and is given by Eq. 8.17.

$$f_{ce} = \frac{P_e}{A} + \frac{P_e \times e \times e}{I} \quad \text{.......................................} (8.17)$$

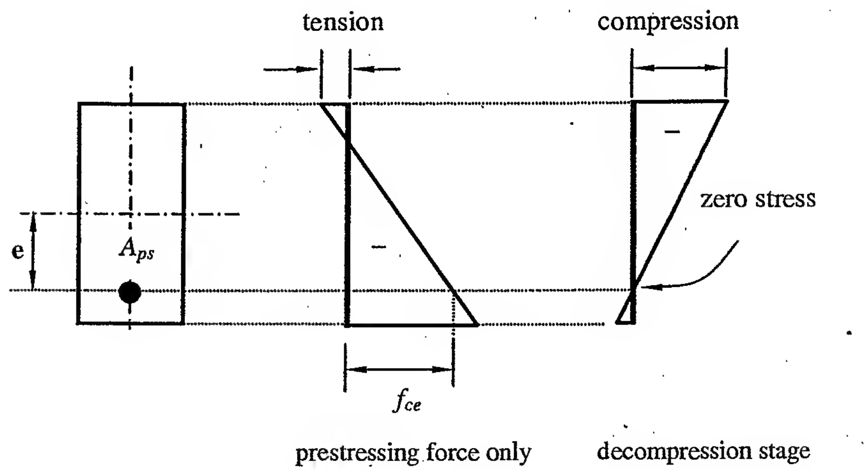


Fig. 8.10 Stresses in concrete due to prestressing and at decompression stage

Calculation of ε_{pc}

After the decompression stage the behavior of the prestressed beam becomes similar to that of an ordinary reinforced concrete beam as shown in Fig. 8.9. From the decompression stage to the ultimate stage, additional strain ε_{pc} starts to develop in the prestressed steel reinforcement. Since the maximum concrete strain at failure stated in the ECP 203 is 0.003, the corresponding strain in the steel above the decompression stage can be calculated from:

$$\varepsilon_{pc} = 0.003 \times \frac{d_p - c}{c} \qquad (8.18)$$

where d_p is the depth of the prestressing steel, c is the depth of the neutral axis obtained from the equilibrium of all forces acting on the section including non-prestressed steel using Eq. 8.11.

The material strength reduction factor for the prestressing steel (γ_{ps}) is taken as 1.15. The corresponding stress at ultimate f_{ps} can be obtained from the idealized stress-strain curve suggested by the code as given by Eq. 8.20 and Fig. 8.11.

$$\frac{f_{ps}}{\gamma_{ps}} = \frac{f_{py}}{\gamma_{ps}} + \frac{(\varepsilon_{ps} - \varepsilon_{py}/\gamma_{ps})}{(\varepsilon_{pu} - \varepsilon_{py}/\gamma_{ps})} \times \left(\frac{f_{pu}}{\gamma_{ps}} - \frac{f_{py}}{\gamma_{ps}}\right) \qquad (8.19)$$

$$f_{ps} = f_{py} + \frac{(\varepsilon_{ps} - \varepsilon_{py}/\gamma_{ps})}{(\varepsilon_{pu} - \varepsilon_{py}/\gamma_{ps})} \times (f_{pu} - f_{py}) \ldots\ldots \quad \varepsilon_{ps} > \varepsilon_{py}/1.15 \quad (8.20a)$$

If the strain ε_{ps} is less than the yielding strain ($\varepsilon_y/\gamma_{ps}$), then f_{ps} equals:

$$f_{ps} = \varepsilon_{ps} \times E_p \quad \ldots\ldots \quad \varepsilon_{ps} \le \varepsilon_{py}/1.15 \quad (8.20b)$$

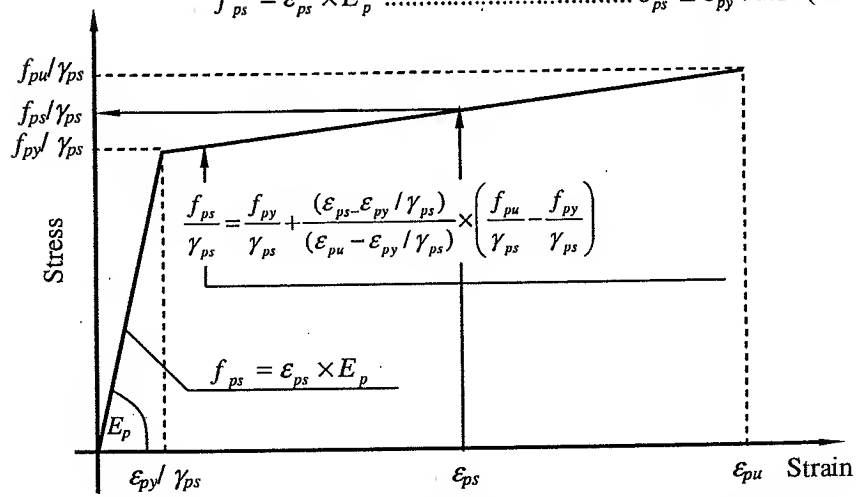


Fig. 8.11 Idealized stress-strain curve for prestressing steel

B: Calculation of f_{ps} Using Empirical Equations

The previous procedure for calculating the prestressing steel stress at failure is reasonably accurate but it is time consuming. The ECP 203 states that the stress in the ***bonded*** prestressing steel at failure can be predicted by the following empirical formula only if f_{pe}>0.5 f_{pu} as follows:

$$f_{ps} = f_{pu}\left[1 - \eta_p\left(\mu_p \frac{f_{pu}}{0.8 f_{cu}} + \frac{d}{d_p}(\omega - \omega')\right)\right] \qquad (8.21)$$

where

$$\omega = \mu\left(\frac{f_y}{0.80 f_{cu}}\right) \qquad \omega' = \mu'\left(\frac{f_y}{0.80 f_{cu}}\right)$$

$$\mu_p = \frac{A_{ps}}{b \times d_p} \qquad \mu = \frac{A_s}{b \times d} \qquad \mu' = \frac{A'_s}{b \times d}$$

η_p is a factor for the type of the prestressing steel

= 0.68 for $f_{py}/f_{pu} \ge 0.80$

= 0.5 for $f_{py}/f_{pu} \ge 0.85$

= 0.35 for $f_{py}/f_{pu} \ge 0.90$

d is the depth of the non-prestressed steel

d_p is the depth of the prestressed steel

b is the width of the compression zone. If the beam has a flange, use the width of the flange (B). However, if the neutral axis falls in the web use the width of the web (b).

For the influence of the compression steel to be considered, two limits imposed by the code must be satisfied:

$$1. \left(\mu_p \frac{f_{pu}}{0.8 f_{cu}}\right) + \frac{d}{d_p}(\omega - \omega') \ge 0.17 \qquad (8.22)$$

$$2. \; d' \le 0.15\, d_p \qquad (8.23)$$

When the term $\left[\left(\mu_p f_{pu}/0.8 f_{cu}\right) + d/d_p(\omega - \omega')\right]$ in Eq. 8.21 is small, the compression reinforcement does not develop its yield strength and Eq. 8.21 becomes un-conservative. This is the reason why the term

$\left[\left(\mu_p\, f_{pu}/0.8 f_{cu}\right)+d/d_p\left(\omega-\omega'\right)\right]$ in Eq. 8.21 may not be taken less than 0.17, if the compression reinforcement is taken into account when computing f_{ps}.

If the conditions given by Eq. 8.22 and Eq. 8.23 are not met, the compression reinforcement contribution is assumed to be zero (ω'=0) and in this case the term $\left[\left(\mu_p\, f_{pu}/0.8 f_{cu}\right)+d/d_p\left(\omega-\omega'\right)\right]$ may be less than 0.17 and an increased value of f_{ps} is obtained. However, the contribution of the compression steel in calculations of the ultimate moment (Eq. 8.13) should be considered.

8.3.3.2 Calculation of f_{ps} for Unbonded Tendons

The grouting of the prestressing ducts is always a recommended practice. However, in some situations such as in two-way slab systems or in voided slabs, it is difficult to perform grouting operation in the ducts because the thickness of the concrete section is small.

Members with unbonded strands lack the bond between the concrete and the prestressing steel and accordingly strain compatibility method cannot be used. Therefore, it is clear that the expressions presented for stress in prestressed bonded steel is not applicable for unbonded steel. However, Eq. 8.12 and Eq 8.13 for the calculation of the ultimate moment M_u are still valid since they are derived from the satisfaction of the equilibrium conditions.

The ECP 203 presents the following set of expressions to estimate f_{ps} in unbonded prestressing steel.

- For members with unbonded tendons having a span-to-depth ratio of 35 or less (applies to most beams):

$$f_{ps}=f_{pe}+70+\left(\frac{f_{cu}}{125\,\mu_p}\right)\quad (\text{N/mm}^2)\ \ldots\ldots\ldots\ldots(8.24a)$$

  but not greater than f_{py} and not greater than (f_{pe}+420 N/mm^2)

- For members with unbonded tendons having a span-to-depth ratio greater than 35 or less (applies to most slabs):

$$f_{ps}=f_{pe}+70+\left(\frac{f_{cu}}{375\,\mu_p}\right)\quad (\text{N/mm}^2)\ \ldots\ldots\ldots\ldots(8.24b)$$

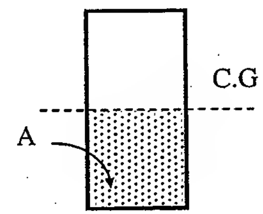
  but not greater than f_{py} and not greater than (f_{pe}+200 N/mm^2)

In order to ensure a good serviceability behavior for members with unbonded tendons, a reasonable amount of non-prestressing steel has to be used. This steel controls the flexural cracks and contributes to the ultimate moment capacity.

The minimum area of non-prestressing steel A_s for a prestressed beam with unbonded steel equals.

$$A_s=0.004\,A\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.25)$$

where A is the area of the part of the section between the tension face and the C.G. of the beam as shown in the figure below.



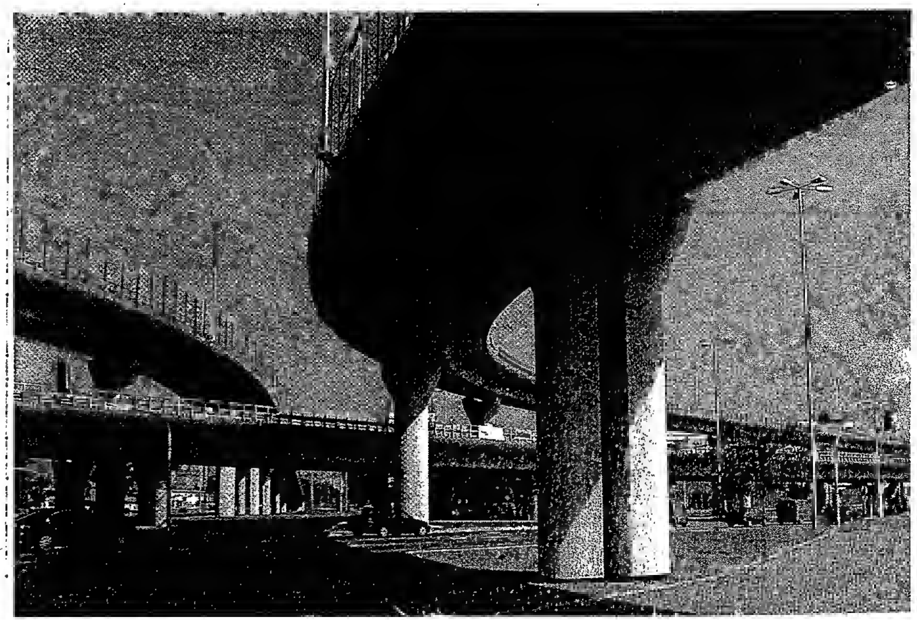


Photo 8.3 Prestressed concrete bridge with multi levels

8.3.4 Maximum Limits for the Areas of Prestressing and non-prestressing Reinforcing Steel

The amount of steel in prestressed members should be limited to ensure ductile failure (similar to c/d limitation for ordinary concrete members). The limitation rarely presents a problem for members with reasonable amount of prestressing steel. The reinforcement index for prestressing and non-prestressing steel shall be limited to:

$$\omega_t = \left(\omega_p + \frac{d}{d_p}(\omega - \omega')\right) \le 0.28 \quad (8.26)$$

Where $\omega_p = \frac{A_{ps}}{b \times d_p} \times \frac{f_{ps}}{0.80 f_{cu}}$, $\omega = \frac{A_s}{b \times d}\left(\frac{f_y}{0.80 f_{cu}}\right)$, $\omega' = \frac{A'_s}{b \times d}\left(\frac{f_y}{0.80 f_{cu}}\right)$

Equations 8.26 can be presented in form of c/d_p, where c is the neutral axis depth obtained using the equilibrium of forces and d_p is the depth of the prestressing steel. For example, for rectangular sections the substitution with the values of ω_p, ω and ω' in Eq. 8.26 gives:

$$\omega_t = \frac{1}{0.80}\left(\frac{A_{ps}}{b \times d_p} \times \frac{f_{ps}}{f_{cu}} + \frac{d}{d_p}\left(\frac{A_s}{b \times d}\left(\frac{f_y}{f_{cu}}\right) - \frac{A'_s}{b \times d}\left(\frac{f_y}{f_{cu}}\right)\right)\right) \quad (8.27)$$

$$\omega_t = \left(\frac{A_{ps} \times f_{ps} + A_s \times f_y - A'_s \times f_y}{0.80 \times b \times d_p \times f_{cu}}\right) \quad (8.28)$$

The compression force in the web equals:

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} = \frac{A_{ps} \times f_{ps}}{1.15} + \frac{A_s \times f_y}{1.15} - \frac{A'_s \times f_y}{1.15} \quad (8.29)$$

Comparison of Eq. 8.28 with Eq. 8.29 gives.

$$\omega_t = \left(\frac{compression\ force\ in\ the\ web \times 1.15}{0.8 \times b \times d_p \times f_{cu}}\right) \quad (8.30)$$

$$\omega_t = \left(\frac{0.67 \times f_{cu} \times b \times (0.8\, c)/1.5 \times 1.15}{0.80 \times b \times d_p \times f_{cu}}\right) = 0.51 \frac{c}{d_p} \quad (8.31)$$

Subsisting with code limit on ω_t of 0.28 gives:

$$\frac{c}{d_p} \le 0.55 \quad (8.32)$$

For T- sections (refer to Fig. 8.12), if the neutral axis lies inside the flange, the flange width (B) is used instead of the web width b as follows:

$$\omega_p = \frac{A_{ps}}{B \times d_p} \times \frac{f_{ps}}{0.80 f_{cu}}, \quad \omega = \frac{A_s}{B \times d}\left(\frac{f_y}{0.80 f_{cu}}\right), \quad \omega' = \frac{A'_s}{B \times d}\left(\frac{f_y}{0.80 f_{cu}}\right)$$

However, if the neutral axis lies in the web, the reinforcement is given by:

$$\omega_t = \left(\omega_{pw} + \frac{d}{d_p}(\omega_w - \omega'_w)\right) \le 0.28 \quad (8.33)$$

in which the ordinary reinforcement indices (ω_w, ω_w') should be based on the web width (b) as follows:

$$\omega_w = \frac{A_s}{b \times d}\left(\frac{f_y}{0.80 f_{cu}}\right), \quad \omega'_w = \frac{A'_s}{b \times d}\left(\frac{f_y}{0.80 f_{cu}}\right)$$

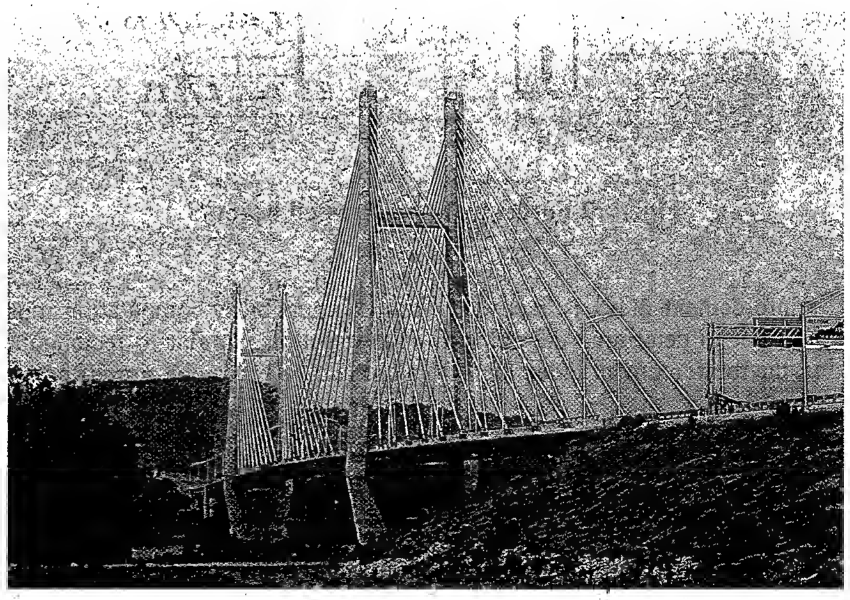


Photo 8.4 Maysville cable stayed bridge, USA.

The prestressing reinforcement index (ω_{pw}) is based on the compressed part of the web as follows:

$$\omega_{pw} = \frac{A_{pw}}{b \times d_p} \times \frac{f_{ps}}{0.80 f_{cu}} \quad \text{.......(8.34a)}$$

$$A_{pw} = A_{ps} - A_{pf} \quad \text{.......(8.34b)}$$

$$\frac{A_{pf} \times f_{ps}}{1.15} = C_f \quad \text{.......(8.34c)}$$

$$C_f = \frac{0.67 \times f_{cu} \, (B - b) \, t_s}{1.5} \quad \text{.......(8.34d)}$$

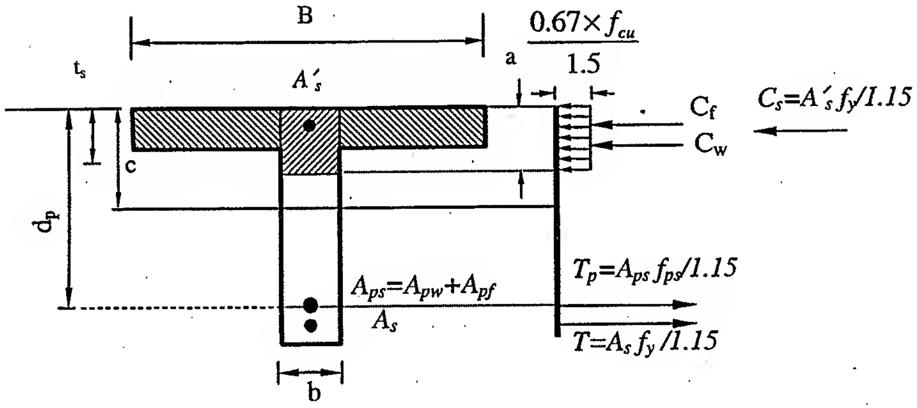


Fig. 8.12 Forces in T- prestressed beam

If the reinforcement index is exceeded, strain compatibility computation must be carried out to determine the strength of the section. Similar to rectangular sections, the reinforcement index for T-sections (Eq. 8.33) can be simplified to:

$$\frac{0.64\, a}{d_p} \le 0.28 \quad \text{.......(8.35)}$$

or $$\frac{c}{d_p} \le 0.55 \quad \text{.......(8.36)}$$

Example 8.6: M_u using the approximate equation (I-section)

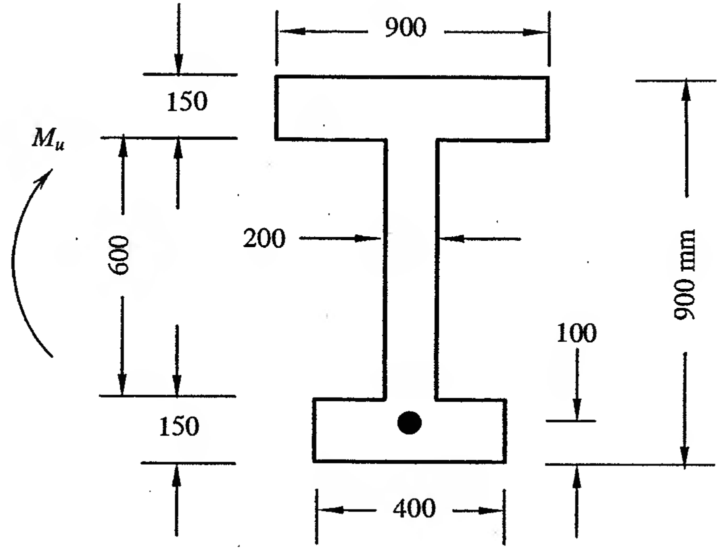
Calculate the ultimate flexural capacity (M_u) for the cross-section shown in the figure below. The beam is pretensioned with bonded tendons having an area of 1780 mm^2.

f_{pe}=1020 N/mm^2

f_{py}=1710 N/mm^2

f_{pu}=1900 N/mm^2

f_{cu}=50 N/mm^2



Beam cross-section

Solution

Step 1: Check the applicability of the approximate equation

To apply the approximate method the following equation must be satisfied

$$f_{pe} > \frac{f_{pu}}{2}$$

$\because 1020 > \frac{1900}{2}$, the approximate equation can be used to calculate f_{ps}.

Step 2: Calculate the ultimate moment

Since the section does not contain any non-prestressed reinforcement, the effective stress f_{ps} equals:

$$f_{ps} = f_{pu}\left[1-\eta_p\left(\mu_p \frac{f_{pu}}{0.80\, f_{cu}}\right)\right]$$

$$d_p = t - cover = (150+600+150)-100 = 800\ mm$$

Assuming that the neutral axis is within the flange, the prestressing reinforcement ratio should be based on the flange width of 900 mm.

$$\mu_p = \frac{A_{ps}}{B \times d_p} = \frac{1780}{900\times 800} = 0.00247$$

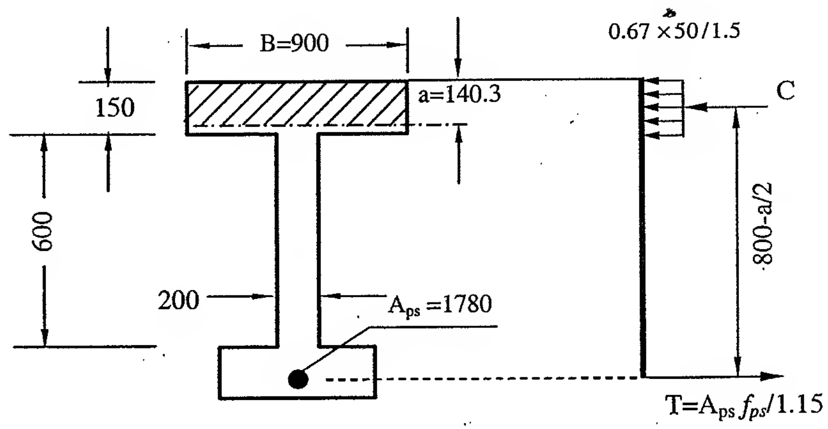
$$For\ \frac{f_{py}}{f_{pu}}\left(\frac{1710}{1900}\right) \geq 0.90 \to\to \eta_p = 0.350$$

$$f_{ps} = 1900\left[1-0.35\left(0.00247\,\frac{1900}{0.80\times 50}\right)\right] = 1821.91\ N/mm^2$$

Applying equilibrium equation, and assuming that the neutral axis within the flange

$$\frac{0.67\times f_{cu}\times B\times a}{1.5} = \frac{A_{ps}\times f_{ps}}{1.15} \quad \rightarrow \quad \frac{0.67\times 50\times 900\times a}{1.5} = \frac{1780\times 1821.91}{1.15}$$

a = 140.3 mm as assumed <150 mm



$$M_u = \frac{A_{ps}\times f_{ps}}{1.15}\left(d_p - \frac{a}{2}\right)$$

$$M_u = \frac{1780\times 1821.91}{1.15}\left(800-\frac{140.3}{2}\right)\times\frac{1}{10^6} = 2058.2\ \text{kN.m}$$

Example 8.7: M_u using the approximate equation (T-section)

Compute the ultimate moment capacity for the T-beam shown in the figure below. The beam is pre-tensioned with bonded normal stress relieved tendons.

Data

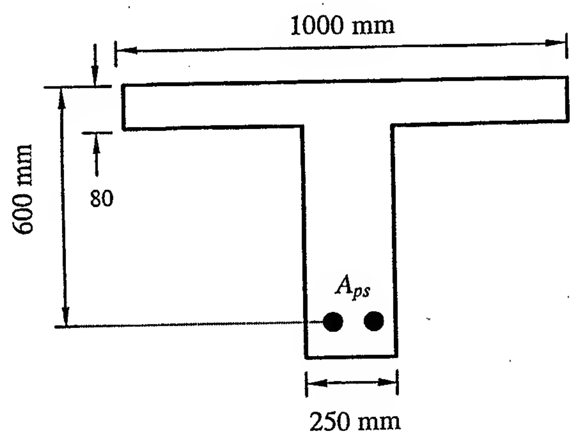
A_{ps} =1500 mm^2

f_{cu} = 40 N/mm^2

f_{pu} =1800 N/mm^2

f_{pe} = 980 N/mm^2

Check that the area of the prestressing steel is less than the maximum allowed by the code.



Beam cross-section

Solution

Step 1: Check the applicability of the approximate equation

To apply the approximate method, $f_{pe} > \frac{f_{pu}}{2}$ must be satisfied.

$\rightarrow 980 > \frac{1800}{2}$, the approximate equation can be used to calculate f_{ps}.

Step 2: Compute f_{ps}

Since no ordinary steel is provided, $\omega = \omega' = 0$

$$f_{ps} = f_{pu}\left[1 - \eta_p\left(\mu_p \frac{f_{pu}}{0.8 f_{cu}}\right)\right]$$

For normal stress relieved strands, the ratio $f_{py}/f_{pu} = 0.85$, thus:

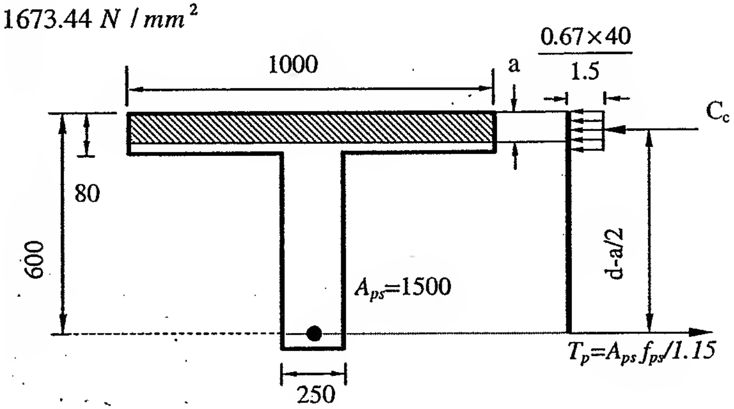
$$For \frac{f_{py}}{f_{pu}} \geq 0.85 \rightarrow\rightarrow \eta_p = 0.50$$

Assuming that the neutral axis is within the flange, the prestressed reinforcement ratio shall be based on the flange width of 1000 mm.

$$\mu_p = \frac{A_{ps}}{B \times d_p} = \frac{1500}{1000 \times 600} = 0.0025$$

$$f_{ps} = f_{pu}\left[1 - \eta_p\left(\mu_p \frac{f_{pu}}{0.80 \times f_{cu}}\right)\right] = 1800\left[1 - 0.5\left(0.0025 \frac{1800}{0.80 \times 40}\right)\right]$$

$$f_{ps} = 1673.44\ N/mm^2$$



Step 3: Check the neutral axis position

Applying the equilibrium equation, the compressed area A_c equals:

$$\frac{0.67\times f_{cu}\times A_c}{1.5}=\frac{A_{sp}\times f_{ps}}{1.15} \quad \rightarrow \quad \frac{0.67\times 40\times A_c}{1.5}=\frac{1500\times 1694.5}{1.15}$$

$$A_c = 122169\ mm^2$$

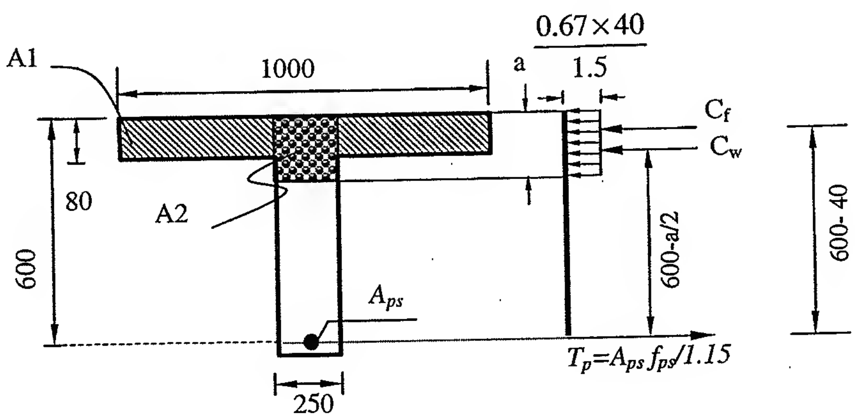
Since the compressed area A_c is greater than the flange area, the neutral axis is located outside the flange ($a>t_s$). Hence, our assumption is not correct and f_{ps} should be recalculated.

Step 4: Recalculate f_{ps}

Since the neutral axis is outside the flange, the reinforcement index should be based on the web thickness of 250 mm.

$$\mu_p=\frac{A_{ps}}{b_w\times d_p}=\frac{1500}{250\times 600}=0.01$$

$$f_{ps}=1800\left[1-0.50\left(0.01\frac{1800}{0.8\times 40}\right)\right]=1293.75\ N/mm^2$$



Step 5: Compute ultimate moment capacity M_u

Application of the equilibrium equation gives:

$$\frac{0.67\times f_{cu}\times A_c}{1.5}=\frac{A_{sp}\times f_{ps}}{1.15} \quad \rightarrow \quad \frac{0.67\times 40\times A_c}{1.5}=\frac{1500\times 1293.75}{1.15}$$

$$A_c = 94450\,mm^2$$

The area of the hatched flange A_1 equals:

$$A_1=(1000-250)\times 80=60000\ mm^2$$

$$a=\frac{94450-60000}{250}=\frac{34450}{250}=137.8\ mm > t_s$$

Summing the moments about the prestressing steel:

$$M_u=C_f\left(d_p-\frac{t_s}{2}\right)+C_w\left(d_p-\frac{a}{2}\right)$$

$$M_u=\frac{0.67\times 40\times(1000-250)\times 80}{1.5}\left(600-\frac{80}{2}\right)+\frac{0.67\times 40\times 250\times 137.8}{1.5}\left(600-\frac{137.8}{2}\right)$$

$$M_u=927.2\ kN.m$$

Step 6: Check the maximum reinforcement ratio

According to the code, the maximum reinforcement index ω_t should be less than 0.28.

$$\omega_t=\left(\omega_{pw}+\frac{d}{d_p}(\omega_w-\omega'_w)\right)\le 0.28$$

Where $\omega_{pw}=\frac{A_{pw}}{b\times d_p}\times\frac{f_{ps}}{0.80 f_{cu}}$ $\qquad A_{pw}=A_{ps}-A_{pf}$

$$C_f=\frac{0.67\times f_{cu}(B-b)\,t_s}{1.5}=\frac{0.67\times 40\ (1000-250)\times 80}{1.5}=1072\times 10^3 N$$

$$\frac{A_{pf}\times f_{ps}}{1.15}=C_f$$

$$\frac{A_{pf}\times 1293.75}{1.15}=1072\times 10^3 \qquad \rightarrow A_{pf}=952.89\ \text{mm}^2$$

$$A_{pw} = A_{ps} - A_{pf} = 1500 - 952.89 = 547.11 \text{ mm}^2$$

$$\omega_{pw} = \frac{A_{pw}}{b \times d_p} \times \frac{f_{ps}}{0.80 f_{cu}} = \frac{547.11}{250 \times 600} \times \frac{1293.75}{0.80 \times 40} = 0.147$$

$$\omega_t = \left(\omega_{pw} + \frac{d}{d_p}(\omega_w - \omega'_w) \right) = (0.147 + 0) = 0.147 \ \langle \ 0.28 \quad o.k.$$

$$a = 137.8\,mm \qquad \rightarrow \qquad c = \frac{137.8}{0.8} = 172.25\ mm$$

$$\frac{c}{d_p} \leq 0.55 \qquad \frac{c}{d_p} = \frac{172.25}{600} = 0.287 < 0.55....o.k$$

Note that the applications of the above two procedures lead to the conclusion that the girder contains reinforcement of 52% of the maximum area of steel.

- The first procedure →→ 0.147/0.28=52%
- The second procedure →→ 0.287/0.55=52%

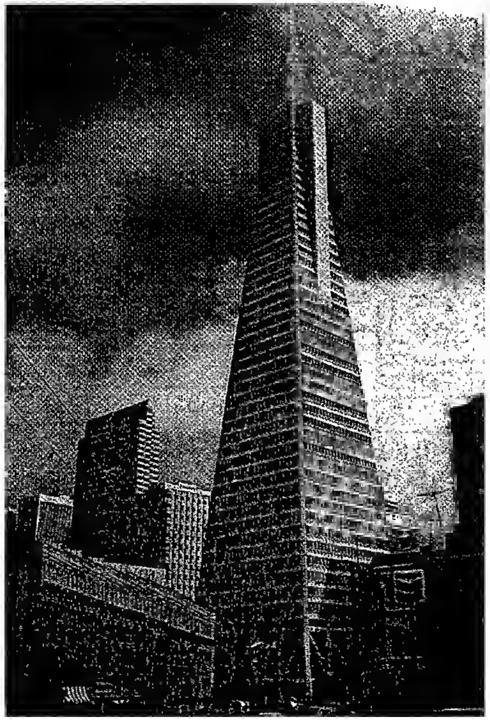


Photo 8.5 Trans-America building, USA

Example 8.8: M_u using the approximate equation

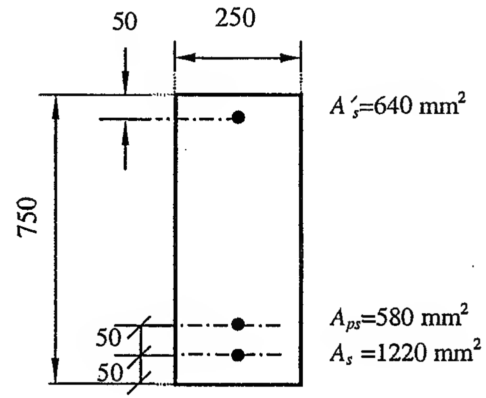
Determine the ultimate moment capacity for the bonded low-relaxation stress relieved strands for the rectangular cross-section shown in the figure below. The material properties are

f_{cu} = 45 N/mm^2

f_{pu} =1900 N/mm^2

f_y = 400 N/mm^2

f_{pe}=1117 N/mm^2.



Beam cross-section

Solution

Step 1: the applicability of the approximate equation

To apply the approximate method $f_{pe} > \dfrac{f_{pu}}{2}$ must be satisfied

$\because 1117 > \dfrac{1900}{2}$, the approximate equation can be used to calculate f_{ps}.

Step 2: Calculate section properties

$d = 750 - 50 = 700\ mm$ $\qquad$ $d_p = 750 - 100 = 650\ mm$

$$\mu = \frac{A_s}{b \times d} = \frac{1220}{250 \times 700} = 0.007$$

$$\mu' = \frac{A_s'}{b \times d} = \frac{640}{250 \times 700} = 0.0037$$

$$\mu_p = \frac{A_{ps}}{b \times d_p} = \frac{580}{250 \times 650} = 0.0036$$

Compute the reinforcement indices ω and ω′

$$\omega = \mu \times \frac{f_y}{f_{cu}} = 0.007 \times \frac{400}{0.8 \times 45} = 0.077$$

$$\omega' = \mu' \times \frac{f_y}{f_{cu}} = 0.0037 \times \frac{400}{0.8 \times 45} = 0.041$$

Step 3: Determine prestressing steel stress at ultimate f_{ps}

To account the compression steel in f_{ps} calculations, two conditions must be satisfied:

1. $$R_p = \left(\frac{\mu_p \times f_{pu}}{f_{cu}} + \frac{d}{d_p}(\omega - \omega') \right) \geq 0.17$$

$$R_p = \left(\frac{0.0036 \times 1900}{0.8 \times 45} + \frac{700}{650}(0.077 - 0.041) \right) = 0.228 > 0.17 \cdots o.k$$

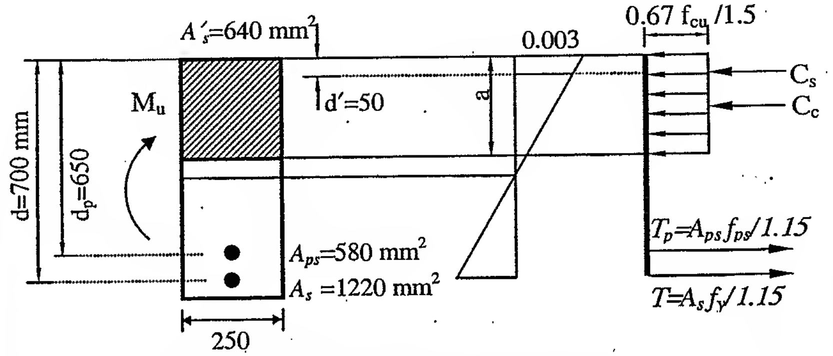
2. $d' \leq 0.15\, d_p \quad \rightarrow \quad 50 < 0.15 \times 650 = 9.75 \cdots o.k$

For low relaxation strands f_{py}/f_{pu}=0.9, the coefficient $\eta_p = 0.35$.

$$f_{ps} = f_{pu}(1 - \eta_p \times R_p)$$

$$f_{ps} = 1900\,(1 - 0.35 \times 0.228) = 1747\ N/mm^2$$

Step 4: Determine the ultimate moment capacity



Strain and stress distributions for the beam

$$\frac{0.67\ f_{cu}\ b\ a}{1.5} + \frac{A_s'\ f_y}{1.15} = \frac{A_s\ f_y}{1.15} + \frac{A_{ps}\ f_{ps}}{1.15}$$

$$\frac{0.67 \times 45 \times 250\ a}{1.5} + \frac{640 \times 400}{1.15} = \frac{1220 \times 400}{1.15} + \frac{580 \times 1747}{1.15}$$

a=215.5 mm $\quad$ and $\quad$ c=269.36 mm

Check yielding of the compression reinforcement

$$f_s' = 600\,\frac{c - d'}{c} = 600\,\frac{269.36 - 50}{269.36} = 488\ N/mm^2 > \frac{400}{1.15} \text{ Compression steel yields}$$

$$M_u = \frac{A_{ps}\ f_{ps}}{1.15}\left(d_p - \frac{a}{2}\right) + \frac{A_s\ f_y}{1.15}\left(d - \frac{a}{2}\right) + \frac{A_s'\ f_y}{1.15}\left(\frac{a}{2} - d'\right)$$

$$M_u = \frac{580 \times 1747}{1.15}\left(650 - \frac{215.5}{2}\right) + \frac{1220 \times 400}{1.15}\left(700 - \frac{215.5}{2}\right) + \frac{640 \times 400}{1.15}\left(\frac{215.5}{2} - 50\right)$$

M_u= 741.96 kN.m

Step 5: Check the maximum reinforcement ratio

According to the ECP 203 the maximum reinforcement index ω_t should be less than 0.28.

$$\omega_t = \left(\frac{\mu_p \times f_{ps}}{f_{cu}} + \frac{d}{d_p}(\omega - \omega')\right) \le 0.28$$

$$\omega_t = \left(\frac{0.0036 \times 1747}{0.8 \times 45} + \frac{700}{650}(0.077 - 0.041)\right) = 0.213 < 0.28 \cdots o.k$$

$$\frac{c}{d_p} \le 0.55$$

$$\frac{c}{d_p} = \frac{269.36}{650} = 0.414 < 0.55 \ldots o.k$$

Example 8.9: M_u using the strain compatibility method

Use strain compatibility method to calculate f_{ps} for the beam given in example 8.8. The ultimate strain for the prestressing steel is equal to 0.04.

Data

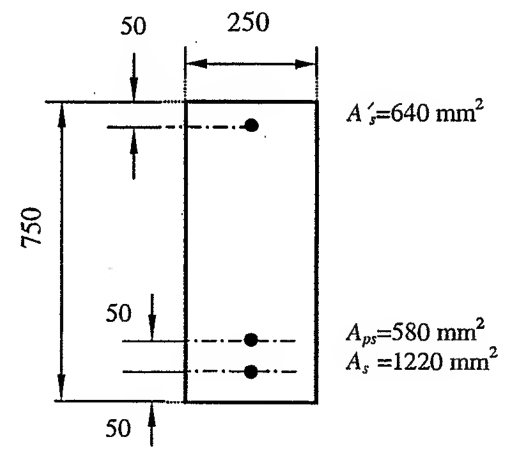
f_{cu} =45 N/mm^2

f_{pu}=1900 N/mm^2

f_y =400 N/mm^2

f_{pe}=1117 N/mm^2

E_p=193000 N/mm^2



Solution

Step 1: Calculate the initial prestressing strain (ε_{pe})

$$P_e = A_{ps} \times f_{pe} = 580 \times 1117 \cong 648000\ N$$

The initial strain due to prestressing equals:

$$\varepsilon_{pe} = \frac{f_{pe}}{E_p} = \frac{1117}{193000} = 0.0058$$

Step 2: Calculate the decompression strain (ε_{ce})

The increase in prestressing steel strain as the concrete decompressed by external loads equals:

$$\varepsilon_{ce} = \frac{f_{ce}}{E_c}$$

The modulus of elasticity of concrete is given by:

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{45} = 29516\ N/mm^2$$

The stress in the concrete due to prestressing only equals:

$$f_{ce} = \frac{P_e}{A} + \frac{P_e \times e \times e}{I}$$

The eccentricity of the cable is given by:

$$e = \frac{t}{2} - cover = \frac{750}{2} - 100 = 275\ mm$$

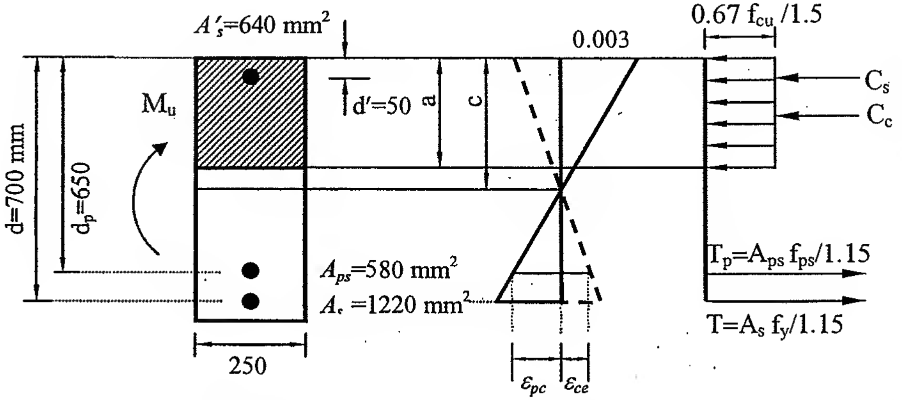
$$I = \frac{b \times t^3}{12} = \frac{250 \times 750^3}{12} = 8.79 \times 10^9\ mm^4$$

Hence, one can get:

$$f_{ce} = \frac{648000}{250 \times 750} + \frac{648000 \times 275 \times 275}{8.79 \times 10^9} = 9.03\ N/mm^2$$

$$\varepsilon_{ce} = \frac{f_{ce}}{E_c} = \frac{9.03}{29516} = 0.000306$$

This increase in the concrete strain (from compression to tension) is balanced by an increase in the steel strain of the same amount (0.000306).



Strain and stress distributions for the beam

Step 3: Calculate strain at ultimate (ε_{pc})

In order to determine the depth of the neutral axis, f_{ps} must be assumed and verified later. Assume f_{ps} between f_{py} and f_{pu}
For low relaxation steel, f_{py}/f_{pu}= 0.9.

$$f_{py} = 0.9 \times f_{pu} = 0.9 \times 1900 = 1710\ N/mm^2$$

Thus, assume f_{ps} = 1750 N/mm^2.

$$\frac{0.67\ f_{cu}\ b\ a}{1.5} + \frac{A'_s\ f_y}{1.15} = \frac{A_s\ f_y}{1.15} + \frac{A_{ps}\ f_{ps}}{1.15}$$

$$\frac{0.67 \times 45 \times 250\,a}{1.5} + \frac{640 \times 400}{1.15} = \frac{1220 \times 400}{1.15} + \frac{580 \times 1750}{1.15}$$

a=215.8 mm

$$c = \frac{a}{0.8} = \frac{215.8}{0.8} = 269.80\ mm$$

The increase in strain from overload to ultimate equals:

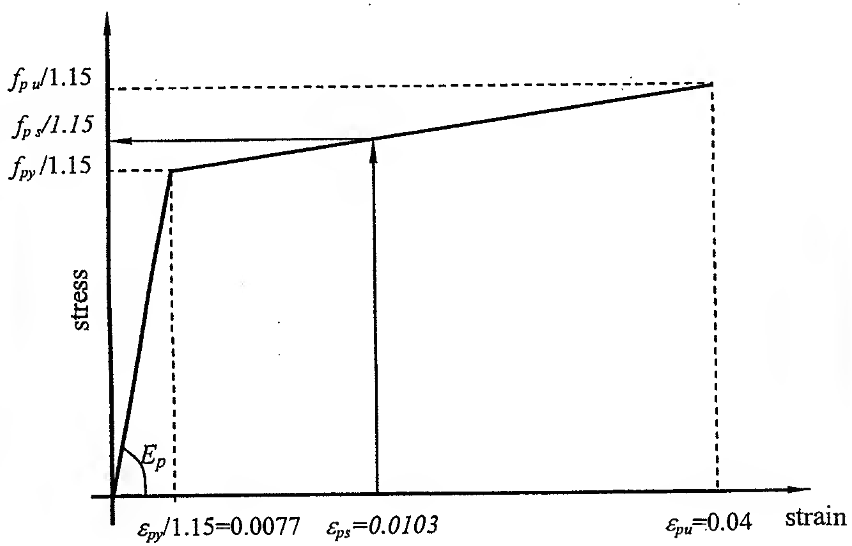
$$\varepsilon_{pc} = 0.003\,\frac{d_p - c}{c} = 0.003\,\frac{650 - 269.80}{269.80} = 0.0042$$

Step 4: Calculate total strain at ultimate (ε_{ps})

$$\varepsilon_{ps} = \varepsilon_{pe} + \varepsilon_{ce} + \varepsilon_{pc}$$

$$\varepsilon_{ps} = 0.0058 + 0.0003 + 0.0042 = 0.0103$$

The stress corresponding to this strain can be obtained using the idealized stress-strain curve specified by the ECP 203. Referring to the figure below and recalling that the ultimate prestressing steel strain is given as 0.04, f_{ps} can be obtained as follows:



$$f_{py} = 0.9 \times f_{pu} = 0.9 \times 1900 = 1710\ N/mm^2$$

$$\frac{\varepsilon_{py}}{\gamma_s} = \frac{f_{py}}{1.15 \times E_p} = \frac{1710}{1.15 \times 193000} = 0.0077$$

$$f_{ps} = f_{py} + \frac{(\varepsilon_{ps} - \varepsilon_{py}/\gamma_{ps})}{(\varepsilon_{pu} - \varepsilon_{py}/\gamma_{ps})} \times (f_{pu} - f_{py})$$

$$f_{ps} = 1710 + \frac{(0.0103 - 0.0077)}{(0.040 - 0.0077)} \times (1900 - 1710) \quad \rightarrow \quad f_{ps} = 1725.3\ \text{N/mm}^2$$

Since it has been assumed that f_{ps}=1750 N/mm^2 , another trial is required.

Step 5: Recalculate the total strain at ultimate

Assume that f_{ps}= 1725N/mm^2

$$\frac{0.67 \times 45 \times 250\, a}{1.5} + \frac{640 \times 400}{1.15} = \frac{1220 \times 400}{1.15} + \frac{580 \times 1725}{1.15}$$

a = 213.3 mm

$$c = \frac{a}{0.8} = \frac{213.3}{0.8} = 266.6\ mm$$

The increase in strain from overload to ultimate equals:

$$\varepsilon_{pc} = 0.003\,\frac{d_p - c}{c} = 0.003\,\frac{650 - 266.6}{266.6} = 0.0043$$

$$\varepsilon_{ps} = \varepsilon_{pe} + \varepsilon_{ce} + \varepsilon_{pc} = 0.0058 + 0.0003 + 0.0043 = 0.0104$$

$$f_{ps} = 1710 + \frac{(0.0104 - 0.0077)}{(0.040 - 0.0077)} \times (1900 - 1710) = 1725.8\ N/mm^2$$

The calculated f_{ps} is very close to the assumed value (usually from 1-2% is close enough), the prestressing steel stress at ultimate and equals=1725 N/mm^2

Check yielding of the compression reinforcement

$$f_s' = 600\,\frac{c - d'}{c} = 600\,\frac{266.6 - 50}{266.6} = 487\ N/mm^2 > \frac{400}{1.15} \quad \text{Compression steel yields}$$

Note: Comparing the value of f_{ps} obtained using the simplified method (1747 N/mm^2) to that obtained using the strain compatibility method, one can notice that the simplified method overestimates f_{ps}.

8.4 Combined Flexure and Axial Loads

Prestressed members subjected to eccentric loading may be encountered in prestressed concrete construction. The analyses of such sections at the service load level and the ultimate stage are outlined in the next two sections.

8.4.1 Stresses at Service Loads

The stresses at service loads for sections subjected to combined flexure and axial loads can be determined according to the following equation:

$$f = -\left(\frac{P}{A} + \frac{N}{A}\right) \pm \frac{P \times e}{I} y \mp \frac{M \times y}{I} \quad \text{......................... (8.37)}$$

Where N is the applied axial load (positive sign indicates a compression force)

8.4.2 Capacity at Ultimate Loads

Prestressed concrete members subjected to combined flexure and axial load are designed using the strain compatibility as outlined in the previous section. The strains in prestressing steel located at the compression and the tension zones are calculated. The stress-strain curve for the prestressing reinforcement is used to determine the stresses. A trial-and-adjustment procedure is carried out to compute the resulting forces and moments. The neutral axis distance c is assumed, and the corresponding forces are calculated. Adjusting c in Eq. 8.38 is repeated until equilibrium is achieved.

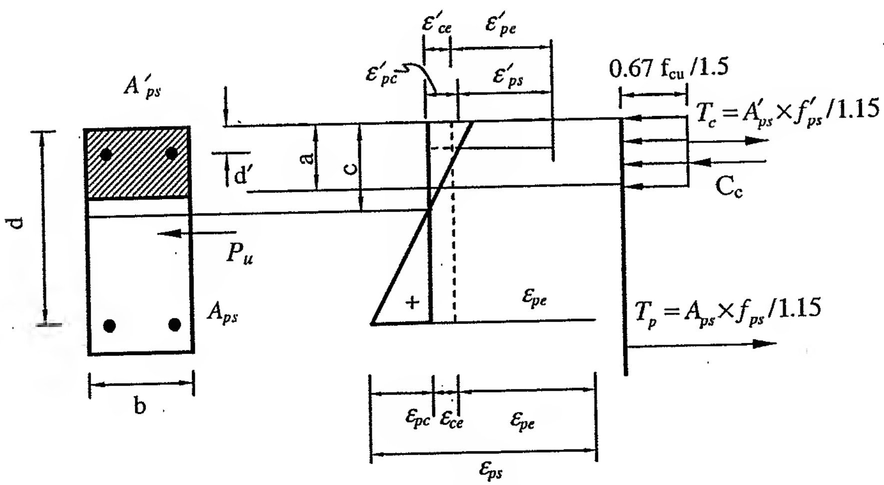


Fig. 8.13 Strain and stress distributions for the beam subject to P_u, M_u

Noting that the force T_c is tension, the equilibrium of the forces gives:

$$P_u = \frac{0.67\, f_{cu}\, b\, a}{1.5} - \frac{A'_{ps}\, f'_{ps}}{1.15} - \frac{A_{ps}\, f_{ps}}{1.15} \quad \text{..................... (8.38)}$$

Having determined c, ε_{pc} and ε'_{pc} are computed using compatibility of stains as follows:

$$\varepsilon_{pc} = 0.003\, \frac{d-c}{c} \quad \text{...(8.39a)}$$

$$\varepsilon'_{pc} = 0.003\, \frac{c-d'}{c} \quad \text{.. (8.39b)}$$

To simplify the calculations of ε_{ce} and ε'_{ce}, it will be assumed that the prestressing forces resulting from A_{ps} and A'_{ps} are such that the effective prestressing is at the C.G. of the section, producing uniform compressive strains as shown in Fig. 8.13.

$$\varepsilon_{ce} = \varepsilon'_{ce} = \frac{P_e}{A_c \times E_c} \quad \text{..(8.39c)}$$

The total prestressing strains ε_{ps} and ε'_{ps} are calculated as follows:

$$\varepsilon'_{ps} = \varepsilon'_{pe} - (\varepsilon'_{pc} - \varepsilon_{ce}) \quad \text{.................................(8.40a)}$$

$$\varepsilon_{ps} = \varepsilon_{pe} + (\varepsilon_{pc} + \varepsilon_{ce}) \quad \text{................................. (8.40b)}$$

The resulting bending moment is then determined by taking moments of all the forces about the centroid of the section (location of P_u) as follows:

$$M_u = C_c \left(\frac{t}{2} - \frac{a}{2}\right) - T_c \left(\frac{t}{2} - d'\right) + T_p \left(\frac{t}{2} - \text{cover}\right) \quad \text{........... (8.41)}$$

Example 8.10 illustrates the calculation procedure.

Example 8.10: Strain compatibility method for combined flexure and axial load

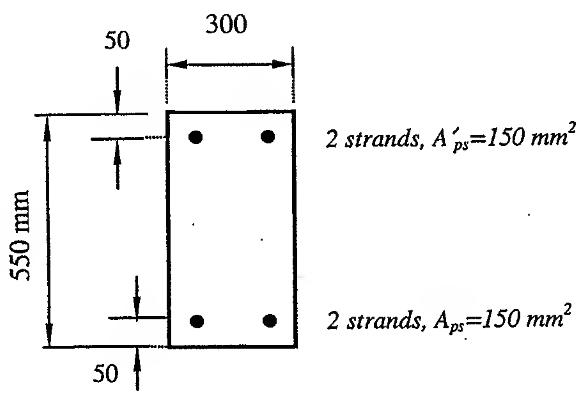
If the cross-section shown in figure is subjected to P_u=360 kN, compute the ultimate flexural capacity. The losses may be assumed as 12%, and the ultimate strain for low relaxation prestressing strands is 0.045.

Data

f_{cu} =45 N/mm^2

f_p =1860 N/mm^2

E_p = 196000 N/mm^2



Beam cross-section

Solution

The approximate code equation can not be used to calculate f_{ps} when prestressing steel is located in the compression zone. In such a case, the strain compatibility procedure must be used.

Step 1: Calculate the initial prestressing strain (ε_{pe})

For low relaxation strands, f_{py}= 0.9 f_{pu}= 0.9 x 1860 =1674 N/mm^2

$$f_{pi} = smaller\ of \begin{cases} 0.70\ f_{pu} = 0.7\times1860 = 1302\ N/mm^2 \\ 0.8\ f_{py} = 0.8\times1674 = 1339\ N/mm^2 \end{cases}$$

f_{pi}=1302 N/mm^2

$$f_{pe} = (1-losses)\ f_{pi} = (1-0.12)\ 1302 = 1145.76\ N/mm^2$$

$$P_e = A_{ps}\times f_{pe} = (150+150)\times1145.76 \cong 343728\ N$$

The initial strain due to prestressing force equals:

$$\varepsilon_{pe} = \varepsilon'_{pe} = \frac{f_{pe}}{E_p} = \frac{1145.76}{196000} = 0.0058$$

Step 2: Calculate the decompression strain (ε_{ce})

The decompression strain equals:

$$\varepsilon_{ce} = \frac{P_e}{A_c\times E_c}$$

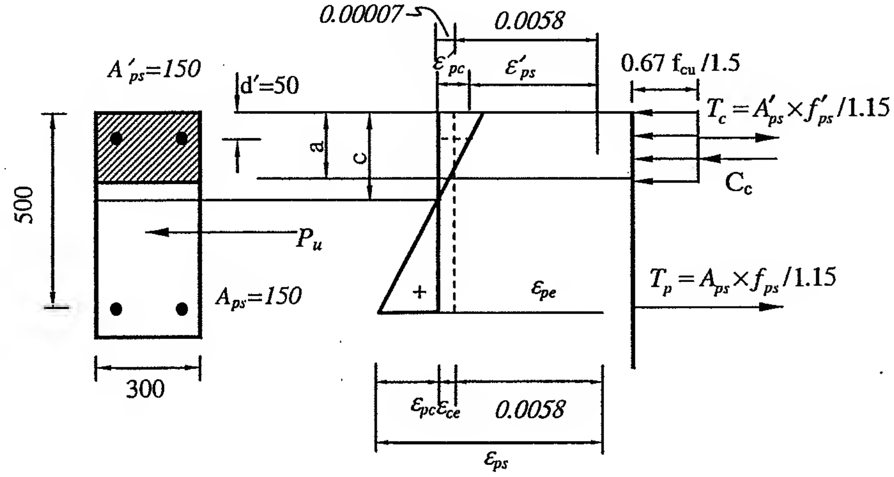
The modulus of elasticity of concrete is given by:

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{45} = 29516\ N/mm^2$$

The eccentricity at of the cable equals:

$$e = \frac{t}{2} - cover = \frac{550}{2} - 50 = 225\ mm$$

$$\varepsilon_{ce} = \varepsilon'_{ce} = \frac{343728}{29516\,(300\times550)} = 0.00007$$



Strain and stress distributions for the beam subject to P_u, M_u

Step 3: Calculate the neutral axis distance (c)

The equilibrium equation is obtained by equating the internal forces to the external forces as follows:

$P_u = C_c - T_c - T_p$ Note: T_p is tension

$$P_u = \frac{0.67\, f_{cu}\, b\, a}{1.5} - \frac{A'_{ps}\, f'_{ps}}{1.15} - \frac{A_{ps}\, f_{ps}}{1.15}$$

$$P_u = \frac{0.67 \times 45 \times 300 \times 0.8\, c}{1.5} - \frac{150 \times f'_{ps}}{1.15} - \frac{150 \times f_{ps}}{1.15}$$

The previous equation is a non-linear equation with one unknown "c". To solve the equation, a trial and adjustment procedure is followed through assuming c and calculating the corresponding strains and stresses in the prestressing steel in the tension and compression zones as follows:

A- Compression force in the concrete

$$C_c = \frac{0.67 f_{cu}\; b \times (0.8\, c)}{1.5} = \frac{0.67 \times 45 \times 300 \times 0.80\, c}{1.5} \times \frac{1}{1000} = 4.824\, c \quad kN$$

B- Strain for the prestressing steel in the tension zone

$$\varepsilon_{pc} = 0.003\, \frac{d_p - c}{c} = 0.003\, \frac{500 - c}{c}$$

$$\varepsilon_{ps} = \varepsilon_{pe} + \varepsilon_{ce} + \varepsilon_{pc} \quad \rightarrow \quad \varepsilon_{ps} = 0.0058 + 0.00007 + 0.003\, \frac{500 - c}{c}$$

$$\frac{\varepsilon_{py}}{\gamma_s} = \frac{f_{py}}{1.15 \times E_p} = \frac{1674}{1.15 \times 196000} = 0.00742$$

$$f_{ps} = f_{py} + \frac{(\varepsilon_{ps} - \varepsilon_{py}/\gamma_{ps})}{(\varepsilon_{pu} - \varepsilon_{py}/\gamma_{ps})} \times (f_{pu} - f_{py})$$

$$f_{ps} = 1674 + \frac{(\varepsilon_{ps} - 0.00742)}{(0.045 - 0.00742)} \times (1860 - 1674)$$

C- Strain and stress for the prestressing steel in the compression zone

$$\varepsilon'_{pc} = 0.003\, \frac{c - d'}{c} = 0.003\, \frac{c - 50}{c}$$

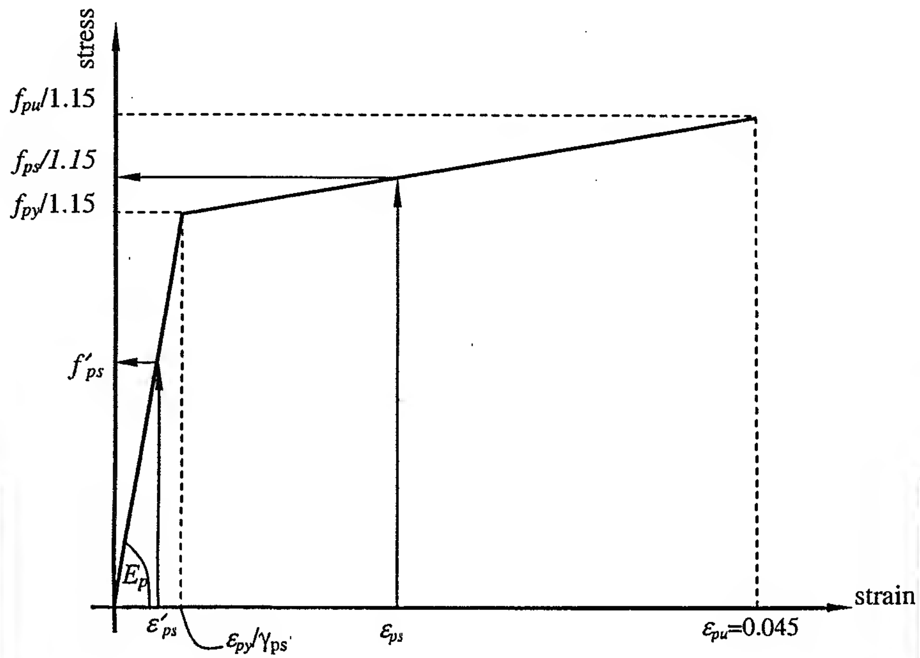
Noting that the strain in the compression zone is negative, the net prestressing steel strain equals:

$$\varepsilon'_{ps} = \varepsilon'_{pe} - (\varepsilon'_{pc} - \varepsilon'_{ce}) = (\varepsilon'_{pe} + \varepsilon'_{ce}) - \varepsilon'_{pc}$$

$$\varepsilon'_{ps} = 0.0058 + 0.00007 - \left(0.003\, \frac{c - 50}{c}\right)$$

$$f'_{ps} = 1674 + \frac{(\varepsilon_{ps} - 0.00742)}{(0.045 - 0.00742)} \times (1860 - 1674) \; \dots\dots\dots \; \varepsilon'_{ps} > \varepsilon_{py}/1.15$$

$$f'_{ps} = \varepsilon'_{ps} \times E_p \; \dots\dots\dots \; \varepsilon'_{ps} \le \varepsilon_{py}/1.15$$



Stress-strain diagram for the prestressing steel

The calculations can be summarized in the following table

Trial	c	ε'_{sp}	ε_{sp}	f'_{sp}	f_{sp}	C_c	T_c	T_p	P_u
1	130	0.0041	0.0145	803.6	1709.0	627.1	104.8	222.9	299.4
2	140	0.0040	0.0136	784.0	1704.6	675.4	102.3	222.3	350.8
3	142	0.0040	0.0135	784.0	1704.1	685.0	102.3	222.3	360.5

It is clear from the table that the neutral axis distance c =142 mm gives very close value (360.5 kN) when compared with the applied force (360 kN).

Step 4: Calculate the flexural strength.

From the previous table it can be determined that C_c = 685 kN, Tc = 102.3 kN, and T_p= 222.3 kN. The moment strength can be calculated by taking moments about Pu, located at the centroid of the section as follows:

$$M_u = C_c\left(\frac{t}{2}-\frac{a}{2}\right) - T_c\left(\frac{t}{2}-d'\right) + T_p\left(\frac{t}{2}-\text{cover}\right)$$

$$M_u = 685\left(\frac{0.55}{2}-\frac{0.8\times 0.142}{2}\right) - 102.3\left(\frac{0.55}{2}-0.05\right) + 222.3\left(\frac{0.55}{2}-0.05\right)$$

$$M_u = 176.4\ kN.m$$

Step 5: Check the maximum reinforcement ratio

Using Eq. 8.36 $\quad \rightarrow\rightarrow \quad \frac{c}{d_p} \le 0.68$

$$\frac{c}{d_p} = \frac{142}{500} = 0.284 < 0.68 \ldots .o.k$$

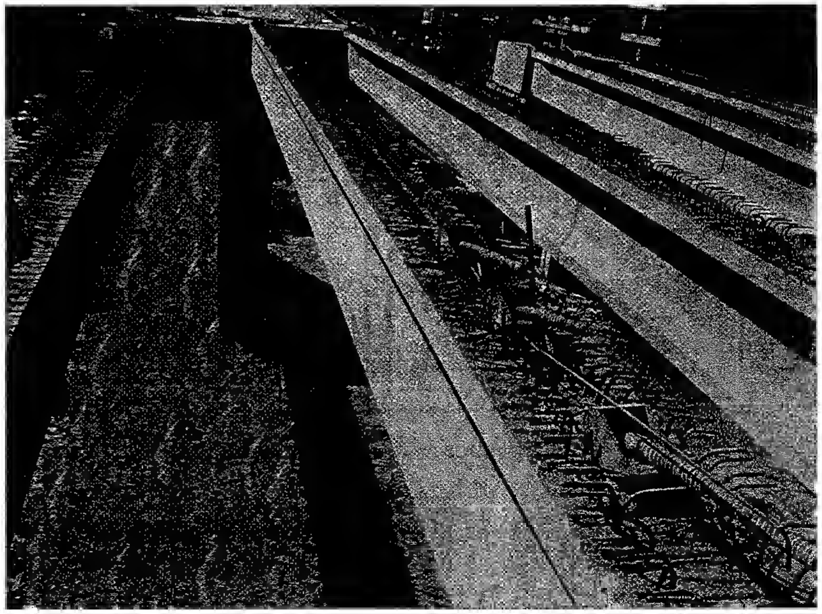
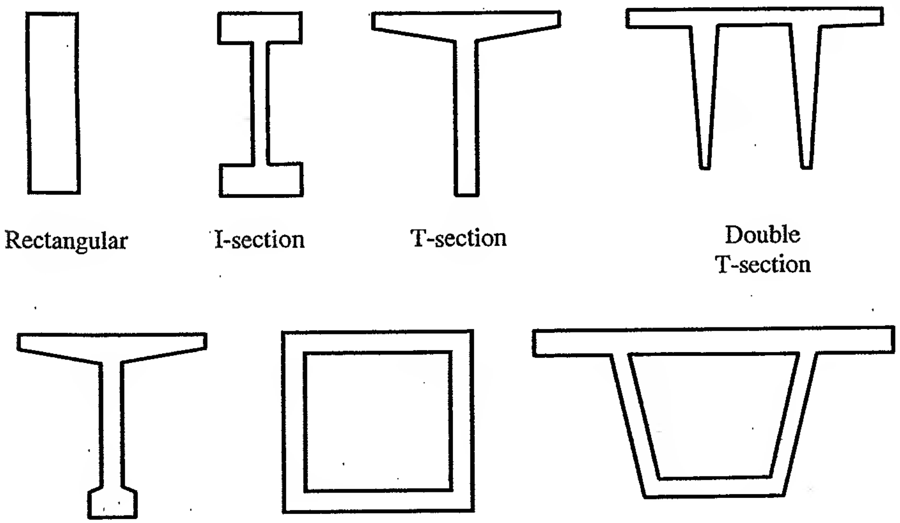


Photo 8.6 Prestressed concrete beams in 15th May bridge befor. the construction of the concrete deck

8.5 Proper Beam Shape Selection

Some of the prestressed concrete beams are fabricated in precast plants that frequently publish tables containing the properties of the cross-section and the uniform load that can support. However, in most cases the designer may have to establish the shape of the cross-section to be used in a special project. This is typically the case in bridge construction. For a simply supported beam, the eccentricity is inversely proportional to the required prestressing force. The larger the eccentricity at mid-span, the smaller the required prestressing force.

A rectangular section is the easiest in fabrication, alignment and the least in form cost. It is frequently used in buildings and parking garages. Bearing in mind that the dead loads may represent a large portion of the total loads on the structure, flanged sections are structurally more efficient because of their high moment of inertia with respect to their self weight.



Rectangular — I-section — T-section — Double T-section

T-section with bottom enlargement — Box section — Box section with cantilever slabs

Fig. 8.14 Different types of prestressed concrete sections

T-sections and wide flange I-sections are appropriate if large eccentricity is required as shown in Fig. 8.14. In such a case, the end section of the beam is usually solid to avoid large eccentricity, and to increase the shear capacity. Double T-sections are also used because of their stability and ease of handling. They are widely used in floor systems in buildings because they eliminate the need for slabs. Long-span parking garages may require I-sections with composite slab topping.

If the self-weight is small compared to the superimposed dead and live loads, a larger lower flange is needed to carry the heavy compressive strength produced by the prestressing force. For long-span bridges, hollow box sections are often more economical. These sections have large torsional capacity. Also, their flexural strength to weight ratio is relatively high compared to other prestressing systems.

8.6 Limiting Eccentricity Envelopes

The tensile stress in the extreme fiber under service load conditions should not exceed the limit specified by the code. Thus, it is important to establish the limiting zone in the concrete section. For example, for a simply supported beam at transfer if no tension is allowed for the top fiber, then:

$$f_{top} = 0 = -\frac{P_i}{A} + \frac{P_i \times e}{Z_{top}} \qquad (8.42)$$

Solving Eq. (8.42) for the eccentricity gives the lower kern point k_b as:

$$k_b = \frac{Z_{top}}{A} \downarrow \text{(below C.G.)} \qquad (8.43)$$

If the eccentricity of the tendon exceeds this kern point it will cause tension at transfer.

At the final stage, if no tension is allowed at the bottom fibers, then:

$$f_{bottom} = 0 = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{bot}} \qquad (8.44)$$

$$k_t = \frac{-Z_{bot}}{A} \text{ (above C.G.)} \qquad (8.45)$$

If the eccentricity exceeds the upper kern k_t, it will cause tension at the final stage.

Similarly, kern point can be established for the right and left parts about the line of symmetry of the section. For rectangular sections, $Z_{top}=Z_{bot}=b\,t^2/6$, giving the kern points as shown in Fig 8.15.

$$k_b = k_t = \frac{t}{6} \quad \text{(for rectangular sections only)} \text{ .... (8.46)}$$

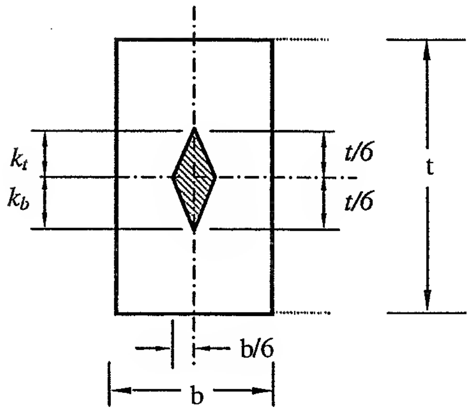


Fig. 8.15 Central kern area for a rectangular section

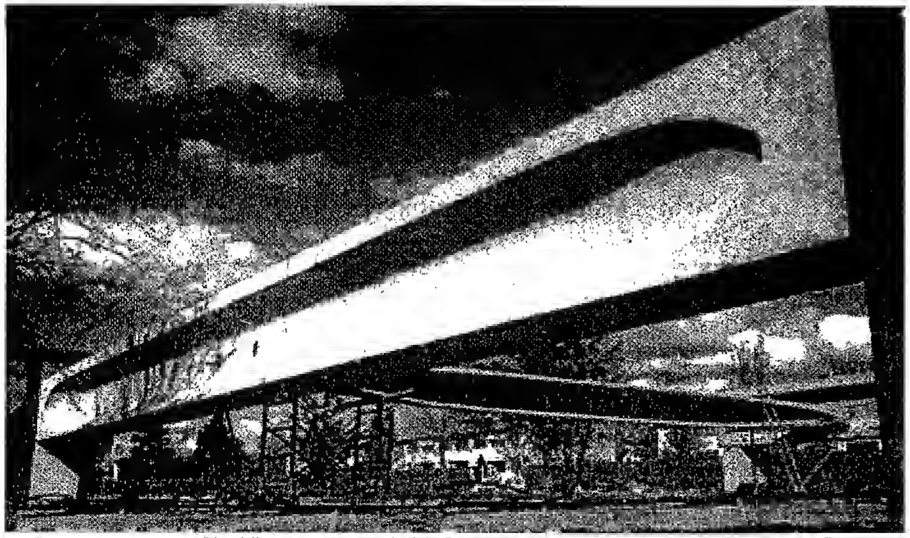


Photo 8.7 Precast-prestressed concrete beam with coved sides

As can be seen from the previous section, any force falling in the kern area will never cause tension at the section. However, many codes, including the ECP 203, allow tension stress at bottom and top fibers (Cases B, C and D). Thus, it is important to establish the limiting envelope at the maximum allowable tension because it is desirable that the designed eccentricities of the tendon along the span fall within these limits.

At transfer, the top fiber is subjected to tension while the bottom fibers are subjected to compression. Thus, the eccentricity e_L at transfer (as shown in Fig. 8.16) at which the top fibers are subjected to the maximum allowable tension is given by:

$$f_{tt,top} = 0.22\sqrt{f_{cui}} = -\frac{P_i}{A} + \frac{P_i \times e_L}{Z_{top}} - \frac{M_{ow}}{Z_{top}} \text{ .... (8.47)}$$

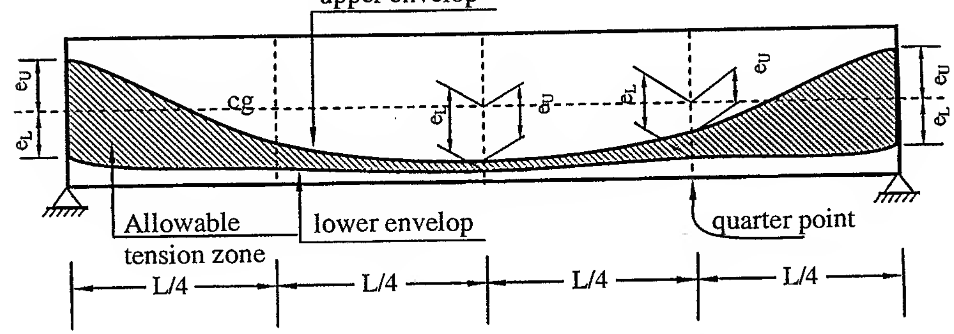


Fig. 8.16 Envelope permitting tension in concrete extreme fibers.

At full service load, the bottom fibers are subjected to tension while top fibers are subjected to compression. The allowable concrete tensile stress should be obtained from Table 8.2. For example, for case B the eccentricity e_U (as shown in Fig. 8.16) at which the bottom fibers are subjected to the maximum allowable tension is given by:

$$f_{te,bottom} = 0.44\sqrt{f_{cu}} = -\frac{P_e}{A} - \frac{P_e \times e_U}{Z_{bot}} + \frac{M_{total}}{Z_{bot}} \text{ .... (8.48)}$$

It is usually sufficient to calculate three points for parabolic tendon (midspan, quarter span, and at support). It should be clear that an envelope that falls outside the section indicates non-economical section. A change in the eccentricity or in the prestressing force improves the design.

Example 8.11: Upper and lower envelopes

The cross-section of a simply supported beam is shown in the figure. The beam is post-tensioned and the prestressing cable is parabolic. Determine the limiting envelopes such that the limiting concrete tensile stress is in accordance with the ECP 203 Case B. Consider the mid-span, the quarter span, and at the support as the controlling points. Assume that:

P_i = 2400 kN

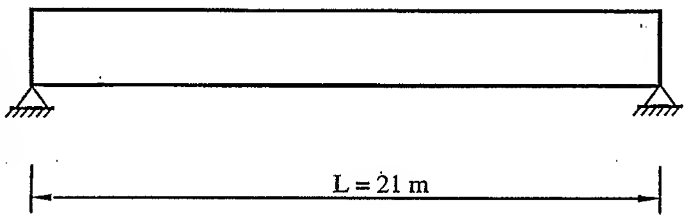
P_e = 2050 kN

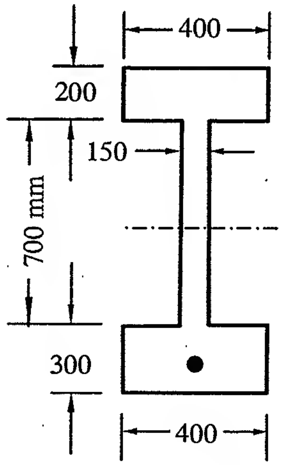
f_{cu} = 40 N/mm^2

f_{cui} = 30 N/mm^2

w_{DL} = 5 kN/m

w_{LL} = 18 kN/m'





Beam cross-section

Solution

Step 1: Calculate section properties

$$A = 400\times200 + 150\times700 + 400\times300 = 305000\ mm^2$$

Since the section is not symmetrical, calculate the location of the center of gravity.

$$y = \frac{400\times200\times100 + 150\times700\times550 + 400\times300\times1050}{305000} = 628.7\ mm$$

$$I = \frac{400\times200^3}{12} + 400\times200\times(628.7-100)^2 + \frac{150\times700^3}{12} + 150\times700\times(628.7-550)^2$$
$$+\frac{400\times300^3}{12} + 400\times300\times(1050-628.7)^2$$

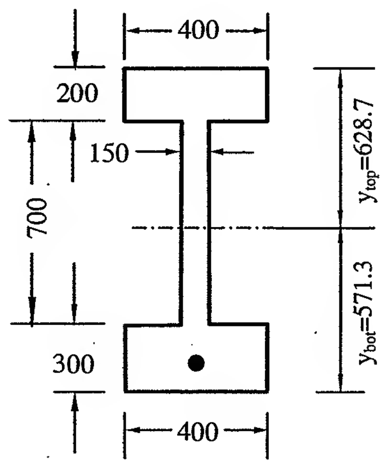
I = 4.98 x 10^{10} mm^4

y_{top} = y = 628.7 mm

$$y_{bot} = t - y = (200+700+300) - 628.7 = 571.3\ mm$$

$$Z_{top} = \frac{I}{y_{top}} = \frac{4.98\times10^{10}}{628.7} = 79.16\times10^6\ mm^3$$

$$Z_{bot} = \frac{I}{y_{bot}} = \frac{4.98\times10^{10}}{571.3} = 87.11\times10^6\ mm^3$$



$$w_{o.w} = \gamma_c \times A = 25 \times \frac{305000}{1000000} = 7.625\, kN/m'$$

$$w_{tot} = w_{ow} + w_{DL} + w_{LL} = 7.625 + 5 + 18 = 30.625\ kN/m'$$

The allowable tensile stresses at transfer f_{ti} and at full service load f_{te} for case B are given by:

$$f_{ti} = 0.22\sqrt{f_{cui}} = 0.22\sqrt{30} = 1.205\ N/mm^2$$

$$f_{te} = 0.44\sqrt{f_{cu}} = 0.44\sqrt{40} = 2.783\ N/mm^2$$

Step 2: Upper and lower envelops at midspan

$$M_{ow} = \frac{w_{ow} \times L^2}{8} = \frac{7.625 \times 21^2}{8} = 420.33\ kN.m$$

$$M_{tot} = \frac{w_{tot} \times L^2}{8} = \frac{30.625 \times 21^2}{8} = 1688.2\ kN.m$$

The limiting condition at transfer is due to condition at top fibers subjected to self-weight only as follows:

$$f_{top} = -\frac{P_i}{A} + \frac{P_i \times e_L}{Z_{top}} - \frac{M_{ow}}{Z_{top}}$$

$$+1.205 = -\frac{2400 \times 1000}{305000} + \frac{2400 \times 1000 \times e_L}{79.16 \times 10^6} - \frac{420.33 \times 10^6}{79.16 \times 10^6}$$

$$e_L = 474.7\ mm \downarrow$$

The limiting condition at full service load is due to condition at bottom fibers subjected to full load as follows:

$$f_{bottom} = -\frac{P_e}{A} - \frac{P_e \times e_U}{Z_{bot}} + \frac{M_{total}}{Z_{bot}}$$

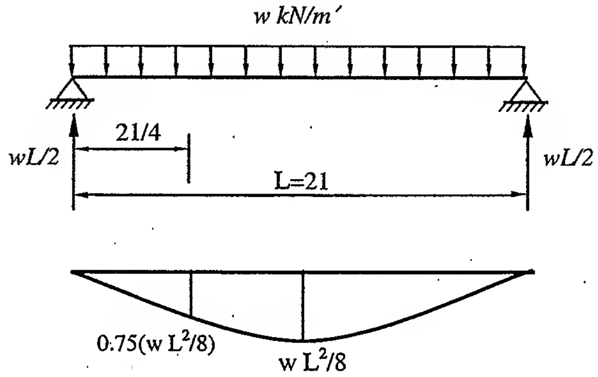
$$+2.783 = -\frac{2050 \times 1000}{305000} - \frac{2050 \times 1000 \times e_U}{87.11 \times 10^6} + \frac{1688.2 \times 10^6}{87.11 \times 10^6}$$

$$e_U = 420\ mm \downarrow$$

Step 3: Upper and lower envelops at quarter point

The moment at the quarter point for a uniformly loaded beam is given by:

$$M = \frac{w \times L}{2} \times \frac{L}{4} - \frac{w}{2} \times \left(\frac{L}{4}\right)^2 = \frac{3}{4} \times \left(\frac{w \times L^2}{8}\right) = \frac{3}{4} M_{at\ mid\ span}$$



Thus, the self-weight moment at quarter point equals:

$$M_{ow} = 0.75 \times 420.33 = 315.25\, kN.m$$

$$M_{tot} = 0.75 \times 1688.2 = 1266.2\ kN.m$$

The limiting condition at transfer is due to condition at top fibers subjected to self-weight only as follows:

$$f_{top} = -\frac{P_i}{A} + \frac{P_i \times e_L}{Z_{top}} - \frac{M_{ow}}{Z_{top}}$$

$$+1.205 = -\frac{2400 \times 1000}{305000} + \frac{2400 \times 1000 \times e_L}{79.16 \times 10^6} - \frac{315.25 \times 10^6}{79.16 \times 10^6}$$

$$e_L = 430.64\ mm \downarrow$$

The limiting condition at full service load is due to condition at bottom fibers subjected to full load as follows:

$$f_{bottom} = -\frac{P_e}{A} - \frac{P_e \times e_U}{Z_{bot}} + \frac{M_{total}}{Z_{bot}}$$

$$+2.783 = -\frac{2050\times1000}{305000} - \frac{2050\times1000\times e_U}{87.11\times10^6} + \frac{1266.2\times10^6}{87.11\times10^6}$$

$$e_U = 213.8\ mm \downarrow$$

Step 4: Upper and lower envelops at support

At support $M_{ow}=M_{total}=0$, *thus the* limiting condition at transfer is due to condition at top fibers subjected to self-weight only as follows:

$$f_{top} = -\frac{P_i}{A} + \frac{P_i \times e_L}{Z_{top}}$$

$$+1.205 = -\frac{2400\times1000}{305000} + \frac{2400\times1000\times e_L}{79.16\times10^6}$$

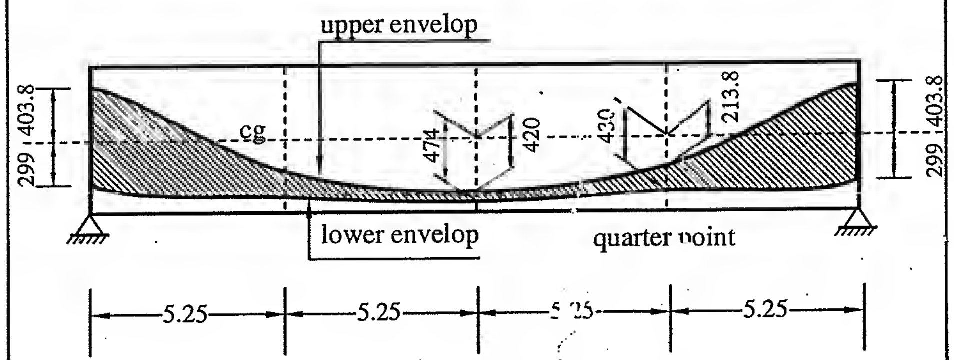
$$e_L = 299.3\ mm \downarrow$$

The limiting condition at full service load is due to condition at bottom fibers subjected to full load as follows:

$$f_{bottom} = -\frac{P_e}{A} - \frac{P_e \times e_U}{Z_{bot}}$$

$$+2.783 = -\frac{2050\times1000}{305000} - \frac{2050\times1000\times e_U}{87.11\times10^6}$$

$$e_U = -403.8\ mm = 403.8\ mm \uparrow$$



Upper and lower envelopes for the prestressing tendons

8.7 Determination of the Prestressing Force and the Eccentricity in Flexural Members

The prestressing force that produces safe stresses at transfer may produce unsafe stresses at full service load. The aim of this section is to find the possible combinations of the prestressing force and the eccentricity that ensure the safety of the beam at the transfer as well as at full service load. The procedure developed by *Magnel* (1948) is very useful as an instructional aid. It illustrates that there are frequently several combinations of prestressing force and eccentricity that will result in compliance with the code requirements of allowable stresses.

The required equations can be obtained by analyzing the stresses at the top and bottom fibers of the beam for the case of transfer and service loads. This will lead to four governing equations. Plotting these equations will produce the zone of the acceptable combinations of the prestressing force and eccentricity. The procedure is illustrated in the following steps.

A- At Transfer (top fibers-tension controls)

The stresses at the top fibers at transfer should be less than the allowable concrete tensile stress at transfer recommended by the code (f_{ti}) as follows:

$$f_{top} = -\frac{P_i}{A} + \frac{P_i \times e}{Z_{top}} - \frac{M_{ow}}{Z_{top}} \le f_{ti} \quad \text{.......... (8.49)}$$

Rearranging the previous equation gives:

$$f_{ti} \ge -\frac{P_i}{A} + \frac{P_i \times e}{Z_{top}} - \frac{M_{ow}}{Z_{top}} \quad \text{.......... (8.50)}$$

The eccentricity that gives the acceptable stress at top should be less than:

$$e \le \frac{1}{P_i}\left[Z_{top}\ f_{ti} + M_{ow}\right] + \frac{Z_{top}}{A} \quad \text{.......... (8.51)}$$

Noting that $K_b = Z_{top} / A$

$$e \le \frac{1}{P_i}\left[Z_{top}\ f_{ti} + M_{ow}\right] + K_b \quad \text{.......... (8.52)}$$

This can be rewritten in terms of the prestressing force as:

$$\frac{1}{P_i} \ge \frac{e - K_b}{\left[Z_{top}\ f_{ti} + M_{ow}\right]} \quad \text{.......... (8.53)}$$

B- At Transfer (bottom fibers) (compression controls)

The stresses at bottom fibers at transfer should be less than the allowable compressive stress at transfer recommended by the code (f_{ci}) as follows:

$$f_{bot} = -\frac{P_i}{A} - \frac{P_i \times e}{Z_{bot}} + \frac{M_{ow}}{Z_{bot}} \geq f_{ci} \text{ (because } f_{ci} \text{ is negative)} \quad \text{........... (8.54)}$$

Thus,

$$f_{ci} \leq -\frac{P_i}{A} - \frac{P_i \times e}{Z_{bot}} + \frac{M_{ow}}{Z_{bot}} \quad \text{.. (8.55)}$$

The eccentricity that gives the acceptable stress at bottom should be less than:

$$e \leq \frac{1}{P_i}\left[-Z_{bot}\ f_{ci} + M_{ow}\right] - \frac{Z_{bot}}{A} \quad \text{.............................. (8.56)}$$

Noting that $K_t = Z_{bot}/A$, the previous equation can be written in terms of the prestressing force P_i:

$$\frac{1}{P_i} \geq \frac{e + K_t}{\left[-Z_{bot}\ f_{ci} + M_{ow}\right]} \quad \text{.................................. (8.57)}$$

C- At Full service load (bottom fibers-tension controls)

The stresses at the bottom fibers at the full service load stage should be less than the allowable concrete tensile stress (f_{te}). Denoting the losses as α, one can obtain:

$$P_e = (1-\alpha)\ p_i = \xi\ P_i \quad \text{.. (8.58)}$$

$$f_{bottom} = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{bot}} + \frac{M_{total}}{Z_{bot}} \leq f_{te} \quad \text{......................... (8.59)}$$

Rearranging the terms:

$$f_{te} \geq -\frac{\xi \times P_i}{A} - \frac{\xi \times P_i \times e}{Z_{bot}} + \frac{M_{total}}{Z_{bot}} \quad \text{........................... (8.60)}$$

The acceptable eccentricity should be greater than:

$$e \geq \frac{1}{\xi \times P_i}\left[-Z_{bot}\ f_{te} + M_{total}\right] - \frac{Z_{bot}}{A} \quad \text{......................... (8.61)}$$

$$\frac{1}{P_i} \leq \frac{e + K_t}{\left[-Z_{bot}\ f_{te} + M_{total}\right]/\xi} \quad \text{......................... (8.62)}$$

D- At Full service load (top fibers) (compression controls)

$$f_{top} = -\frac{P_e}{A} + \frac{P_e \times e}{Z_{top}} - \frac{M_{total}}{Z_{top}} \geq f_{ce} \text{ (because } f_{ce} \text{ is negative)} \quad \text{............ (8.63)}$$

$$f_{ce} \leq -\frac{\xi \times P_i}{A} + \frac{\xi \times P_i \times e}{Z_{top}} - \frac{M_{total}}{Z_{top}} \quad \text{........................... (8.64)}$$

$$e \geq \frac{1}{\xi \times P_i}\left[Z_{top}\ f_{ce} + M_{total}\right] + \frac{Z_{top}}{A} \quad \text{........................... (8.65)}$$

The eccentricity that gives the acceptable stress at top should be greater than:

$$e \geq \frac{1}{\xi \times P_i}\left[Z_{top}\ f_{ce} + M_{total}\right] + K_b \quad \text{............................ (8.66)}$$

The minimum prestressing force that stratify code requirements is given by

$$\frac{1}{P_i} \leq \frac{e - K_b}{\left[Z_{top}\ f_{ce} + M_{total}\right]/\xi} \quad \text{.......................... (8.67)}$$

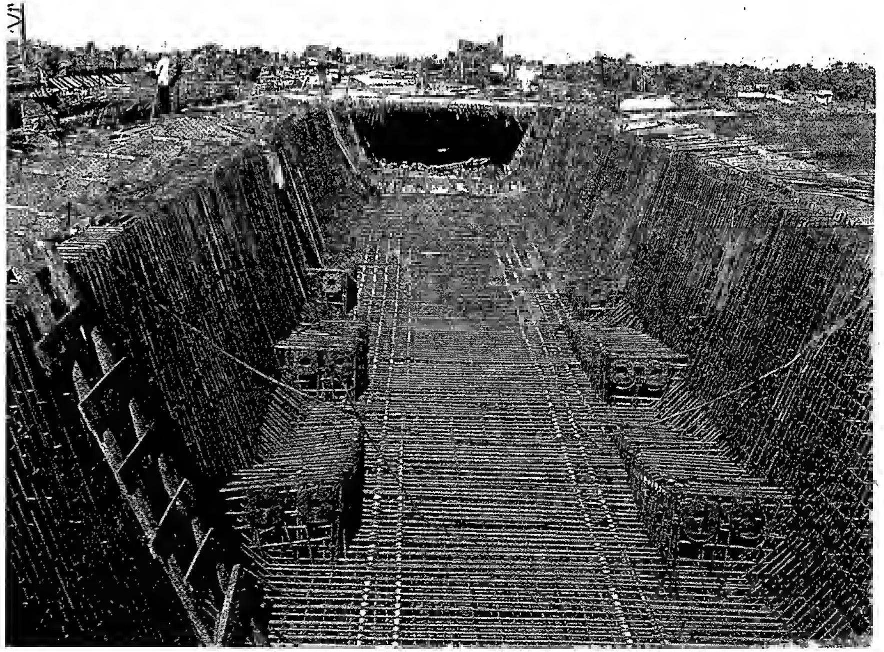


Photo 8.8 Cable placement in a box girder

The four governing equations can be summarized in the following table.

$$\frac{1}{P_i} \geq \frac{e - K_b}{\left[Z_{top}\; f_{ti} + M_{ow}\right]} \qquad (1)$$

$$\frac{1}{P_i} \geq \frac{e + K_t}{\left[-Z_{bot}\; f_{ci} + M_{ow}\right]} \qquad (2)$$

$$\frac{1}{P_i} \leq \frac{e + K_t}{\left[-Z_{bot}\; f_{te} + M_{total}\right]/\xi} \qquad (3)$$

$$\frac{1}{P_i} \leq \frac{e - K_b}{\left[Z_{top}\; f_{ce} + M_{total}\right]/\xi} \qquad (4)$$

All the terms appearing in the previous equations are known and can be determined except P_i and e. there are a number of combinations of these terms satisfy all the equations. The right combination can be determined by plotting each equation as shown in Fig. 8.17. In this graph, the horizontal axis represents the eccentricity, while the vertical axis represents the inverse of the initial prestressing force. The hatched area in the figure indicates the acceptable combinations of ($1/p_i$ and e) that satisfy all code requirements of allowable stress in transfer and full service load stages. It should be noted that some of the possible eccentricities may not be attainable because they might lie outside the limits of the section. Therefore, these points should be excluded from the acceptable zone as shown in Fig. 8.17.

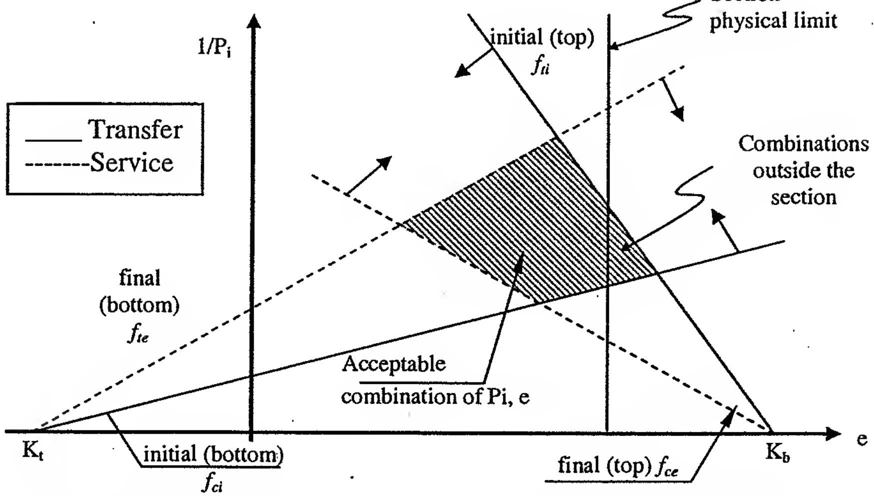


Fig. 8.17 Graphical representation for the four governing equations

Example 8.12: Determination of P and e combinations

The cross-section of a simply supported beam with unbonded tendons is shown in figure. It is required to:

A. Determine the acceptable combinations of P_i and e at mid-span according to the ECP 203 allowable stresses for case B.

B. Check the combinations of the following prestressing force and eccentricity:

Case	P_i (kN)	e (mm)
1	2700	400.00
2	2000	250.00
3	3000	250.00
4	3000	450.00

Data

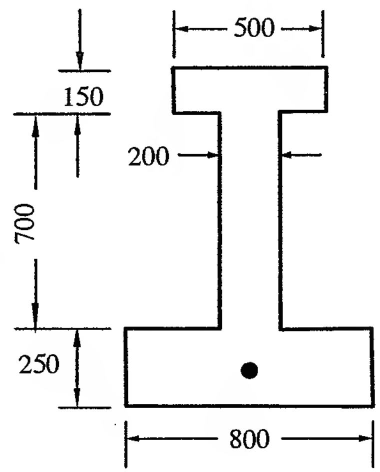
Losses =15%

f_{cu} = 45 N/mm^2

f_{cui} = 31.5 N/mm^2

w_{LL} = 21 kN/m′

span = 20.0 m



Beam cross-section

Solution

Step 1: Calculate section properties

$$A = 500\times150 + 200\times700 + 250\times800 = 415000 \ mm^2$$

Since the section is not symmetrical, calculate the location of the center of gravity.

$$y = \frac{500\times150\times75 + 200\times700\times500 + 250\times800\times975}{415000} = 652.1 \ mm$$

$I = 5.6 \times 10^{10} \ mm^4$

$y_{top} = y = 652.1$ mm

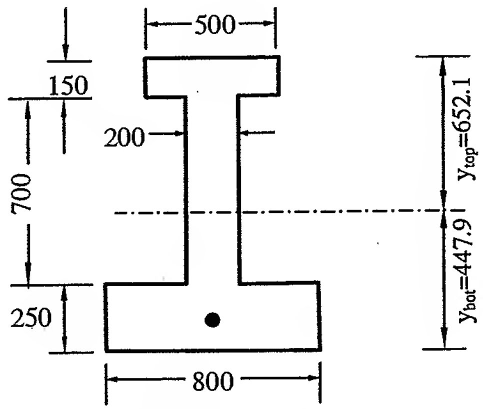
$$y_{bot} = t - y = (150+700+250) - 652.1 = 447.9 \ mm$$

$$Z_{top} = \frac{I}{y_{top}} = \frac{5.6\times10^{10}}{652.1} = 85.83\times10^6 \ mm^3$$

$$K_b = \frac{Z_{top}}{A} = \frac{85.83\times10^6}{415000} = 206.81 \ mm$$

$$Z_{bot} = \frac{I}{y_{bot}} = \frac{5.6\times10^{10}}{447.9} = 125\times10^6 \ mm^3$$

$$K_t = \frac{Z_{bot}}{A} = \frac{125\times10^6}{415000} = 301.2 \ mm$$



beam cross section

$$w_{o.w} = \gamma_c \times A = 25\times\frac{415000}{1000000} = 10.375 \ kN/m'$$

$$w_{tot} = w_{ow} + w_{LL} = 10.375 + 21 = 31.375 \ kN/m'$$

$$M_{ow} = \frac{10.375\times20^2}{8} = 518.75 \ kN.m$$

$$M_{total} = \frac{31.375\times20^2}{8} = 1568.8 \ kN.m$$

Step 2: Governing equations

Step 2.1: Equation at transfer (top fibers-tension controls)

The allowable tension stress at transfer f_{ti} is given by:

$$f_{ti} = 0.22\sqrt{f_{cui}} = 0.22\sqrt{31.5} = 1.235 \ N/mm^2$$

$$\frac{1}{P_i} \geq \frac{e - K_b}{[Z_{top} \ f_{ti} + M_{ow}]}$$

$$\frac{1}{P_i} \geq \frac{e - 206.8}{[85.8\times10^6 \times 1.235/1000 + 518.75\times1000]}$$

$$\frac{1}{P_i} \geq 1.6\times10^{-6} \ (e - 206.8)$$

e	206.8	700
$1/p_i$	0	7.9×10^{-4}

Step 2.2 At transfer (bottom fibers-compression controls)

$$f_{ci} = -0.45 \ f_{cui} = -0.45\times31.5 = -14.175 \ N/mm^2$$

$$\frac{1}{P_i} \geq \frac{e + K_t}{[-Z_{bot} \ f_{ci} + M_{ow}]}$$

$$\frac{1}{P_i} \geq \frac{e + 301.1}{[-125\times10^6 \times -14.175/1000 + 518.75\times1000]}$$

$$\frac{1}{P_i} \geq 0.436\times10^{-6} \ (e + 301.1)$$

e	-301.1	700
1/pi	0	4.37 x 10^{-4}

Step 2.3: Equation at full service load (bottom fibers-tension controls)

Since the beam has un-bonded tendons, it is classified as case B (refer to Table 8.2). Thus, the allowable tension is given by:

$$f_{te} = 0.44\sqrt{f_{cu}} = 0.44\sqrt{45} = 2.95\ N/mm^2$$

$$\xi = 1 - losses = 1 - 0.15 = 0.85$$

$$\frac{1}{P_i} \leq \frac{e + K_t}{[-Z_{bot}\ f_{te} + M_{total}]/\xi}$$

$$\frac{1}{P_i} \leq \frac{e + 301.1}{[-125\times10^6 \times 2.95/1000 + 1568.8\times1000]/0.85}$$

$$\frac{1}{P_i} \leq 0.708\times10^{-6}\ (e + 301.1)$$

e	-301.1	700
1/pi	0	7.09 x 10^{-4}

Step 2.4 Equation at full service load (top fibers-compression controls)

$$f_{ci} = -0.40\ f_{cu} = -0.40\times45.0 = -18\ N/mm^2$$

$$\frac{1}{P_i} \leq \frac{e - K_b}{[Z_{top}\ f_{ce} + M_{total}]/\xi}$$

$$\frac{1}{P_i} \leq \frac{e - 206.8}{[85.8\times10^6 \times -18/1000 + 1568.8\times1000]/0.85}$$

$$\frac{1}{P_i} \leq 34.8\times10^{-6}\ (e - 206.8)$$

e	206.8	225
1/pi	0	6.4 x 10^{-4}

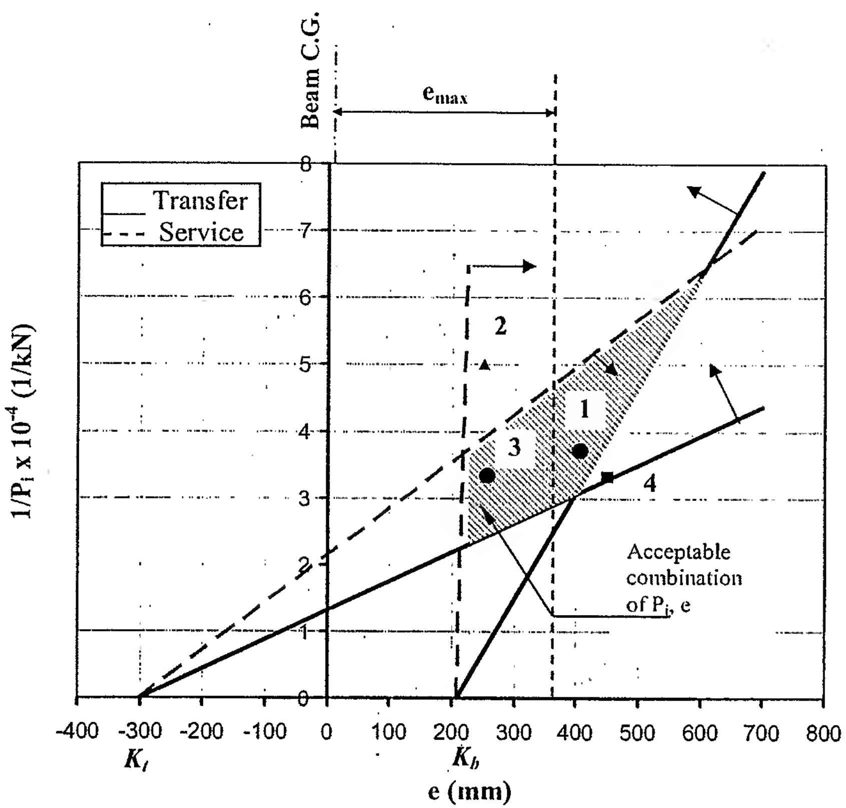
Step 3: Acceptable P_i-e diagram

From the points calculated in Step 2, the acceptable combinations diagram can be plotted as shown in figure. Assuming concrete cover of 70 mm, the maximum acceptable eccentricity e_{max} equals:

$$e_{max} = y_{bot} - cover = 414.3 - 70 = 344.3\ mm$$

Locating the points inside the diagram and realizing that any point falling inside the hatched area is considered safe and vice versa, the following table can be established.

Case	P(kN)	e (mm)	1/P (x10^{-4})	Status	Reason
1	2700	400.0	3.70	Unsafe	inside the acceptable area but outside the section
2	2000	250.0	5	Unsafe	outside the acceptable area
3	3000	250.0	3.33	Safe	inside the acceptable area
4	3000	450.0	3.33	Unsafe	outside the acceptable area



Acceptable combinations of P_i and e

Example 8.13: Determination of P and e combinations

The cross-section of a simply supported beam is shown in the figure below. The beam is a part of the structural system of a chemical factory. It is required to:

A. Determine the acceptable combinations of Pi and e at mid-span according to the ECP 203 allowable stresses for case B. Adjust the cross-section dimensions if necessary.

Data

Losses =20%

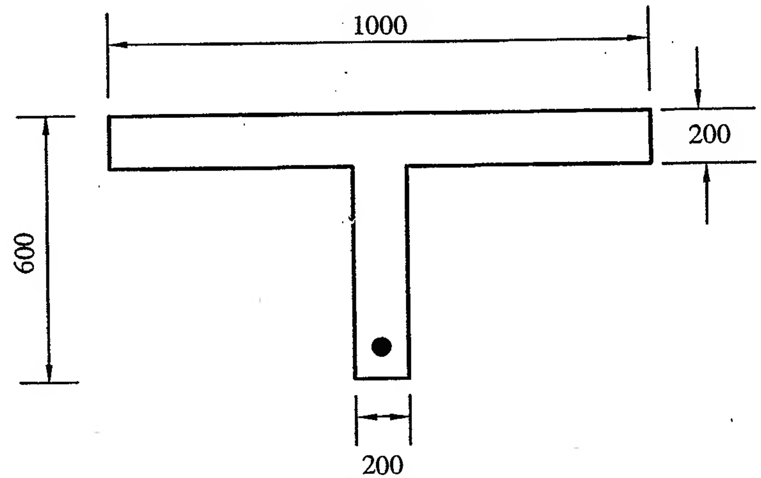
f_{cu} = 50 N/mm^2

f_{cui} = 35 N/mm^2

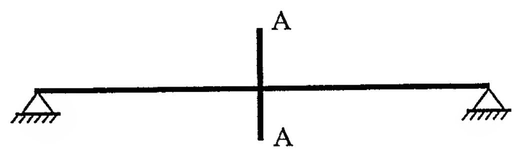
w_{DL} = 5 kN/m′

w_{LL} = 10 kN/m′

span = 12 m



Section A-A



Solution

Step 1: Calculate section properties

$$A = 1000 \times 200 + 200 \times 400 = 280000 \ mm^2$$

Since the section is not symmetrical, calculate the location of the center of gravity.

$$y = \frac{1000 \times 200 \times 100 + 200 \times 400 \times 400}{280000} = 185.7 \ mm$$

$I = 6.88 \times 10^9 \ mm^4$

$y_{top} = y = 185.7$ mm

$$y_{bot} = t - y = 600 - 185.7 = 414.3 \ mm$$

$$Z_{top} = \frac{I}{y_{top}} = \frac{6.88 \times 10^9}{185.7} = 37.03 \times 10^6 \ mm^3$$

$$K_b = \frac{Z_{top}}{A} = \frac{37.03 \times 10^6}{280000} = 132.2 \ mm$$

$$Z_{bot} = \frac{I}{y_{bot}} = \frac{6.88 \times 10^9}{414.3} = 16.6 \times 10^6 \ mm^3$$

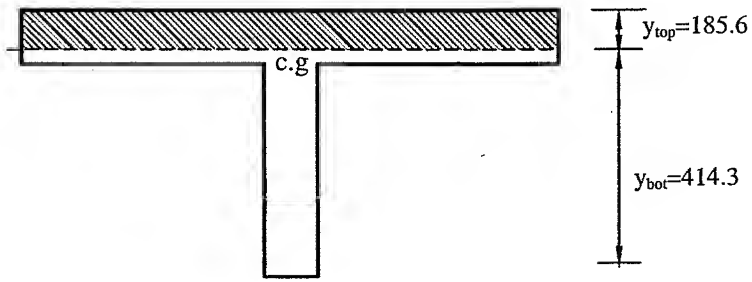
$$K_t = \frac{Z_{bot}}{A} = \frac{16.6 \times 10^6}{280000} = 59.27 \ mm$$

$$w_{o.w} = \gamma_c \times A = 25 \times \frac{280000}{1000000} = 7.0 \, kN/m'$$

$$w_{tot} = w_{ow} + w_{DL} + w_{LL} = 7.0 + 5 + 10 = 22 \ kN/m'$$

$$M_{ow} = \frac{7.0 \times 12^2}{8} = 126 \ kN.m$$

$$M_{total} = \frac{22 \times 12^2}{8} = 396 \ kN.m$$



Step 2. Governing equations

Step 2.1 Equation at transfer (top fibers-tension controls)

The allowable tension stress at transfer f_{ti} is given by:

$$f_{ti} = 0.22\sqrt{f_{cui}} = 0.22\sqrt{35} = 1.302 \ N/mm^2$$

$$\frac{1}{P_i} \geq \frac{e - K_b}{[Z_{top} \ f_{ti} + M_{ow}]}$$

$$\frac{1}{P_i} \geq \frac{e - 132.2}{[37.02 \times 10^6 \times 1.302/1000 + 126 \times 1000]}$$

$$\frac{1}{P_i} \geq 5.74 \times 10^{-6} \ (e - 132.2)$$

e	132.2	600
1/pi	0	26.8 x 10^{-4}

Step 2.2 At transfer (bottom fibers-compression controls)

$$f_{ci} = -0.45 \ f_{cui} = -0.45 \times 35 = -15.75 \ N/mm^2$$

$$\frac{1}{P_i} \geq \frac{e + K_t}{[-Z_{bot} \ f_{ci} + M_{ow}]}$$

$$\frac{1}{P_i} \ge \frac{e+59.27}{\left[-16.6\times10^6 \times -15.75/1000 + 126\times1000\right]}$$

$$\frac{1}{P_i} \ge 2.58\times10^{-6}\ (e+59.27)$$

e	-59.27	600
1/pi	0	17.01 x 10^{-4}

Step 2.3: Equation at full service load (bottom fibers-tension controls)

$$f_{te} = 0.44\sqrt{f_{cu}} = 0.44\sqrt{50} = 3.11\ N/mm^2$$

$$\xi = 1 - losses = 1 - 0.20 = 0.80$$

$$\frac{1}{P_i} \le \frac{e+K_t}{\left[-Z_{bot}\ \ f_{te} + M_{total}\right]/\xi}$$

$$\frac{1}{P_i} \le \frac{e+59.2}{\left[-16.6\times10^6 \times 3.11/1000 + 396\times1000\right]/0.80}$$

$$\frac{1}{P_i} \le 2.32\times10^{-6}\ (e+59.2)$$

e	-59.2	600
1/pi	0	15.3 x 10^{-4}

Step 2.4 Equation at full service load (top fibers-compression controls)

$$f_{ci} = -0.40\ f_{cu} = -0.40\times50.0 = -20\ N/mm^2$$

$$\frac{1}{P_i} \le \frac{e-K_b}{\left[Z_{top}\ \ f_{ce} + M_{total}\right]/\xi}$$

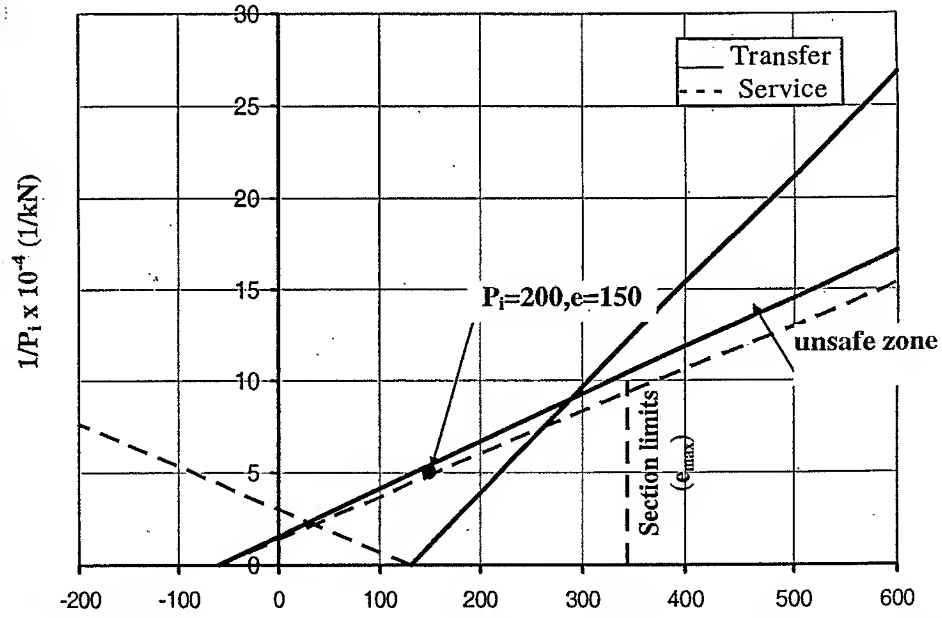
$$\frac{1}{P_i} \le \frac{e-132.2}{\left[37.02\times10^6 \times -20/1000 + 396\times1000\right]/0.80}$$

$$\frac{1}{P_i} \le -2.322\times10^{-6}\ (e-132.2)$$

e	132.2	600	-200
1/p$_i$	0	-10.8 x 10^{-4}	7.7 x 10^{-4}

Step 3: Acceptable P$_i$-e diagram

From the points calculated in step 2, the following diagram can be plotted:



Assuming a concrete cover of 70 mm, the maximum acceptable eccentricity e_{max} equals:

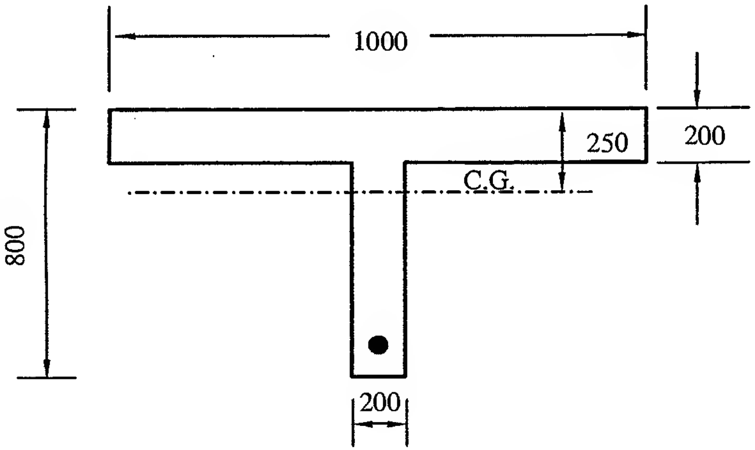
$$e_{max} = y_{bot} - cover = 414.3 - 70 = 344.3\ mm$$

It is clear from the diagram that the acceptable zone does not exist. For example an initial prestressing force of 200 kN (1/P$_i$=5) and an eccentricity of 150 mm give the following stresses:

At Transfer			
f_{top}	-2.44	<-15.75	Safe
f_{bot}	-17.62	>-15.75	unsafe
At Final Stage			
f_{bot}	3.685	>3.11	Unsafe
f_{top}	-9.928	<-20	Safe

Step 4: Adjusting section dimensions

The reason for the unsatisfactory performance of this section is attributed to the fact that its size is not sufficient to resist the applied loads. One of the solutions is to increase the size of the section. Increasing the section dimension by 200 mm gives the following section properties:



$$A = 1000 \times 200 + 200 \times 600 = 320000 \ mm^2$$

Since the section is not symmetrical, calculate the location of the center of gravity.

$$y = \frac{1000 \times 200 \times 100 + 200 \times 600 \times 500}{320000} = 250 \ mm$$

I =1.63 x 10 ^{10} mm^4

$y_{top} = y = 250$ mm

$$y_{bot} = t - y = 800 - 250 = 550 \ mm$$

$$Z_{top} = \frac{I}{y_{top}} = \frac{1.63 \times 10^{10}}{250} = 65.06 \times 10^6 \ mm^3$$

$$K_b = \frac{Z_{top}}{A} = \frac{65.06 \times 10^6}{320000} = 203.3 \ mm$$

$$Z_{bot} = \frac{I}{y_{bot}} = \frac{1.63 \times 10^{10}}{550} = 29.57 \times 10^6 \ mm^3$$

$$K_t = \frac{Z_{bot}}{A} = \frac{29.57 \times 10^6}{2320000} = 92.4 \ mm$$

$$w_{o.w} = \gamma_c \times A = 25 \times \frac{320000}{1000000} = 8.0 kN/m' \qquad M_{ow} = 144 \ kN.m$$

$$w_{tot} = 23 kN/m' \qquad M_{tot} = 414 \ kN.m$$

Step 4.1 Governing equations

Step 4.1.1 Equation at transfer (top fibers-tension controls)

The allowable tension stress at transfer f_{ti} is given by:

$$f_{ti} = 0.22\sqrt{f_{cui}} = 0.22\sqrt{35} = 1.302 \ N/mm^2$$

$$\frac{1}{P_i} \geq \frac{e - K_b}{[Z_{top} \ f_{ti} + M_{ow}]}$$

$$\frac{1}{P_i} \geq \frac{e - 203.3}{[65.06 \times 10^6 \times 1.302/1000 + 144 \times 1000]}$$

$$\frac{1}{P_i} \geq 4.37 \times 10^{-6} \ (e - 203.3)$$

e	203.3	600
$1/p_i$	0	17.35 x 10^{-4}

Step 4.1.2 At transfer (bottom fibers-compression controls)

$$\frac{1}{P_i} \geq \frac{e+92.42}{\left[-29.57\times10^6 \times -15.75/1000+144\times1000\right]}$$

$$\frac{1}{P_i} \geq 1.64\times10^{-6}\,(e+92.42)$$

e	-92.42	600
1/pi	0	11.35 x 10^{-4}

Step 4.1.3 Equation at full service load (bottom fibers-tension controls)

$$\frac{1}{P_i} \leq \frac{e+92.42}{\left[-29.57\times10^6 \times 3.11/1000+414\times1000\right]/0.80}$$

$$\frac{1}{P_i} \leq 2.48\times10^{-6}\,(e+92.42)$$

e	-92.42	600
1/pi	0	17.2 x 10^{-4}

Step 4.1.4 Equation at full service load (top fibers-compression controls)

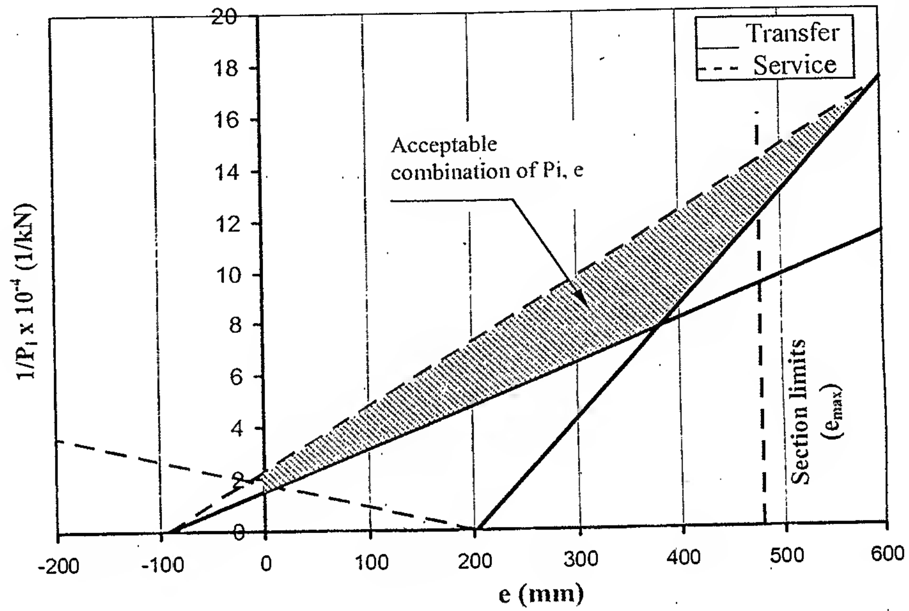
$$\frac{1}{P_i} \leq \frac{e-203.3}{\left[65.07\times10^6 \times -20/1000+414\times1000\right]/0.80}$$

$$\frac{1}{P_i} \leq -9.015\times10^{-6}\,(e-203.3)$$

e	203.3	600	-200
1/pi	0	-3.58 x 10^{-4}	3.64 x 10^{-4}

Step 4.2 Acceptable P_i-e diagram

From the points calculated in step 4.1,the following diagram can be plotted:



8.8 Reduction of Prestressing Force near Supports

Sections with straight tendons at supports are subjected to zero moment and may suffer from high tensile stresses resulting from the prestressing force. There are two practical methods of reducing the high stresses near the supports:

1. Reducing the eccentricity of some cables as the reach the support zone as shown in Fig. 8.18.a.
2. Sheathing some of the cables by plastic tubing as shown in Fig. 8.18.b. This eliminates the transfer of the prestressing force to the concrete at this area.

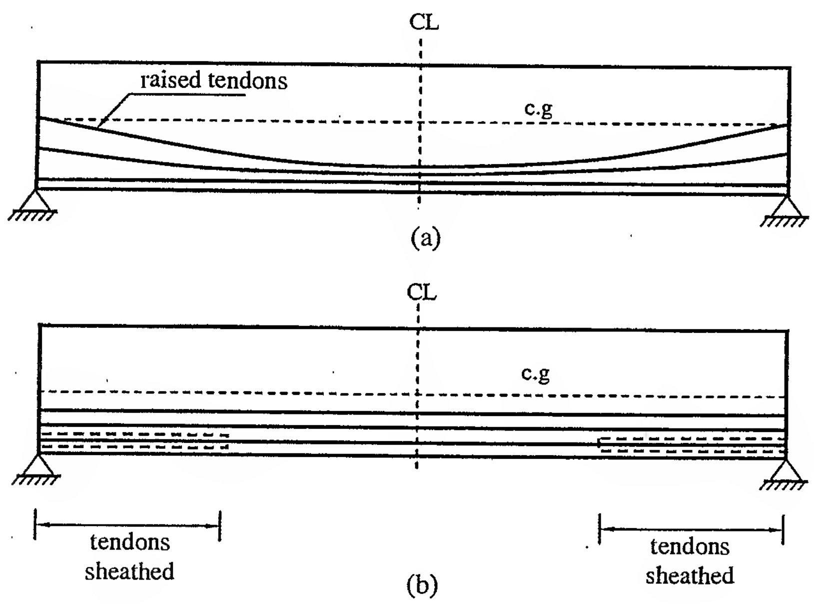


Fig. 8.18 Reduction of prestressing force near support (a) raising part of the tendons. (b) sheathing part of the tendons

8.9 Deflection of Prestressed Beams

8.9.1 Introduction

Deflection of prestressed concrete beams is of a great importance because they are more slender than ordinary reinforced concrete beams.

At transfer, prestressed concrete beams are subjected mainly to an eccentric compression force that produces reverse deflection (*camber*). The amount of camber should be controlled for proper drainage of roofs in buildings. Moreover, in projects involving prestressed beams and precast slabs, excessive camber may prohibit proper alignment of the precast members.

At the final stage where all the working loads are applied, the upward deflection of the beam (camber) becomes downward deflection. Excessive deflections of beams may cause excessive vibrations, damage to the appearance of the structure, poor roof drainage, and uncomfortable feelings for the occupants. Also, such deflections may damage partitions and cause poor fitting of doors and windows.

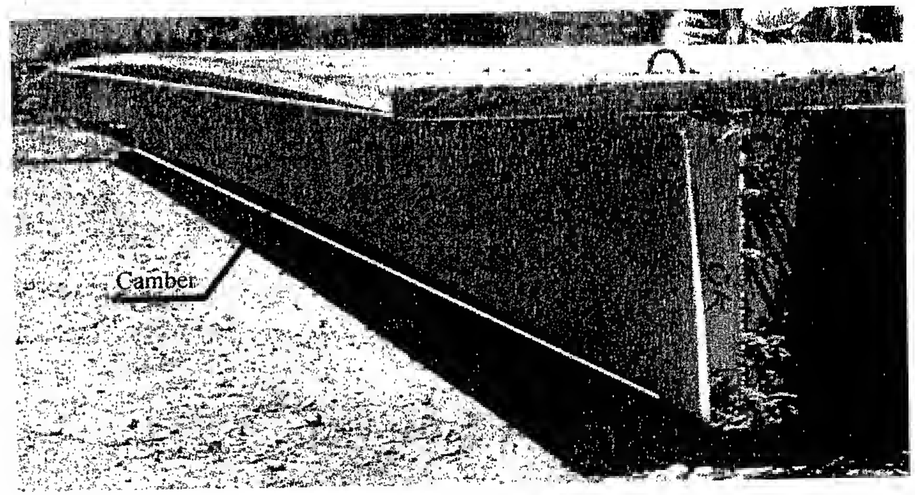


Photo 8.9 Camber of a prestressed beam at transfer

8.9.2 Calculations of Deflections - ECP 203

The procedures for calculating the deflections of prestressed concrete beams are summarized as follows:

1- The calculation of the expected camber of prestressed beams at transfer should be carried out using the gross moment of inertia of the cross-section I_g. Such a camber should be limited to the values that do not cause problems to the project under consideration. It is the task of the designer to judge the allowable value of the camber.

2- When calculating the immediate (short-term) deflection, the gross moment of inertia of the cross-section I_g is used for cases A, B, and C. For case D, the effective moment of inertia of the cross-section I_e is used. Limits of the short-term deflection for prestressed concrete beams are the same as those for reinforced concrete beams.

3- Long-term deflection of prestressed concrete beams is calculated taking into account all the permanent loads, in addition to the effect of shrinkage, creep and relaxation of prestressing steel. Limits of the long-term deflection of prestressed concrete beams are the same as those for reinforced concrete beams.

Young's modulus of concrete E_c is used in deflection computations. The value of E_c can be computed using the ECP 203 equation as follows:

$$E_c = 4400\sqrt{f_{cu}} \quad \text{.......... (8.68)}$$

The values of the deflections can be calculated using the theory of structures with the appropriate value of the moment of inertia according to the case under consideration. Examples of the deflection expressions are given in Eq. 8.69 and the rest can be found in then Appendix.

$$\Delta = \left|\begin{array}{ll} \dfrac{w\,L^4}{384\,E_c I} & \text{for fixed end beam with uniform load} \\ \dfrac{5w\,L^4}{384\,E_c I} & \text{for simple beam with uniform load} \\ \dfrac{P\,L^3}{48\,E_c I} & \text{for simple beam with point load at midspan .... (8.69)} \\ \dfrac{w\,L^4}{8\,E_c I} & \text{for cantilever beam with uniform load} \\ \dfrac{P\,L^3}{3\,E_c I} & \text{for cantilever beam with point load at edg} \end{array}\right.$$

A-Calculation of the effective moment of inertia (I_e)

The ECP 203 gives the following expression for calculating the effective moment of inertia:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \quad \text{.......... (8.70)}$$

$$i.e \left(\frac{M_{cr}}{M_a}\right)^3 \le 1$$

The previous equation can be simplified as follows:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \quad \text{.......... (8.71)}$$

where

I_{cr} the cracked moment of inertia (mm^4)

I_g the gross moment of inert

M_a maximum unfactored moment (kN.m)

M_{cr} cracking moment (kN.m)

$$M_{cr} = \frac{I_g}{y_t}\left(0.45\sqrt{f_{cu}} + f_{pce} - f_{cd}\right) \quad \text{.......... (8.72)}$$

Where y_t is the distance from the neutral axis to the outermost tension fibers as shown in Fig. 8.19.

$$f_{pce} = \frac{P_e}{A} + \frac{P_e \times e \times y_t}{I_g}$$

$$f_{cd} = \frac{M_d\, y_t}{I_g}$$

In which M_d= unfactored moment due to dead load.

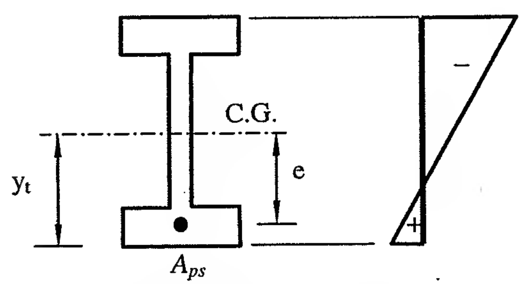
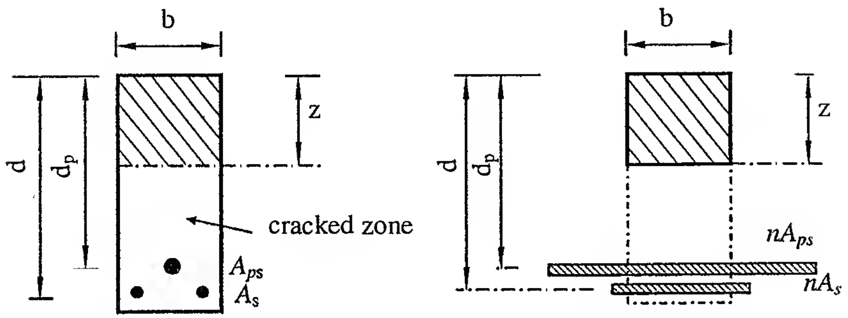


Fig. 8.19 Calculation of the cracking moment

B-Calculation of the cracked moment of inertia (I_{cr})

If we assume that the neutral axis is located at a distance z from the compression face, the location of the neutral axis can be easily determined by taking the first moment of area about the center of gravity of the section (C.Gg.) as shown in Fig. 8.20 and as given by Eq. 8.73. It should be noted that the center of gravity coincides with neutral axis *(no normal force)*.

$$b \times z^2/2 - n\,A_s\,(d-z) - n\,A_{ps}(d_p - z) \;=0 \quad \text{.............. (8.73)}$$



Cracked section　　　　Transformed section

Fig. 8.20 Determination of the neutral axis

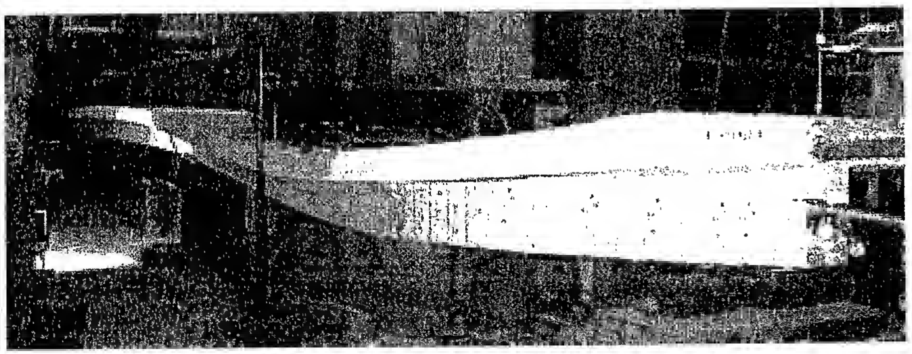
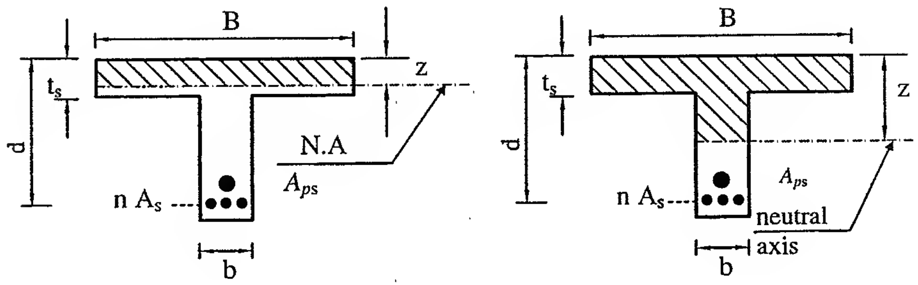


Fig. 8.10 Deflection of a prestressed beam during testing

Having determined the neutral axis distance z, the cracked moment of inertia I_{cr} can be computed as:

$$I_{cr} = \frac{b \times z^3}{3} + n\,A_s\,(d-z)^2 + n\,A_{ps}\,(d_p - z)^2 \quad \text{................ (8.74)}$$

In T-sections, the location of the neutral axis may lie inside or outside the flange as shown in Fig. 8.21. Therefore, hand calculations must be carried out as explained in the illustrative examples.



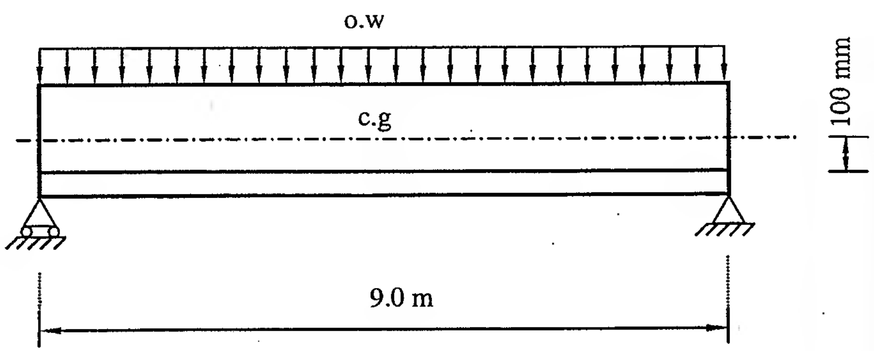
Neutral axis inside the flange $z<t_s$　　　　Neutral axis outside the flange $z>t_s$

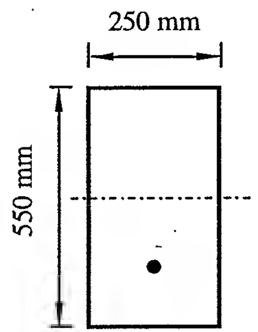
Fig. 8.21 Determination of the neutral axis for T-sections

Example 8.14

The figure shows a simply supported full prestressed beam with straight tendons at an eccentricity of 100 mm. The initial tendon force is 1200 kN and the effective prestressing force is 950 kN. Compute the initial camber at mid-span due to prestressing and self weight of the beam. If the beam is left without being subjected to any additional loads for a long period of time, calculate the long term camber.

f_{cu} =40 N/mm^2





Beam Section

Solution

Step 1: Calculation of immediate deflection

Step 1.1: Deflection due to self weight

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{40} = 27828\,N/mm^2 = 27.828\,kN/mm^2$$

$$w_{ow} = (250\times550)\times\frac{25}{1000\times1000} = 3.44\ kN/m'$$

$$M_{ow} = \frac{w_{ow}\times L^2}{8} = \frac{3.44\times9^2}{8} = 34.83\ kN.m$$

Since the beam is fully prestressed (case A), the gross moment of inertia is used in deflection calculations.

$$I_g = \frac{b\,t^3}{12} = \frac{250\times550^3}{12} = 3.47\times10^9\ mm^4$$

The deflection of simply supported beam subjected to uniform load equals:

$$\Delta_{ow} = \frac{5\,w_{ow}\times L^4}{384\times E_c\times I_g} = \frac{5\times(3.44/1000)\times(9000)^4}{384\times27.828\times3.47\times10^9} = 3.043\,mm\downarrow$$

Step 1.2: Camber due to prestressing

The immediate deflection at mid span due to initial prestressing only equals:

$$\Delta_p = \frac{M_i\,L^2}{8\,E_c\,I_g}$$

The initial moment due to prestressing at the supports M_i equals:

$$M_i = P_i\times e = 1200\times100 = 120000\,kN.mm$$

$$\Delta_p = \frac{M_i\,L^2}{8\,E_c\,I_g} = \frac{120000\times(9000)^2}{8\times27.828\times3.47\times10^9} = 12.6\,mm\uparrow\text{(camber)}$$

Step 1.3: Immediate camber/deflection

The immediate camber equals:

$$\Delta_{ow+p} = \Delta_p - \Delta_{ow} = 12.6 \uparrow - 3.043 \downarrow = 9.55\ mm \uparrow$$

Step 2: Long-term deflection

Step 2.1 Long-term deflection due to self-weight

Since no compression steel is present, the creep coefficient equals 2.

$$\Delta_{ow\ (long-term)} = (1+\alpha)\times\Delta_{ow} = (1+2)\times 3.04 = 9.13\ mm \downarrow$$

Step 2.2 Long-term camber due to prestressing

The long-term deflection at mid span due to final prestressing only equals

$$\Delta_{p(long-term)} = (1+\alpha)\frac{M_e\ L^2}{8E_c I_g}$$

For long term calculations, the effective prestressing force (P_e)is used. Thus, the moment at the supports M_e equals:

$$M_e = P_e \times e = 950\times 100 = 95000\ \ kN.mm$$

$$\Delta_{p(long-term)} = (1+\alpha)\frac{M_e\ L^2}{8E_c I_g} = (1+2)\frac{95000\times(9000)^2}{8\times 27.828\times 3.47\times 10^9} = 29.88\,mm \uparrow$$

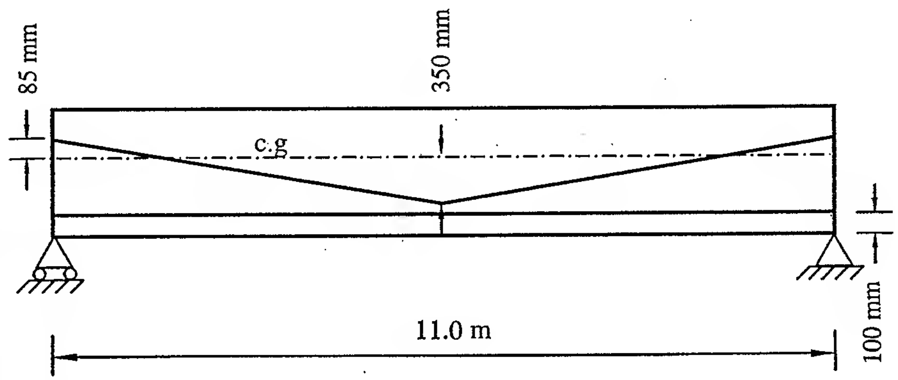
Step 2.3: Long-term camber/deflection

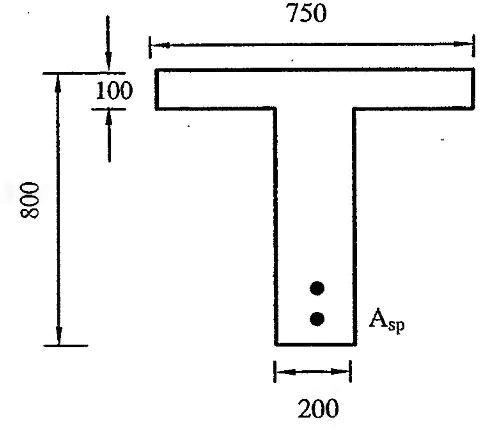
The long-term camber equals:

$$\Delta_{d+p} = \Delta_p - \Delta_d = 29.88 - 9.13 = 20.75\,mm \uparrow \text{ (camber)}$$

Example 8.15

Compute the immediate camber at midspan for the beam shown in figure. The initial prestressing force in the broken tendon is 1500 kN and in the straight cable is 300 kN. The beam may be classified as zone B. f_{cu} =45 N/mm^2



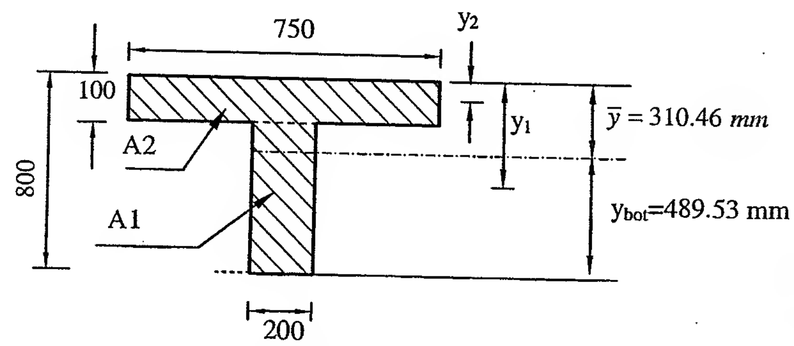


Beam Section

Solution

Step 1: Calculate uncracked section properties

Since the beam is categorized in zone B, the gross moment of inertia is used in the calculations. Since the section is not symmetrical, calculate the C.G.



uncracked section

$A_1 = 200 \times 700 = 140000\ mm^2$ $\quad$ y_1=450 mm

$A_2 = 750 \times 100 = 75000\ mm^2$ $\quad$ y_2=50 mm

$$\bar{y} = \frac{140000 \times 450 + 75000 \times 50}{140000 + 75000} = 310.465\ mm$$

$$y_{bot} = 800 - 310.46 = 489.53\ mm$$

$$I_g = \frac{200 \times 700^3}{12} + 140000 \times (450 - 310.46)^2 + \frac{750 \times 100^3}{12} + 75000 \times (310.46 - 50)^2$$

I_g=13.6 x10^9 mm^4

Step 2: Deflection due to self weight

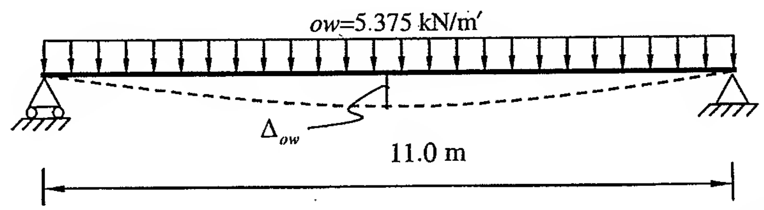
The concrete modulus of elasticity equals:

$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{45} = 29516\ N/mm^2 = 29.516\ kN/mm^2$$

$$w_{ow} = (75000 + 140000) \times \frac{25}{1000 \times 1000} = 5.375\ kN/m'$$

The deflection of simply supported beam subjected to uniform load equals:

$$\Delta_{ow} = \frac{5\, w_{ow} \times L^4}{384 \times E_c \times I_g} = \frac{5 \times (5.375/1000) \times (11000)^4}{384 \times 29.516 \times 13.6 \times 10^9} = 2.55\, mm \downarrow$$



Step 3: Camber due to prestressing

The camber due to prestressing is the sum of two components:

- End moment (due to eccentricity of the two cables at support)
- Due to the concentrated load developed from the tendon slope change

$$\Delta_p = \Delta_{pm} + \Delta_{pw}$$

Deflection due to end moments Δ_{pm}

The moment at the supports due to the broken cable M_{i1} equals

$M_{i1} = P_{i1} \times e_1$

$e_1 = 85\ mm \uparrow$

$M_{i1} = 1500 \times 85 = 127500\ kN.mm$ (positive moment)

The moment at the supports due to the straight cable Mi_2 equals:

$M_{i2} = P_{i2} \times e_2$

$e_2 = y_{bot} - \text{cover} = 489.53 - 100 = 389.53\ mm \downarrow$

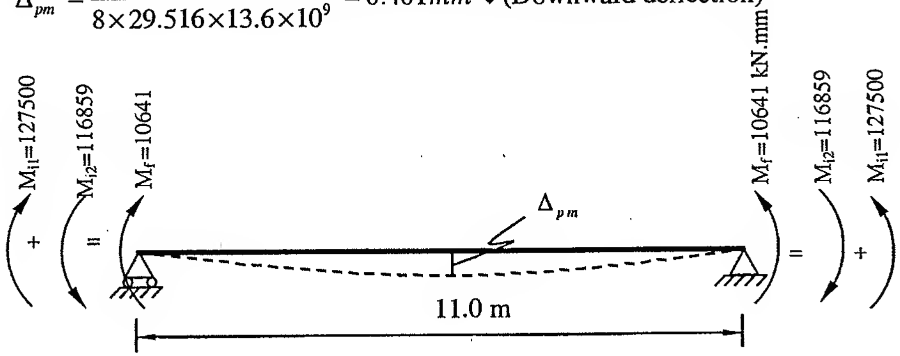
$M_{i2} = 300 \times 389.53 = 116859\, kN.mm$ (negative moment)

The resultant moment M_f=M_{i1} –M_{i2}=127500-116859= 10641 kN.mm (positive)

The immediate deflection at mid span due to the positive end moments equals:

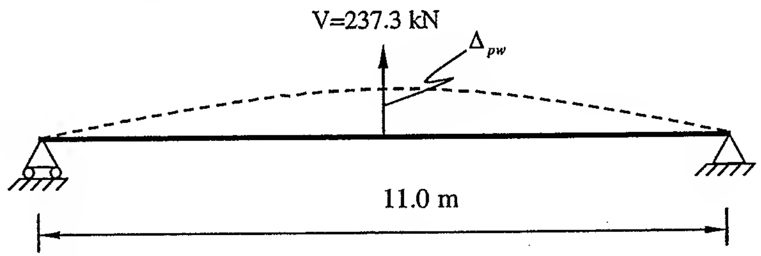
$$\Delta_{pm} = \frac{M_f \; L^2}{8 E_c \, I_g} \downarrow$$

$$\Delta_{pm} = \frac{10641 \times (11 \times 1000)^2}{8 \times 29.516 \times 13.6 \times 10^9} = 0.401 mm \downarrow \text{(Downward deflection)}$$



Deflection due to concentrated load

The camber due to the broken cable can be calculated using an equivalent concentrated load usually called the *balanced load*. The calculations of such load are as follows:



The effective sage (e_t)at midspan equals:

$$e_t = 85 + 350 = 435 mm$$

$$\frac{V \times L}{4} = P_i \times e_t$$

$$\frac{V \times 11}{4} = 1500 \times 0.435$$

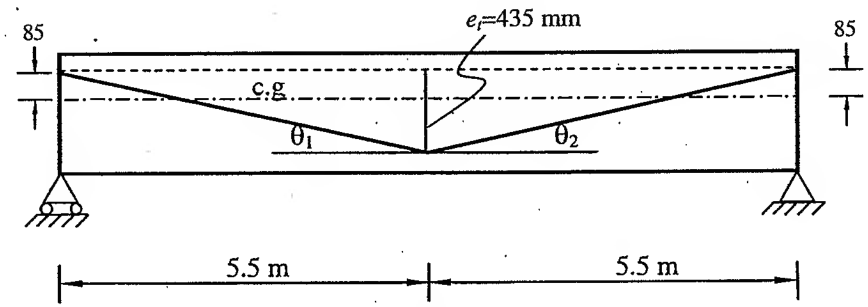
$$V = 237.3 \; kN$$

Alternatively, the equivalent force at midspan may be calculated as follows:

$$V = P_i \; (\tan\theta_1 + \tan\theta_2) = P_i \; (\theta_1 + \theta_2) \qquad \text{(true for small angles)}$$

$$\theta_1 = \theta_2 = \frac{(85 + 350)/1000}{5.5} = 0.0791$$

$$V = 1500 \times (0.0791 + 0.0791) = 237.3 \; kN$$



The immediate camber at midspan due to the equivalent concentrated load V equals:

$$\Delta_{pv} = \frac{V \; L^3}{48 E_c \, I} \uparrow$$

$$\Delta_{pv} = \frac{237.3 \times (11000)^3}{48 \times 29.516 \times 13.6 \times 10^9} = 16.4 \; mm \uparrow$$

$$\Delta_p = \uparrow \Delta_{pv} - \downarrow \Delta_{pm} = 16.4 - 0.401 \cong 16 mm \uparrow$$

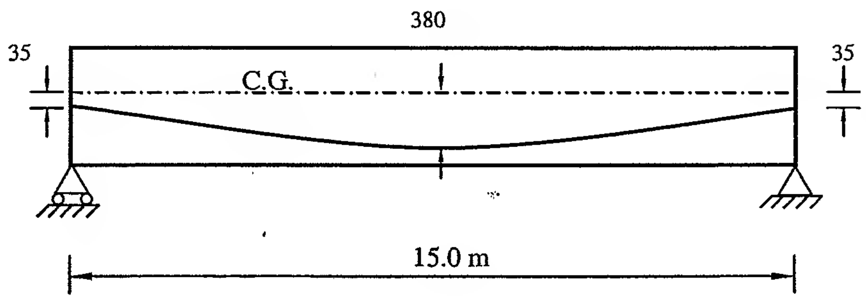
Step 4: Immediate camber/deflection

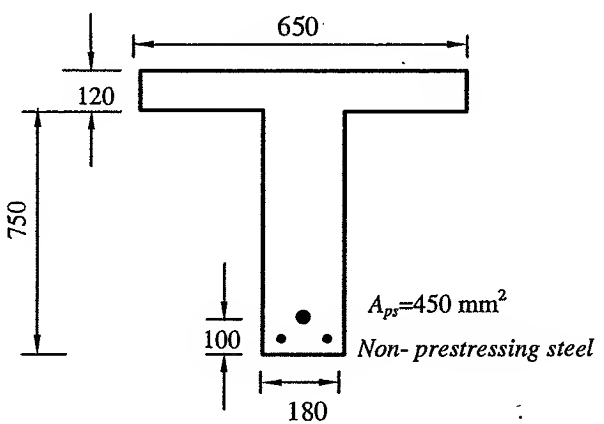
The immediate camber equals:

$$\Delta_{P+ow} = \uparrow \Delta_p - \downarrow \Delta_{ow} = 16 - 2.55 = 13.45 \; mm \uparrow$$

Example 8.16

The figure given below shows a simply supported partially prestressed beam (zone D). The beam is subjected to an initial prestressing force of 900 kN and an effective prestressing force of 740 kN. Calculate the immediate deflection, the long-term deflection and check code limits for deflection. The beam is located on a typical floor and support walls that are not likely to be damaged by deflection. w_{LL}=10 kN/m′, w_{SD}=3 kN/m′ (superimposed load), f_{cu}=35 N/mm^2, and n=10.



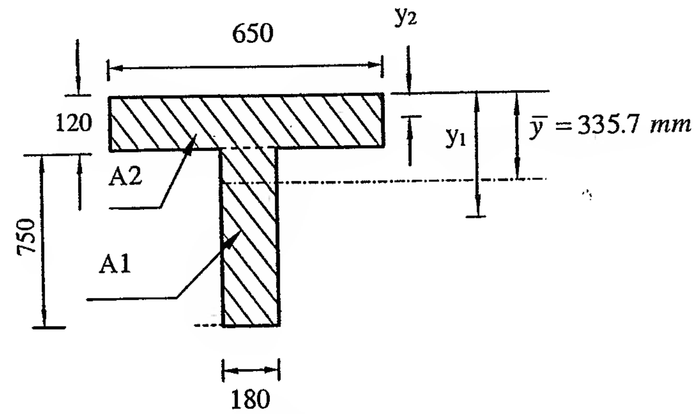


Beam cross-section

Solution

Step 1: Calculate uncracked section properties

The calculation of the gross moment of inertia shall be carried out by considering the concrete section and neglecting the prestressing steel as well as the non-prestressingd steel. Since the section is not symmetrical, the center of gravity is calculated as follows:



uncracked section

$A_1 = 180 \times 750 = 135000\ mm^2$ y1=495 mm

$A_2 = 650 \times 120 = 78000\ mm^2$ y2=60 mm

$A = A_1 + A_2 = 213000\ mm^2$

$$\bar{y} = \frac{135000 \times 495 + 78000 \times 60}{213000} = 335.7\ mm$$

$$I_g = \frac{180 \times 750^3}{12} + 135000 \times (495 - 335.7)^2 + \frac{650 \times 120^3}{12} + 78000 \times (335.7 - 60)^2$$

I_g=15.77 x10^9 mm^4

$$y_{bot} = (120 + 750) - 335.7 = 534.3\ mm$$

$$z_{bot} = \frac{I_g}{y_{bot}} = \frac{15.77 \times 10^9}{534.3} = 29.53 \times 10^6\ mm^3$$

Step 2: Calculate immediate deflection /camber
Step 2.1: Deflection due to self-weight

Since at transfer only the self weight is applied, the gross moment of inertia may be used for deflection calculations

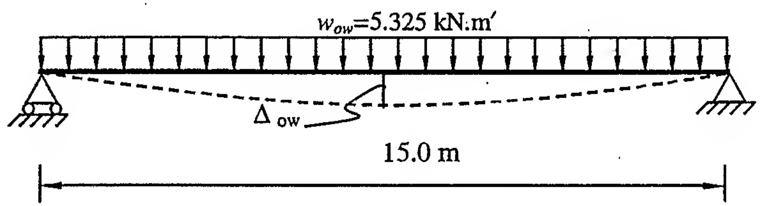
$$E_c = 4400\sqrt{f_{cu}} = 4400\sqrt{35} = 26030\,N/mm^2 = 26.03\,kN/mm^2$$

The self weight of the beam equals

$$w_{ow} = 213000 \times \frac{25}{1000\times1000} = 5.325\ kN/m'$$

The deflection of simply supported beam subjected to uniform load equals:

$$\Delta_{ow} = \frac{5\,w_{ow}\times L^4}{384\times E_c\times I_g} = \frac{5\times(5.325/1000)\times(15000)^4}{384\times26.03\times15.77\times10^9} = 8.55\,mm \downarrow (downward)$$



Step 2.2 Camber due to prestressing

The camber due to prestressing is the sum of two components:

- End moment (due to eccentricity of the cable at support) Δ_{pm}
- Due to the uniform load developed from change of curvature of the tendon Δ_{pw} $\Delta_p = \Delta_{pm} + \Delta_{pw}$

Camber due to eccentricity of the cable

The immediate deflection at midspan due to the negative end moments equals:

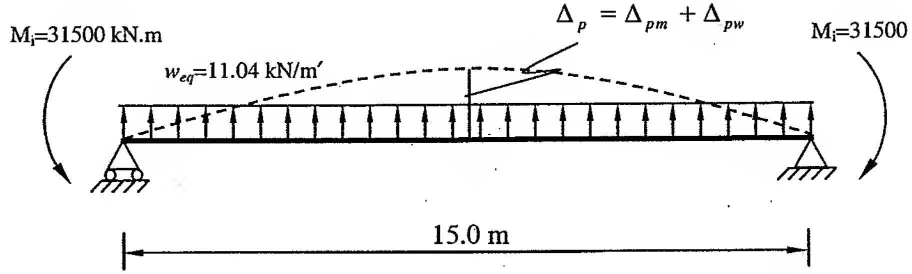
$$\Delta_{pm\,(due\ to\ end\ mment)} = \frac{M_i\,L^2}{8E_c\,I_g}$$

The moment at the supports M_i equals:

$$M_i = P_i \times e = 900\times35 = 31500\ kN.m$$

$$\Delta_{pm} = \frac{M_i\,L^2}{8E_c\,I_g} = \frac{31500\times(15000)^2}{8\times26.03\times15.77\times10^9} = 2.15\,mm \uparrow (\text{upward})$$

Camber due to curved tendon



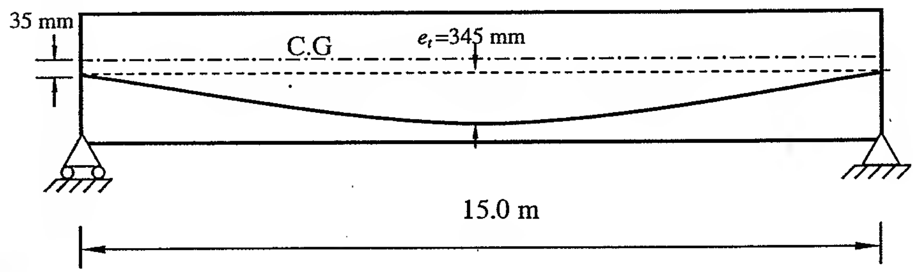
The effective eccentricity of the curved tendon at midspan (e_t) equals:

$$e_t = 380 - 35 = 345\,mm$$

The camber due to the curvature of the cable can be calculated using an equivalent uniform load usually called the *balanced load*. The calculations of such load are as follows:

$$\frac{w_{eq}\times L^2}{8} = P_i \times e_t$$

$$\frac{w_{eq}\times 15^2}{8} = 900\times0.345$$



$w_{eq} = 11.04\ kN/m' \uparrow$

$$\Delta_{pw} = \frac{5\, w_{eq} \times L^4}{384 \times E_c \times I_g} = \frac{5 \times (11.04/1000) \times (15000)^4}{384 \times 26.03 \times 15.77 \times 10^9} = 17.72\, mm \uparrow$$

$$\Delta_p = \Delta_{pm} + \Delta_{pw} = 2.15 \uparrow + 17.72 \uparrow = 19.87\ mm \uparrow$$

Step 2.3: Immediate camber/deflection

The immediate camber equals:

$$\Delta_{p+ow} = \Delta_p + \Delta_{ow} = 19.87 \uparrow - 8.55 \downarrow = 11.33\, mm \uparrow$$

Step 3: Service load deflection

Since the beam is partially prestressed, one should calculate the cracking moment and the effective moment of inertia.

Step 3.1: Calculate cracking moment M_{cr}

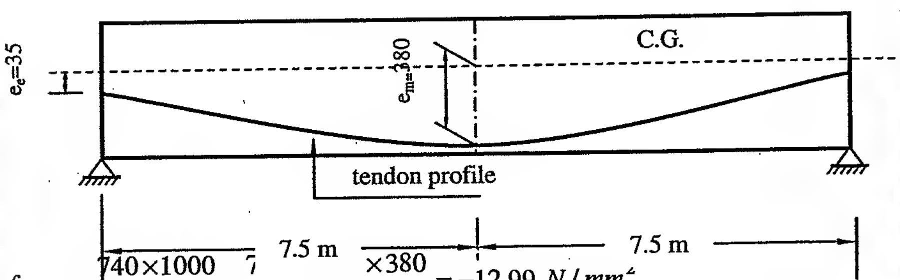
According the ECP 203 the cracking moment of prestressed beams equals:

$$M_{cr} = \frac{I}{y_t}\left(0.45\sqrt{f_{cu}} + f_{pce} - f_{cd}\right)$$

where f_{pce} is the compressive at the extreme fibers(bottom in this case) due to effective prestressing force only.

$$f_{pce} = -\frac{P_e}{A} - \frac{P_e \times e}{Z_{bot}}$$

The eccentricity at midspan equals 380 mm.



$$f_{pce} = -\frac{740 \times 1000}{213000} - \frac{740 \times 1000 \times 380}{29.53 \times 10^6} = -12.99\ N/mm^2$$

$$w_{DL} = w_{ow} + w_{SD} = 5.325 + 3 = 8.325\ kN/m'$$

$$M_{DL} = \frac{w_{DL}\, L^2}{8} = \frac{8.325 \times 15^2}{8} = 234.14\ kN.m$$

$$f_{cd} = \frac{M_{DL}}{Z_{bot}} = \frac{234.14 \times 10^6}{29.53 \times 10^6} = 7.93\, N/mm^2$$

Noting that $Z_{bot} = I/y_t$, thus,

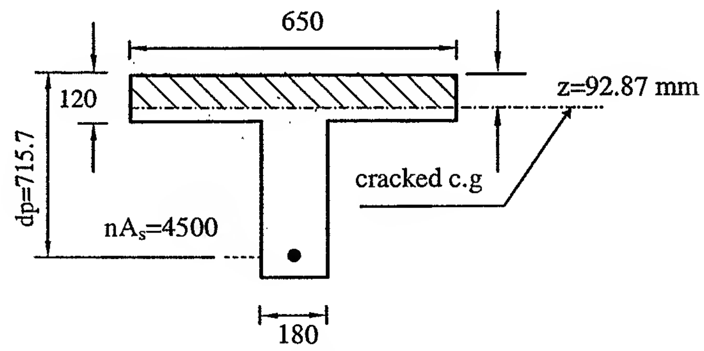
$$M_{cr} = 29.53 \times 10^6 \left(0.45\sqrt{35} + 12.99 - 7.93\right)/10^6 = 228.25\ kN.m$$

$$w_{total} = w_{DL} + w_{LL} = 8.325 + 10 = 18.325\ kN/m'$$

$$M_a = \frac{w_{total}\, L^2}{8} = \frac{18.325 \times 15^2}{8} = 515.39\ kN.m$$

Since $M_a > M_{cr}$ then calculate I_e

Step 3.2: Calculate cracked moment of inertia I_{cr}



I_{cr} calculations

It is customary to neglect the area of the non-prestressing steel in deflection calculations. The transformed area of prestressing steel equals:

$n\ A_{ps} = 10 \times 450 = 4500\ mm^2$

$$d_p = \bar{y} + e = 335.7 + 380 = 715.70\ mm$$

Assume that the c.g. is located inside the flange. Taking the first moment of area for the transformed section about the C.G. gives,

$$B\frac{z^2}{2} = n\,A_{ps}\,(d_p - z)$$

$$650 \times z \times \frac{z}{2} = 4500\,(715.70 - z)$$

$$325\,z^2 + 4500\,z - 3220650 = 0$$

Solving for z $\qquad z = 92.87$ mm < 120 mm *(inside the flange as assumed)*

Cracked moment of inertia $I_{cr} = \dfrac{B\,z^3}{3} + n\,A_{ps}(d_p - z)^2$

$$I_{cr} = \frac{650 \times 92.87^3}{3} + 4500 \times (715.7 - 92.87)^2 = 1.92 \times 10^9\ mm^4$$

Step 3.3: Calculate effective moment of inertia

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3$$

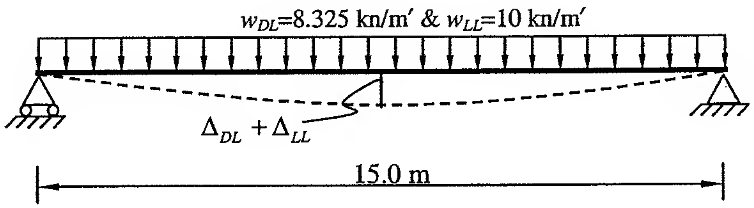
$$I_e = 1.92 \times 10^9 + \left(15.77 \times 10^9 - 1.92 \times 10^9\right)\left(\frac{228.25}{515.39}\right)^3 = 3.122 \times 10^9\ mm$$

Step 3.4: Deflection due to total load

The deflection of simply supported beam subjected to dead loads only equals:

$$\Delta_{DL} = \frac{5\,w_{DL} \times L^4}{384 \times E_c \times I_e} = \frac{5 \times (8.325/1000) \times (15 \times 1000)^4}{384 \times 26.03 \times 3.122 \times 10^9} = 67.5\,mm \downarrow$$

$$\Delta_{LL} = \frac{w_{LL}}{w_{DL}} \times \Delta_{DL} = \frac{10}{8.325} \times 67.5 = 81.09\ mm \downarrow$$



Step 3.5: Camber due to effective prestressing

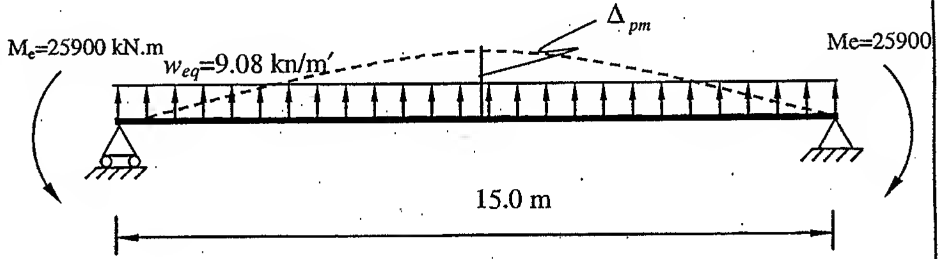
The service load deflection at mid-span due to the negative end moments only equals:

$$\Delta_{pm} = \frac{M_e\,L^2}{8\,E_c\,I_e} \uparrow$$

The moment at the supports equals:

$$M_e = P_e \times e = 740 \times 35 = 25900\ kN.m$$

$$\Delta_{pm} = \frac{M_e\,L^2}{8\,E_c\,I_e} = \frac{25900 \times (15 \times 1000)^2}{8 \times 26.03 \times 3.122 \times 10^9} = 8.96\,mm \uparrow$$



The eccentricity (e_t)at midspan equals, $e_t = 380 - 35 = 345\,mm$

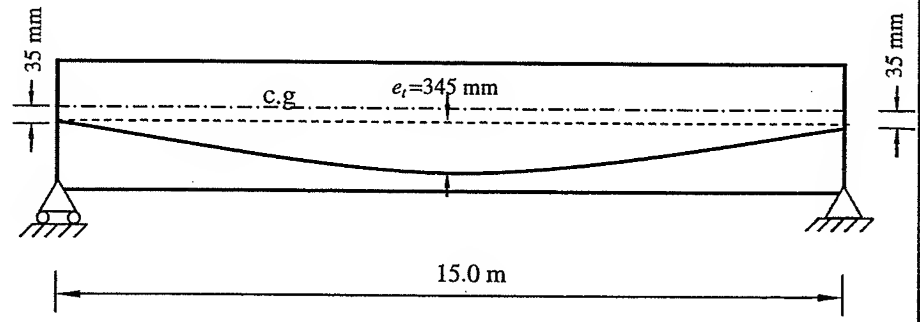
$$\frac{w_{eq} \times L^2}{8} = P_e \times e_t$$

$$\frac{w_{eq} \times 15^2}{8} = 740 \times 0.345$$

$$w_{eq} = 9.08\ kN/m' \uparrow$$

$$\Delta_{pw} = \frac{5\,w_{eq} \times L^4}{384 \times E_c \times I_e} = \frac{5 \times (9.08/1000) \times (15 \times 1000)^4}{384 \times 26.03 \times 3.122 \times 10^9} = 73.61\,mm \uparrow$$

$$\Delta_p = \Delta_{pm} + \Delta_{pw} = 8.96 \uparrow + 73.61 \uparrow = 82.57\ mm \uparrow$$



Step 3.6: Immediate deflection

The final deflection equals:

$$\Delta_{total} = (\Delta_{DL} + \Delta_{LL}) - \Delta_p = (67.507 + 81.09) \downarrow -82.57 \uparrow = 66.03\,mm \downarrow$$

Step 4: Long-term deflection

The long term factor α=2

$$\Delta_{long-term} = (1+\alpha)\ (\Delta_{DL} + \Delta_P) + \Delta_{LL}$$

$$\Delta_{long-term} = (1+2)\ (67.507 \downarrow -82.57 \uparrow) + 81.09 \downarrow = 35.905\,mm \downarrow$$

$$< \frac{L}{250} < \frac{1500}{250} \ldots . o.k$$

The beam satisfies the limits of the total deflection. However, the live load deflection should be checked as follows

- The beam is in a floor that supports walls that are not likely to be damaged by large deflections. Hence, the limiting live load deflection is given by:

$$\Delta_{LL} \le \frac{L}{360} \le \frac{15 \times 1000}{360} = 41.67\ mm$$

Since Δ_{LL} (81.09 mm)< $\Delta_{allowable}$ (41.67 mm) the beam does not meet code requirements for deflections.

9

SHEAR AND TORSION IN PRESTRESSED CONCRETE BEAMS

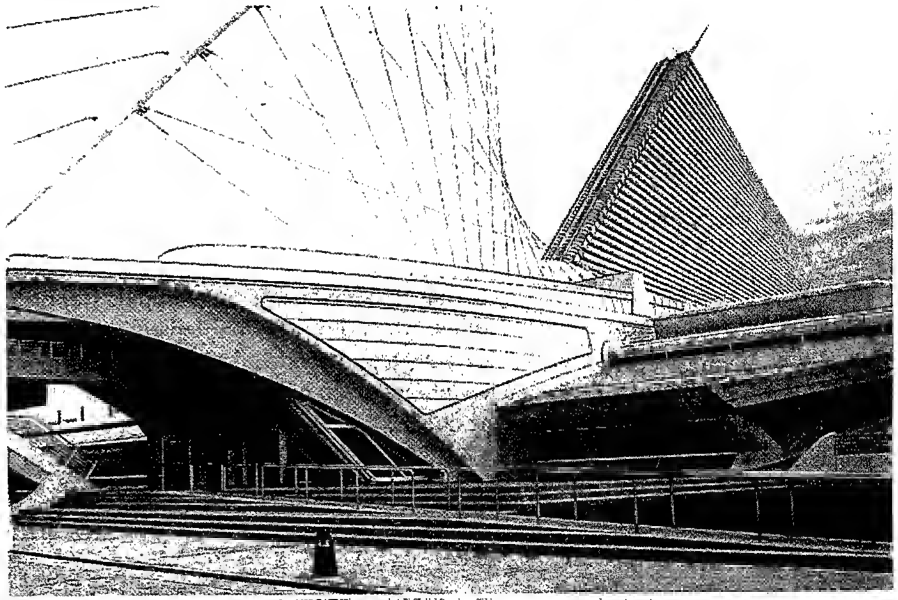


Photo 9.1 Milwaukee art museum, USA

9.1 Introduction

This chapter presents procedures for the design of prestressed concrete sections to resist shear and torsion resulting from externally applied loads. Since the strength of concrete in tension is considerably lower than its strength in compression, design for shear and torsion becomes of major importance in all types of concrete structures.

The behavior of prestressed concrete beams in shear or combined shear and torsion is different from their behavior in flexure: they may fail abruptly

without sufficient advance warning, and the diagonal cracks that develop are considerably wider than the flexural cracks. Both shear and torsion forces result in shear stress. Such a stress can result in principal tensile stresses at the critical section which can exceed the tensile strength of concrete.

9.2 Shear in prestressed Beams

9.2.1 Inclined Cracking

Cracking in prestressed concrete beams depends on the magnitude of moment and shear as shown in Fig. 9.1. At locations where the moment is large and shear is small, vertical flexural cracks form when the normal tensile strength is exceeded.

Two types of inclined cracking occur in prestressed concrete beams; *web shear cracking* and *flexure-shear cracking*. These two types of inclined cracking are illustrated in Fig. 9.1.

Web-shear cracking begins at the centroidal axis of the cross section when the principal tensile stresses due to shear exceeds the tensile strength of concrete. Web shear cracking occurs in the regions where moment is small and shear is large.

Flexure-shear cracking is essentially an extension of a vertical flexural cracking. The flexure-shear crack develops when the principal tensile stress due to combined shear and flexural tensile stresses exceed the tensile strength of concrete. It should be mentioned that web-shear cracks usually occur in thin-walled I-beams near the C.G. where the shear stresses in the web are high while the flexural stresses are low.

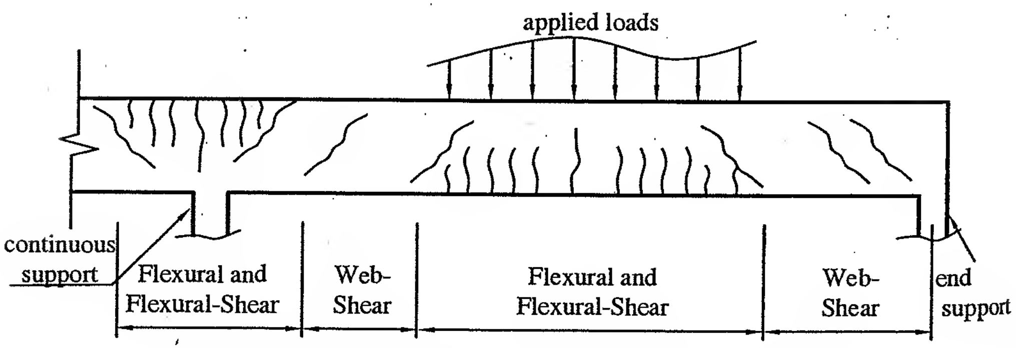


Fig. 9.1 Type of cracking in prestressed concrete beams

9.2.2 Effect of Prestressing

The shear web cracks in prestressed beams are attributed to the developing of diagonal tension stresses as shown in Fig 9.2. The maximum web shear q_{cw} occurs at the C.G. of the section where the actual diagonal tension cracks develops. Referring to Mohr's circle shown in Fig. 9.2, the principle tensile stress in concrete due to both compression stress f_{pcc} and shear stress q_{cw} equals:

$$f_t = \sqrt{\left(\frac{f_{pcc}}{2}\right)^2 + q_{cw}^2} - \left(\frac{f_{pcc}}{2}\right) \quad \text{.......... (9.1)}$$

$$\tan 2\theta = \frac{q_{cw}}{(f_{pcc}/2)} \quad \text{.......... (9.2)}$$

where f_{pcc} is the concrete compressive stress due to effective prestressing at the C.G. level. However, if the neutral axis falls inside the flange for flanged sections, the stress is calculated at the intersection of the flange and the web.

It is clear from Mohr's circle that the normal compressive stress f_{pcc} reduces the maximum principle tension f_t and the angle θ. Therefore if cracking occurs, the inclined crack is flatter and the effectiveness of the stirrups increases.

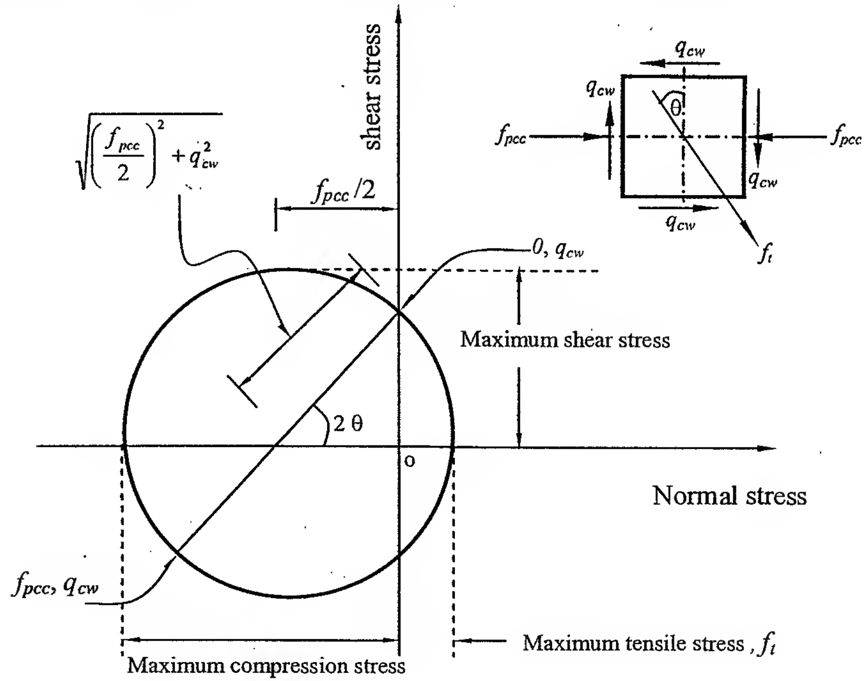


Fig. 9.2 Principle tensile stresses in prestressed beam

Solving Eq. 9.2 for the web shear strength q_{cw} gives:

$$q_{cw} = f_t \sqrt{\left(\frac{f_{pcc}}{f_t}\right) + 1} \quad \text{......} (9.3)$$

9.2.3 Shear Strength According to ECP 203

9.2.3.1 Upper Limit for Design Shear Stress q_{umax}

The critical section for shear in prestressed reinforced concrete beams is at t/2 from the face of the support.
The applied shear stress is given by:

$$q_u = \frac{Q_u}{b \times d_p} \quad \text{......} (9.4a)$$

$$d_p \geq 0.80t \quad \text{......} (9.4b)$$

where b is the width of the section and d_p is the distance from the compression fiber, to the centroid of the cables but not less than $0.8\ t$.

For prestressed concrete beams containing grouted ducts with diameter (ϕ) more than $b_w/8$ (where b_w is the width of the web), as shown in Fig. 9.3, the effective width of the web (b) in Eq. 9.4a should be taken as:

$$b = b_w - 0.5\sum\phi \qquad for\ \phi > \frac{b_w}{8} \quad \text{......} (9.4c)$$

$$b = b_w \qquad for\ \phi \leq \frac{b_w}{8} \quad \text{......} (9.4d)$$

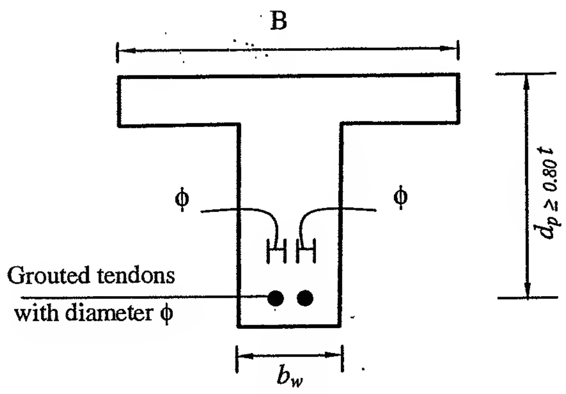


Fig. 9.3 Effective width for prestressed beams with grouted ducts

To ensure that shear failures occur in a ductile manner by yielding of the shear reinforcement, the ECP 203 specifies that the shear stress q_u should not exceed the maximum shear stress $q_{u,max}$ given by:

$$q_{u,max} = 0.75 \sqrt{\frac{f_{cu}}{\gamma_c}} \leq 4.5 N/mm^2 \quad \text{......} (9.5)$$

It is clear from Fig. 9.4 that the maximum shear strength $q_{u,max}$ for prestressed members is slightly higher than that for ordinary reinforced concrete members. However, the concrete shear strength q_{cu} of prestressed members is much higher than that of ordinary reinforced concrete members.

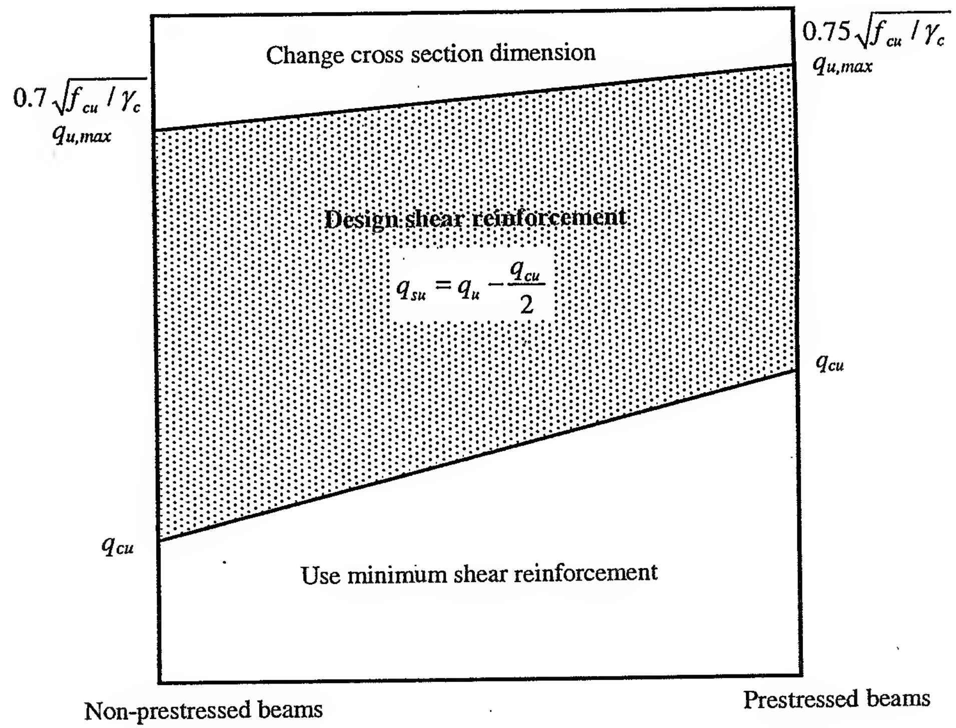


Fig. 9.4 Shear reinforcement requirements in prestressed and non-prestressed beams

9.2.3.2 Shear Strength Provided by Concrete q_{cu}

The ECP 203 gives two procedures for calculating concrete shear strength q_{cu} of prestressed beams as follows:

- Simplified procedure
- Detailed procedure

A: Concrete Shear Strength Using the Simplified Procedure

Several empirical expressions that predict shear strength of concrete have been developed from experimental studies of prestressed beams. For prestressed members in which the tendons are stressed to at least 40% of their ultimate tensile strength f_{pu}, the nominal concrete shear strength can be conservatively estimated by the following ECP 203 expression:

$$q_{cu} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + \frac{3.6\times Q_u \times d_p}{M_u} \geq 0.24\sqrt{f_{cu}/\gamma_c} \quad \ldots\ldots\ldots\ldots\ldots (9.6)$$

$$\leq 0.375\sqrt{f_{cu}/\gamma_c}$$

with the following condition $\left(\frac{Q_u \times d_p}{M_u}\right) \leq 1.0$

where Q_u and M_u are the values of shear and bending moment, respectively, produced by the ultimate loads at the section under consideration. d_p is the distance from the prestressing reinforcement to the outermost compression fibers.

The term d_p used in the previous equation is the actual cable depth. The previous equation does not give a valid result when either Q_u or M_u is small.

For simply supported beams subject to uniform loads, the term $Q_u\, d_p/M_u$ can be expressed as:

$$\frac{Q_u \times d_p}{M_u} = \frac{d_p\ (L-2x)}{x\ (L-x)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.7)$$

where L is the span length and x is the distance from the support to the critical section.

B: Concrete Shear Strength Using Detailed Procedure

Although the use of the simplified method for calculating concrete shear strength is quite easy, it may produce very conservative results. This is especially true for I-sections. The shear strength calculated using the detailed procedure can be as high as 150% of that calculated using the simplified procedure. Therefore, when the tendon stress is below 0.4 f_{pu}, or when the full concrete shear strength need to be utilized, the shear strength can be evaluated using *the detailed procedure*.

For a thin-walled section (I-beam), with small shear spans, the shear stresses in the web are high while the flexural stresses are low. The principal stresses at the neutral axis may exceed those at the bottom flange causing cracking to start at the web. This is called web-cracking shear. On the other hand, for beams with relatively large shear span, vertical flexural cracks occur first and extend diagonally due stress redistribution. This is called the flexural-shear cracking.

According to the ECP 203, the concrete shear strength q_{cu} is the smaller value of the flexural shear strength q_{ci} and the web-cracking shear strength q_{cw}.

$$q_{cu} = \text{smaller of} \begin{cases} q_{ci} \\ \\ q_{cw} \end{cases}$$

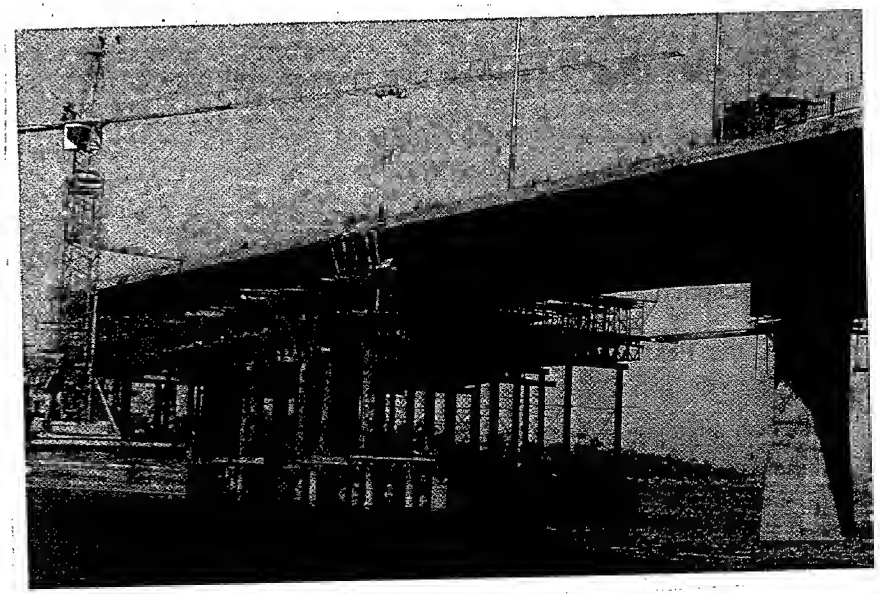


Photo 9.2 Prestressed concrete bridge during construction

Flexural-Shear Strength q_{ci}

The flexural-shear strength is the shear strength of the beam at the time of developing the first flexure-shear crack. Flexure-shear cracking occurs when flexural cracks, which are initially vertical, become inclined under the influence of shear. Because the flexural-shear strength cannot be predicted by calculating the principal stresses in an uncracked beam, equations for estimating flexural shear strength are based on experimental tests. This is attributed to the redistribution of stresses that occur at the tip of the flexural crack.

The code specifies the following formula to predict the concrete flexural shear strength q_{ci}:

$$q_{ci} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + 0.8\left(q_d + q_i \times \frac{M_{cr}}{M_{\max}}\right) \geq 0.24\sqrt{f_{cu}/\gamma_c} \quad \text{............ (9.8)}$$

Where

$Q_i = Q_u - Q_d$ $\qquad$ $M_{\max} = M_u - M_d$

$$q_i = \frac{Q_i}{b \times d}$$

$$q_d = \frac{Q_d}{b \times d}$$

q_d is the **unfactored** shear stress due to dead load only at the critical section.

q_i is **factored** shear stress at the critical section due to externally applied loads occurring simultaneously with M_{max}

M_{max} is the **factored** moment at the critical section due to the external applied loads (Refer to Fig. 9.5).

M_{cr} is the cracking moment.

d is the depth of the cross section but not less than $0.8t$

In Eq. 9.8, the flexural shear strength is assumed to be the sum of three components:

1. The shear stress required to transform a vertical flexural crack into inclined crack $0.045\sqrt{f_{cu}/\gamma_c}$
2. The un-factored shear stress $0.8\ q_d$, and
3. The portion of the remaining factored shear stress that will cause a flexural crack to initially occur $0.8 \times q_i \times M_{cr}/M_{\max}$

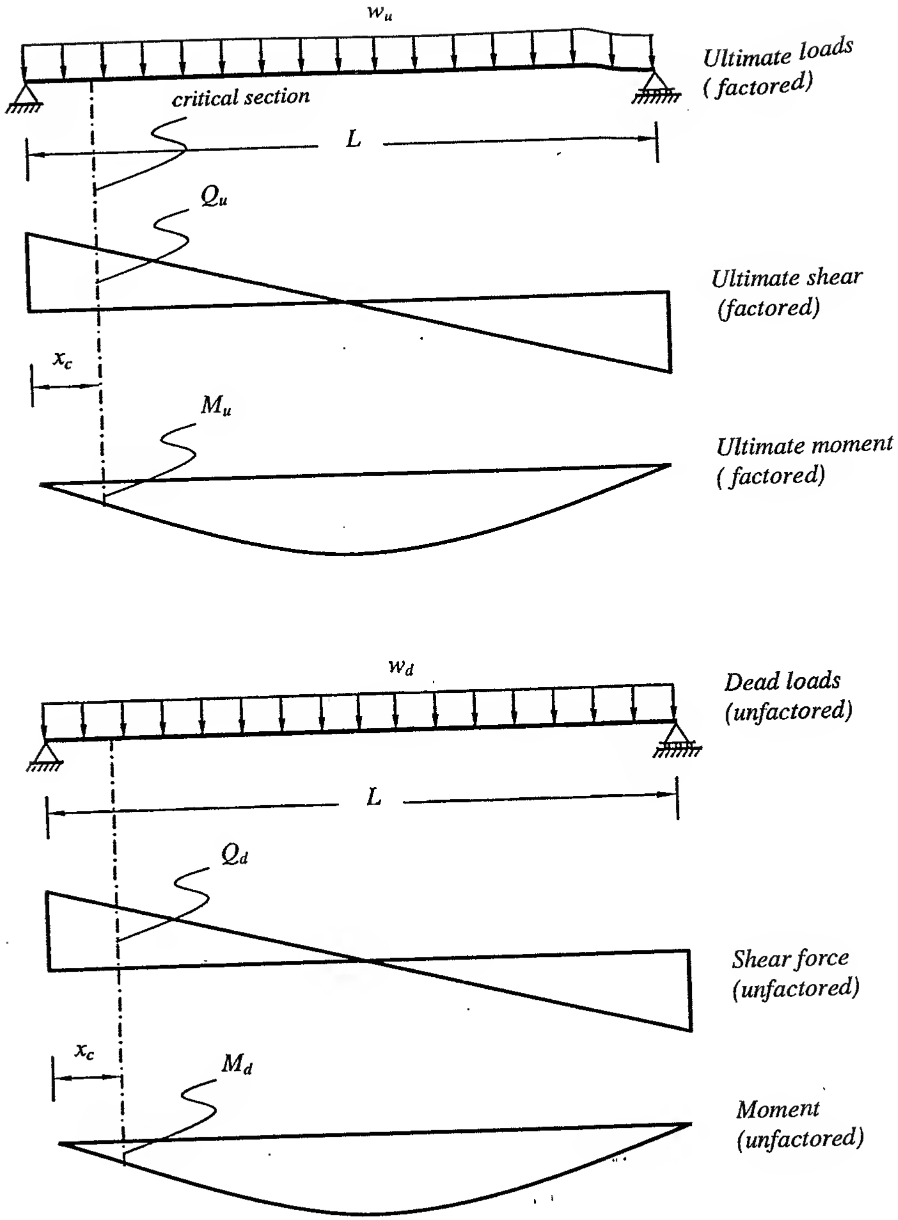


Fig. 9.5 Shear and bending moments for calculating q_{ci}

Cracking Moment M_{cr}

Concrete beams are assumed to behave elastically as long as the maximum tensile strength of the concrete is not exceeded. A simply supported prestressed beam will crack at the bottom when the tensile strength is exceeded. Thus the tensile stress at the bottom fibers for simply supported beam equals:

$$f_{ctr} = -\frac{P_e}{A} - \frac{P_e \times e \times y_t}{I} + \frac{M_{cr}\, y_t}{I} \quad \text{.............................. (9.9)}$$

where

y_t is the distance from the neutral axis to the tension surface
M_{cr} is the cracking moment
P_e is the effective prestressing force
f_{ctr} is the maximum allowable tension

The term $f_{ctr} = 0.45\sqrt{f_{cu}}$ is the tensile strength of concrete for web-shear cracking. Subsisting with this value in the previous equation gives:

$$M_{cr} = \frac{I}{y_t}\left(0.45\sqrt{f_{cu}} + \frac{P_e}{A} + \frac{P_e \times e \times y_t}{I}\right) \quad \text{................ (9.10)}$$

The code specifies that the value of the **unfactored** dead load moment at the critical section should be subtracted from the cracking moment. Hence,

$$M_{cr} = \frac{I}{y_t}\left(0.45\sqrt{f_{cu}} + \frac{P_e}{A} + \frac{P_e \times e \times y_t}{I}\right) - M_d \quad \text{.......... (9.11)}$$

The code rewrites the previous equation in terms of stress as follows:

$$M_{cr} = \frac{I}{y_t}\left(0.45\sqrt{f_{cu}} + f_{pce} - f_{cd}\right) \quad \text{........................ (9.12)}$$

where

f_{pce} is the compression stress in concrete due to prestressing force after considering all losses calculated at the extreme tension fibers as follows:

$$f_{pce} = \frac{P_e}{A} + \frac{P_e \times e \times y_t}{I}$$

f_{cd} is the concrete stress due to dead loads and any sustained live loads calculated at the extreme tension fibers.

$$f_{cd} = \frac{M_d\, y_t}{I}$$

Because most of the prestressed beams are designed to remain uncracked during the lifetime of the structure, the cracking moment is normally higher than the applied moment at full service loads. Furthermore, the ECP 203 requires that prestressed should be designed to withstand at least 1.2 M_{cr}.

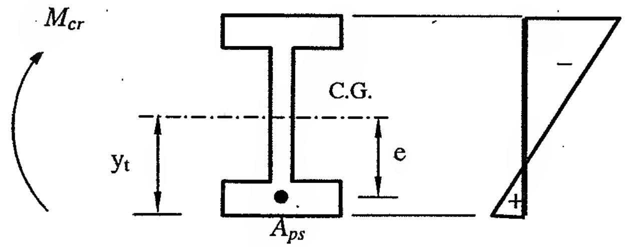


Fig. 9.6 Cracking moment calculations

Web Shear Strength q_{cw}

In the Egyptian code, the value of the web concrete shear strength is given by:

$$q_{cw} = 0.24\left(\sqrt{\frac{f_{cu}}{\gamma_c}} + f_{pcc}\right) + q_{pv} \quad \text{.............................. (9.13)}$$

where

f_{pcc} is the concrete stress at the C.G. of the section due to effective prestressing after considering all losses.

$$f_{pcc} = \frac{P_e}{A_c}$$

q_{pv} is un-factored shear stress due to the vertical component of prestressing Q_{pv}.

$$q_{pv} = \frac{Q_{pv}/\gamma_{ps}}{b \times d_p} = \frac{(P_e \times \sin\theta)/\gamma_{ps}}{b \times d_p} = \frac{(f_{pe}/\gamma_{ps} \times A_{ps}) \times \sin\theta}{b \times d_p}$$

Where $\gamma_{ps} = 1.15$. Referring to Fig. 9.7, the vertical component of the prestressing force Q_{pv} at the critical section for straight tendons equals:

$$Q_{pv} = P_e \sin(\theta) \cong P_e \tan(\theta) \quad \text{.................................... (9.14)}$$

The approximation of using (*tan*) instead of (*sin*) is justified because the eccentricity is very small compared to the span. For beams with straight cables, the vertical prestressed component equals:

$$Q_{pv} = P_e \frac{e_m - e_e}{x} \quad \text{.. (9.15)}$$

where e_m is the eccentricity inside the beam and e_e is the eccentricity at the end.

If the tendon eccentricity is located above the C.G., a negative value of e_e should be used.

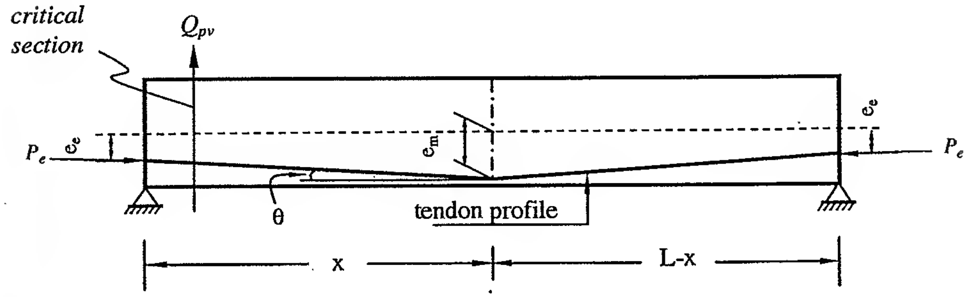


Fig. 9.7 Vertical prestressing component Q_{pv} for case of straight tendons

In case of using a parabolic tendon, the angle θ may be obtained by differentiating the equation of the parabola as follows:

$y = ax^2$, taking the first derivative gives

$y' = \tan\theta = \sin\theta = 2ax$

Where a is the parabola constant and x is the location of the section measured from the center of the parabola as shown in Fig. 9.8. It can be easily shown for the parabola in Fig. 9.8 that the following expressions are valid:

$$a = \frac{4e_m}{L^2}, \quad y = \frac{4e_m}{L^2} x^2 \quad \text{and} \quad \tan\theta = \frac{8e_m}{L^2} x$$

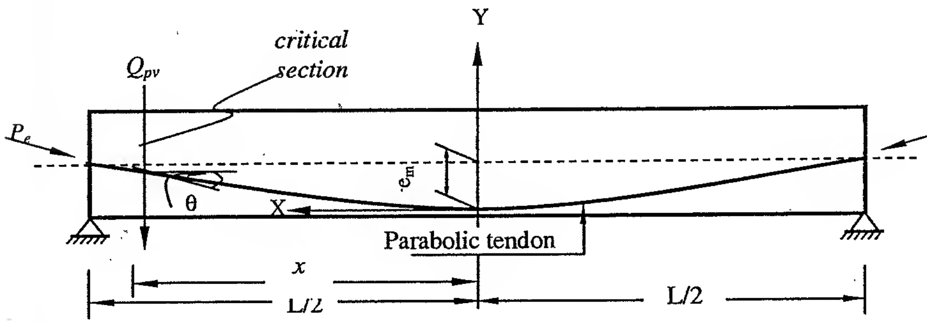


Fig. 9.8 Vertical prestressing component Q_{pv} for case of parabolic tendons

An alternate method for determining the web shear strength q_{cw} is to limit the principal tensile stresses at the C.G. of the rectangular sections or at the intersection of the web and the flange for flanged sections if the neutral axis falls inside the flange. The code states that this limit may be taken as $0.25\sqrt{f_{cu}}$. Substituting with this value for the diagonal tension stress f_t given by Eq. 9.3, the web shear strength q_{cw} can be obtained by solving the resulting equation as follows:

$$q_{cw} = 0.25\sqrt{f_{cu}} \sqrt{\left(\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}\right) + 1} \quad \text{............................ (9.16)}$$

9.2.4 Shear Reinforcement Calculations

If the applied shear stress q_u exceeds concrete shear strength q_{cu}, web reinforcement must be used according to the following equation:

$$q_{su} = q_u - \frac{q_{cu}}{2} \quad \text{.. (9.17)}$$

Shear reinforcement is obtained in a similar fashion like ordinary reinforced concrete beam. The required vertical stirrups spacing (s) is given by :

$$s = \frac{A_{st} \times f_y / 1.15}{b \times q_{su}} \quad \text{...................................... (9.18)}$$

Where A_{st} is the area of the stirrups according to the number of branches as shown in Fig. 9.9. The area of the provided stirrups should not be less than:

$$A_{st,\min} = \frac{0.4}{f_y}\, b\, s \quad \text{... (9.19)}$$

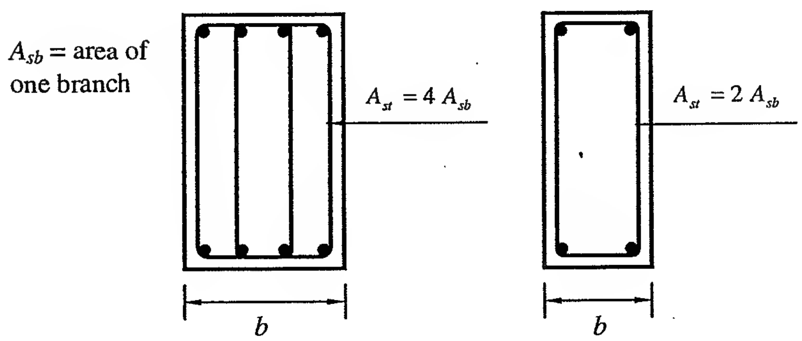


Fig. 9.9 Stirrups for shear

Summary of the Design for Shear

Step 1: Calculate the ultimate shear stresses due to Q_u

$$q_u = \frac{Q_u}{b\ d_p}$$

Where Q_u is calculated at t/2 from the support as follows:

$$Q_u = \text{Shear at column axis} - w_u \times \left(\frac{t}{2} + \frac{column\ width}{2}\right)$$

Step 2: Check that section size is adequate

The developed shear stresses due tot shear should stratify the following equation:

$$q_{u\max} \leq 0.75\sqrt{\frac{f_{cu}}{\gamma_c}}$$
$$\leq 4.5\ N/mm^2$$

If $q_u < q_{u\max}$, the concrete dimensions of the section are adequate.
If the above condition is not satisfied, one has to increase the dimensions.

Step 3: Determine the shear stress carried by concrete

Calculate the concrete contribution to the shear resistance, q_{cu} using:

- The simplified procedure (in case of $f_{pe} \geq 0.40 f_{pu}$)

$$q_{cu} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + \frac{3.6 \times Q_u \times d_p}{M_u} \geq 0.24\sqrt{f_{cu}/\gamma_c}$$
$$\leq 0.375\sqrt{f_{cu}/\gamma_c}$$

where Q_u and M_u are calculated at the critical section.

- The detailed procedure, q_{cu} is taken as the smaller of the two values:

1. $q_{ci} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + 0.8\left(q_d + q_i \times \frac{M_{cr}}{M_{\max}}\right) \geq 0.24\sqrt{f_{cu}/\gamma_c}$

2. $q_{cw} = 0.24\left(\sqrt{\frac{f_{cu}}{\gamma_c}} + f_{pcc}\right) + q_{pv}$

Where

$$M_{cr} = \frac{I}{y_t}\left(0.45\sqrt{f_{cu}} + f_{pce} - f_{cd}\right)$$

$$q_i = \frac{Q_i}{b \times d} \qquad q_d = \frac{Q_d}{b \times d} \qquad M_{\max} = M_u - M_d$$

$$f_{pcc} = \frac{P_e}{A} \qquad q_{pv} = \frac{Q_{pv}}{b\ d_p} \qquad f_{cd} = \frac{M_d\ y_t}{I}$$

$$f_{pce} = \frac{P_e}{A} + \frac{P_e \times e \times y_t}{I}$$

Step 4: Design the web reinforcement

If $q_u \leq q_{cu}$, then provide minimum stirrups.

If $q_u > q_{cu}$, then design shear reinforcement to resists q_{su} given by:

$$q_{su} = q_u - 0.5 q_{cu}$$

The spacing of stirrups needed for shear is obtained from:

$$s = \frac{A_{st} \times f_y / 1.15}{b \times q_{su}}$$

Check minimum shear reinforcement

$$A_{st,\min} = \frac{0.4}{f_y}\ b\ s$$

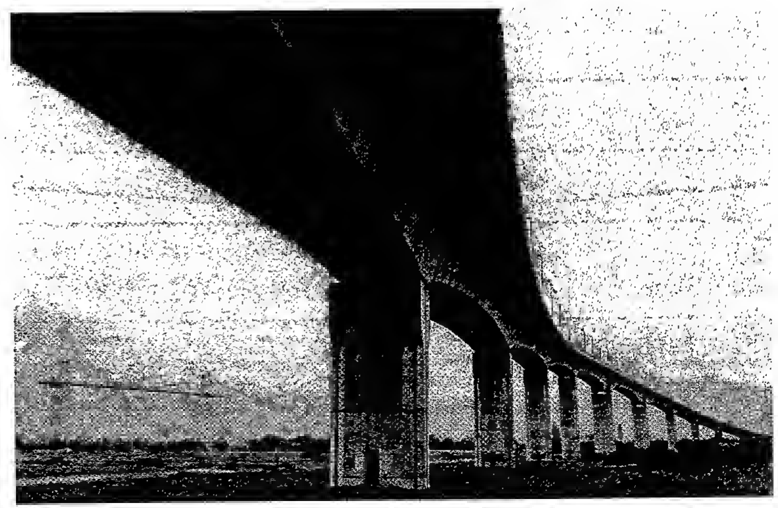


Photo 9.3 Multi-span prestressed box-girder bridge

Example 9.1: Shear design using the simplified procedure

The cross-section of a simply supported beam is shown in Figure Ex 9.1. Using the code-simplified method, determine the required shear reinforcement at the critical section knowing that the stirrups diameter is 10 mm (f_{yst}=360 N/mm^2). Assume that the column width is 500 mm. The material properties are f_{pu}=2000 N/mm^2 for normal stress relieved strands, f_{pe}= 1080 N/mm^2 and f_{cu}=35 N/mm^2. Assume that the applied live load and superimposed dead load are 22 kN/m', 2 kN/m', respectively.

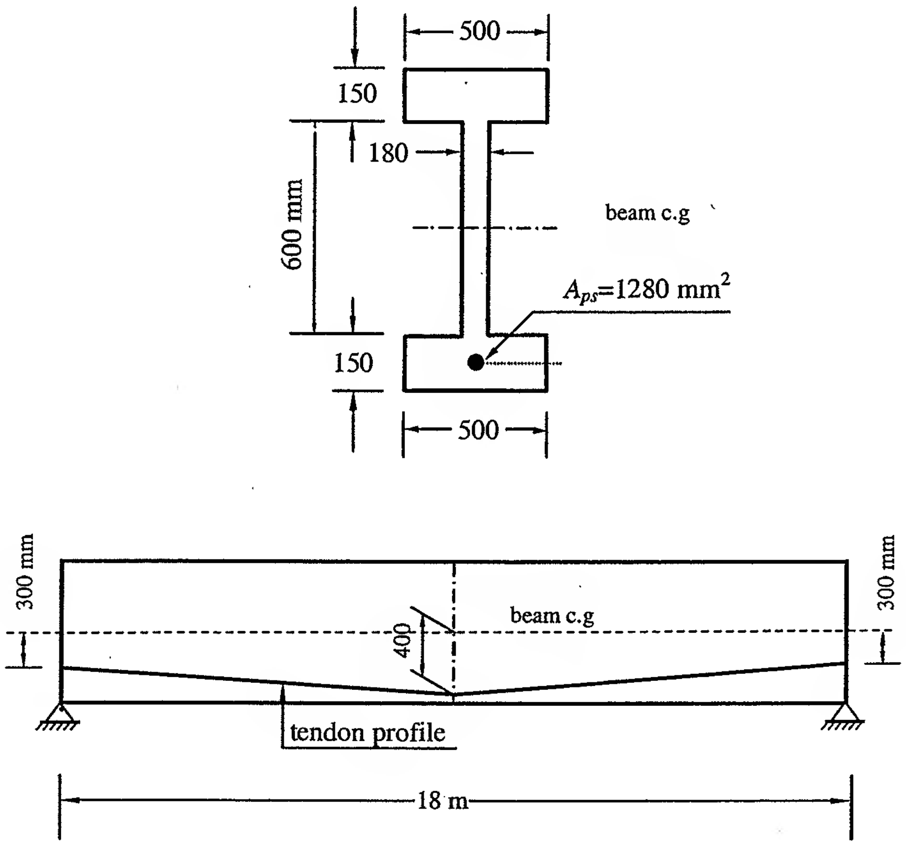


Fig. Ex. 9.1 Tendon profile for Example 9.1

Solution

Step 1: Verify the use of the simplified method

Since the effective prestressing stress f_{pe} is greater than 40% of the ultimate tendon strength f_{pu} (1080 > 0.40 x 2000=800 N/mm^2), the code simplified expression can be used.

Step 2: Calculate section properties

$$A = 2\times 500\times 150 + 180\times 600 = 258000 \ mm^2$$

$$w_{ow} = \gamma_c \times A = 25\times\frac{258000}{10^6} = 6.45\,kN\,/m'$$

Step 3: Calculate forces at the critical section

$$w_u = 1.4\ w_{DL} + 1.6\ w_{LL}$$

$$w_u = 1.4(6.45+2) + 1.6\times 22 = 47.03\,kN\,/\,m'$$

The critical section is at $t/2$ from the face of the support. Since the column width (c) is 500 mm, the critical section is at distance x_c from the centerline of the support. x_c equals:

$$x_c = \frac{t}{2}+\frac{c}{2} = \frac{900}{2}+\frac{500}{2} = 700\ mm = 0.70\ m$$

The reaction at the support equals:

$$R_u = \frac{w_u\times L}{2} = \frac{47.03\times 18}{2} = 423.27\ kN$$

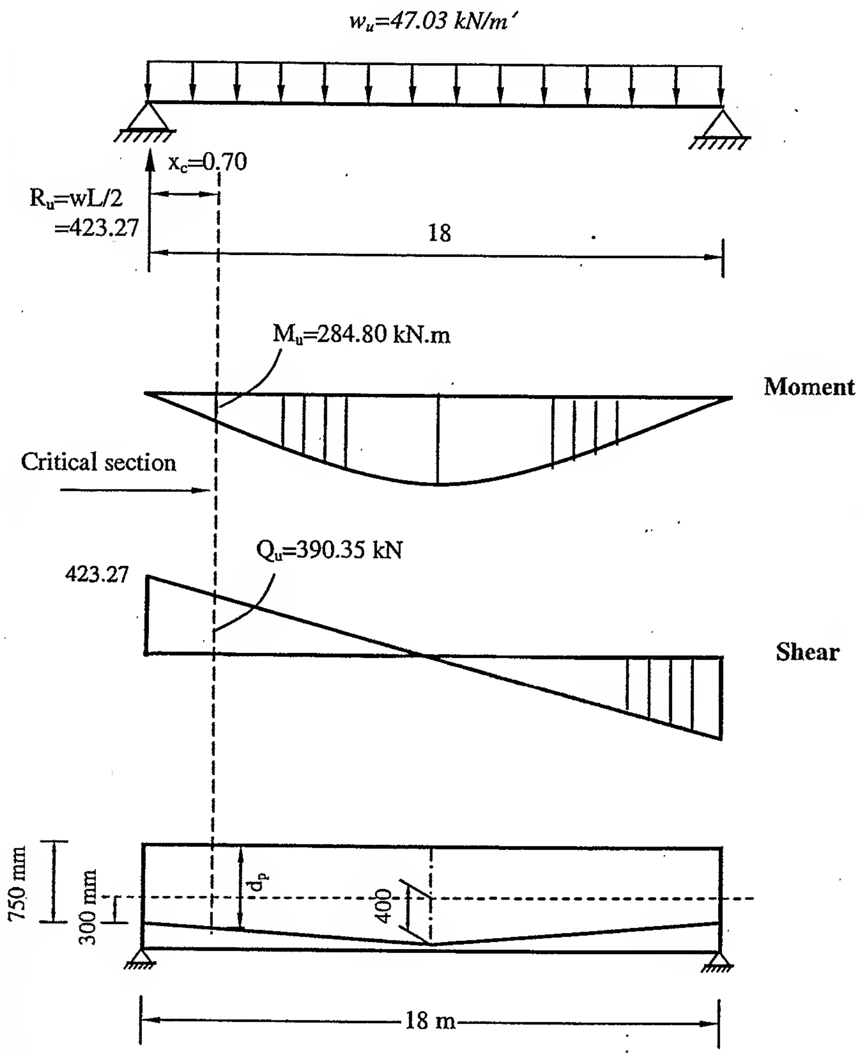
The shear at the critical section equals:

$$Q_u = R_u - w_u\cdot x_x = 423.27 - 47.03(0.70) = 390.35\ kN$$

The moment at the critical section for shear equals:

$$M_u = R_u\cdot x_c - \frac{w_u\cdot x_c^{\,2}}{2}$$

$$M_u = 423.27\times 0.7 - \frac{47.03\times 0.70^2}{2} = 284.8\ kN.m$$



The depth of the prestressing steel at the critical section equals:

$$d_p = 750 + \frac{0.70}{9} \times (400 - 300) = 757.8\,mm$$

d_p=757.8 mm > 0.8 t ...o.k

$$q_u = \frac{Q_u}{b\ d_p} = \frac{390.35 \times 1000}{180 \times 757.8} = 2.86\ N\,/\,mm^2$$

Step 4: Check the maximum shear stress q_{umax}

The maximum shear strength is given by:

$$q_{u\,\max} = 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.75\sqrt{\frac{35}{1.5}} = 3.62\ N\,/\,mm^2 \le 4.5\ N\,/\,mm^2$$

$$q_{u,\max} = 3.62\ N\,/\,mm^2$$

Since q_u is less than $q_{u,max}$ the concrete dimensions of the section are acceptable.

Step 5: Calculate concrete shear strength q_{cu}

$$q_{cu} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + \frac{3.6 \times Q_u \times d_p}{M_u} \ge 0.24\sqrt{f_{cu}\,/\,\gamma_c}$$

$$\le 0.375\sqrt{f_{cu}\,/\,\gamma_c}$$

$$\frac{Q_u \times d_p}{M_u} = \frac{390.35 \times (757.8\,/\,1000)}{284.77} = 1.03 > 1.0$$

$$or........\frac{Q_u \times d_p}{M_u} = \frac{d_p\ (L - 2x)}{x\ (L - x)} = \frac{(757.8\,/\,1000)\,(18.0 - 2 \times 0.70)}{0.70\ (18 - 0.7)} = 1.03$$

Use $\frac{Q_u \times d_p}{M_u} = 1.0$

$$q_{cu,\min} = 0.24\sqrt{35\,/\,1.5} = 1.16\ N\,/\,mm^2$$

$$q_{cu,\max} = 0.375\sqrt{35\,/\,1.5} = 1.81\ N\,/\,mm^2$$

$$q_{cu} = 0.045\sqrt{\frac{35}{1.5}} + 3.6 \times 1.0 = 3.82\ N\,/\,mm^2 > q_{cu,\min}$$

Since $q_{cu} > q_{cu,max}$ use $q_{cu} = q_{cu,max}$

q_{cu}=1.81 N/mm^2

Step 6: Calculate shear reinforcement

Since q_u (2.86) > q_{cu} (1.81), shear reinforcement is required.

The shear stress that needs to be carried by web reinforcement equals:

$$q_{su} = q_u - \frac{q_{cu}}{2} = 2.86 - \frac{1.81}{2} = 1.955\ N/mm^2$$

For 10 mm stirrups, the total shear reinforcement area for two branches equals:

$$A_{st} = 2 \times 78.5 = 157\ mm^2$$

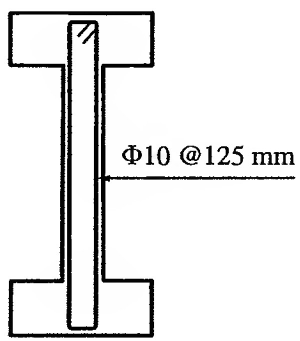
The required spacing is given by:

$$s = \frac{A_{st} \times f_y / 1.15}{b \times q_{su}} = \frac{157 \times 360 / 1.15}{180 \times 1.955} = 139.7\ mm$$

Take s =125mm

$$A_{st,min} = \frac{0.4}{f_y} b \times s = \frac{0.4}{360} \times 180 \times 125 = 25\ mm^2 < A_{st}\ (157) \ldots o.k$$

Use Φ10 @125 mm (8Φ10/m′)



Shear reinforcement details

Example 9.2: Shear design using the detailed procedure

The cross-section of a simply supported beam is shown in Figure Ex 9.2. Using the code-detailed method, determine the required stirrup spacing (f_{yst}=360 N/mm^2) at the critical section. Assume column width of 500 mm. Given that f_{pu}=2000 N/mm^2 for normal stress relived strands, f_{pe}= 1080 N/mm^2 and f_{cu}=35 N/mm^2. Assume that the applied live load is 22 kN/m′ and the superimposed dead load is 2 kN/m′.

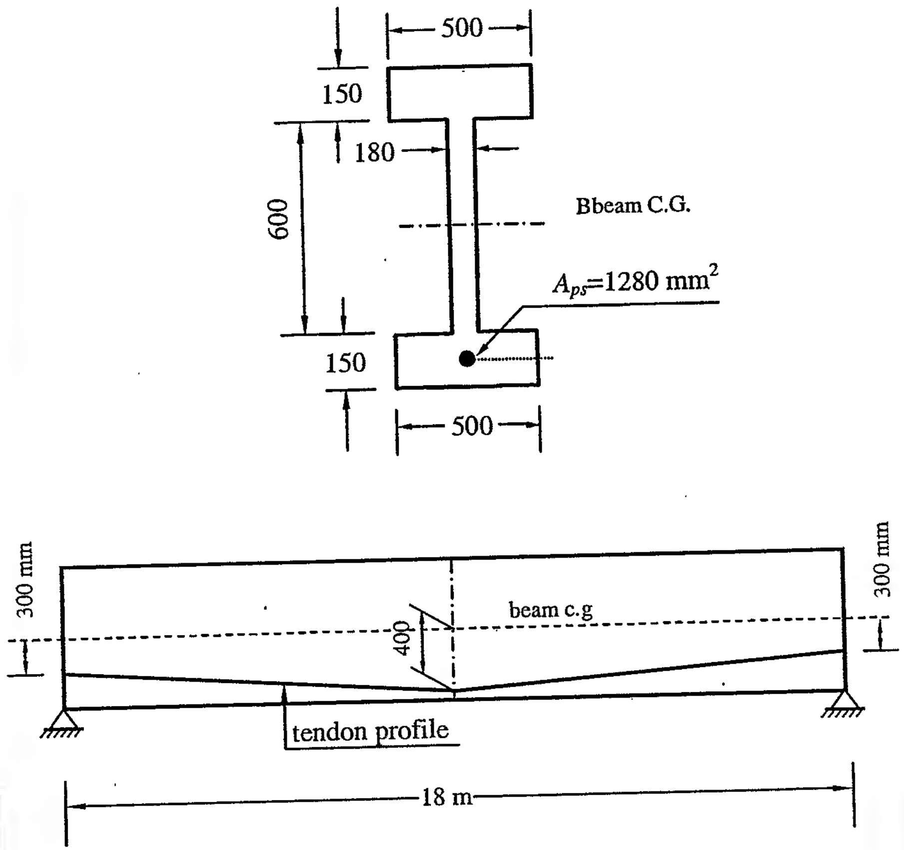


Fig. EX.9.2

Solution

In the detailed procedure, the concrete shear strength is taken as the smaller of two vales q_{ci} (flexural-shear strength) and q_{cw} (web-cracking shear strength).

Step 1: Calculate q_{ci}

$$q_{ci} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + 0.8\left(q_d + q_i \times \frac{M_{cr}}{M_{\max}}\right) \geq 0.24\sqrt{f_{cu}/\gamma_c}$$

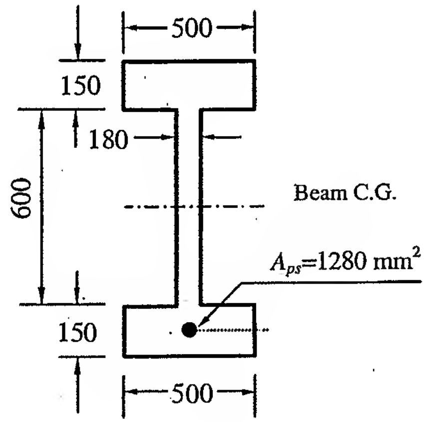
Step 1.1: Calculate M_{cr}

$$I = 2\times\left[\frac{500\times150^3}{12} + 500\times150\times(450-75)^2\right] + \frac{180\times600^3}{12}$$

I =2.46 x 10^{10} mm^4

Since the section is symmetrical, $y_{top} = y_{bot} = 450$ mm

$$Z_{bot} = \frac{I}{y_{bot}} = \frac{2.46\times10^6}{450} = 54.7\times10^6\ mm^3$$



$$M_{cr} = \frac{I}{y_b}\left(0.45\sqrt{f_{cu}} + f_{pce} - f_{cd}\right)$$

where f_{pce} is the compressive stresses at the extreme fibers (bottom in this case) due to the prestressing steel only.

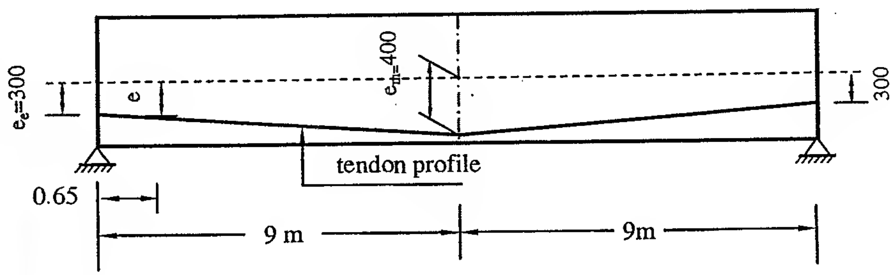
$$f_{pce} = -\frac{P_e}{A} - \frac{P_e\times e}{Z_{bot}}$$

The eccentricity at the critical section equals:

$$e = 300 + (400-300)\times\frac{0.7}{(18/2)} = 307.8\ mm$$

$$d_p = 750 + \frac{0.70}{9}\times(400-300) = 757.8\,mm$$

$$P_e = f_{pe}\times A_{ps} = 1080\times1280/1000 = 1382.4\ kN$$



$$f_{pce} = -\frac{1382.4\times1000}{258000} - \frac{1382.4\times1000\times307.8}{54.7\times10^6} = -13.14N/mm^2$$

It should be clear that the absolute value of f_{pce} is used. Hence, it is given by:

f_{pce}=*13.14 N/mm*^{2}

f_{cd} is the unfactored concrete stress under dead load moment only.

$$w_d = 6.45 + 2 = 8.45\ kN/m'$$

$R_d = w_d\ L/2 = 76.05$ kN

The dead load moment at the critical section equals:

$$M_d = R_d\cdot x_c - \frac{w_d\cdot x_c^2}{2} = 76.05\times0.70 - \frac{8.45\times0.70^2}{2} = 51.2\ kN.m$$

$$f_{cd} = \frac{M_d}{Z_{bot}} = \frac{51.2\times10^6}{54.7\times10^6} = 0.93\,N/mm^2$$

Noting that Z_{bot}=I /y_t, thus

$$M_{cr} = 54.7\times10^6\left(0.45\sqrt{35} + 13.14 - 0.93\right)/10^6 = 813.22\ kN.m$$

Step 1.2: Calculate q_d and q_i

The unfactored shear due to dead loads at the critical section equals:

$$Q_d = R_d - w_d \cdot x_c = 76.05 - 8.45(0.70) = 70.135\ kN$$

$$q_d = \frac{Q_d}{b \times d_p} = \frac{70.135 \times 1000}{180 \times 757.8} = 0.514\ N/mm^2$$

$$w_u = 1.4 \times w_d + 1.6 \times w_{LL} = 1.4 \times 8.45 + 1.6 \times 22 = 47.03\, kN\ /m'$$

The factored reaction at the support equals:

$$R_u = \frac{w_u \times L}{2} = \frac{47.03 \times 18}{2} = 423.27\ kN$$

The factored shear Q_u at the critical section equals:

$$Q_u = R_u - w_u \cdot x_x = 423.27 - 47.03(0.70) = 390.35\ kN$$

The factored moment M_u at the critical section equals:

$$M_u = R_u \cdot x_c - \frac{w_u \cdot x_c^2}{2} = 423.7 \times 0.7 - \frac{47.03 \times 0.70^2}{2} = 284.8\ kN.m$$

The factored shear Q_i at the critical section equals:

$$Q_i = Q_u - Q_d = 390.35 - 70.135 = 320.21\ kN$$

$$q_i = \frac{Q_i}{b \times d_p} = \frac{320.21 \times 1000}{180 \times 757.8} = 2.35\ N/mm^2$$

The factored moment M_{max} at the critical section equals:

$$M_{\max} = M_u - M_d = 284.8 - 51.2 = 233.6\ kN.m$$

Hence,

$$q_{ci} = 0.045\sqrt{\frac{35}{1.5}} + 0.8\left(0.514 + 2.35 \times \frac{813.22}{233.64}\right) = 7.17\ \text{N/mm}^2$$

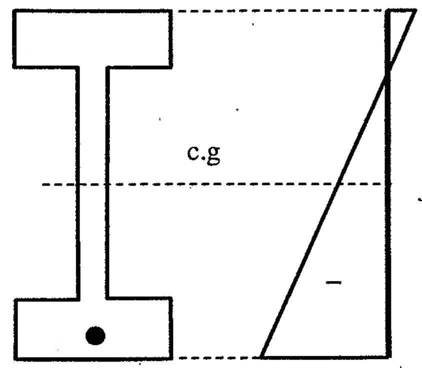
$$q_{ci,\min} = 0.24\sqrt{35/1.5} = 1.16\ N/mm^2$$

Since $q_{ci} > q_{cmin}$, q_{ci}=7.17 N/mm^2

Step 2: Calculate q_{cw}

$$q_{cw} = 0.24\left(\sqrt{\frac{f_{cu}}{\gamma_c}} + f_{pcc}\right) + q_{pv}$$

f_{pcc} represents the stress at the C.G. of the section after losses. At the C.G. only normal stress exists as shown the in figure.



$$f_{pcc} = \frac{P_e}{A} = \frac{1382.4 \times 1000}{258000} = 5.36\ N/mm^2$$

q_{pv} represents the stress due to the vertical component of the section. The vertical component was determined previously as:

$$q_{pv} = \frac{Q_{pv}/\gamma_{ps}}{b \times d_p} = \frac{(P_e \times \sin\theta)/\gamma_{ps}}{b \times d_p} = \frac{(f_{pe}/\gamma_{ps} \times A_p) \times \sin\theta}{b \times d_p}$$

Since θ is small we can assume that sin θ= tan θ.

$$Q_{pv} = P_e \times \tan(\theta) = P_e \frac{e_m - e_e}{L/2} = 1382.4 \times \frac{400 - 300}{18/2 \times 1000} = 15.36\ kN$$

$$q_{pv} = \frac{Q_{pv}/\gamma_{ps}}{b \times d_p} = \frac{15.36/1.15 \times 1000}{180 \times 757.8} = 0.098\ N/mm^2$$

$$q_{cw} = 0.24\left(\sqrt{\frac{35}{1.5}} + 5.36\right) + 0.098 = 2.54\ N/mm^2$$

Alternatively

q_{cw} may be taken as the shear stress that produces a principle tension stress of $f_t = 0.25\sqrt{35} = 1.48\, N/mm^2$ at the centroid of the web.

$$f_t = \sqrt{\left(\frac{f_{pcc}}{2}\right)^2 + q_{cw}^2} - \left(\frac{f_{pcc}}{2}\right)$$

$$q_{cw} = 0.25\sqrt{f_{cu}}\sqrt{\left(\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}\right) + 1}$$

Since f_{pcc}=5.36 N/mm^2 and $0.25\sqrt{f_{cu}} = 1.48$, the previous equation gives:

$$q_{cw} = 1.48\sqrt{\left(\frac{5.36}{1.48}\right) + 1} \qquad q_{cw}=3.18\ \text{N/mm}^2$$

There is about 25% difference in the value of the web-cracking shear strength determined from both methods. We shall take the conservative value of q_{cw} = 2.54 N/mm^2.

Step 3: Calculate q_{cu}

The concrete shear strength is the smaller of the flexural-shear strength (q_{ci}=7.17 N/mm^2) and the web-cracking shear strength q_{cw}=2.54 N/mm^2. Hence, it is given by:

$$q_{cu} = 2.54\ N/mm^2$$

Step 4: Check the maximum shear stress q_{umax}

The applied ultimate shear stress at the critical section equals:

$$q_u = \frac{Q_u}{b\ d_p} = \frac{390.35}{180 \times 757.8} = 2.86\ N/mm^2$$

The maximum shear strength is given by:

$$q_{u\max} \le 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.75\sqrt{\frac{35}{1.5}} = 3.62\ N/mm^2 \le 4.5\ N/mm^2$$

$$q_{u,\max} = 3.62\ N/mm^2$$

Since q_u is less than $q_{u,max}$ the concrete dimensions of the section are adequate.

Step 5: Calculate shear reinforcement

Since q_u (2.86) > q_{cu} (2.54), shear reinforcement is required. Assume that the stirrups diameter is 10 mm, thus:

$$A_{st} = 2 \times 78.5 = 157\ mm^2$$

$$q_{su} = q_u - \frac{q_{cu}}{2} = 2.86 - \frac{2.54}{2} = 1.59\ N/mm^2$$

For 10 mm stirrups, the total shear reinforcement area for two branches equals:

$$A_{st} = 2 \times 78.5 = 157\ mm^2$$

The required spacing is given by:

$$s = \frac{A_{st} \times f_y / 1.15}{b \times q_{su}} = \frac{157 \times 360 / 1.15}{180 \times 1.59} = 172\ mm \quad \text{Use } \Phi 10\ @166\ \text{mm } (6\Phi 10/\text{m}')$$

$$A_{st,\min} = \frac{0.4}{f_y}\, b \times s = \frac{0.4}{360} \times 180 \times 166 = 33.3\, mm^2 < A_{st}\ (157)....o.k$$

It is clear form the previous two examples that the simplified method is very conservative. The shear strength calculated using the detailed method (2.54 N/mm^2) is about 40 percent more than the simplified procedure (1.81 N/mm^2).

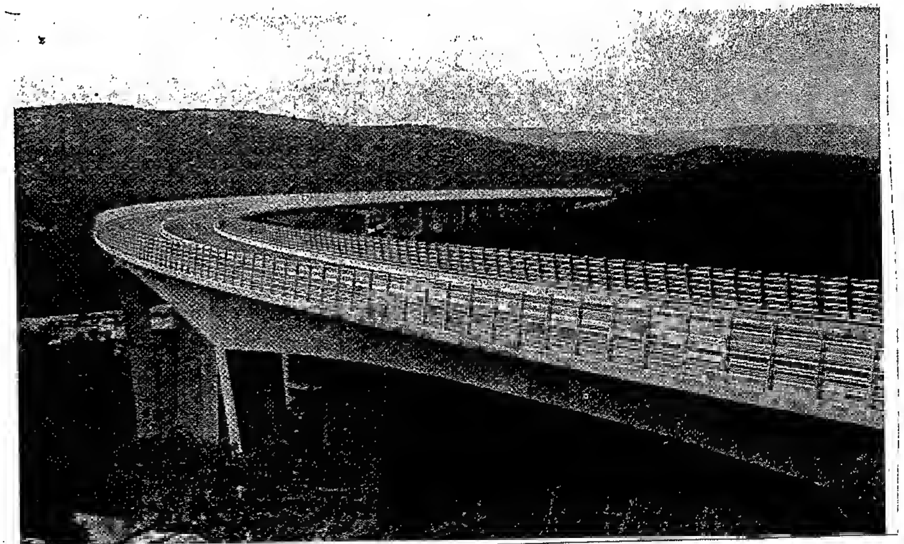


Photo 9.4 Curved reinforced concrete bridge

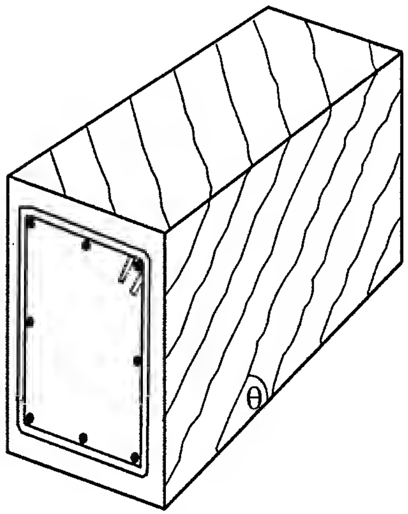
9.3 Torsion in Prestressed Concrete

9.3.1 General

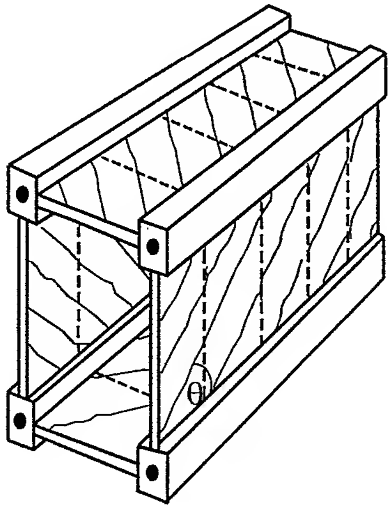
When subjected to torsion, a cracked prestressed concrete beam as the one shown in Fig. (9.10a) can be idealized as shown in Fig. (9.10b). The cracked beam resists the applied torsional moment through acting as *a space-truss* as shown in Fig. 9.10. The space truss consists of:

- Longitudinal reinforcement concentrated at the corners.
- Closed stirrups
- Diagonal concrete compression members between the cracks which spiral around the beam.

The angle of the inclination of the compression diagonals with respect to the beam axis, θ, depends on the ratio of the force carried by the longitudinal reinforcement to that carried by the stirrups and also on the value of the prestressing force.



a) Section of the actual beam



b) Idealized section of the truss

Fig. 9.10 Idealized cross-section for torsion

9.3.2 The Design for Torsion in the Egyptian Code

9.3.2.1 Introduction

The ECP 203 torsion design procedure for prestressed beams is based on the space truss model with some simplifications. The assumptions are the same as those for ordinary reinforced concrete with minor modifications and can be summarized in the following:

- The angle of inclination of the compression diagonals (which is the angle of inclination of the cracks) is set equal to:

  45° for cases in which $f_{pe} \leq 40\%\ of\ f_{pu}$

  37.5° for cases in which $f_{pe} > 40\%\ of\ f_{pu}$

- The thickness of the walls of the truss model, t_e, and the area enclosed by the shear flow, A_o, are calculated using the expressions given in the ECP 203.
- A limiting value for the allowed shear stresses developed due to torsion is given to ensure prevention of crushing failure of concrete in the struts.

In the ECP 203 torsion design procedure, the following three strength criteria are considered:

- First, a limitation on the shear stress developed due torsion is established such that the stirrups and the longitudinal reinforcement will yield before the crushing of the concrete struts.
- Second, closed stirrups are provided to resist the applied torsional moment.
- Third, the longitudinal steel distributed around the perimeter of the stirrups should be adequate to resist the longitudinal force due to torsion.

9.3.2.2 Calculation of the Shear Stress due to Torsion

The ECP 203 adopts a thin-walled section analysis, to predict the shear stress due to torsion in hollow as well as in solid sections.

The ultimate shear stress developed due to the ultimate torque M_{tu} is given by:

$$q_{tu} = \frac{M_{tu}}{2A_o\, t_e} \quad \ldots\ldots (9.20)$$

For simplicity, the following expressions are suggested by the code to compute the area enclosed by the shear flow path, A_o, and the equivalent thickness of the shear flow zone, t_e:

$$A_o = 0.85 A_{oh} \quad \ldots\ldots (9.21)$$

$$t_e = A_{oh}/P_h \quad \ldots\ldots (9.22)$$

where

A_{oh} is the gross area bounded by the centerline of the outer closed stirrups.

P_h is the perimeter of the stirrups.

The area A_{oh} is shown in Fig. 9.11 for cross-sections of various shapes.

For hollow sections, the actual thickness of the walls of the section should be used if it is less than t_e .

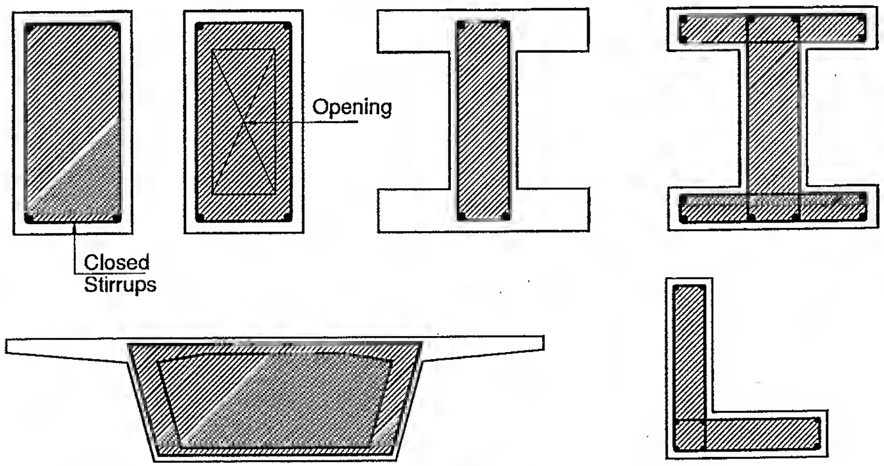


Fig. 9.11 Definition of A_{oh}

9.3.2.3 Consideration of Torsion

According to the Egyptian code ECP 203, torsional moments should be considered in design if the factored torsional stresses calculated from Eq. 9.20 exceed $q_{tu\,min}$, given by:

$$q_{tu\,min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}\sqrt{1+\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}} \quad \ldots\ldots (9.23)$$

Where f_{pcc} is the average normal stress at the C.G. of the section $(f_{pcc} = \frac{P_e}{A})$.

The previous equation is similar to that of ordinary reinforced concrete except for the magnification factor $\sqrt{1+f_{pcc}/0.25\sqrt{f_{cu}}}$ that presents the added concrete strength due to prestressing as shown in Fig. 9.12.

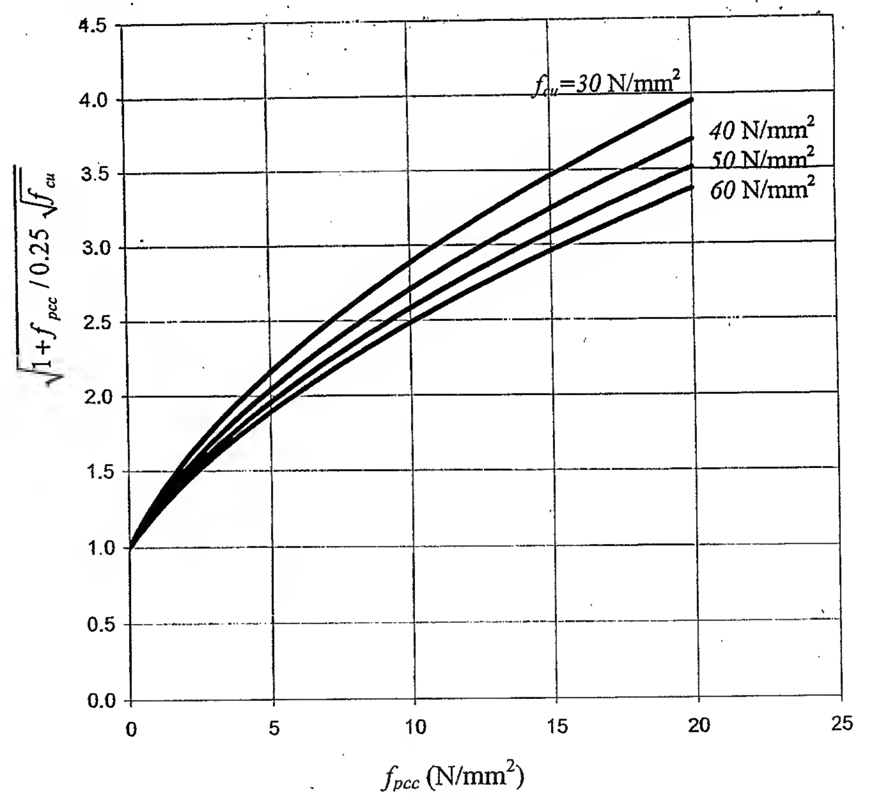


Fig. 9.12 Magnification factor according to the applied prestressing stress and concrete strength

9.3.2.4 Check the Adequacy of the Concrete Section

The concrete compression diagonals carry the diagonal forces necessary for the equilibrium of the space-truss model. Preventing crushing failure of the compression diagonals can be achieved either by limiting the compressive stresses in the concrete struts or by limiting the maximum shear stress. The ECP 203 limits the shear stress calculated by Eq. 9.20 to the value given by:

$$q_{tu\,max} = 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.5\ N/mm^2 \quad \text{.....................} (9.24)$$

If $q_u > q_{tu,max}$, the concrete dimensions of the cross-section must be increased.

9.3.2.5 Design of Torsional Reinforcement

A-Closed Stirrups

The ECP 203 uses the expression that was derived from the space-truss model with the angle θ set equal to either 37.5° or 45° depending on the amount of prestressing. Hence, the area of one branch of closed stirrups A_{str} is given by:

$$A_{str} = \frac{M_{tu} \cdot s}{2\,A_o \left(\frac{f_{yst}}{\gamma_s}\right) \cot\theta} \quad \text{..................................} (9.25)$$

In case of rectangular sections, Eq. 9.25 takes the form:

$$A_{str} = \frac{M_{tu} \cdot s}{1.7\,(x_1 \cdot y_1)\left(\frac{f_{yst}}{\gamma_s}\right)\cot\theta} > A_{str,min} \quad \text{..........} (9.26)$$

where x_1 and y_1 are the shorter and the longer center-to-center dimensions of closed stirrups. The angle θ is taken as

- 45° for cases in which $f_{pe} \le 40\%\ of\ f_{pu}$
- 37.5° for cases in which $f_{pe} > 40\%\ of\ f_{pu}$

$$A_{str\,min} = \frac{0.40}{f_{yst}}\, b \times s$$

B-Longitudinal Reinforcement

The area of longitudinal reinforcement required for torsion A_{sl} is given by:

$$A_{sl} = \frac{M_{tu}\ P_h}{2\,A_o\,\frac{f_y}{\gamma_s}}\cot^2\theta \quad \text{..} (9.27)$$

Substituting the value of M_{tu} from Eq. 9.25, the area of the longitudinal reinforcement can be expressed in terms of A_{str} as follows:

$$A_{sl} = \frac{A_{str}\ P_h}{s}\,\frac{f_{yst}}{f_y}\cot^2\theta \quad \text{......................................} (9.28)$$

where f_y and f_{yst} are the yield strength of the longitudinal reinforcement and the yield strength of the stirrups, respectively.

The area of the longitudinal reinforcement should not be less than:

$$A_{sl\,min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}}\,A_{cp}}{f_y/\gamma_s} - \left(\frac{A_{str}}{s}\right)P_h\left(\frac{f_{yst}}{f_y}\right) \quad \text{..................} (9.29)$$

where A_{cp} is the area enclosed by outside perimeter of the section including area of openings.

In the previous equation $\frac{A_{str}}{s}$ should not be less than $\frac{b}{6\times f_{yst}}$

9.3.2.6 Related Code Provisions

The Egyptian Code sets the following requirements with respect to the arrangements and the detailing of reinforcement for torsion as follows:

1- Stirrups must be closely spaced with maximum spacing (s) such that:

$$s = smaller\ of \begin{cases} 200\ mm \\ \frac{p_h}{8} \end{cases}$$

2- Only the outer two legs are utilized for torsion plus shear, and the interior legs are utilized for vertical shear only.

3- For box sections, transversal and longitudinal reinforcement arranged along the outside and the inside perimeter of the section may be considered effective in resisting torsion provided that the wall thickness t_w is less or equal to b/6 where b is the shorter side length of the section. If the wall thickness is thicker, torsion shall be resisted by reinforcement arranged along the outside perimeter only.

4- Stirrups proportioned for torsion must be closed as shown in Fig. 9.13.

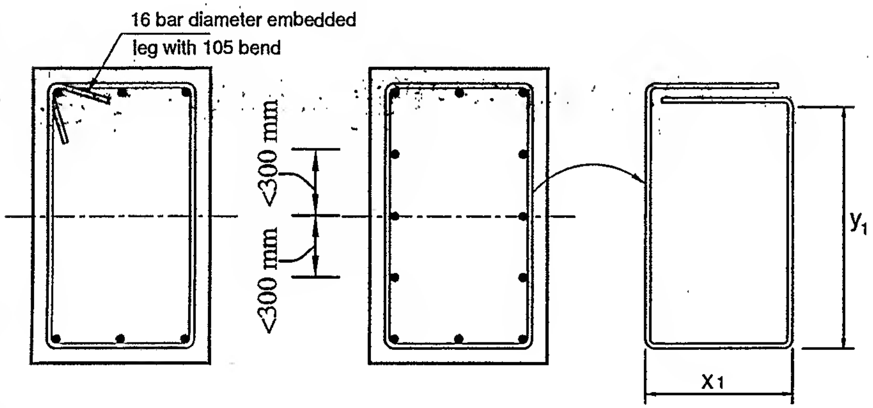


Fig. 9.13 Torsion stirrup details

5- It is permitted to neglect the effective part of the slab in T- and L- sections when calculating the nominal shear stresses due to torsion.

6- In case of considering the effective part of the slab in T- and L- sections when calculating the nominal shear stresses due to torsion, the following measures are taken refer to Fig. 9.14.

- The effective part of the slab in T and L sections measured from the outer face of the beam should not be more than 3 times the slab thickness.
- The effective part of the slab should be provided with web reinforcement.

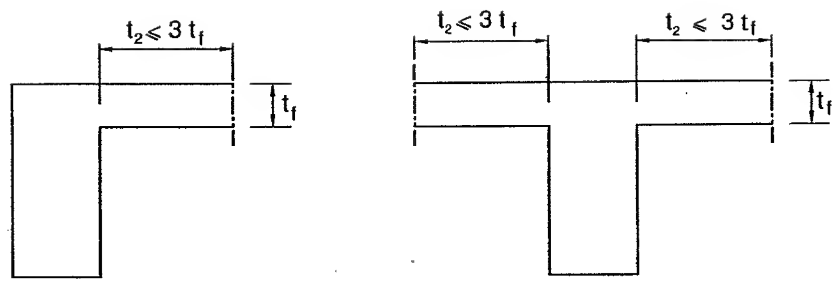


Fig. 9.14 Effective flange width for torsion

7- The spacing of the longitudinal bars should not exceed 300 mm and they should be uniformly distributed along the perimeter as shown in Fig. 9.13. At least one bar must be placed in each corner of the section (i.e. in each corner of stirrup). The minimum bar diameter shall be 12 mm or 1/15 of the spacing between stirrups whichever is larger.

8- Enough anchorage of longitudinal torsional reinforcement should be provided at the face of the supporting columns, where torsional moments are often the maximum.

Summary of Torsion Design

Step 1: Determine cross-sectional parameters

A_{oh} = area enclosed by the centerline of the closed stirrups.

P_h = Perimeter of the centerline of the closed stirrups.

Step 2: Calculate the shear stress due to the ultimate torsion

$$q_{tu} = \frac{M_{tu}}{2A_o t_e} \qquad A_o = 0.85\ A_{oh} \qquad t_e = \frac{A_{oh}}{P_h}$$

Note: If the actual thickness of the wall of the hollow section is less than A_{oh}/P_h , then the actual wall thickness should be used.

Step 3: Check the need for considering torsion

$$q_{tu\,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}\sqrt{1+\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}}$$

If $q_{tu} \geq q_{tu\,\min}$, one has to consider the shear stresses due to torsion.

Step 4: Check that section size is adequate

$$q_{tu\,\max} = 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} \leq 4.5\ N/mm^2$$

If $q_{tu} < q_{tu\,\max}$, the concrete dimensions of the section are adequate.

If $q_{tu} > q_{tu\,\max}$, one has to increase concrete dimensions.

Step 5: Design the closed stirrups

The amount of closed stirrups required to resist the torsion is:

$$A_{str} = \frac{M_{tu} \cdot s}{2\,A_{oh}\left(\frac{f_{yst}}{\gamma_s}\right)\cot\theta}$$

Check that the provided area is more than $A_{str\,\min} = \frac{0.40}{f_{yst}}\,b \times s$

Check that the provided spacing is less than the code requirement.

Step 6: Design longitudinal reinforcement

$$A_{sl} = A_{str}\left(\frac{P_h}{s}\right)\left(\frac{f_y}{f_{yst}}\right)\cot^2\theta$$

Check that the provided longitudinal reinforcement is not less than $A_{sl\,\min}$

$$A_{sl\,\min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}}\,A_{cp}}{f_y/\gamma_s} - \left(\frac{A_{str}}{s}\right)P_h\left(\frac{f_{yst}}{f_y}\right)$$

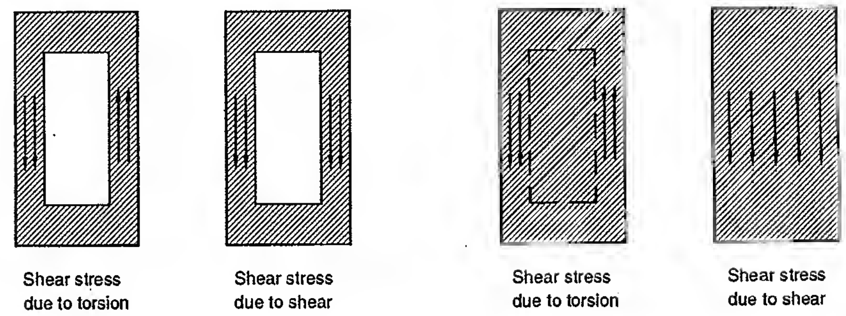
In the previous equation $\frac{A_{str}}{s}$ should not be less than $\frac{b}{6 \times f_{yst}}$

9.4 Combined Shear and Torsion

9.4.1 Introduction

When a hollow section is subjected to a direct shear force and a torsional moment, the shear stresses on one side of the cross section are additive and on the other side are subtractive as shown in Figs. 9.15a.

When a solid section is subjected to combined shear and torsion, the shear stresses due to shear are resisted by the entire section, while the shear stresses due torsion are resisted by the idealized hollow section as shown in Fig. 9.15b.



a) Hollow section b) Solid section

Fig. 9.15 Addition of torsional and shear stresses

9.4.2 Design for Shear and Torsion in ECP 203

9.4.2.1 Consideration of Torsion

In prestressed members, the Egyptian code ECP 203 requires considering the torsional moments in design if the factored torsional stresses calculated from Eq. 9.20 exceed $q_{tu\,\min}$, given by:

$$q_{tu\,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}\sqrt{1+\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}} \quad \text{.............................. (9.30)}$$

Where $f_{pcc} = \frac{P_e}{A}$

9.4.2.2 Adequacy of the Concrete Cross-Section

The shear stresses q_u due to direct shear and shear stress q_{tu} due to torsional moment are given by:

$$q_u = \frac{Q_u}{b_w\, d} \qquad\qquad q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

The Egyptian Code concentrates on the side of the hollow section where the shear and torsional stresses are additive. On that side:

$$q_u + q_{tu} \le 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.5\ N/mm^2 \quad .... (9.31)$$

In a solid section, the shear stresses due to direct shear are assumed to be uniformly distributed across the width of the section, while the torsional shears only exist in the walls of the assumed thin-walled tube, as shown in Fig. (9.15b). The direct summation of the two terms tends to be conservative and a root-square summation is used

$$\sqrt{(q_u)^2 + (q_{tu})^2} \le 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.5\ N/mm^2 \quad .... (9.32)$$

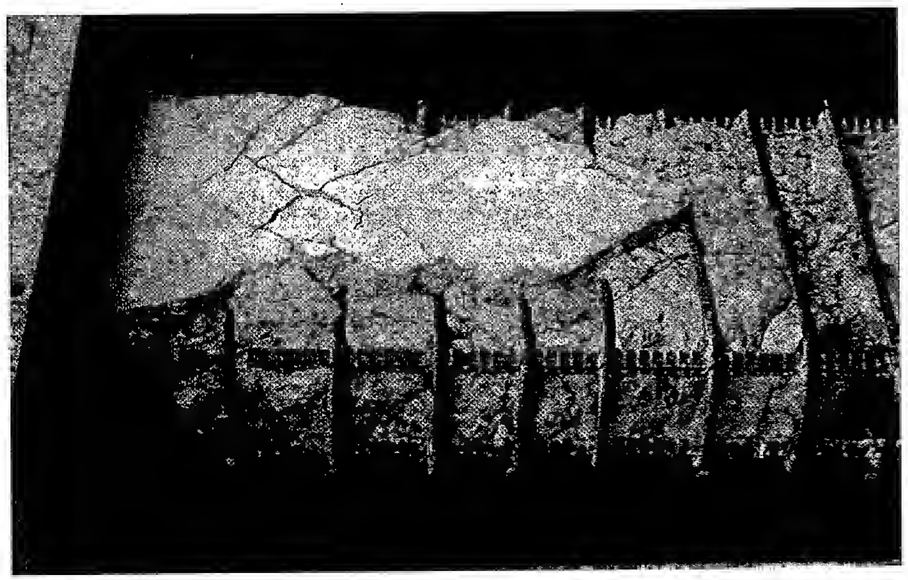


Photo 9.5 Beam failure due to combined shear and torsion

9.4.2.3 Design of Transverse Reinforcement

For prestressed members under combined shear and torsion, the Egyptian Code requires adding the transverse steel due to torsion to that due to shear. Concrete is assumed to contribute to the shear strength of the beam. It does not, however, contribute to the torsional strength of the beam. The transverse reinforcement for combined shear and torsion is obtained according to Table 9.1.

Table 9.1: Transverse reinforcement requirements according to ECP 203

Case	$q_{tu} \le 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}\sqrt{1+\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}}$	$q_{tu} > 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}\sqrt{1+\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}}$
$q_u \le q_{cu}$	Provide minimum reinforcement given by Eq. 9.33	Provide reinforcement to resist q_{tu}, given by Eq. 9.25
$q_u > q_{cu}$	Provide reinforcement to resist $q_u - q_{cu}/2$	Provide reinforcement to resist $q_u - q_{cu}/2$ and q_{tu}

In Table (9.1), q_{cu} is the concrete contribution to the shear strength and is obtained from either

- The simplified procedure

$$q_{cu} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + \frac{3.6 \times Q_u \times d_p}{M_u} \ge 0.24\sqrt{f_{cu}/\gamma_c}$$
$$\le 0.375\sqrt{f_{cu}/\gamma_c}$$

- The detailed procedure, where q_{cu} is taken as the smaller of :

1. $q_{ci} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + 0.8\left(q_d + q_i \times \frac{M_{cr}}{M_{max}}\right) \ge 0.24\sqrt{f_{cu}/\gamma_c}$
2. $q_{cw} = 0.24\left(\sqrt{f_{cu}} + f_{pcc}\right) + q_{pv}$

The total amount of stirrups needed for shear and torsion should satisfy the following equation:

$$2A_{str} + A_{st})_{min} \geq \frac{0.40\, b\ \ s}{f_{yst}} \qquad (9.33)$$

9.4.2.4 Design of Longitudinal Reinforcement.

The longitudinal steel is not required for shear. However, longitudinal steel for torsion should be obtained using Eq. 9.34.

$$A_{sl} = A_{str} \left(\frac{P_h}{s}\right)\left(\frac{f_y}{f_{yst}}\right) \cot^2 \theta \qquad (9.34)$$

The area of the longitudinal reinforcement should not be less than:

$$A_{sl\,min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}}\, A_{cp}}{f_y / \gamma_s} - \left(\frac{A_{str}}{s}\right) P_h \left(\frac{f_{yst}}{f_y}\right) \qquad (9.35)$$

Photo 9.6 Curved prestressed box-girder bridge

Design Summary for Combined Shear and Torsion

Step 1: Determine cross-sectional parameters

The cross-sectional parameters for combined shear and torsion design are b, d_p, A_{oh} and, P_h.

Step 2: Calculate the ultimate shear stresses due to Q_u and M_t

$$q_u = \frac{Q_u}{b\ d_p}$$

$$q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

$$A_o = 0.85\, A_{oh} \qquad t_e = \frac{A_{oh}}{P_h} \qquad d_p \geq 0.80t$$

Note: If the actual thickness of the wall of the hollow section is less than A_{oh}/P_h , then the actual wall thickness should be used. d_p is should not be less *0.8 t.*

Step 3: Check the need for considering torsion

Calculate the minimum shear stress below which torsion can be neglected.

$$q_{tu\,min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}\sqrt{1+\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}}$$

If $q_{tu} \rangle\, q_{tu\,min}$, one has to consider the shear stresses due to torsion

Step 4: Check that section size is adequate

The developed shear stresses due to shear and torsions should stratify the following equations

For Hollow sections

$$q_u + q_{tu} \leq 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} \leq 4.5\ N/mm^2$$

For solid sections

$$\sqrt{(q_u)^2 + (q_{tu})^2} \leq 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} \leq 4.5\ N/mm^2$$

If $q_{tu} \langle q_{tu\,max}$ and $q_u \langle q_{u\,max}$, the concrete dimensions of the section are adequate.
If the above condition is not satisfied, one has to increase the dimensions.

Step 5: Calculate the concrete shear strength q_{cu}

Calculate the concrete contribution to the shear resistance, q_{cu} using one of the following two procedures:

- The simplified procedure (use only if f_{pe}>0.4 f_{pu})

$$q_{cu} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + \frac{3.6 \times Q_u \times d_p}{M_u} \geq 0.24\sqrt{f_{cu}/\gamma_c}$$

$$\leq 0.375\sqrt{f_{cu}/\gamma_c}$$

- The detailed procedure, where q_{cu} is taken as the smaller of the flexural shear strength and the web shear strength.

1. $$q_{ci} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + 0.8\left(q_d + q_i \times \frac{M_{cr}}{M_{\max}}\right) \geq 0.24\sqrt{f_{cu}/\gamma_c}$$

2. $$q_{cw} = 0.24\left(\sqrt{\frac{f_{cu}}{\gamma_c}} + f_{pcc}\right) + q_{pv}$$

Step 6: Design the closed stirrups

If $q_u \rangle q_{cu}$, calculate the stirrups needed for shear

$$q_{su} = q_u - 0.5\, q_{cu}$$

$$A_{st} = \frac{q_{su}\ b\ s}{f_{yst}/\gamma_s}$$

The area of one branch of stirrups needed for torsion is obtained from:

$$A_{str} = \frac{M_{tu}\ s}{1.7 A_{oh}\,(f_{yst}/\gamma_s)\ \cot\theta}$$

The area of one branch of stirrups needed for resisting shear and torsion = $A_{str} + \frac{A_{st}}{n}$
where n is the number of branches determined from shear calculations as shown in Fig. 9.16.

Check that the chosen area of stirrups satisfies the minimum requirements.

$$(A_{st} + 2A_{str})_{chosen} \rangle \frac{0.40 \times s \times b}{f_{yst}}$$

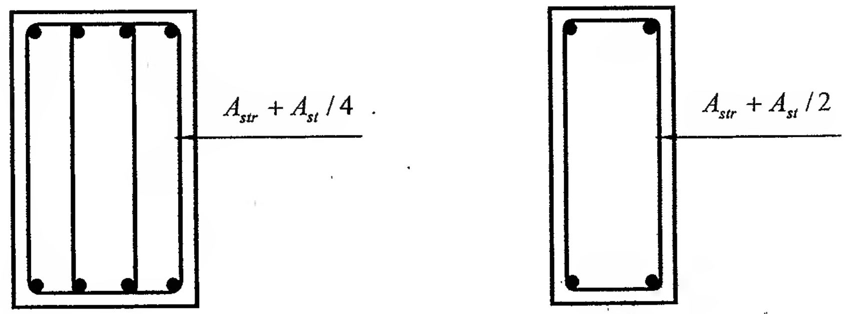


Fig. 9.16 Stirrups for shear and torsion

Step 7: Design longitudinal reinforcement

$$A_{sl} = A_{str}\left(\frac{P_h}{s}\right)\left(\frac{f_y}{f_{yst}}\right)\cot^2\theta$$

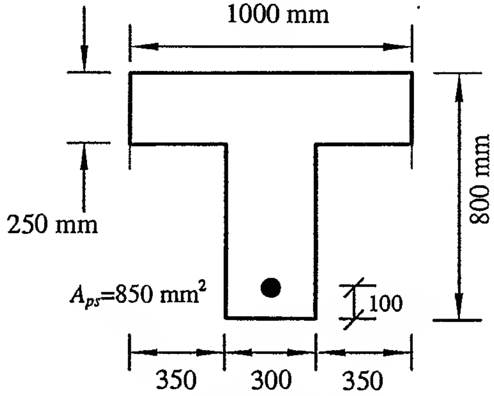
Check that the provided longitudinal torsional reinforcement is more than the minimum requirement $A_{sl,min}$, where:

$$A_{sl\,\min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}}\,A_{cp}}{f_y/\gamma_s} - \left(\frac{A_{str}}{s}\right)P_h\left(\frac{f_{yst}}{f_y}\right)$$

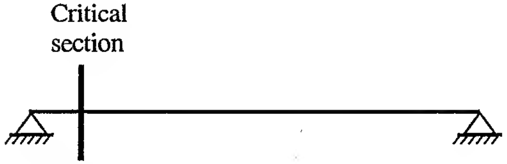
In the previous equation $\frac{A_{str}}{s}$ should not be less than $\frac{b}{6 \times f_{yst}}$

Example 9.3: Combined shear and torsion design (1)

The cross section of a simply supported prestressed beam is shown in Fig. Ex 9.3. The beam is subjected to a factored shear force of 510 kN, a factored bending moment of 80 kN.m, and a factored torsional moment of 34 kN.m at the critical section. Design the required reinforcement to resist the applied shear and torsion. The material properties are f_{pu}=1860 N/mm^2 for normal stress relieved strands, f_{pe} = 960 N/mm^2, f_y =360 N/mm^2, f_{yst} =360 N/mm^2 and f_{cu}=40 N/mm^2.



Beam cross section



Q_u=510 kN
M_u=80 kN.m
M_{tu}=34 kN.m

Straining actions at the critical section

Fig. Ex. 9.3

Solution

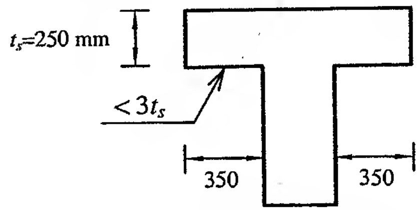
Step 1: Calculate section properties

To design a T-section for torsion, one has two options:

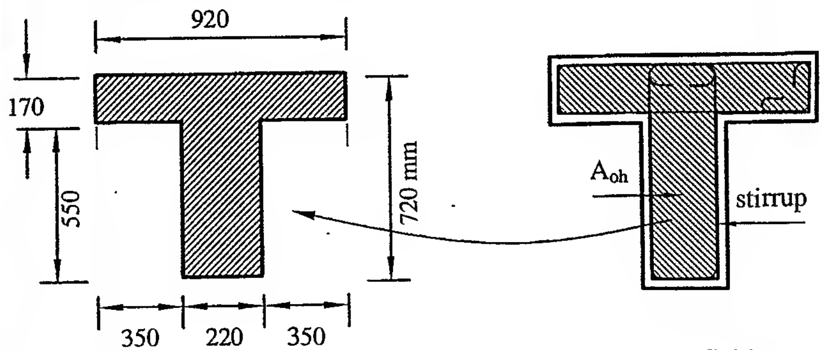
1- **Consider** the slab in the calculations and reinforce **both** the slab and the beam for torsion.

2- **Do not** consider the slab contribution in torsion design, and provide stirrups and longitudinal reinforcement in the **web only** (easier and more practical for thin slabs).

In this example, the contribution of the slab is considered in the calculations. Note that the flanges must be less than 3 t_s as shown in the figure below.



Assume concrete cover of 40 mm to the *centerline* of the stirrup.



A_{oh} definition

$$p_h = 2\times(720+920) = 3280\ mm$$

$$A_{oh} = 220\times550+170\times920 = 277400\ mm^2$$

$$A_o = 0.85A_{oh} = 0.85\times277400 = 235790\ mm^2$$

$$t_e = \frac{A_{oh}}{p_h} = \frac{277400}{3280} = 84.6\ mm$$

$$A = 1000\times250+300\times550 = 415000\ mm^2$$

Step 2: Calculate the ultimate shear stresses due to Q_u and M_t

1. Shear stress:

The depth of the prestressing steel at the critical section equals

$$d_p = 800 - 100 = 700\,mm$$

$d_p = 700$ mm > 0.8 t ...o.k

Only the web width is effective in resisting shear force, thus b is taken as 300 mm.

$$q_u = \frac{Q_u}{b\ d_p} = \frac{510\times1000}{300\times700} = 2.429\ N/mm^2$$

2. Torsional stresses

$$q_{tu} = \frac{M_{tu}}{2\times A_o \times t_e} = \frac{34\times10^6}{2\times235700\times84.6} = 0.852\ N/mm^2$$

Step 3: Check the need for considering torsion

The value of $q_{tu,min}$ equals

$$q_{tu\ min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}\sqrt{1+\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}}$$

The prestressing force P_e is obtained by multiplying the effective prestressing stress after considering all losses f_{pe} by the area of prestressing steel A_{ps}

$$P_e = f_{pe} \times A_{ps} = 960\times850/1000 = 816\ kN$$

$$f_{pcc} = \frac{P_e}{A} = \frac{816000}{415000} = 1.97\,N/mm^2$$

$$q_{tu\ min} = 0.06\sqrt{\frac{40}{1.5}}\sqrt{1+\frac{1.97}{0.25\sqrt{40}}} = 0.424\ N/mm^2$$

Since $q_{tu} > q_{tu\ min}$, we have to consider the shear stresses due to torsion.

Step 4: Check that section size is adequate

For solid sections, the developed shear stresses due to combined shear and torsion should stratify the following equation:

$$\sqrt{(q_u)^2 + (q_{tu})^2} \le q_{u\max}$$

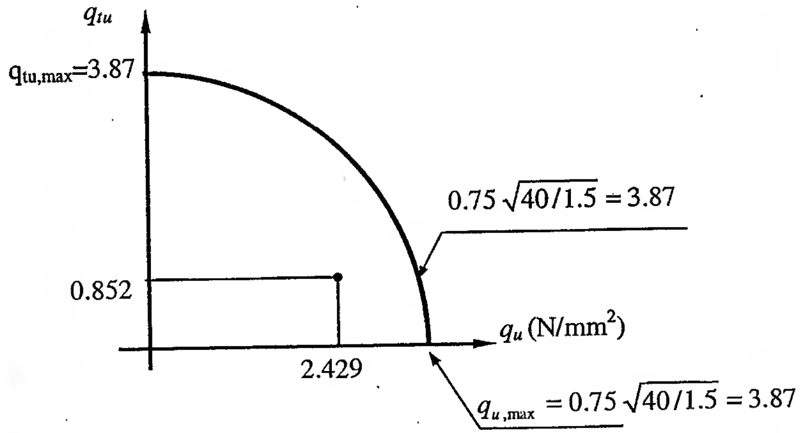
The maximum shear strength q_{umax} is given by:

$$q_{u\max} \le 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.75\sqrt{\frac{40}{1.5}} = 3.87\ N/mm^2 \le 4.5\ N/mm^2$$

$$q_{u,\max} = 3.87\ N/mm^2$$

$$\sqrt{(q_u)^2 + (q_{tu})^2} = \sqrt{(2.429)^2 + (0.852)^2} = 2.57\ N/mm^2$$

Since $\sqrt{(q_u)^2 + (q_{tu})^2} \le q_{u\max}$, the concrete dimensions of the section are adequate as shown in the graphical representation below.



Step 5: Calculate concrete shear strength q_{cu}

To simplify the calculation of q_{cu}, the simplified method is used. However, verification needs to be made as shown in step 5.1.

Step 5.1: Verify the use of the simplified method

Since the effective prestressing stress f_{pe} is greater than 40% of the ultimate tendon strength f_{pu} (960 > 0.40 x 1860=744 N/mm^2), the code simplified expression can be used. To use the simplified equation for evaluating concrete shear strength, the term $Q_u\,d_p/M_u$ equals:

$$\frac{Q_u \times d_p}{M_u} = \frac{510\times(700/1000)}{80} = 4.46 > 1.0$$

$$\text{use } \frac{Q_u \times d_p}{M_u} = 1.0$$

1. $$q_{cu,\min} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{40}{1.5}} = 1.24\ N/mm^2$$

2. $$q_{cu,\max} = 0.375\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.375\sqrt{\frac{40}{1.5}} = 1.94\ N/mm^2$$

$$q_{cu} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + 3.6\times\left(\frac{Q_u \times d_p}{M_u}\right)$$

$$q_{cu} = 0.045\sqrt{\frac{40}{1.5}} + 3.6\times1.0 = 3.83\ N/mm^2 > q_{cu,min}(1.24)\ \ldots\text{o.k}$$

However, since $q_{cu} > q_{cu,max}$ use $q_{cu} = q_{cu,max}$

q_{cu} =1.94 N/mm^2

Step 6: Design of stirrups for shear and torsion

Step 6.1: Area of stirrups for shear

Since the applied shear q_u is greater than q_{cu}, shear reinforcement is needed.

$$q_{su} = q_u - \frac{q_{cu}}{2} = 2.429 - \frac{1.94}{2} = 1.459\ N/mm^2$$

The spacing of the stirrups should be smaller of ph/8 (410) mm or 200 mm, **try a spacing of 100 mm**

$$A_{st} = \frac{q_{su}\times b\times s}{f_y/1.15} = \frac{1.459\times300\times100}{360/1.15} = 139.8\ mm^2$$

Area for one branch of the stirrup equals $A_{st}/2$=69.91 mm^2

Step 6.2: Area of stirrups for torsion

Since f_{pe} (960 N/mm^2) is greater than 0.40 f_{pu}, use θ=45

Using the same stirrup spacing of 100 mm, the area of one branch A_{str}

$$A_{str} = \frac{M_{tu}\times s}{2\times A_o \times f_{yst}/\gamma_s \cot\theta} = \frac{34\times10^6\times100}{2\times235790\times360/1.15 \cot 45^o} = 23.03\ mm^2$$

Step 6.3: Stirrups for combined shear and torsion

A: Web

Area of one branch for combined shear and torsion

$= A_{str} + A_{st}/2$= 23.03 + 69.97 = 93 mm^2

Choose ϕ 12 mm (113 mm^2)

$$A_{st,\min} = \frac{0.40}{f_y} b\times s = \frac{0.40}{360} 300\times100 = 33.3\ mm^2$$

Total area chosen $= 2\times113 > A_{st,\min}$ ......*o.k*

Final design use Φ 12/100 mm

B: Flanges

The flanges only resists torsion thus the area of one branch=A_{str}

A_{str} = 23.03 mm^2

use Φ 8/100 mm

Step 7 Design of longitudinal reinforcement for torsion

$$A_{sl} = \frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)\cot^2\theta = \frac{23.03\times3280}{100}\left(\frac{360}{360}\right)\cot^2 45 = 755\ mm^2$$

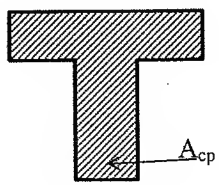
Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

$$A_{sl,\min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y/\gamma_s} - \frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$

There is a condition on this equation that $\frac{A_{str}}{s} \ge \frac{b}{6\times f_{yst}}$

$$\frac{23.03}{100} \ge \frac{300}{6\times360}\ldots o.k$$

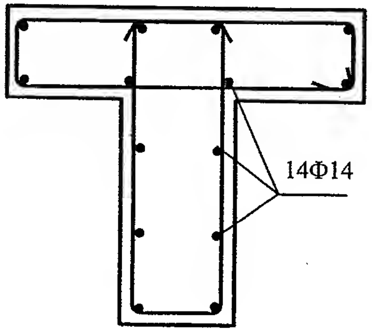
$$A_{cp} = 300\times550 + 250\times1000 = 415000\ mm^2$$



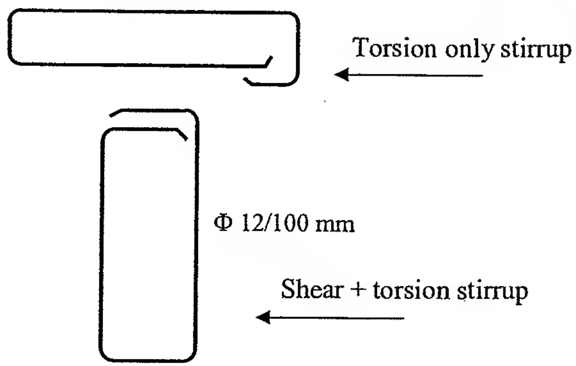
$$A_{sl,\min} = \frac{0.40\sqrt{\frac{40}{1.5}} \times 415000}{360/1.15} - \frac{23.03 \times 3280}{100}\left(\frac{360}{360}\right) = 1983\ mm^2$$

Since $A_{sl} < A_{sl,min}$ ...use $A_{sl,min}$

Choose 14 ϕ14 (2155.1 mm^2). Note that the maximum spacing between longitudinal bars is 300 mm



Torsional reinforcement details



Stirrup detail

Example 9.4: Combined shear and torsion design (Box-section)

Figure Ex. 9.4 shows a box section that constitutes the cross-section of the girder of a road-way bridge. Structural analysis of the bridge revealed that the critical section of the girder near the support is subject to the following straining actions:

Q_u = 13000 kN

M_{tu}=32000 kN.m

M_u=60200 kN.m

At this section the girder has been post-tensioned with 32 tendons arranged in 8 ducts 60 mm diameter. The total prestressing steel A_{ps} = 17120 mm^2. The low-relaxation strands have f_{pu} = 1860 N/mm^2 and f_{pe} =1080 N/mm^2. It is required to carry out a design for the combined shear and torsion for that section. The material properties are as follows:

f_{cu}= 40 N/mm^2 and f_y = 400 N/mm^2

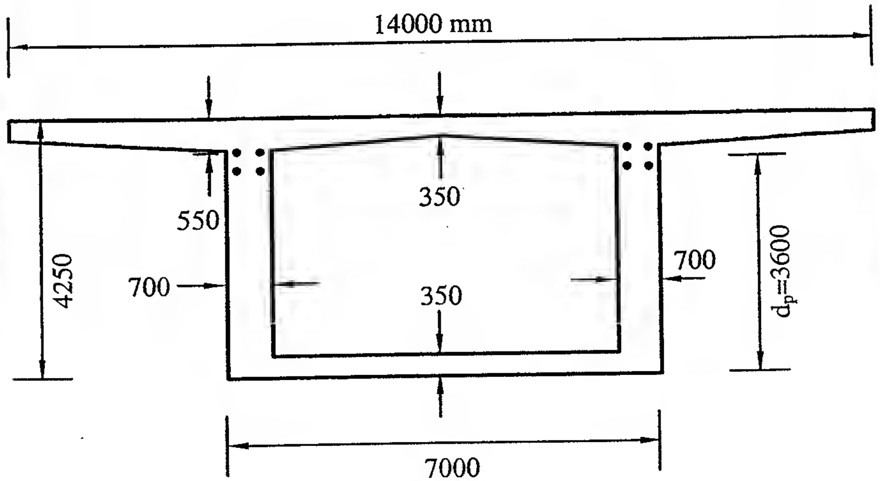


Fig. EX. 9.4 Cross section of the road-way bridge

Solution

Step 1: Calculate section properties

Assume a clear concrete cover of 40 mm and the diameter of the bars used is 22 mm as illustrated in the figure shown below

$$A_{oh} = [4250 - 2\times(40+11)]\times[7000 - 2\times(40+11)]$$

$$= 4148\times6898 = 28612904\ mm^2$$

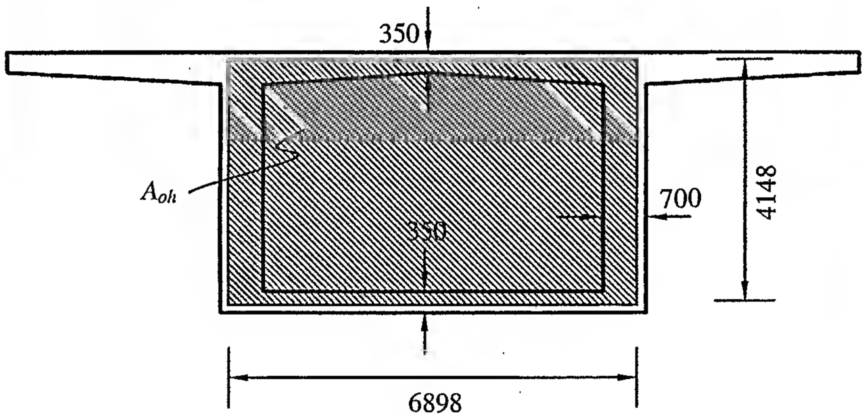
$$p_h = 2\times(4148+6898) = 22092\ mm$$

$$t_e = \frac{A_{oh}}{p_h} = \frac{28612904}{22092} = 1295.1mm$$

Since the effective thickness (t_e) is less than both the web thickness (700 mm) and the flange thickness (350 mm), use the actual thickness.

Use $t_e = t_{actual} = 350$ mm for flanges

Use $t_e = t_{actual} = 700$ mm for webs



Step 2: Calculate the ultimate shear stresses due to Q_u and M_t

1. Shear Stress:

The applied vertical shear force is resisted by the internal shear stresses developed in each web. From the figure one gets:

$$d_p = 3600\ mm > (0.8t = 0.80\times4250 = 3400mm)....ok$$

$$q_u = \frac{Q_u}{b\times d} = \frac{13000\times1000}{(2\times700)\times3600} = 2.58N/mm^2$$

2. Torsional Stresses

The torsional stresses in the webs equal:

$$q_{tu\,(web)} = \frac{M_{tu}}{2\times A_o\times t_e} = \frac{32000\times10^6}{2\times(0.85\times28612904)\times700} = 0.94\ N/mm^2$$

The top flange is more critical because its thickness is smaller than the bottom one. Thus the torsion stress in the slab (flange) equals:

$$q_{tu\,(flange)} = \frac{M_{tu}}{2\times A_o\times t_e} = \frac{32000\times10^6}{2\times(0.85\times28612904)\times350} = 2.23\ N/mm^2$$

Step 3: Check the need for considering torsion

The value of $q_{tu,min}$ equals:

$$q_{tu\,min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}\sqrt{1+\frac{f_{pcc}}{0.25\sqrt{f_{cu}}}}$$

The prestressing force P_e is obtained by multiplying the effective prestressing stress after considering all losses f_{pe} by the area of prestressing steel A_{ps}.

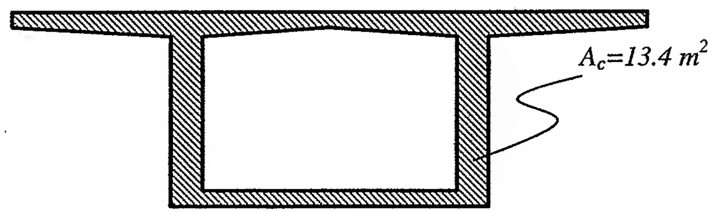
$$P_e = f_{pe}\times A_{ps} = 1080\times17120/1000 = 18489.6\ kN$$

It can be computed that the total concrete cross sectional area A_c=13.4 m^2

$$f_{pcc} = \frac{P_e}{A_c} = \frac{18489.6\times1000}{13.4\times10^6} = 1.38N/mm^2$$

$$q_{tu\,min} = 0.06\sqrt{\frac{40}{1.5}}\sqrt{1+\frac{1.38}{0.25\sqrt{40}}} = 0.4\ N/mm^2$$

Since $q_{tu} > q_{tu\,min}$, we have to consider the shear stresses due to torsion.



Step 4: Check that section size is adequate

The maximum shear strength q_{umax} is given by:

$$q_{u\max} \le 0.75\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.75\sqrt{\frac{40}{1.5}} = 3.87\ N/mm^2 \le 4.5\ N/mm^2$$

For the flanges (top or bottom flanges)

Since $q_{tu(\text{flange})} = 2.23\ N/mm^2 < q_{u,max}$, the flange thickness is adequate.

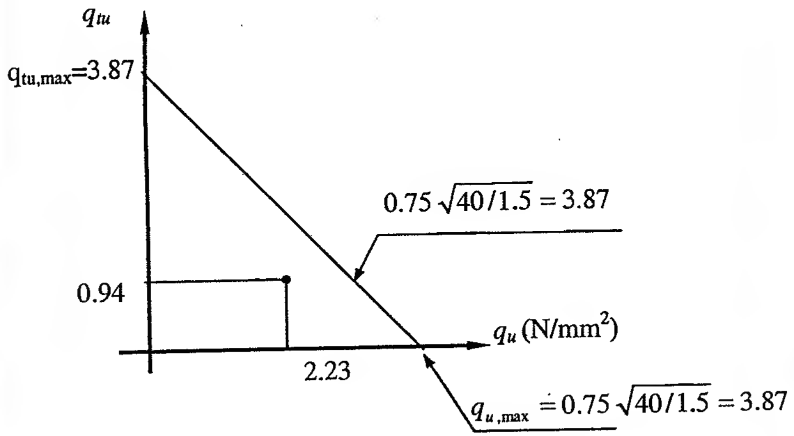
For the webs

For hollow sections, the developed shear stresses due to combined shear and torsion should stratify the following equation:

$$q_u + q_{tu} \le q_{u\max}$$

$$q_u + q_{tu} \le q_{u\max}\ q_u + q_{tu} = 0.94 + 2.23 = 3.17\ N/mm^2$$

Since , the concrete dimensions of the section are adequate as shown in the graphical representation below.



Step 5: Calculate concrete shear strength q_{cu}

To simplify the calculation of q_{cu}, the simplified method is used. However, verification needs to be made as shown in step 5.1.

Step 5.1: Verify the use of the simplified method

Since the effective prestressing stress f_{pe} is greater than 40% of the ultimate tendon strength f_{pu} (1080 > 0.40 x 1860=744 N/mm^2), the code simplified expression can be used. To use the simplified equation for evaluating concrete shear strength, the term $Q_u\ d_p/M_u$ equals:

$$\frac{Q_u \times d_p}{M_u} = \frac{13000\times(3600/1000)}{60200} = 0.77 < 1.0 .... ok$$

- $q_{cu,\min} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{40}{1.5}} = 1.24\ N/mm^2$
- $q_{cu,\max} = 0.375\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.375\sqrt{\frac{40}{1.5}} = 1.94\ N/mm^2$

$$q_{cu} = 0.045\sqrt{\frac{f_{cu}}{\gamma_c}} + 3.6\times\left(\frac{Q_u \times d_p}{M_u}\right)$$

$$q_{cu} = 0.045\sqrt{\frac{40}{1.5}} + 3.6\times 0.77 = 3.51\ N/mm^2 > q_{cu,min}(1.24) ....\text{o.k}$$

Since $q_{cu} > q_{cu,max}$ use $q_{cu} = q_{cu,max}$ $\quad q_{cu}$=1.94 N/mm^2

Step 6: Design of stirrups for shear and torsion

Step 6.1: Area of stirrups for shear

Since the applied shear q_u is greater than q_{cu}, shear reinforcement is needed.

$$q_{su} = q_u - \frac{q_{cu}}{2} = 2.58 - \frac{1.94}{2} = 1.61\ N/mm^2$$

The spacing of the stirrups should be smaller of
$P_h/8$ (2716) mm or 200 mm, **try a spacing of 100 mm**

$$A_{st} = \frac{q_{su} \times b \times s}{f_y/1.15} = \frac{1.61\times(2\times700)\times100}{400/1.15} = 648\ mm^2$$

For one web = 648/2=324 mm^2 and area of one branch=324/2=162 mm^2

Step 6.2: Area of stirrups for torsion

Since f_{pe} (1080 N/mm^2) is greater than 0.40 f_{pu}, use θ=45.

Using s of 100 mm, the area of one branch A_{str}, one gets:

$$A_{str} = \frac{M_{tu} \times s}{2 \times A_o \times f_{yst} / \gamma_s \cot\theta} = \frac{32000 \times 10^6 \times 100}{2 \times (0.85 \times 28612904) \times 400/1.15 \cot 45^o} = 189.1\ mm^2$$

For box sections, the code permits the use of reinforcement along the interior and exterior sides of each web if the wall thickness t_w is less or equal to the section width/6.

$$\because t_w\ (700) < \frac{b}{6} < \frac{7000}{6}$$

The area of the stirrups for torsion can be divided on the two sides

Area of one branch A_{str}=189.1/2 =94.55 mm^2

Step 6.3: Stirrups for combined shear and torsion

A: Web

Area of one branch for combined shear and torsion

$$= A_{str} + A_{st}/2 = 94.55 + 162 = 256.6\ mm^2$$

Choose Φ 20 mm (314 mm^2)

$$A_{st,\min} = \frac{0.4}{f_y} b \times s = \frac{0.40}{400} 700 \times 100 = 70\ mm^2 \text{(for one web)}$$

$$Total\ area\ chosen = 2 \times 314 > A_{st,\min} ......o.k$$

Final design use Φ 18/100 mm

B: Flanges

The flanges only resists torsion thus the area of one branch=A_{str}

A_{str} = 94.55 mm^2

use Φ 12/100 mm

$$2A_{str} + A_{st})_{\min} = \frac{0.40}{f_y} b \times s = \frac{0.40}{400}(350) \times 100 = 35\ mm^2$$

$$A_{st,chosen} = 2 \times 113 = 226\ mm^2 > 35.2\ .....ok$$

Final design two stirrups ϕ20/100 mm (two branches) in the webs and two stirrups ϕ12/100 mm (two branches) in the flanges

Step 7 Design of longitudinal reinforcement for torsion

$$A_{sl} = \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)\cot^2\theta = \frac{189.1 \times 22092}{100}\left(\frac{400}{400}\right)\cot^2 45 = 41784.mm^2$$

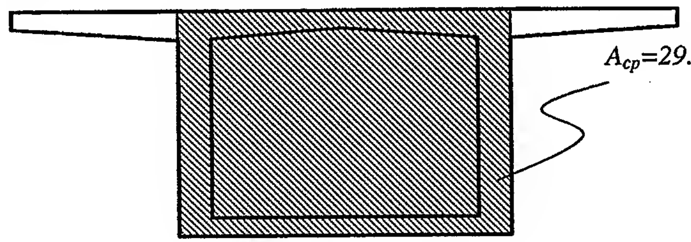
Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

$$A_{sl,\min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}} A_{cp}}{f_y / \gamma_s} - \left(\frac{A_{str}}{s}\right) \times p_h \left(\frac{f_{yst}}{f_y}\right)$$

There is a condition on this equation that $\frac{A_{str}}{s} \geq \frac{b}{6 \times f_{yst}}$

$$\frac{189}{100} < \frac{7000}{6 \times 400} \text{ thus use } \frac{b}{6 \times f_{yst}}$$

$$A_{cp} = 7 \times 4.25 = 29.75\ m^2$$



$$A_{sl,\min} = \frac{0.4\sqrt{\frac{40}{1.5}} \times 29.75 \times 10^6}{400/1.15} - \left(\frac{7000}{6 \times 400}\right) \times 22092 \times \left(\frac{400}{400}\right) = 112237\ mm^2$$

Since $A_{sl} < A_{sl,min}$ ...use $A_{sl,min}$

Choose 184 Φ 28 such that the maximum spacing between longitudinal steel is less than 300 mm

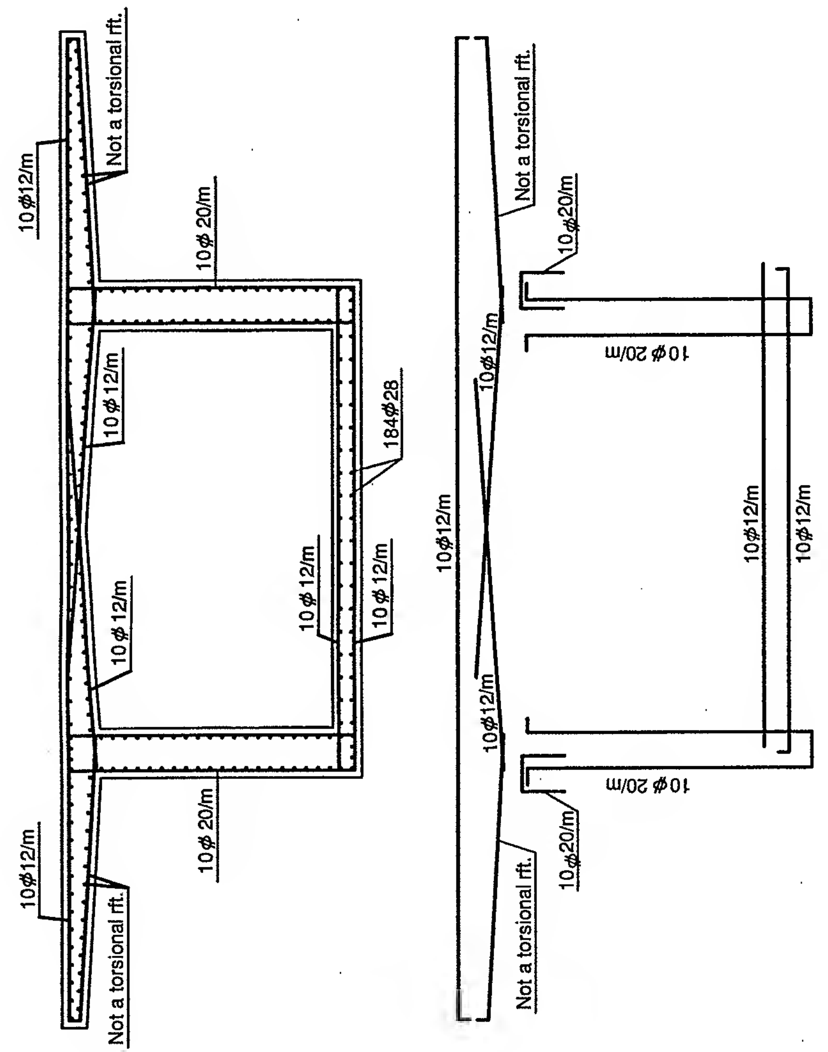


Fig. EX.9.4 Reinforcement details.

10

CONTINUOUS PRESTRESSED BEAMS

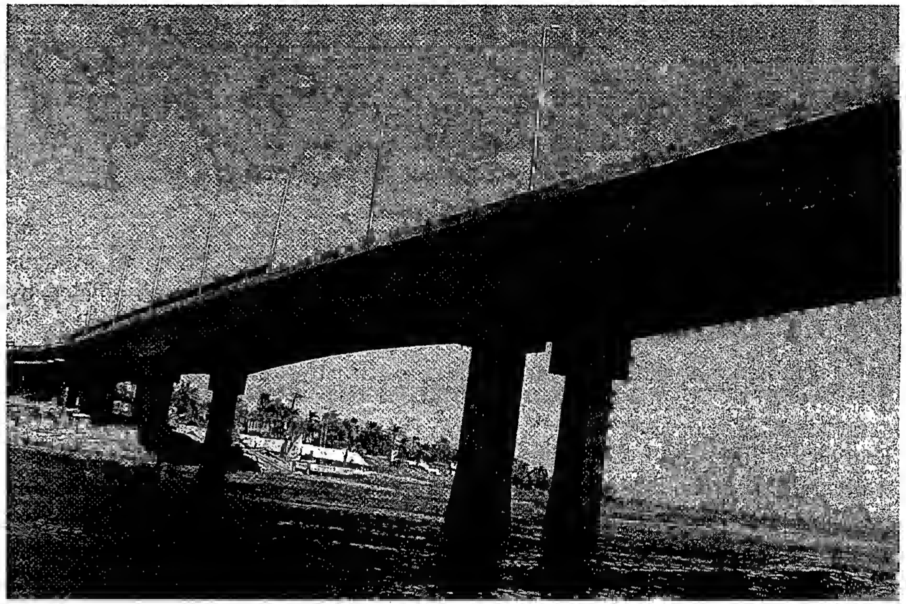


Photo 10.1 Sohag bridge over the River Nile

10.1 Introduction

Continuity is frequently used because of the several benefits that can be achieved. Continuity reduces the bending moments resulting in more economic designs. It also permits tensioning of the tendons over several supports with a great reduction in the number of anchorage and labor cost in the prestressing operation. Finally, the deflection of continuous members is greatly reduced when compared to that of the simple span.

Continuous prestressed concrete beams are widely used in Egypt in the construction of bridges particularly those constructed using post-tensioning technique.

The disadvantages of continuity can be summarized in the following:

- Higher frictional losses due to the larger number of bends and longer path.
- The sections over the interior supports are subjected to combined effect of high bending moments and high shear forces whereas the section at mid-span of a simple beam is subjected to zero shear.
- Development of horizontal forces and moments in the supporting columns. These forces are produced by elastic shortening of the beams.
- Formation of secondary stresses due to shrinkage, creep and temperature.
- Moment reversal may occur due to alternate loading of spans.
- Formation of secondary moment due to induced reactions at the middle supports caused by prestressing force *(to be discussed later).*

Despite of all these disadvantages, the use of continuous prestressed concrete beams is an attractive solution when compared to other structural systems especially in bridge construction. Most of these factors can be eliminated by following the appropriate design considerations.

10.2 Tendon Profile for Continuous Beams

Tendon profiles for continuous beams vary according to the span length, structural integrity, and the construction method. Figure 10.1a shows how continuity is achieved in a post-tensioned beam. This profile is used extensively in slabs and short span beams. The tendon is located below the C.G. at mid-span while it is located above the C.G. at the supports. The major advantage of such a system is the simplicity of formwork. However, high frictional losses due to the length of the cable and the large number of bends are considered as major disadvantage.

To overcome this disadvantage, a non-prismatic section is used through varying the depth over the support as shown in Fig. 10.1b. The added cost of formwork is justified by the use of almost a straight cable that reduces the frictional losses. The system shown in Fig. 10.1c has the advantage of small frictional losses due to shorter cable length in addition to the simplicity of the formwork. Additional cost may be incurred due to the use of several anchorages.

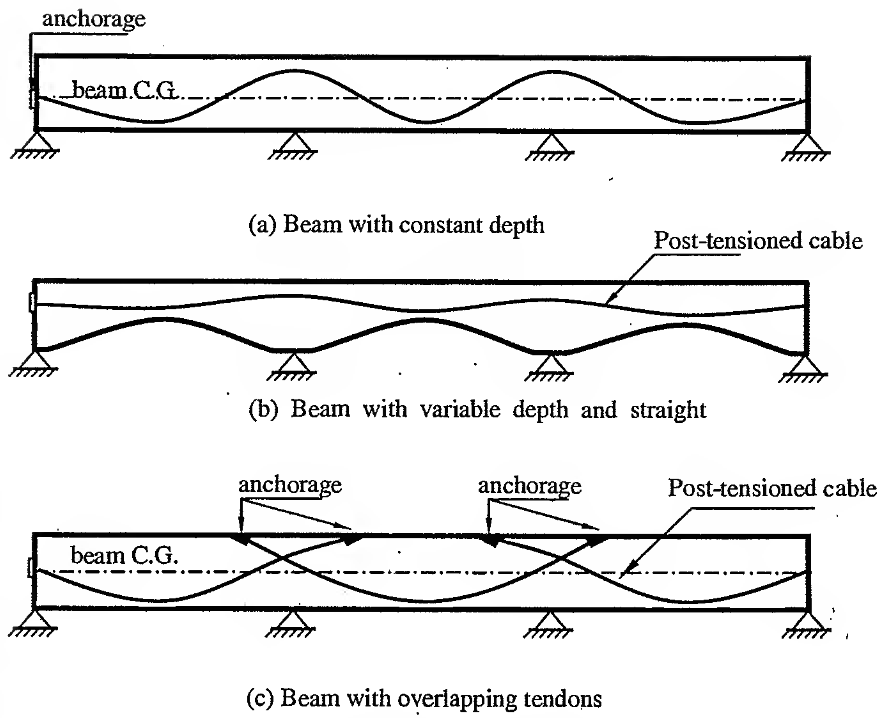


Fig. 10.1 Tendon profile in continuous post-tensioned beams

Continuity may also be achieved by pre-tensioning some cables in the beam and post-tensioning other cables. In Fig. 10.2, the pre-tensioned precast elements are arranged over the support as simple beams. Then post-tensioning is applied longitudinally to provide continuity over the supports. Concrete is poured over the support between the beams. Example of continuous prestressed box section with various cable profiles is shown in Fig. 10.3.

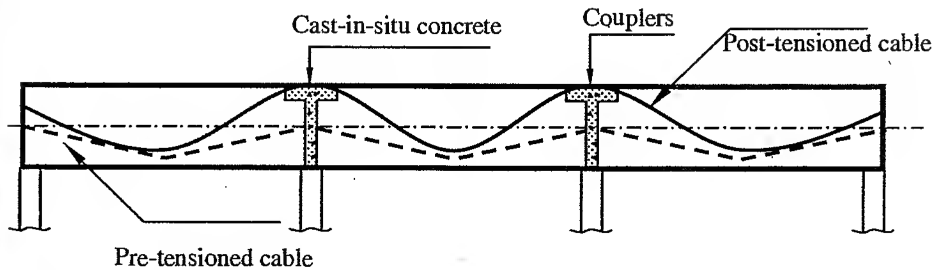


Fig. 10.2 Continuity through pre-tensioning and post-tensioning

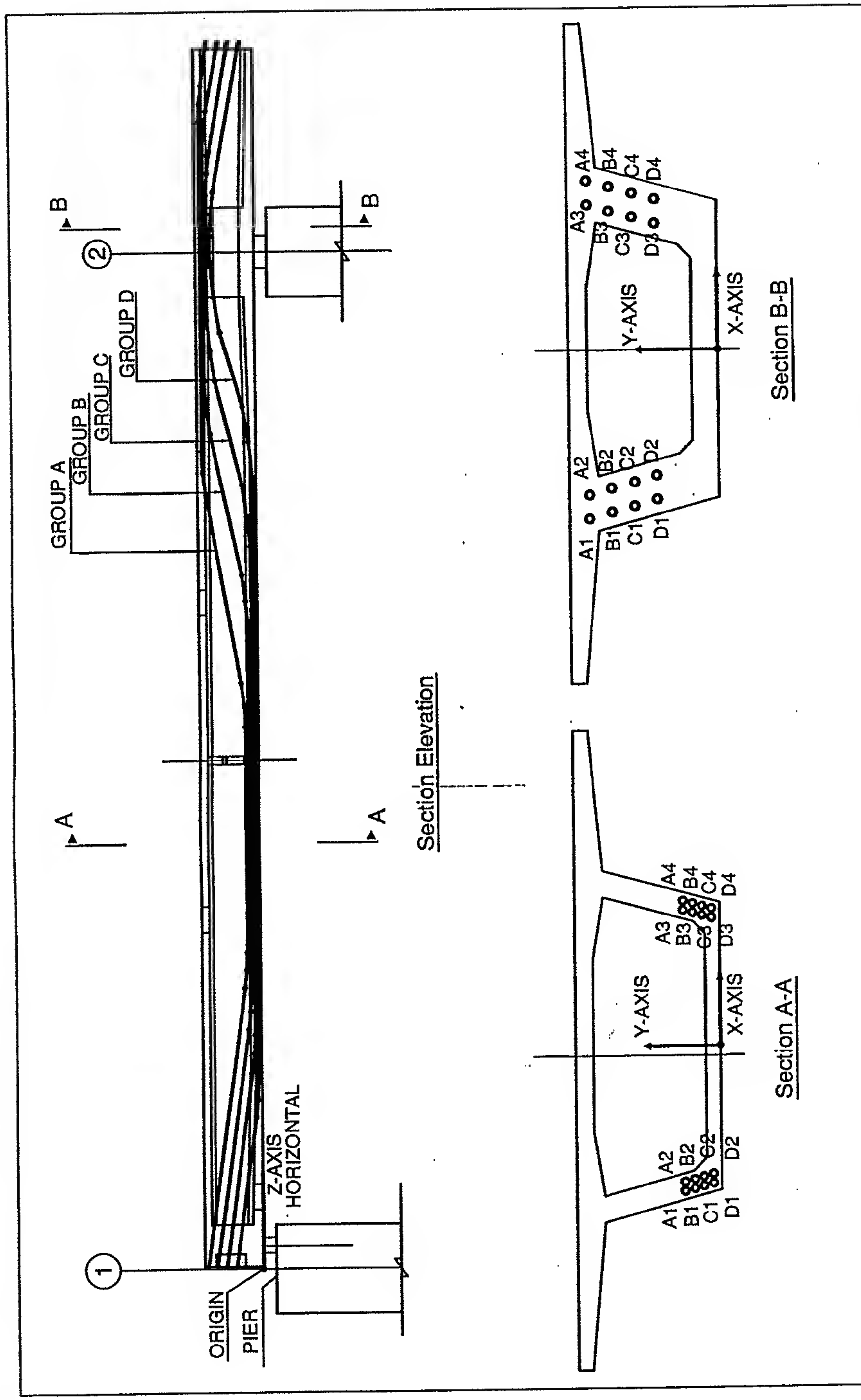


Fig. 10.3 Tendon profile for a continuous box-section.

10.3 Elastic Analysis of Continuous Beams

10.3.1 Effects of the Prestress

The deformation caused by prestressing in a statically determinate member is free to take place without any restraint from supports. In statically indeterminate members, however, this is not the case. The intermediate supports impose additional geometric constraint which is zero deflection at the intermediate supports. During the stressing operation, the geometric constrains of zero deflection at the intermediate supports cause additional reactive forces to develop at the locations of the intermediate supports, which in turn change the distribution and magnitude of the moments and shears in the members.

In continuous prestressed concrete beams, the moment induced by prestressing on a particular cross-section in a statically indeterminate structure may be considered to be made of two components:

a- The first component is the product of the prestressing force (P) and its eccentricity from the centroidal axis (e). This is the moment that acts on the cross-section when the geometric constraints imposed by the intermediate supports are removed. The moment (*P*.e) is called the *primary moment.*

b- The second component is the moment caused by the reactions developed at the intermediate supports. As mentioned before, such reactions are required in order to achieve zero deflection at the intermediate supports due to prestressing. This moment is called *secondary moment.*

Elastic analysis of continuous beams can be carried out using one of two methods:

- Support displacement method.
- Equivalent load method.

10.3.2 Support Displacement Method

10.3.2.1 Background

Let us consider a cantilevered beam with a roller support at the end as shown in Fig. 10.4. If this support is removed, the beam becomes determinate and it will deflect downward. To maintain its previous position, a force *R* is required. This force causes the secondary moment. The original (primary) moment due to prestressing equals $M_{10}=P.e$ *(negative in this case).* The final moment any point equals:

$$M_{final} = M_{primary} + M_{secondary} \qquad (10.1)$$

To determine the unknown reaction R, the method of consistent deformation is used. The unknown reaction is replaced by a unit load and the structure becomes determinate. The deflection δ_{11} due to this unit load (P =1) can be obtained by integrating the moment M_{11} over the span as follows:

$$\delta_{11} = \frac{1}{EI} \int M_{11} \times M_{11}\, d_x \quad \text{.............................. (10.2)}$$

Furthermore, for simple structures, the deflection δ_{11} can be obtained using the expressions given in Appendix A. For example, the deflection of a cantilever beam with concentrated load at the end equals:

$$\delta_{11} = \frac{P\,L^3}{3E\,I} = \frac{L^3}{3E\,I} \quad \text{....................................... (10.3)}$$

The deflection δ_{10} due to the primary moment M_{10} can be computed by as follows:

$$\delta_{10} = \frac{1}{E\,I} \int M_{11} M_{10}\, d_x \quad \text{................................ (10.4)}$$

Since, the deflection at the actual support should equal to zero, the unit load deflection δ_{11} should be equal and opposite to the deflection caused by the primary moment δ_{10}. Hence, compatibility of deformations gives:

$$R\,\delta_{11} = -\delta_{10} \quad \text{.. (10.5)}$$

R is a factor required for consistency of deformations and is given by:

$$R = -\frac{\delta_{10}}{\delta_{11}} \quad \text{.. (10.6)}$$

The secondary moment has the same shape as the unit force moment, but with a modified value given by:

$$M_{sec} = R \times M_{11} \quad \text{.. (10.7)}$$

It should be clear that the secondary moment is added to the primary moment to produce the final moment. Also, the value of the secondary moment is dependent on the prestressing force and the tendon profile.

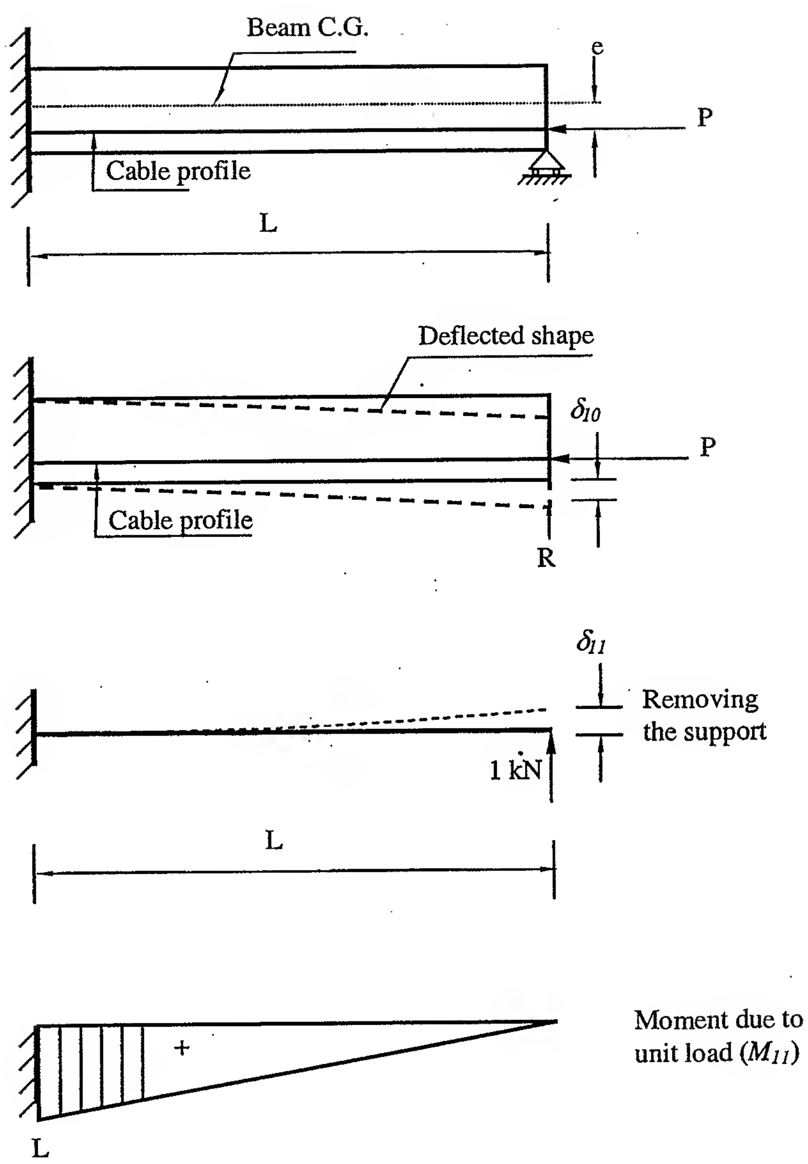


Fig. 10.4 Application of support displacement method to continuous prestressed beam

Applying of the method of consistent deformation to the beam shown in Fig. 10.5, results in the following expressions:

$$\delta_{11} = \frac{1}{EI}\int M^{2}_{11} d_x = \frac{1}{EI}\frac{L \times L}{2} \times \frac{2L}{3} = \frac{L^3}{3EI} \uparrow$$

$$\delta_{10} = \frac{1}{EI}\int M_{11} M_{10}\, d_x = \frac{1}{EI}(-P \cdot e) \cdot L \times \frac{L}{2} = \frac{-P \cdot e \cdot L^2}{2EI} \downarrow$$

To facilitate the calculations of the above integrals, the integrations of some typical shapes are given in Table 10.1. Thus, R equals:

$$R = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-P \cdot e \cdot L^2/2EI}{L^3/3EI}$$

$$\boxed{R = \frac{3P \cdot e}{2L} \uparrow}$$

The secondary moment at the fixed support equals:

$$M_{sec} = R \times M_{11} = \frac{3P \cdot e}{2L} \times L = \frac{3P \cdot e}{2}$$

The final moment at the fixed support $= M_{primary} + M_{secondary}$

$$= -P \cdot e + \frac{3P \cdot e}{2} = +\frac{P \cdot e}{2}$$

The effective eccentricity at the fixed end = +e/2 (above C.G.)

The Moment at the free end $= M_{primary} + M_{secondary}$

$$= -P \cdot e + 0 = -P \cdot e$$

The effective eccentricity at the free end = -e (below C.G.)

Figure 10.5 shows the primary, the secondary and the final moments for the beam. The final moment can be represented by an effective cable profile or line of pressure or C-line. The profile is obtained by dividing the final moment by the prestressing force.

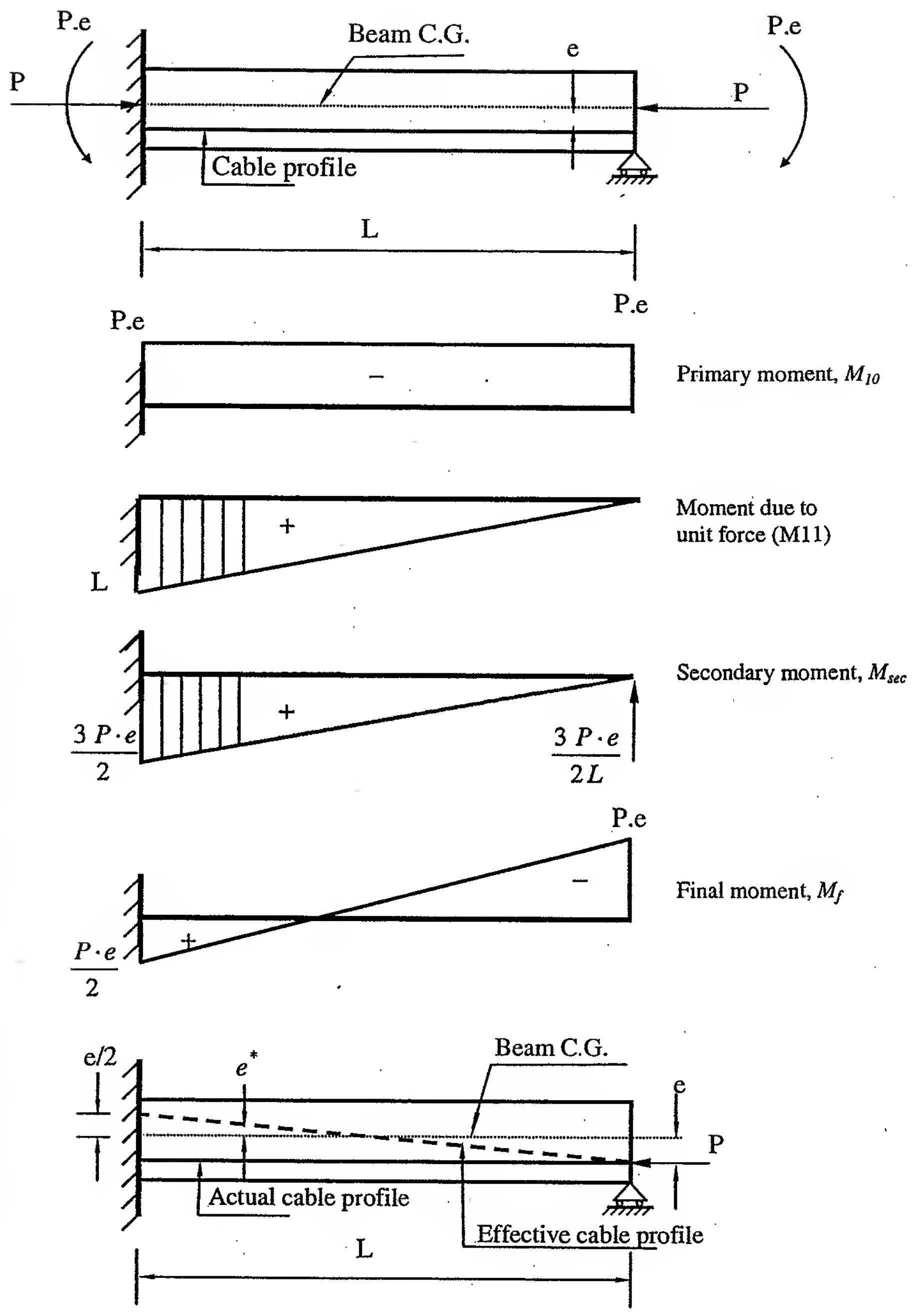


Fig. 10.5 C-line, or pressure line, or effective cable profile

The stress distribution in a statically determinate beam is given by considering the actual cable profile, whereas in a continuous beam, the stress distribution is obtained by using the effective cable profile which gives the effective eccentricity e^* or the final moment due to prestressing ($P\,e^*$). Hence, the stresses at the top and bottom fibers of a section in a continuous beam can be obtained using the following equation:

$$f = -\frac{P}{A} \mp \frac{P \times e^*}{Z} \pm \frac{M} {Z} \quad \text{.............................. (10.8)}$$

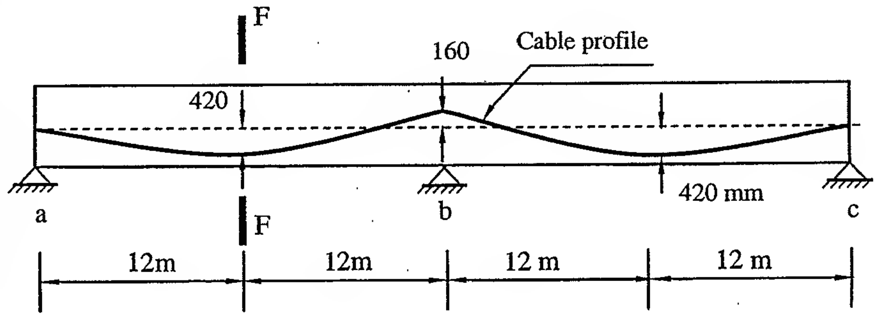
where M is the applied moment at the critical section.

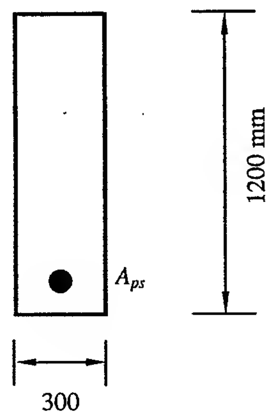
Table 10.1 Values of product integral $\int M_{11}\, M_{10}\, dx$

	M_1, L	M_1, M_2, L	M_1, L	M_1, L
M_3, L	$\frac{L}{2} M_1 M_3$	$\frac{L}{2} (M_1 + M_2) M_3$	$\frac{L}{2} M_1 M_3$	$\frac{2L}{3} M_1 M_3$
M_3, L	$\frac{L}{3} M_1 M_3$	$\frac{L}{6} (M_1 + 2M_2) M_3$	$\frac{L}{4} M_1 M_3$	$\frac{L}{3} M_1 M_3$
M_3, L	$\frac{L}{6} M_1 M_3$	$\frac{L}{6} (2M_1 + M_2) M_3$	$\frac{L}{4} M_1 M_3$	$\frac{L}{3} M_1 M_3$
M_3, M_4, L	$\frac{L}{6} M_1 (M_3 + 2M_4)$	$\frac{L}{6} M_1 (2M_3 + M_4) +$ $\frac{L}{6} M_2 (M_3 + 2M_4)$	$\frac{L}{4} M_1 (M_3 + M_4)$	$\frac{L}{3} M_1 (M_3 + M_4)$
M_3, L	$\frac{L}{4} M_1 M_3$	$\frac{L}{4} M_3 (M_1 + M_2)$	$\frac{L}{3} M_1 M_3$	$\frac{5L}{12} M_1 M_3$

Example 10.1

The figure given below shows a post-tensioned continuous beam. The tendon profile is shown in figure. The effective prestressing force after losses is 1500 kN. Compute the primary, the secondary and the final moments using the support displacement method. Calculate the stresses at section F due to prestressing and self-weight knowing that the cross-section of the beam is rectangular with dimensions 300 mm x1200 mm.





Beam section F-F

Solution

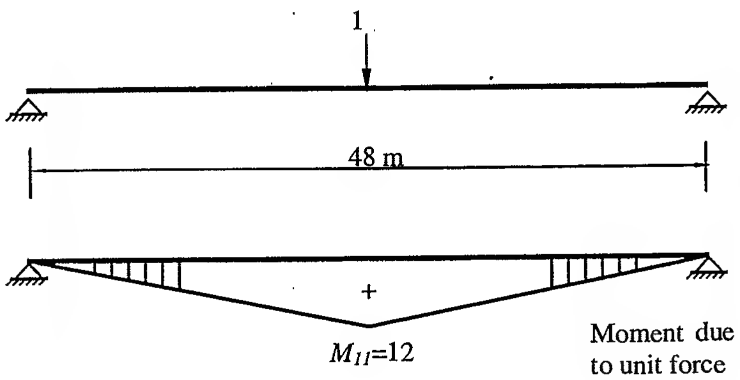
Step 1: Calculate the secondary moment at support b

The primary moment due to prestressing causes camber at the middle support. By removing the middle support and assuming a unit downward force of 1 kN, the bending moment equals:

$$M_{11} = \frac{P \times L}{4} = \frac{1 \times (24+24)}{4} = 12\ kN.m$$

The displacement at the middle support δ_{11} due to the unit force equals (refer to Table 10.1):

$$\delta_{11} = \frac{1}{EI}\int M_{11} M_{11} dx = \frac{L}{3EL} \times M_{11} \times M_{11} = \frac{12 \times 12 \times 48}{3\ EI} = \frac{2304}{E\ I} \downarrow$$



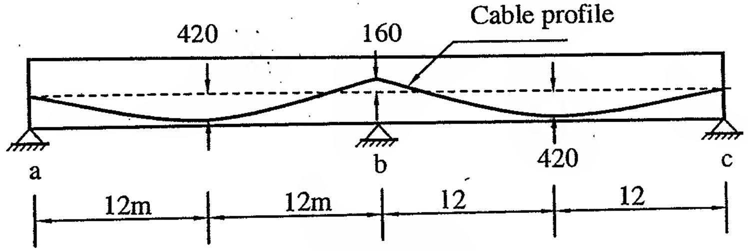
δ_{11} can also be determined using the deflection equation as follows:

$$\delta_{11} = \frac{P\ L^3}{48 \times E\ I} = \frac{1 \times (24+24)^3}{48 \times E\ I} = \frac{2304}{E\ I} \downarrow$$

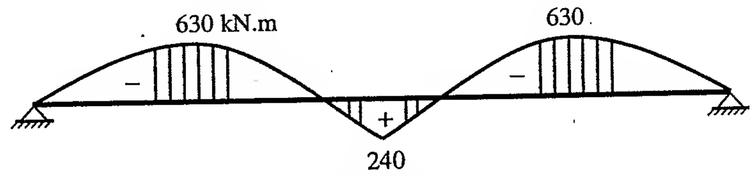
The displacement due to the original prestressing force can be obtained by applying the method of consistent deformation. Referring to Table 10.1, the deflection at intermediate support equals:

$$\delta_{12} = \frac{1}{E\ I}\int M_{11} M_o d_x = \frac{2}{E\ I} \times \left(\frac{24 \times 240 \times 12}{3} - \frac{750 \times 24 \times 12}{3}\right) = -\frac{97920}{EI}$$

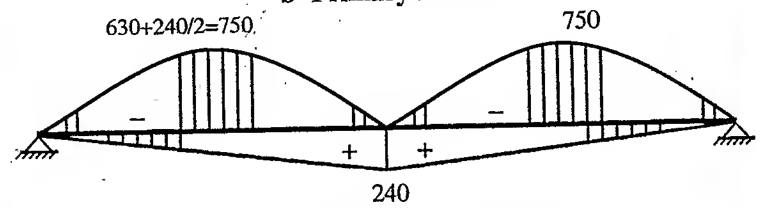
The condition at the support is given by $R\,\delta_{11} + \delta_{10} = 0$



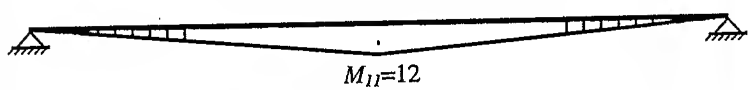
a- Cable profile



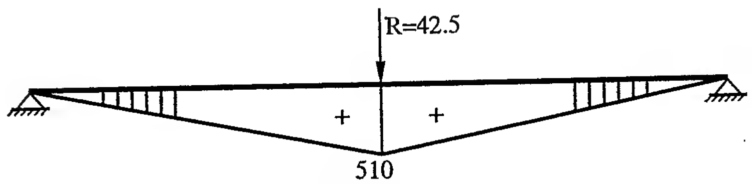
b- Primary moment



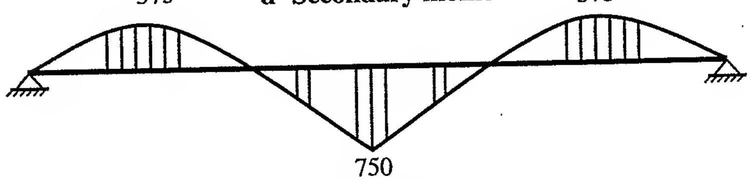
c- Simplified primary moment



Moment due to 1 kN



d- Secondary moment



e- Final moment

$$R = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-97920/EI}{2304/EI} = 42.5\downarrow$$

Thus, the secondary moment at the intermediate support equals:

$$M_{sec} = R \times M_{11} = 42.5 \times 12 = 510\, kN.m$$

Step 2: Calculate final moment and effective eccentricity

The final moment (due to prestressing) at the intermediate support is given by:

$$M_b + M_{sec} = 240 + 510 = 750\, kN.m$$

The effective eccentricity at the intermediate support equals:

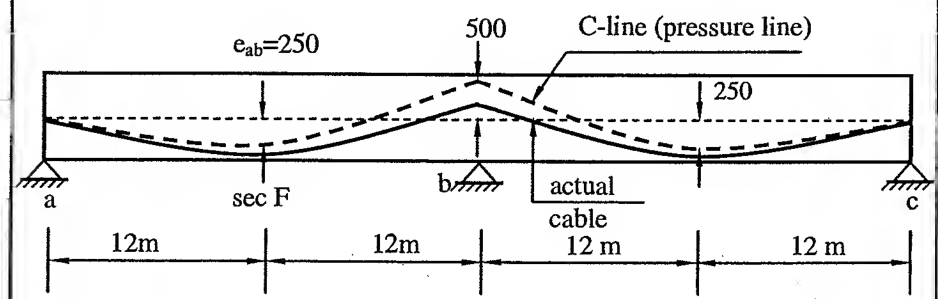
$$e_b = \frac{750 \times 1000}{1500} = 500\ mm \uparrow$$

The midspan final moment $= primary\ moment - \frac{M_{sec}}{2}$

$$= -630 + \frac{510}{2} = -375\ kN.m$$

The effective eccentricity at mid-span equals:

$$e_{ab} = \frac{-375 \times 1000}{1500} = 250\ mm \downarrow$$



Effective cable profile (C-line)

Step 3: Concrete stress due to prestressing and self-weight at section F

The effective eccentricity e* at section F equals:

$$e^* = e_{ab} = 250\ mm$$

$$A = 300 \times 1200 = 360000\ mm^2$$

$$w_{ow} = 25 \times \frac{360000}{10^6} = 9\ kN/m'$$

The maximum positive bending for a continuous beam with two equal spans and equal loading can be obtained as follows:

$$M_{ow} = \frac{w_{ow} \times L^2}{11} = \frac{9 \times 24^2}{11} = 471\ kN.m$$

$$Z_{bot} = Z_{top} = \frac{b \times t^2}{6} = \frac{300 \times 1200^2}{6} = 72 \times 10^6\ mm^3$$

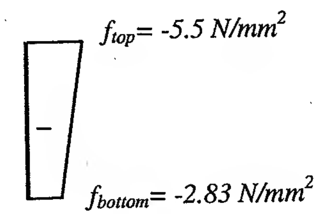
The stresses at bottom and top fibers due to prestressing and self-weight equal:

$$f_{bot} = -\frac{P}{A} - \frac{P \times e^*}{Z_{bot}} + \frac{M_{ow}}{Z_{bot}}$$

$$f_{bot} = -\frac{1500 \times 1000}{360000} - \frac{1500 \times 1000 \times 250}{72 \times 10^6} + \frac{471.27 \times 10^6}{72 \times 10^6} = -2.83\ N/mm^2$$

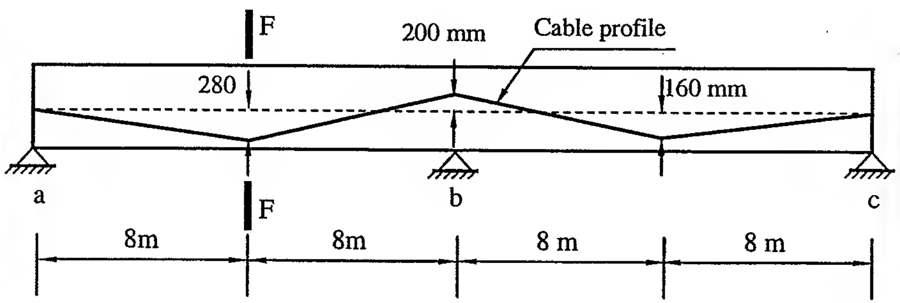
$$f_{top} = -\frac{P}{A} + \frac{P \times e^*}{Z_{top}} - \frac{M_{ow}}{Z_{bot}}$$

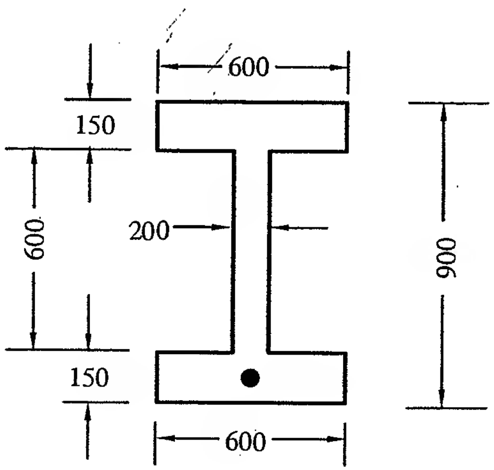
$$f_{top} = -\frac{1500 \times 1000}{360000} + \frac{1500 \times 1000 \times 250}{72 \times 10^6} - \frac{471.27 \times 10^6}{72 \times 10^6} = -5.5\ N/mm^2$$



Example 10.2

A continuous pretensioned beam with two equal spans of 16.0m each is shown in the figure below. It is required to compute the primary, the secondary and the final moments using the support displacement method. The effective prestressing force after losses can be estimated as 1800 kN. Calculate the stresses at the section F due to additional live loads of 32 kN/m′.





Beam cross section

Solution

Step 1: Calculate section properties

$A = 2\times 600\times 150 + 600\times 200 = 300000\ mm^2$

Since the section is symmetrical; $y_{top} = y_{bottom} = 450$ mm

$$I = 2\times\left(\frac{600\times150^3}{12} + 600\times150\times(450-75)^2\right) + \frac{200\times600^3}{12} = 2.925\times10^{10}\ mm^4$$

$$Z_{bot} = \frac{I}{y_{bottom}} = \frac{2.925\times10^{10}}{450} = 65\times10^6\ mm^3$$

$Z_{top} = Z_{bot} = 65\text{x}10^6$ mm^3

$$w_{o.w} = \gamma_c\times A = 25\times\frac{300000}{1000000} = 7.5\,kN/m'$$

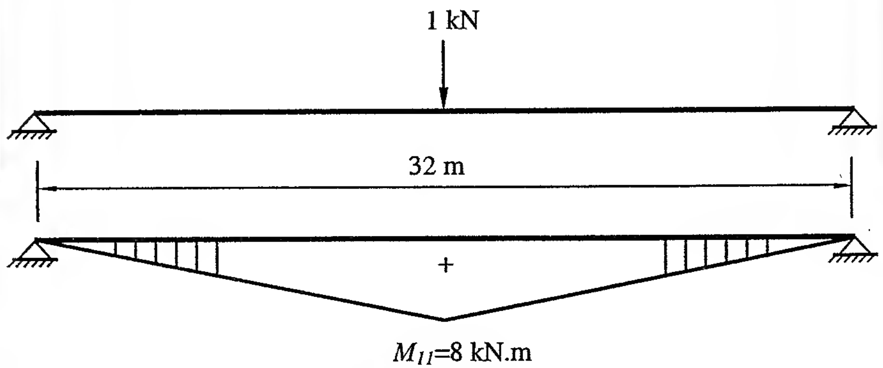
Step 2: Calculate the secondary moment at support b

The primary moment due to prestressing causes camber at the middle support b. By removing the middle support and applying a unit force of 1 kN, the bending moment equals:

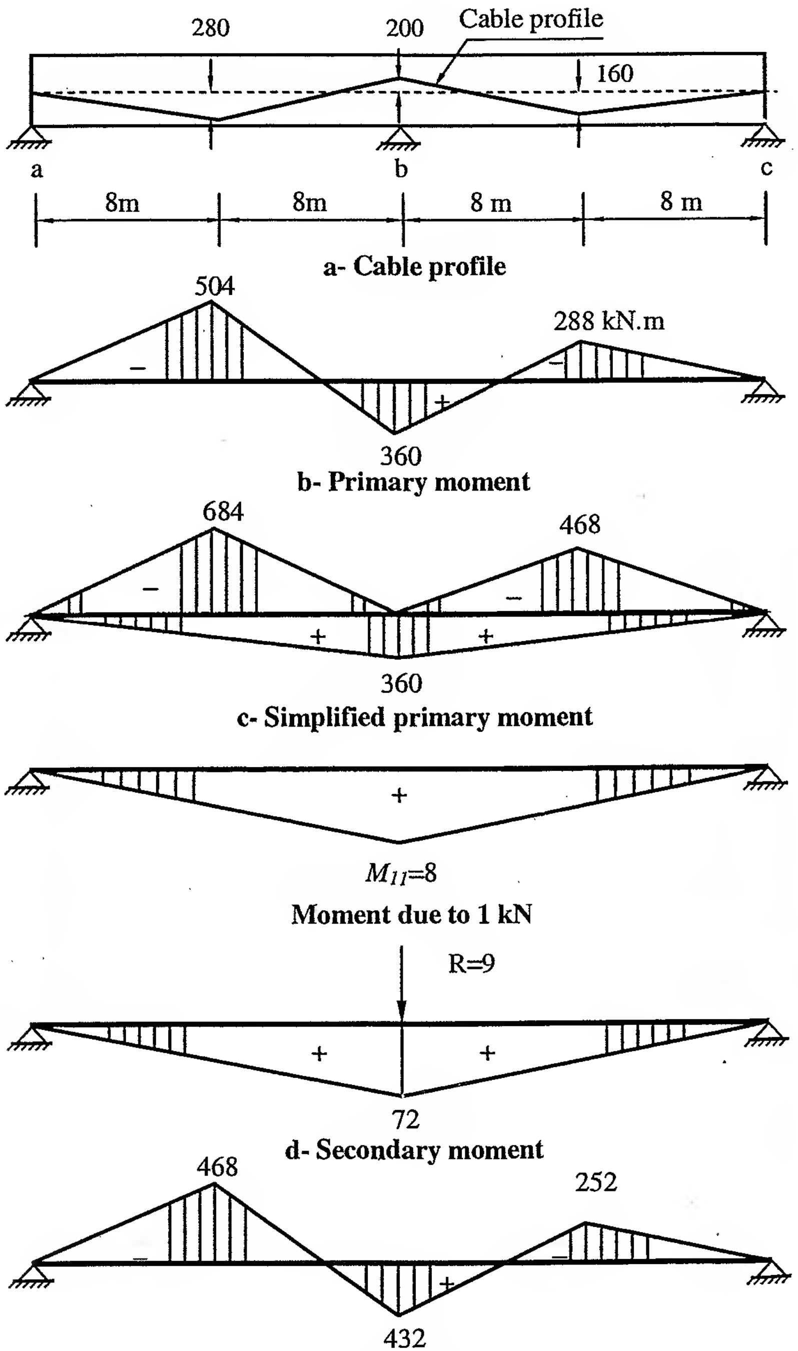
$$M_{11} = \frac{P\times L}{4} = \frac{1\times(16+16)}{4} = 8\ kN.m$$

The displacement at the middle support δ_{11} due to the unit force equals:

$$\delta_{11} = \frac{1}{EI}\int M_{11}M_{11}d_x = \frac{L}{3EI}M_{11}\times M_{11} = \frac{32}{3\ EI}\times8\times8 = \frac{682.667}{E\ I}\downarrow$$



Bending Moment due to a unit force



a- Cable profile

b- Primary moment

c- Simplified primary moment

Moment due to 1 kN

d- Secondary moment

e- Final moment

The displacement due to the primary moment (prestressing force) equals:

$$\delta_{10} = \frac{1}{EI}\int M_{11} M_o d_x = \frac{1}{EI}\left(\frac{32\times8\times360}{3} - \frac{16\times684\times8}{4} - \frac{16\times468\times8}{4}\right) = -\frac{6144}{EI}$$

The condition at the support is given by $R\,\delta_{11} + \delta_{10} = 0$

$$R = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-6144/EI}{682.667/EI} = 9\downarrow$$

Thus, the secondary moment at the intermediate support equals:

$$M_{sec} = R\times M_{11} = 9\times8 = 72\,kN.m$$

Step 3: Calculate final moment and effective eccentricity

The final moment (due to prestressing) at the intermediate support is given by:

$$M_b + M_{sec} = 360 + 72 = 432\ kN.m$$

The effective eccentricity at intermediate support equals:

$$e_b = \frac{432\times1000}{1800} = 240\ mm\uparrow$$

The midspan final moment (span a-b) $= primary\ moment - \dfrac{M_{sec}}{2}$

$$= -504 + \frac{72}{2} = -468\ kN.m$$

The effective eccentricity at left midspan equals:

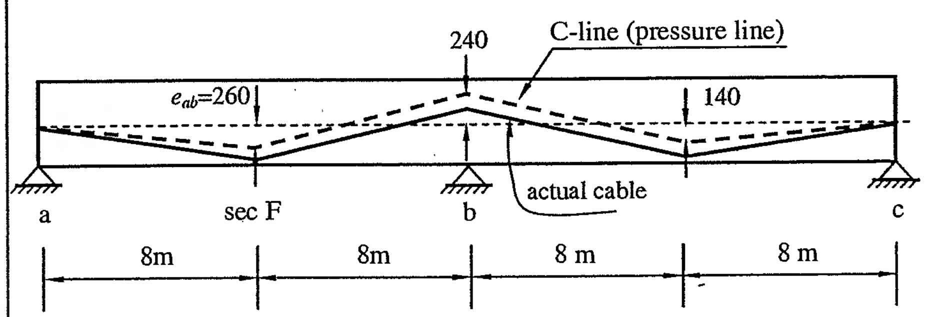
$$e_{ab} = \frac{-468\times1000}{1800} = 260\ mm\downarrow$$

The midspan final moment (span b-c) $= primary\ moment - \dfrac{M_{sec}}{2}$

$$= -288 + \frac{72}{2} = -252\ kN.m$$

The effective eccentricity at the midspan of the right span equals:

$$e_{bc} = \frac{-252\times1000}{1800} = 140\ mm\downarrow$$



Effective cable profile (C-line)

Step 3: Concrete stress due to service loads at section F

The effective eccentricity e* at section F equals:

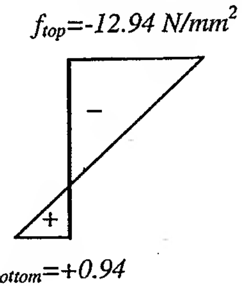
$$e^* = e_{ab} = 260\ mm$$

The total weight w_{tot} equals:

$$w_{tot} = w_{ow} + (w_{DL} + w_{LL}) = 7.5 + 32 = 39.5\ kN\ /m'$$

The maximum positive bending for a continuous beam with two equal spans and equal loading is given as:

$$M = \frac{w_{tot} \times L^2}{11} = \frac{39.5 \times 16^2}{11} = 919.27\ kN.m$$



$$f_{bot} = -\frac{P}{A} - \frac{P \times e^*}{Z_{bot}} + \frac{M}{Z_{bot}}$$

$$f_{bot} = -\frac{1800 \times 1000}{300000} - \frac{1800 \times 1000 \times 260}{65 \times 10^6} + \frac{919.27 \times 10^6}{65 \times 10^6} = +0.94\ N\ /mm^2$$

$$f_{top} = -\frac{P}{A} + \frac{P \times e^*}{Z_{top}} - \frac{M}{Z_{top}}$$

$$f_{top} = -\frac{1800 \times 1000}{300000} + \frac{1800 \times 1000 \times 260}{65 \times 10^6} - \frac{919.27 \times 10^6}{65 \times 10^6} = -12.94\ N\ /mm^2$$

10.3.3 Equivalent Load Method

The equivalent load method is another approach for computing the secondary and the final moments. In this method, the prestressing force is replaced by an equivalent load produced by the primary moments. Solving the continuous beam under the effect of the equivalent loads gives the final moments directly. The secondary moments are obtained by subtracting the final moment from the primary moments as follows:

$$M_{Secondary} = M_{final} - M_{Primary} \quad \text{.........................} (10.9)$$

The method of moment distribution is usually used to calculate the indeterminate moments at the supports. The equivalent loads can be computed for both linear and curved tendons. The equivalent loads for straight tendon profiles are concentrated forces while those for parabolic tendon profile are uniform loads.

A-Straight Tendon Profile

The horizontal and the vertical components of the tendon forces shown in Fig. 10.6 can be expressed as functions of the cable slope angle as follows:

$$H = P\ \cos\theta \qquad V = P\ \sin\theta$$

For small angles, it can be assumed that cos θ=1 and sin θ =θ. Thus,

$$H = P \qquad V = P\ \theta$$

At points D and D′ where the tendon changes direction, a summation of forces indicates the existence of an upward force in the vertical direction. The changes in tendon slope at points D and D' equal:

$$\Delta_\theta = \theta_1 + \theta_2 \qquad \Delta'_\theta = \theta'_1 + \theta'_2$$

where

$$\theta_1 = \frac{e_1 + e_2}{x_1} \quad \text{and} \quad \theta_2 = \frac{e_2 + e_3}{L - x_1} \quad \text{..........................} (10.10a)$$

$$\theta'_1 = \frac{e_3 + e_4}{x_2} \quad \text{and} \quad \theta_2 = \frac{e_4 - e_5}{L' - x_2} \quad \text{......................} (10.10b)$$

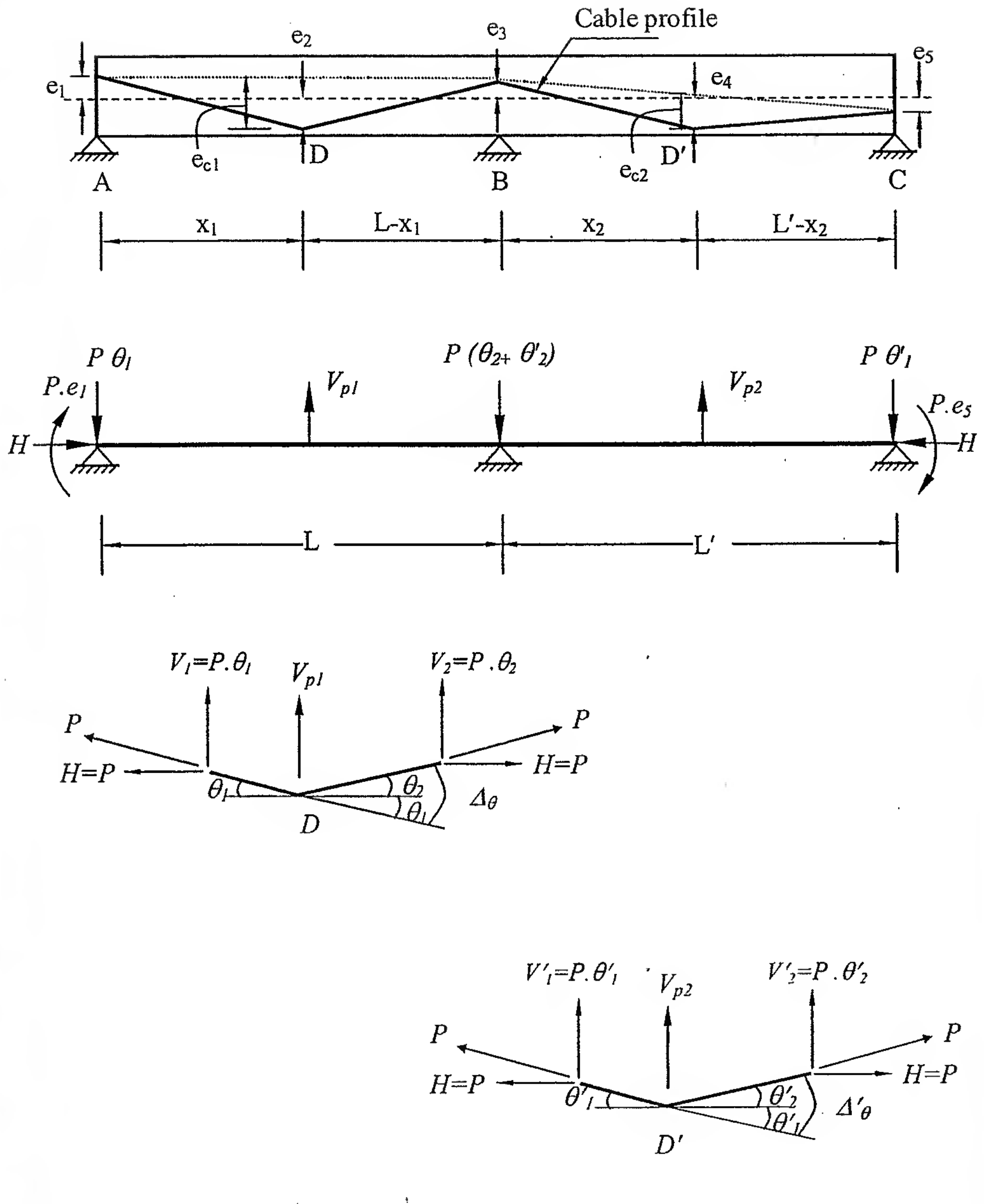


Fig. 10.6 Calculation of the equivalent loads for straight cable profile

The equivalent concentrated loads V_{P1} and V_{P2} are given by:

$$V_{p1} = P \cdot \Delta_{\theta} = P\left(\frac{e_1 + e_2}{x_1} + \frac{e_2 + e_3}{L - x_1}\right) \quad \text{.....................} (10.11)$$

$$V_{p2} = P \cdot \Delta'_{\theta} = P\left(\frac{e_3 + e_4}{x_2} + \frac{e_4 - e_5}{L' - x_2}\right) \quad \text{.....................} (10.12)$$

The equivalent concentrated loads can also be obtained by equating the moment due to prestressing force to the moment caused by the equivalent concentrated loads as follows:

$$V_{p1} \frac{x_1(L - x_1)}{L} = P\, e_{c1} \quad \text{...............................} (10.13a)$$

$$V_{p2} \frac{x_2(L' - x_2)}{L'} = P\, e_{c2} \quad \text{...........................} (10.13b)$$

where e_{c1} and e_{c2} are the eccentricities at point of change in slop at spans ab and bc, respectively.

The eccentricities of the cable cause concentrated positive moment equals ($P.e_1$) at support A and a concentrated negative moment equals ($P.e_5$) at support C.

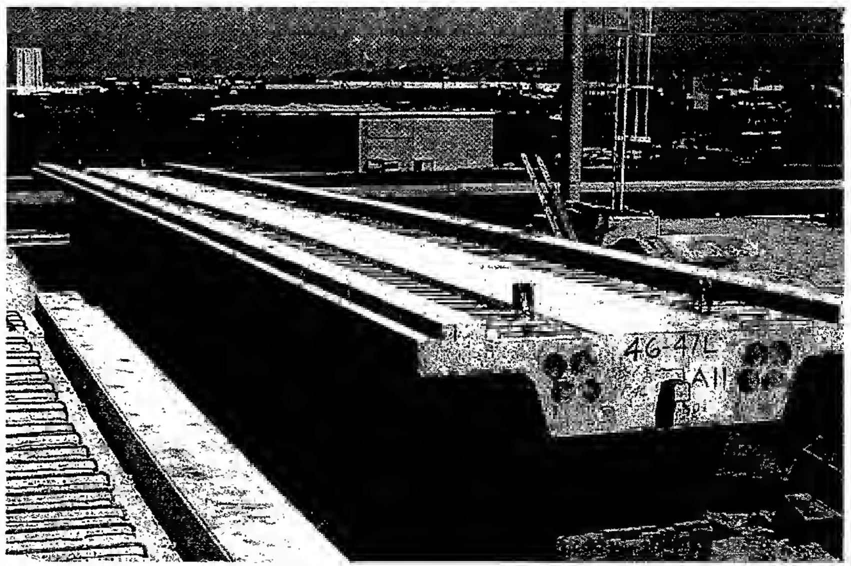


Photo 10.2 Prestressed concrete box section

B-Curved Tendon Profile

The equivalent load caused by a curved tendon configuration can be established by considering a simple beam with parabolic tendon profile. The bending moment at any point due to this prestressing force is given by (M_1=P.e). If we assume an equivalent uniform upward load $w_{eq\ acting}$ on the beam as shown in Fig. 10.7, the maximum moment at mid-span equals:

$$M_2 = \frac{w_{eq} \times L^2}{8} \qquad (10.14)$$

Since the equation of the bending moment for uniformly loaded simply supported beam is also parabolic, the moments must be equal in the two beams at any point. Thus,

$$M_1 = M_2 \qquad (10.15)$$

Application of this equation at the mid-span gives:

$$P \cdot e_m = \frac{w_{eq} \times L^2}{8} \qquad (10.16)$$

$$w_{eq} = \frac{8\ P \cdot e_m}{L^2} \qquad (10.17)$$

where e_m is the mid-span deflection.

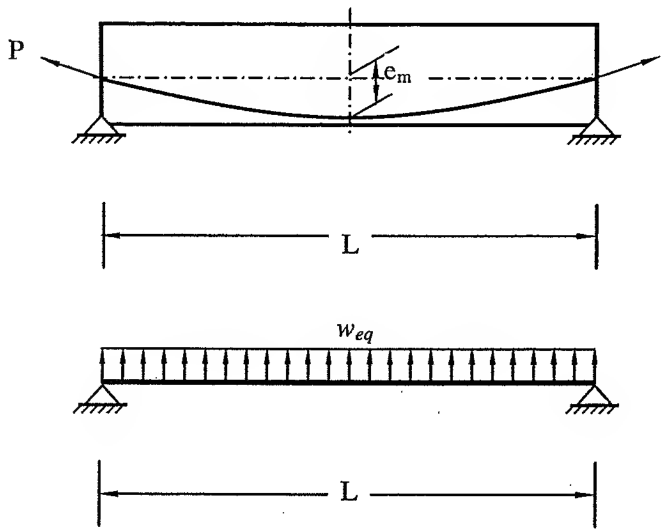


Fig. 10.7 Calculation of the equivalent load for curved tendons

It should be noted that for eccentric cables, the eccentricity e_m is measured at mid-span from the cable to the line connecting the two ends as shown in Fig. 10.8.

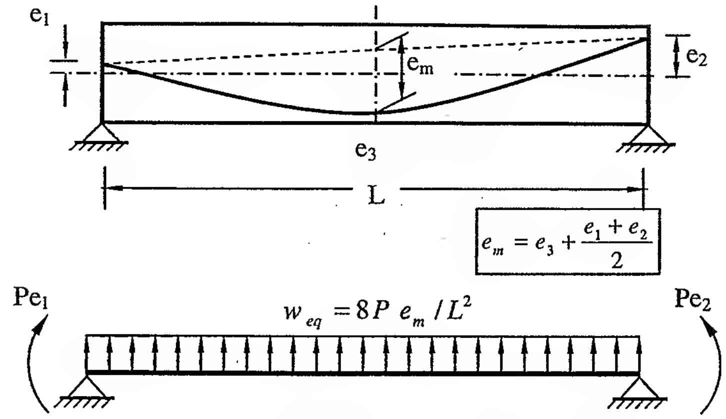


Fig. 10.8 Determination of e_m for cable with eccentricity e_1 and e_2

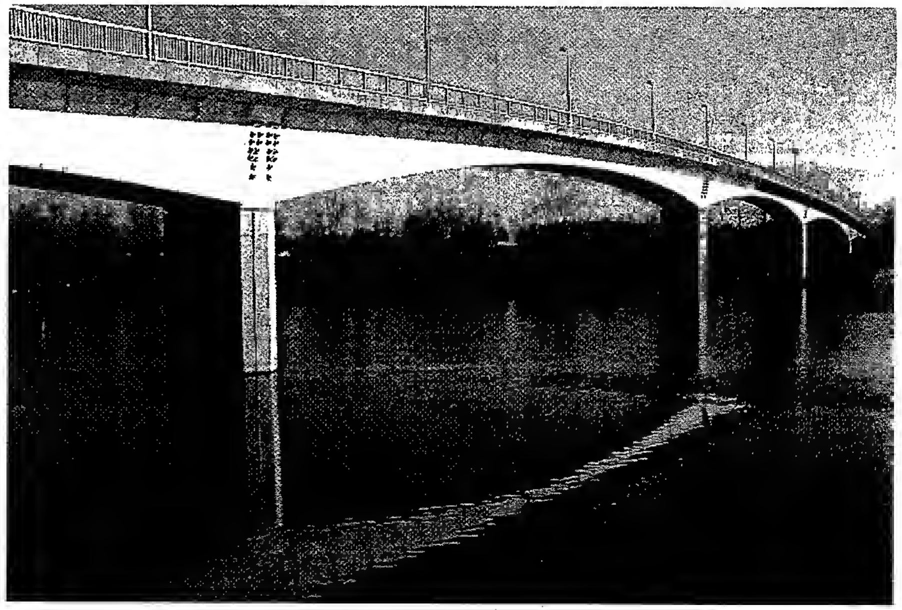
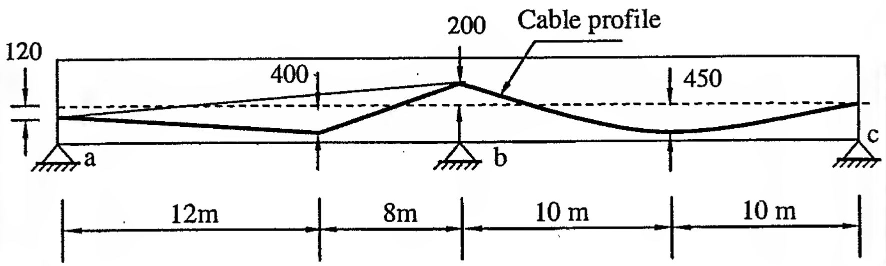
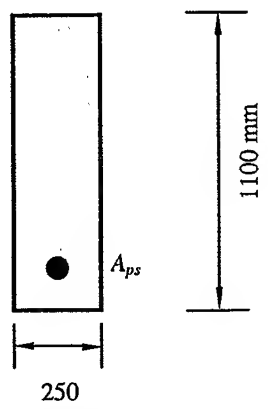


Photo 10.3 Continuous prestressed box-girder bridge

Example 10.3

A continuous prestressed beam having a rectangular cross-section (250 mm x1100 mm) is shown in the figure below. Also shown is the cable profile. The span a-b has linear cables while span B-C has a parabolic curve. Locate the line of pressure. Calculate the final stresses at support b due to the prestressing as well as a uniformly distributed load of 15 kN/m′ . (P_e=1200 kN)





Beam cross section

Solution

Step 1: Calculate the secondary moment at b

The equivalent concentrated load acting on the beam equals:

$$V_p = P\times(\theta_1+\theta_2) = 1200\times\left(\frac{0.4-0.12}{12}+\frac{0.4+0.2}{8}\right) = 118\ kN\uparrow$$

or $e_{m1} = (400-120)+(120+200)\times 12/20 = 472\ mm$

Using Eq. 10.13: $$V_p\,\frac{x_1(L-x_1)}{L} = P\ e_{m1}$$

$$V_p\,\frac{8\times12}{20} = 1200\times\frac{472}{1000} \qquad V_p = 118\ kN\uparrow$$

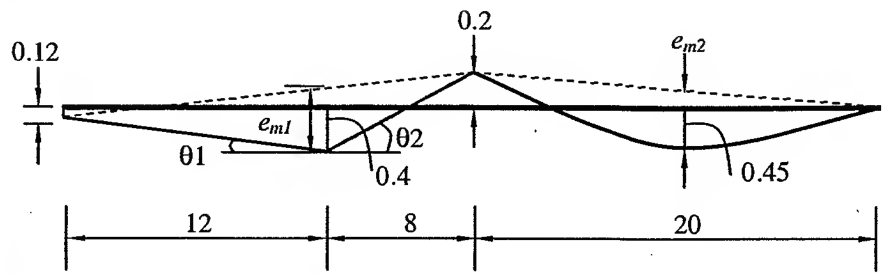
The equivalent uniform load on span b-c equals $P\cdot e_{m2} = \dfrac{w_{eq}\times L^2}{8}$

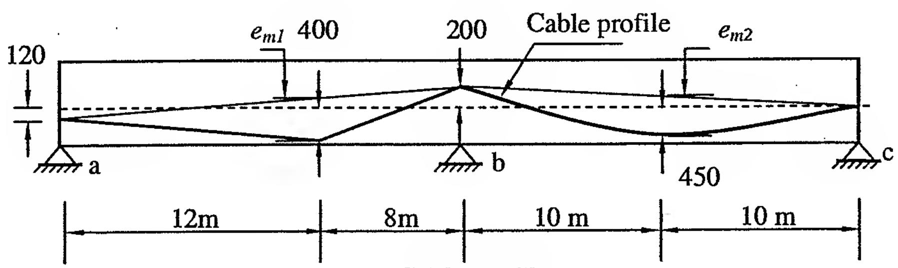
The eccentricity at mid-span $e_{m2} = 0.45+\dfrac{0.2}{2} = 0.55\ m$

$$w_{eq} = \frac{8\cdot P\cdot e_{m2}}{L^2} = \frac{8\cdot1200\cdot0.55}{20^2} = 13.2\ kN/m'\uparrow$$

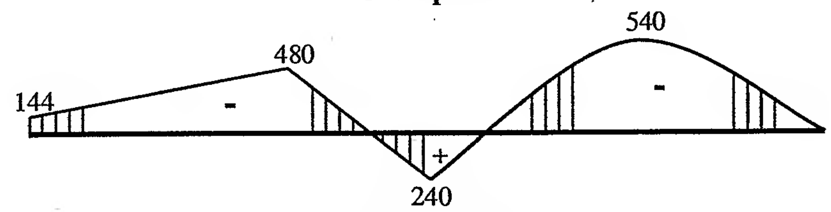
The concentrated moment at point a equals

$$M_a = P.e = 1200\times-0.12 = -144\ \ kN.m$$

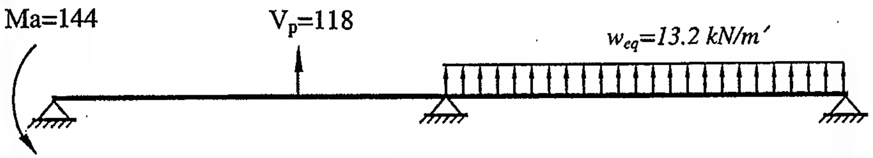




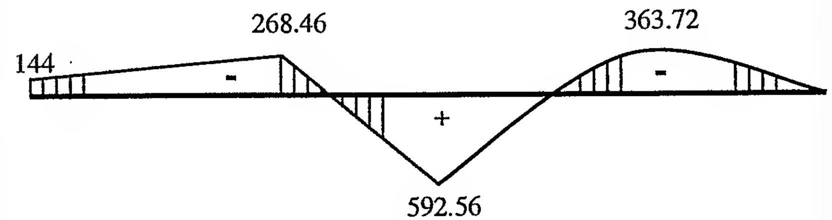
a- Cable profile



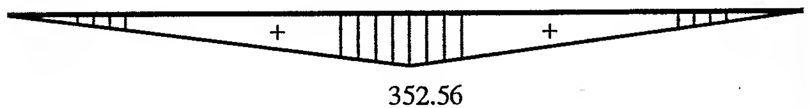
b- Primary moment



c- Equivalent loading



d- Final moment



e-Secondary moment

Primary, secondary and final moments

The beam is once statically indeterminate. We shall use the three-moment equation to calculate the moment at the middle support.

$$M_a L_1 + 2M_b(L_1 + L_2) + M_c L_2 = -6(R_{ab} + R_{bc})$$

In the previous equation, M_a=-144 (negative bending) and M_c=0

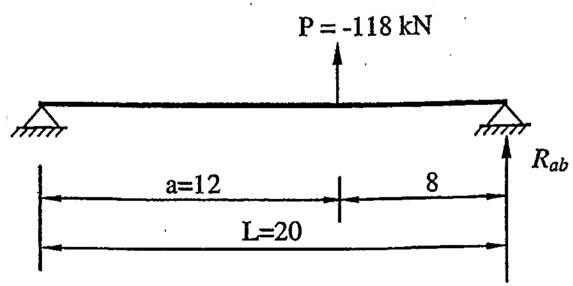
The elastic reaction R_{bc} due to uniform load is given by:

$$R_{bc} = \frac{w_{eq} \times L^3}{24} = \frac{-13.2 \times 20^3}{24} = -4400\ kN.m^2$$

The elastic reaction R_{ab} due to concentrated load not in the middle is given by

$$R_{ab} = \frac{P \times a \times b\ (L + a)}{6\ L} = \frac{-118 \times 8 \times 12 \times (20 + 12)}{6 \times 20} = -3020.8\ kN.m^2$$

The reader should be aware that the negative sign is due to upward force direction.



$$-144 \times 20 + 2M_b(20 + 20) + 0 = -6\ (-4400 - 3020.8)$$

M_b = +592.56 kN.m (positive bending)

Step 2: Calculate the C-line

The secondary moment at the support (b) equals:

$M_{secondary} = M_{final}$ -$M_{primary}$=592.56-240 =352.56 kN.m

Thus, the eccentricity of C-line at the support (b) equals:

$$e_b = \frac{M_b}{P} = \frac{592.56}{1200} = 0.494\ m \uparrow$$

The negative bending at the cable broken point of beam *a-b* equals:

$$M_{ab,final} = M_{ab,primary} + M_{ab,secondary} = -480 + 352.56 \times \frac{12}{20} = -268.46\ kN.m$$

Thus, the eccentricity of C-line at the cable broken point of beam a-b equals:

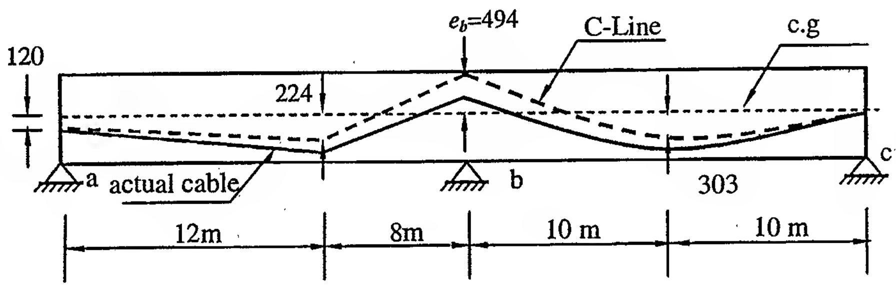
$$e_{ab} = \frac{M_{ab}}{P} = \frac{-268.64}{1200} = 0.224\ m \downarrow$$

The bending moment at mid-span of span b-c equals:

$$M_{bc} = \frac{592.56}{2} - \frac{13.2 \times 20^2}{8} = -363.72\ kN.m$$

Thus, the eccentricity of C-line at mid-span of span b-c equals:

$$e_{bc} = \frac{M_{bc}}{P} = \frac{-363.72}{1200} = 0.303\,m \downarrow$$



Effective cable profile or C-line

Step 3: Concrete stress due to prestressing at support (b)

The effective eccentricity e* equals:

$$e^* = e_b = 0.494\ m$$

$$A = 250 \times 1100 = 275000\ mm^2$$

$$Z_{bot} = Z_{top} = \frac{b \times t^2}{6} = \frac{250 \times 1100^2}{6} = 50.416 \times 10^6\ mm^3$$

The beam self weight is given by:

$$w_{ow} = 25 \times 0.25 \times 1.1 = 6.875\ kN/m'$$

$$w_{total} = w_{ow} + (w_{LL} + w_{DL}) = 6.875 + 15 = 21.875\ kN/m'$$

The negative moment at support equals:

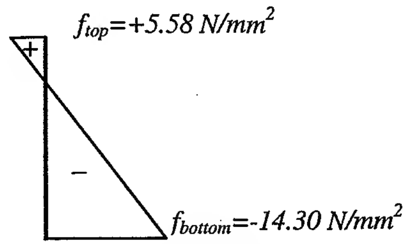
$$M = \frac{w_{total} \times L^2}{8} = \frac{21.875 \times 20^2}{8} = 1093.8\ kN.m$$

$$f_{top} = -\frac{P}{A} - \frac{P \times e^*}{Z_{top}} + \frac{M}{Z_{top}}$$

$$f_{top} = -\frac{1200 \times 1000}{275000} - \frac{1200 \times 1000 \times 494}{50.416 \times 10^6} + \frac{1093.8 \times 10^6}{50.41 \times 10^6} = +5.58\ N/mm^2$$

$$f_{bottom} = -\frac{P}{A} + \frac{P \times e^*}{Z_{bot}} - \frac{M}{Z_{top}}$$

$$f_{bottom} = -\frac{1200 \times 1000}{275000} + \frac{1200 \times 1000 \times 494}{50.416 \times 10^6} - \frac{1093.8 \times 10^6}{50.41 \times 10^6} = -14.30\ N/mm^2$$



10.4 Linear Transformation and Concordant Profiles

If the original cable profile was selected as the effective cable profile, the secondary moment will be equal to zero and the final moment will be equal to the primary moment. This is called a *concordant profile*.

The concordant profile induces no reactions due to the prestressing at the intermediate supports *(secondary moment = 0)*. The choice of a concordant profile or a non-concordant profile is governed by the concrete cover. In addition, any tendon profile can be linearly transformed without affecting the C-line position.

When the position of a cable is moved over the interior supports without changing the curvature and the shape of the cable within each individual span, the line is said to be linearly transformed. It is possible to linearly transform any pressure line by rising or lowering the eccentricity at the interior support without altering the exterior eccentricity. The resulting effective cable profile is the same in both cases. However, the amount of the secondary moment and the induced reactions at the middle support is different for each profile, but the sum of the primary and the secondary moment is the same. For example the beam shown in Fig. 10.9 has two tendon profiles, each produce different primary moment. However, the equivalent loads and hence the final moments in both cases are the same.

For the cable profile 1

$$e_{m1} = (400-120)+(120+200)\times 12/20 = 472\ mm$$

$$e_{m2} = 0.45 + \frac{0.2}{2} = 0.55\ m$$

For the cable profile 2

$$e'_{m1} = (280-120)+(120+400)\times 12/20 = 472\ mm$$

$$e'_{m2} = 0.35 + \frac{0.4}{2} = 0.55\ m$$

For both profiles

$$V_p \frac{8\times 12}{20} = 1200\times\frac{472}{1000} \qquad V_p = 118\ kN\uparrow$$

$$w_{eq} = \frac{8\cdot P\cdot e_{m2}}{L^2} = \frac{8\times 1200\times 0.55}{20^2} = 13.2\ kN/m'\uparrow$$

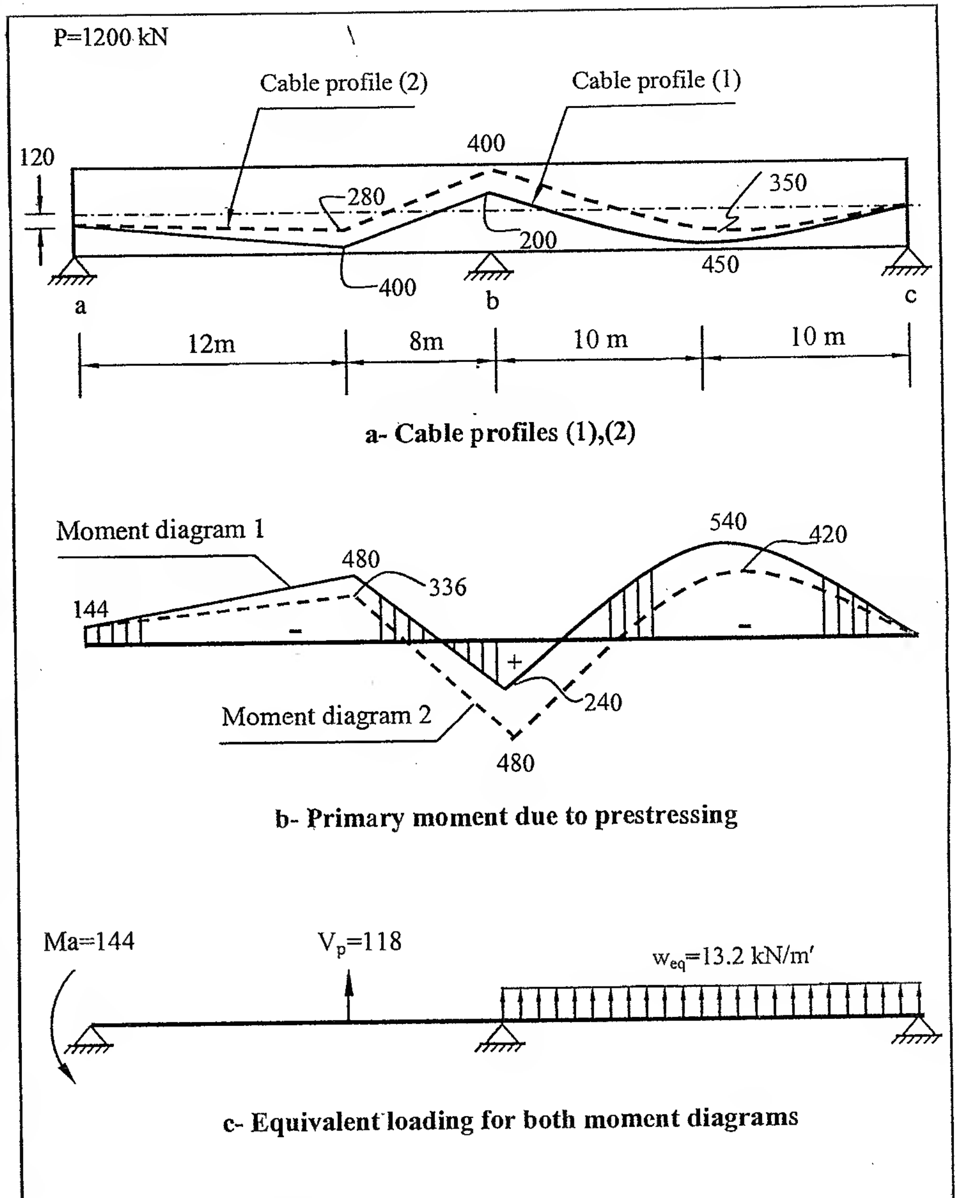


Fig. 10.9 Linear transformation of cables

APPENDIX A

Design Charts for Sections Subjected to Flexure

Area of Steel Bars in cm ^{2} (used in Egypt)

Φ	Weight	Cross sectional area (cm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	0.28	0.57	0.85	1.13	1.41	1.70	1.98	2.26	2.54	2.83	3.11	3.39
8	0.395	0.50	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	5.03	5.53	6.03
10	0.617	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.42
12	0.888	1.13	2.26	3.39	4.52	5.65	6.79	7.92	9.05	10.18	11.31	12.44	13.57
14	1.208	1.54	3.08	4.62	6.16	7.70	9.24	10.78	12.32	13.85	15.39	16.93	18.47
16	1.578	2.01	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11	22.12	24.13
18	1.998	2.54	5.09	7.63	10.18	12.72	15.27	17.81	20.36	22.90	25.45	27.99	30.54
20	2.466	3.14	6.28	9.42	12.57	15.71	18.85	21.99	25.13	28.27	31.42	34.56	37.70
22	2.984	3.80	7.60	11.40	15.21	19.01	22.81	26.61	30.41	34.21	38.01	41.81	45.62
25	3.853	4.91	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.18	49.09	54.00	58.90
28	4.834	6.16	12.32	18.47	24.63	30.79	36.95	43.10	49.26	55.42	61.58	67.73	73.89
32	6.313	8.04	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	80.42	88.47	96.51
38	8.903	11.34	22.68	34.02	45.36	56.71	68.05	79.39	90.73	102.1	113.4	124.8	136.1

Area of Other Steel Bars in cm ^{2}

Φ	Weight	Cross sectional area (cm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	0.28	0.57	0.85	1.13	1.41	1.70	1.98	2.26	2.54	2.83	3.11	3.39
8	0.395	0.50	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	5.03	5.53	6.03
10	0.617	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.42
13	1.042	1.33	2.65	3.98	5.31	6.64	7.96	9.29	10.62	11.95	13.27	14.60	15.93
16	1.578	2.01	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11	22.12	24.13
19	2.226	2.84	5.67	8.51	11.34	14.18	17.01	19.85	22.68	25.52	28.35	31.19	34.02
22	2.984	3.80	7.60	11.40	15.21	19.01	22.81	26.61	30.41	34.21	38.01	41.81	45.62
25	3.853	4.91	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.18	49.09	54.00	58.90
28	4.834	6.16	12.32	18.47	24.63	30.79	36.95	43.10	49.26	55.42	61.58	67.73	73.89
32	6.313	8.04	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	80.42	88.47	96.5
38	8.903	11.34	22.68	34.02	45.36	56.71	68.05	79.39	90.73	102.1	113.4	124.8	136.1

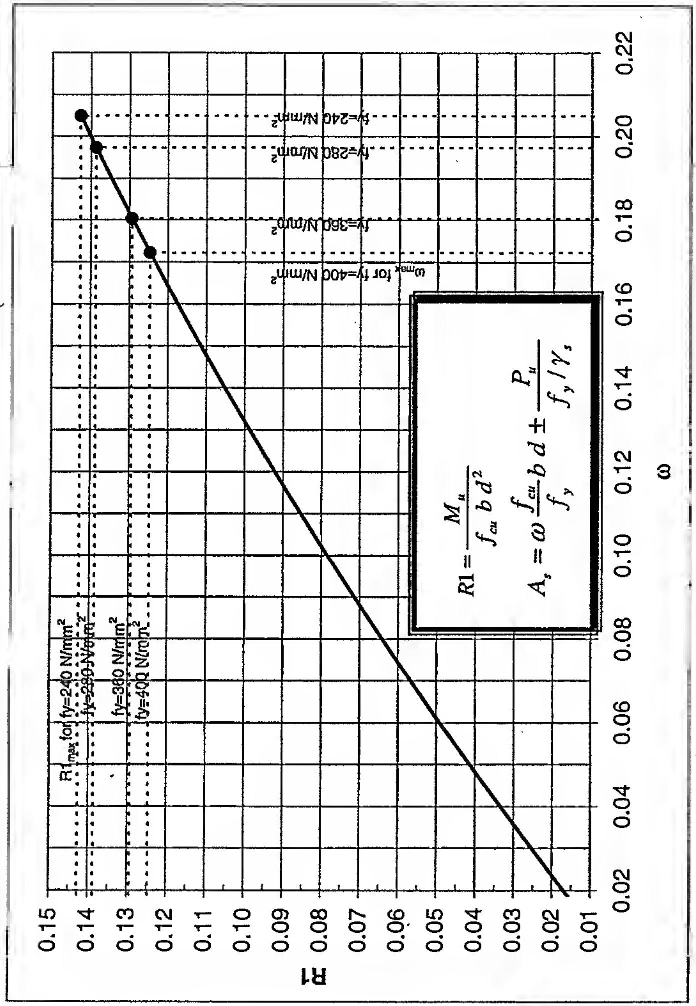
Area of Steel Bars in mm ^{2} (used in Egypt)

Φ	Weight	Cross sectional area (mm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	28.3	56.5	84.8	113	141	170	198	226	254	283	311	339
8	0.395	50.3	101	151	201	251	302	352	402	452	503	553	603
10	0.617	78.5	157	236	314	393	471	550	628	707	785	864	942
12	0.888	113	226	339	452	565	679	792	905	1018	1131	1244	1357
14	1.208	154	308	462	616	770	924	1078	1232	1385	1539	1693	1847
16	1.578	201	402	603	804	1005	1206	1407	1608	1810	2011	2212	2413
18	1.998	254	509	763	1018	1272	1527	1781	2036	2290	2545	2799	3054
20	2.466	314	628	942	1257	1571	1885	2199	2513	2827	3142	3456	3770
22	2.984	380	760	1140	1521	1901	2281	2661	3041	3421	3801	4181	4562
25	3.853	491	982	1473	1963	2454	2945	3436	3927	4418	4909	5400	5890
28	4.834	616	1232	1847	2463	3079	3695	4310	4926	5542	6158	6773	7389
32	6.313	804	1608	2413	3217	4021	4825	5630	6434	7238	8042	8847	9651
38	8.903	1134	2268	3402	4536	5671	6805	7939	9073	10207	11341	12475	13609

Area of Other Steel Bars in mm ^{2}

Φ	Weight	Cross sectional area (mm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	28.3	56.5	84.8	113.1	141.4	170	198	226	254	283	311	339
8	0.395	50.3	100.5	151	201	251	302	352	402	452	503	553	603
10	0.617	79	157	236	314	393	471	550	628	707	785	864	942
13	1.042	133	265	398	531	664	796	929	1062	1195	1327	1460	1593
16	1.578	201	402	603	804	1005	1206	1407	1608	1810	2011	2212	2413
19	2.226	284	567	851	1134	1418	1701	1985	2268	2552	2835	3119	3402
22	2.984	380	760	1140	1521	1901	2281	2661	3041	3421	3801	4181	4562
25	3.853	491	982	1473	1963	2454	2945	3436	3927	4418	4909	5400	5890
28	4.834	616	1232	1847	2463	3079	3695	4310	4926	5542	6158	6773	7389
32	6.313	804	1608	2413	3217	4021	4825	5630	6434	7238	8042	8847	9651
38	8.903	1134	2268	3402	4536	5671	6805	7939	9073	10207	11341	12475	13609

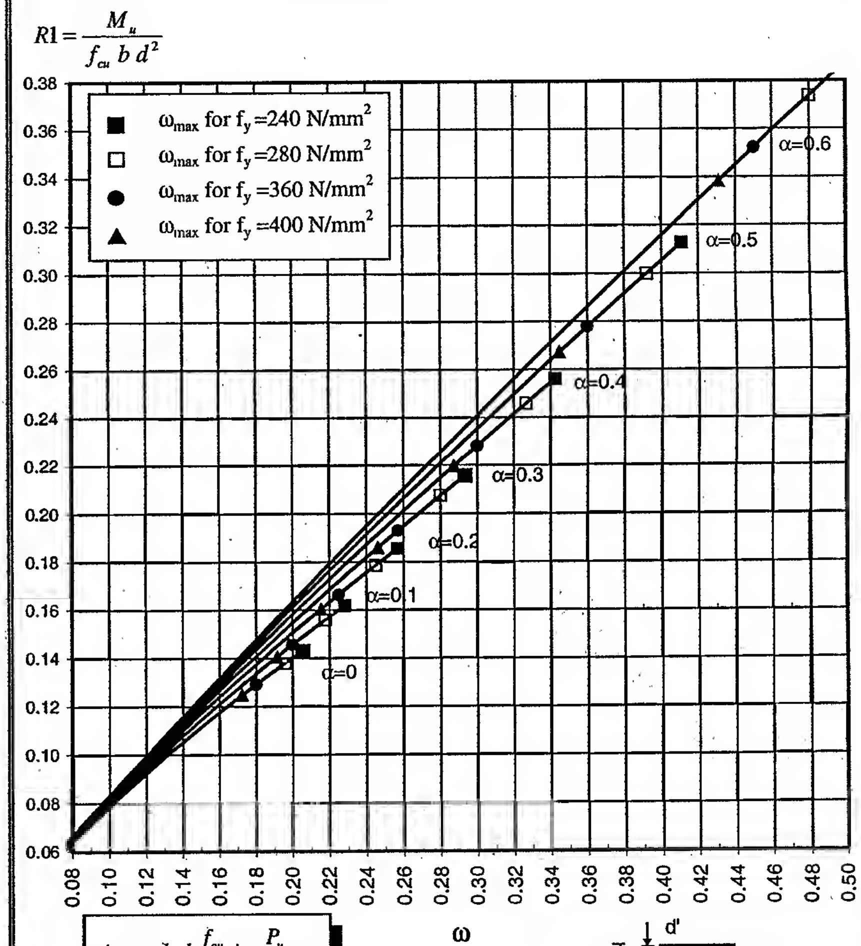
DESIGN CHART FOR SECTIONS SUBJECTED TO SIMPLE BENDING (Table 4-1)



	R1	ω
	0.015	0.018
	0.020	0.024
	0.025	0.030
	0.030	0.036
	0.035	0.042
	0.040	0.048
	0.045	0.055
	0.050	0.061
	0.055	0.068
	0.060	0.074
	0.065	0.081
	0.070	0.088
	0.075	0.095
	0.080	0.102
	0.085	0.109
	0.090	0.117
	0.095	0.124
	0.100	0.132
	0.105	0.140
	0.110	0.148
	0.115	0.156
$R1_{max}$	0.120	0.164
fy=400	0.125	0.173
fy=360	0.129	0.180
fy=280	0.139	0.198
fy=240	0.143	0.206

DESIGN CHART FOR DOUBLY REINFORCED SECTIONS SUBEJCED TO SIMPLE BENDING.

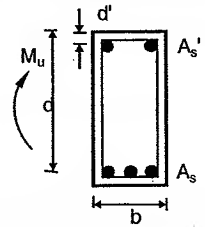
All types of steel (table 4-1). d'/d=0.05



$$A_s = \omega\, b\, d\, \frac{f_{cu}}{f_y} \pm \frac{P_u}{f_y/\gamma_s}$$

$$A'_s = \alpha\, \omega\, b\, d\, \frac{f_{cu}}{f_y}$$

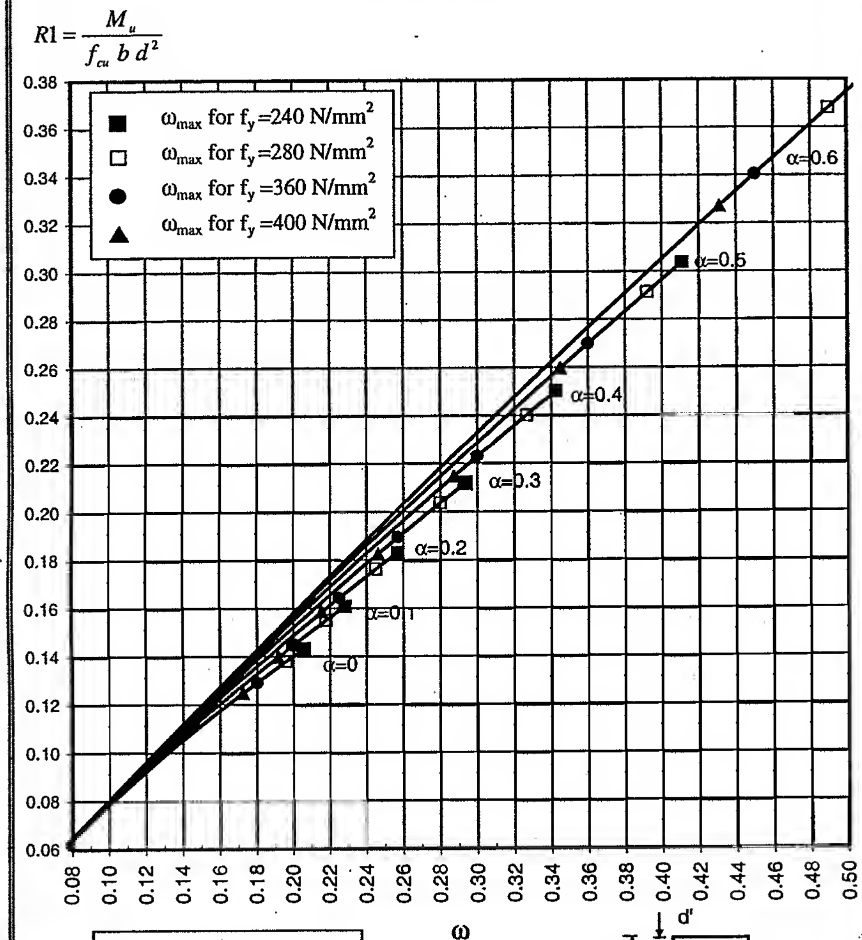
In case P_u=0, $A'_s = \alpha \times A_s$



DESIGN CHART FOR DOUBLY REINFORCED SECTIONS SUBEJCED TO SIMPLE BENDING.

All types of steel (table 4-1). d'/d=0.10

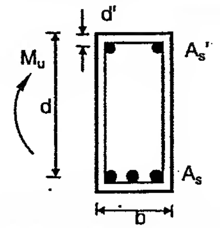
$$R1 = \frac{M_u}{f_{cu}\, b\, d^2}$$



$$A_s = \omega\, b\, d\, \frac{f_{cu}}{f_y} \pm \frac{P_u}{f_y / \gamma_s}$$

$$A'_s = \alpha\, \omega\, b\, d\, \frac{f_{cu}}{f_y}$$

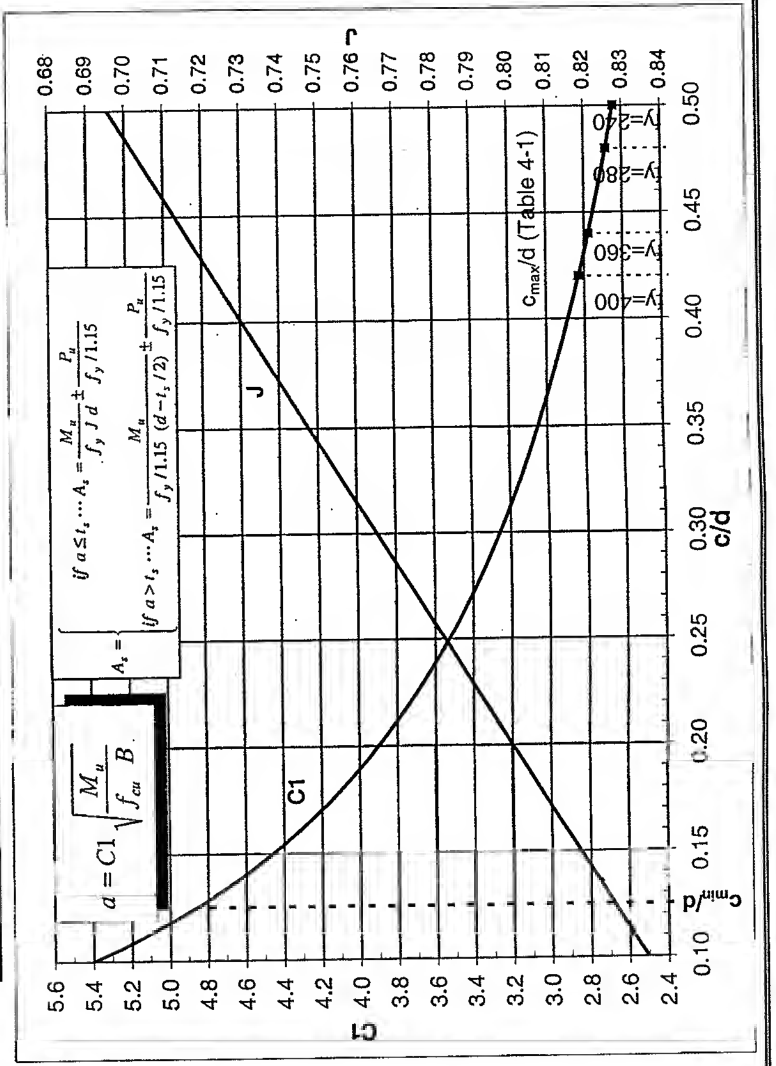
In case P_u=0, $A'_s = \alpha \times A_s$



DESIGN CHART FOR SECTIONS SUBJECTED TO SIMPLE BENDING (R and T-sections) FOR ALL GRADES OF STEEL AND CONCRETE

$$d = C1 \sqrt{\frac{M_u}{f_{cu}\, B}}$$

$$A_s = \begin{cases} \text{if } a \le t_s \cdots A_s = \dfrac{M_u}{f_y\, J\, d} \pm \dfrac{P_u}{f_y/1.15} \\ \text{if } a > t_s \cdots A_s = \dfrac{M_u}{f_y/1.15\,(d - t_s/2)} \pm \dfrac{P_u}{f_y/1.15} \end{cases}$$

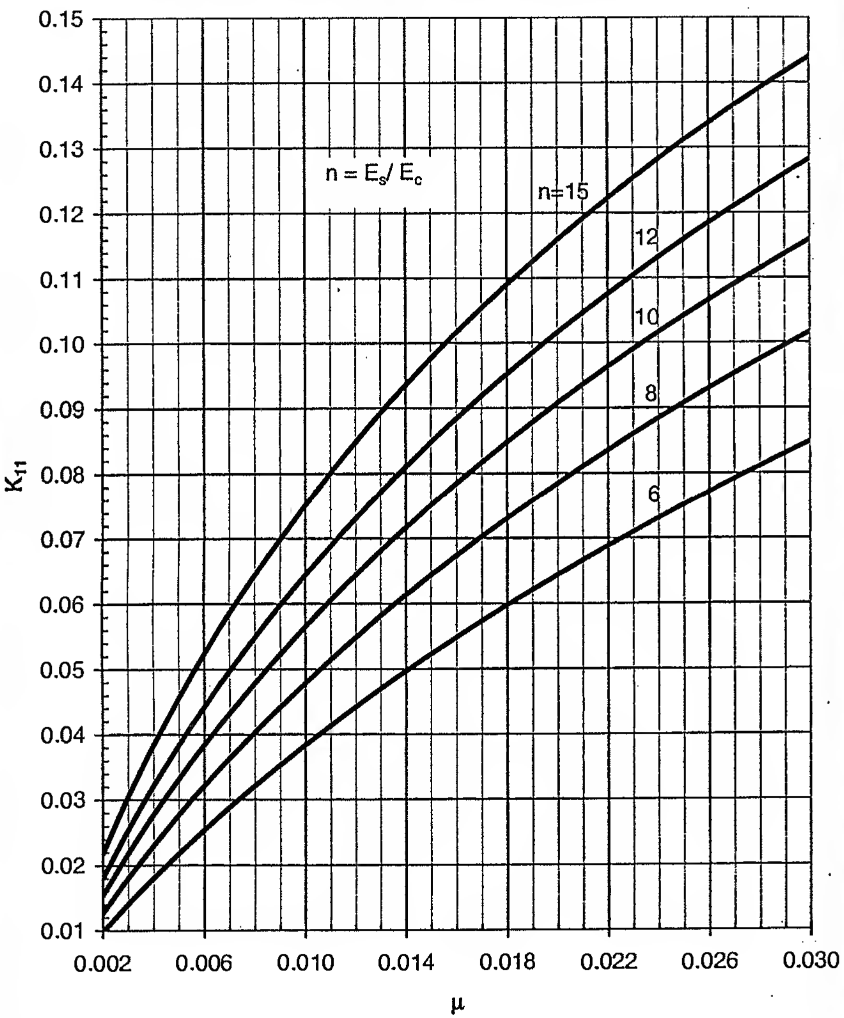


C1	J	c/d	c_{max}/d
2.65	0.696	0.500	fy=240
2.69	0.703	0.480	fy=280
2.78	0.717	0.440	fy=360
2.83	0.723	0.420	fy=400
2.90	0.732	0.395	
2.95	0.738	0.379	
3.00	0.743	0.364	
3.05	0.748	0.350	
3.10	0.753	0.337	
3.15	0.757	0.324	
3.20	0.761	0.312	
3.25	0.765	0.301	
3.30	0.768	0.291	
3.35	0.772	0.281	
3.40	0.775	0.272	
3.45	0.778	0.263	
3.50	0.781	0.254	
3.55	0.784	0.246	
3.60	0.787	0.239	
3.65	0.789	0.231	
3.70	0.791	0.225	
3.75	0.794	0.218	
3.80	0.796	0.212	
3.85	0.798	0.206	
3.90	0.800	0.200	
3.95	0.802	0.194	
4.00	0.804	0.189	
4.05	0.806	0.184	
4.10	0.807	0.179	
4.15	0.809	0.175	
4.20	0.810	0.170	
4.25	0.812	0.166	
4.30	0.813	0.162	
4.35	0.815	0.158	
4.40	0.816	0.154	
4.45	0.817	0.150	
4.50	0.818	0.147	
4.55	0.820	0.143	
4.60	0.821	0.140	
4.65	0.822	0.137	
4.70	0.823	0.134	
4.75	0.824	0.131	
4.80	0.825	0.128	
4.85	0.826	0.125	

APPENDIX B

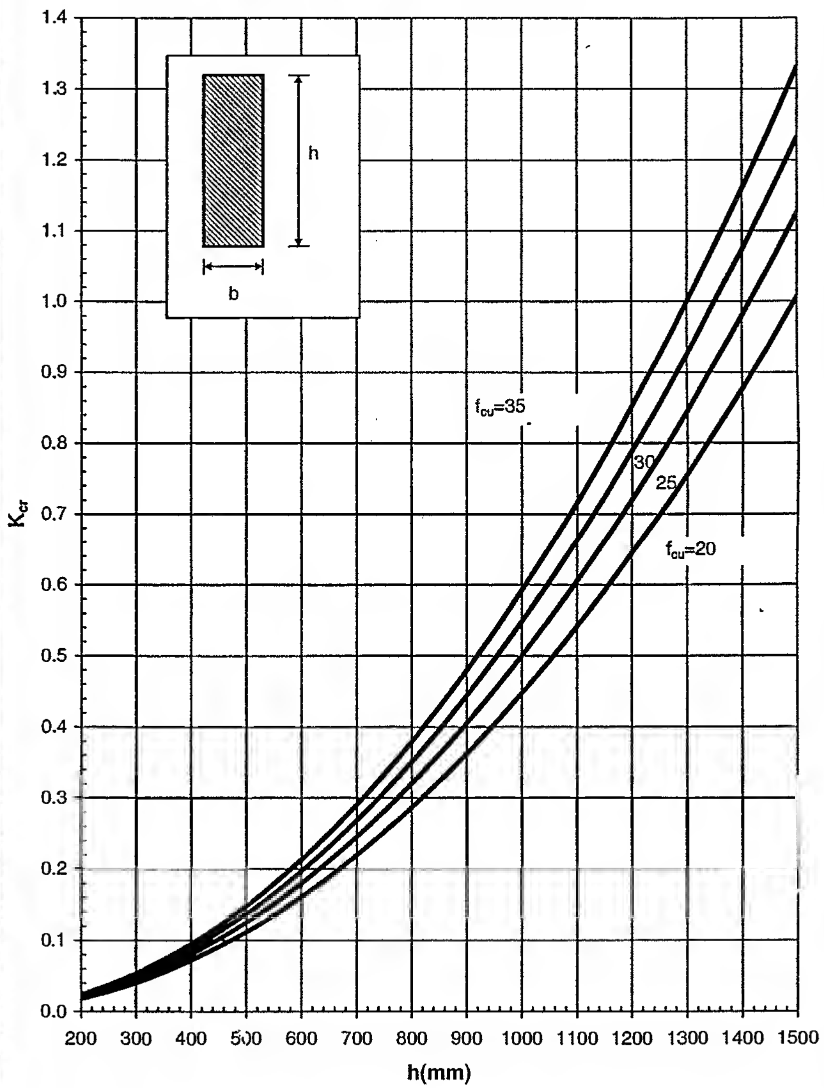
Design Charts for Calculating I_{cr} and ω_k

Cracked Section Moment of Inertia I_{cr} for Rectangular Sections with Tension Reinforcement only



$I_{cr} = K_{11}\ b\ d^3$

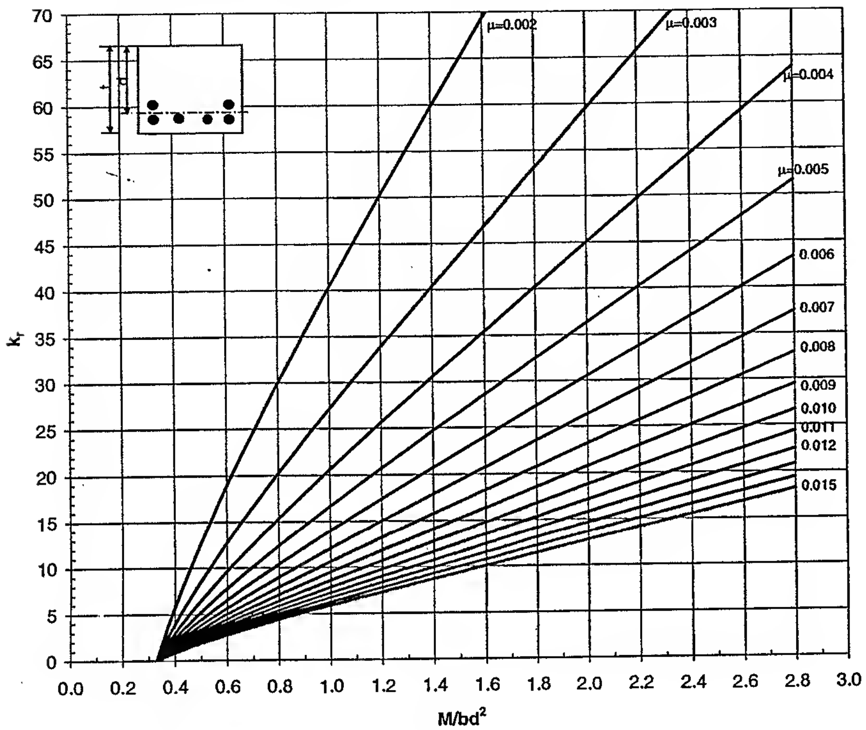
Cracking Moment M_{cr} for rectangular



$M_{cr} = b(mm).\ K_{cr}$ (kN.m)

w_k factor for sections subjected to bending only

f_{cu}=25 N/mm^2, t/d=1.15, smooth bars, n=10



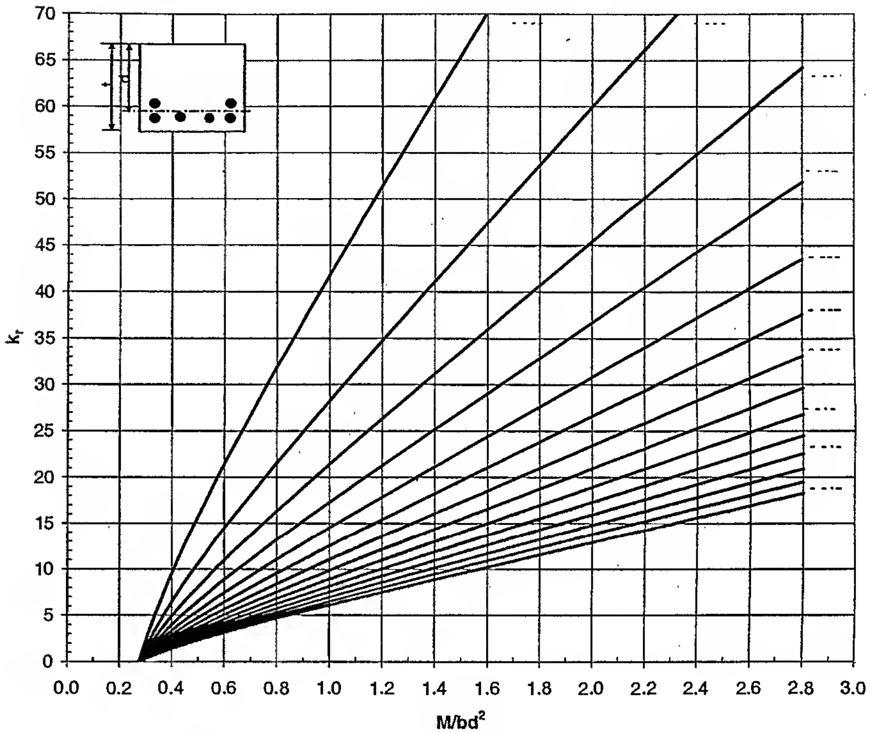
Values of S_m

μ	Bar Diameter									
	10	12	14	16	18	20	22	25	28	32
0.001	425	500	575	650	725	800	875	987	1100	1250
0.002	238	275	313	350	388	425	463	519	575	650
0.003	175	200	225	250	275	300	325	363	400	450
0.004	144	163	181	200	219	238	256	284	313	350
0.005	125	140	155	170	185	200	215	238	260	290
0.006	113	125	138	150	163	175	188	206	225	250
0.007	104	114	125	136	146	157	168	184	200	221
0.008	97	106	116	125	134	144	153	167	181	200
0.009	92	100	108	117	125	133	142	154	167	183
0.010	88	95	103	110	118	125	133	144	155	170
0.011	84	91	98	105	111	118	125	135	145	159
0.012	81	88	94	100	106	113	119	128	138	150
0.013	79	85	90	96	102	108	113	122	131	142
0.014	77	82	88	93	98	104	109	117	125	136
0.015	75	80	85	90	95	100	105	113	120	130

$$w_k = S_m \times k_r \times 10^{-4}$$

w_k factor for sections subjected to bending only

f_{cu}=25 N/mm^2, t/d=1.05, smooth bars, n=10



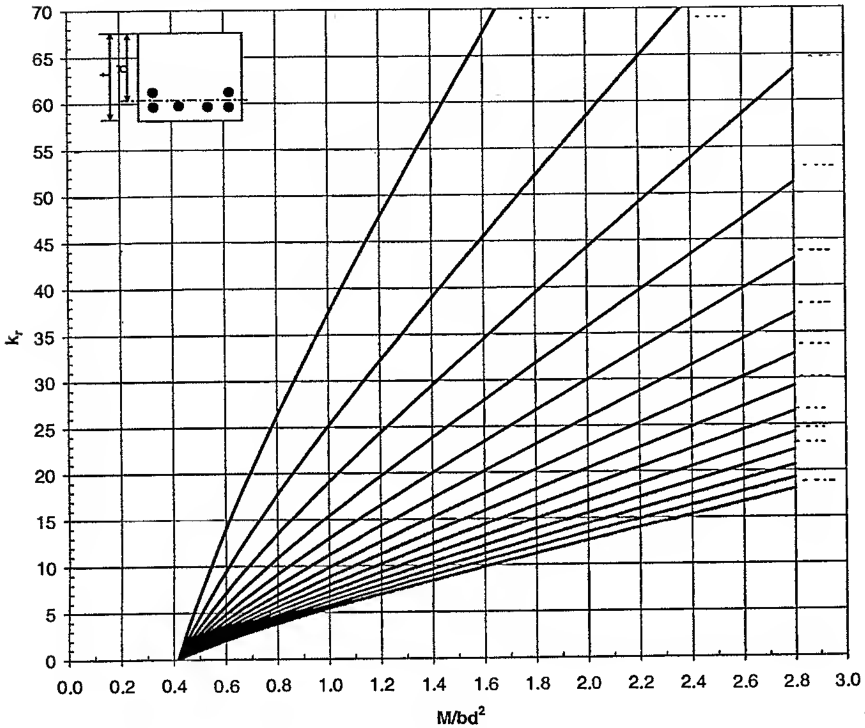
Values of S_m

μ	Bar Diameter									
	10	12	14	16	18	20	22	25	28	32
0.001	175	200	225	250	275	300	325	363	400	450
0.002	113	125	138	150	163	175	188	206	225	250
0.003	92	100	108	117	125	133	142	154	167	183
0.004	81	88	94	100	106	113	119	128	138	150
0.005	75	80	85	90	95	100	105	113	120	130
0.006	71	75	79	83	88	92	96	102	108	117
0.007	68	71	75	79	82	86	89	95	100	107
0.008	66	69	72	75	78	81	84	89	94	100
0.009	64	67	69	72	75	78	81	85	89	94
0.010	63	65	68	70	73	75	78	81	85	90
0.011	61	64	66	68	70	73	75	78	82	86
0.012	60	63	65	67	69	71	73	76	79	83
0.013	60	62	63	65	67	69	71	74	77	81
0.014	59	61	63	64	66	68	70	72	75	79
0.015	58	60	62	63	65	67	68	71	73	77

$$w_k = S_m \times k_r \times 10^{-4}$$

w_k factor for sections subjected to bending only

f_{cu}=25 N/mm^2, t/d=1.15, ribbed bars, n=10



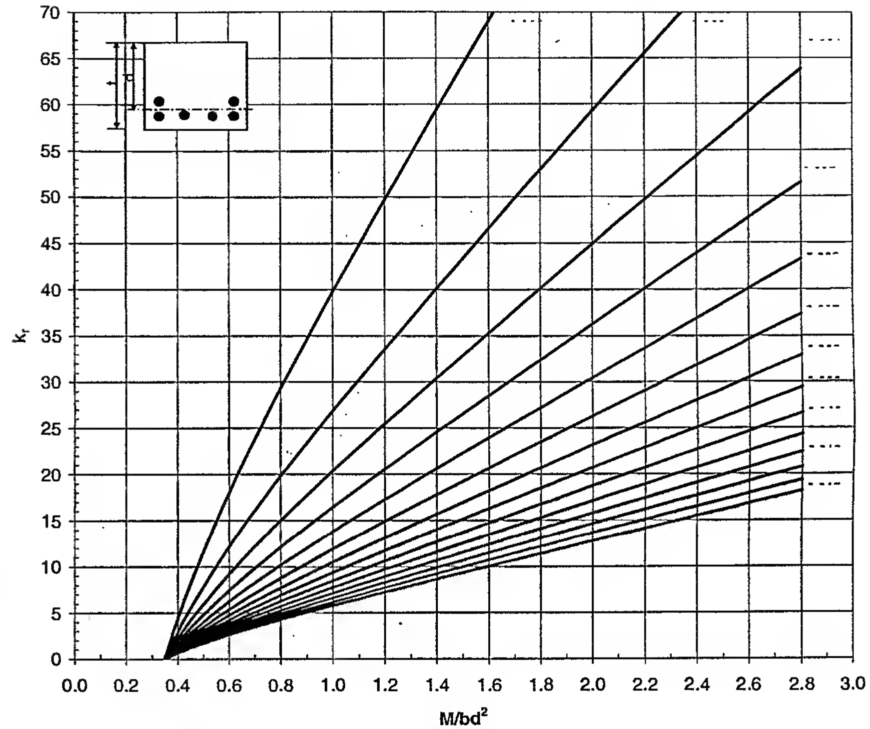
Values of S_m

μ	Bar Diameter									
	10	12	14	16	18	20	22	25	28	32
0.001	425	500	575	650	725	800	875	987	1100	1250
0.002	238	275	313	350	388	425	463	519	575	650
0.003	175	200	225	250	275	300	325	363	400	450
0.004	144	163	181	200	219	238	256	284	313	350
0.005	125	140	155	170	185	200	215	238	260	290
0.006	113	125	138	150	163	175	188	206	225	250
0.007	104	114	125	136	146	157	168	184	200	221
0.008	97	106	116	125	134	144	153	167	181	200
0.009	92	100	108	117	125	133	142	154	167	183
0.010	88	95	103	110	118	125	133	144	155	170
0.011	84	91	98	105	111	118	125	135	145	159
0.012	81	88	94	100	106	113	119	128	138	150
0.013	79	85	90	96	102	108	113	122	131	142
0.014	77	82	88	93	98	104	109	117	125	136
0.015	75	80	85	90	95	100	105	113	120	130

$$w_k = S_m \times k_r \times 10^{-4}$$

w_k factor for sections subjected to bending only

f_{cu}=25 N/mm^2, t/d=1.05, ribbed bars, n=10



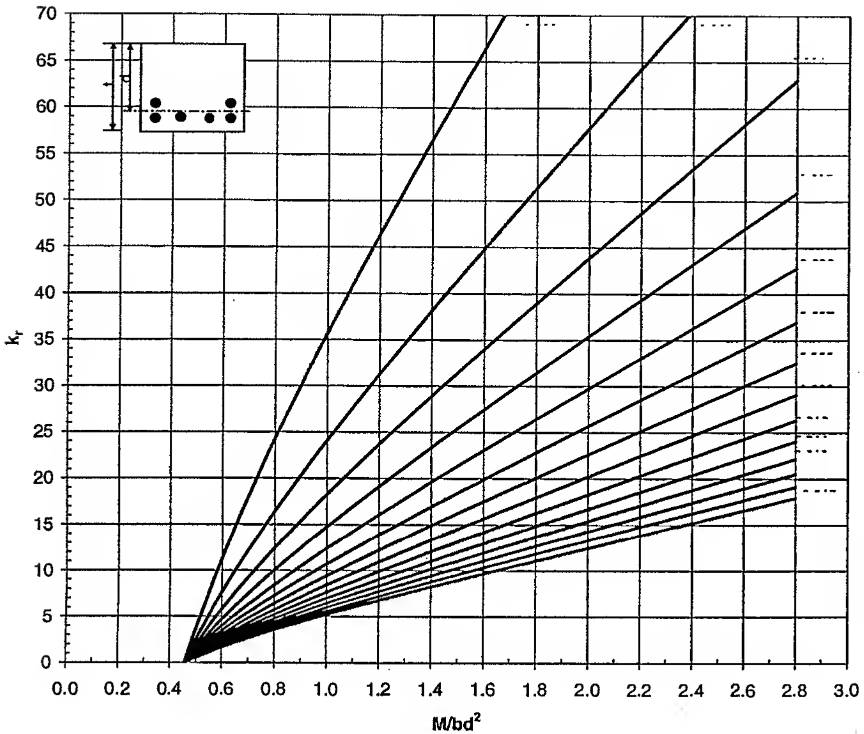
Values of S_m

μ	Bar Diameter									
	10	12	14	16	18	20	22	25	28	32
0.001	175	200	225	250	275	300	325	363	400	450
0.002	113	125	138	150	163	175	188	206	225	250
0.003	92	100	108	117	125	133	142	154	167	183
0.004	81	88	94	100	106	113	119	128	138	150
0.005	75	80	85	90	95	100	105	113	120	130
0.006	71	75	79	83	88	92	96	102	108	117
0.007	68	71	75	79	82	86	89	95	100	107
0.008	66	69	72	75	78	81	84	89	94	100
0.009	64	67	69	72	75	78	81	85	89	94
0.010	63	65	68	70	73	75	78	81	85	90
0.011	61	64	66	68	70	73	75	78	82	86
0.012	60	63	65	67	69	71	73	76	79	83
0.013	60	62	63	65	67	69	71	74	77	81
0.014	59	61	63	64	66	68	70	72	75	79
0.015	58	60	62	63	65	67	68	71	73	77

$$w_k = S_m \times k_r \times 10^{-4}$$

w_k factor for sections subjected to bending only

f_{cu}=30 N/mm², t/d=1.15, ribbed bars, n=10



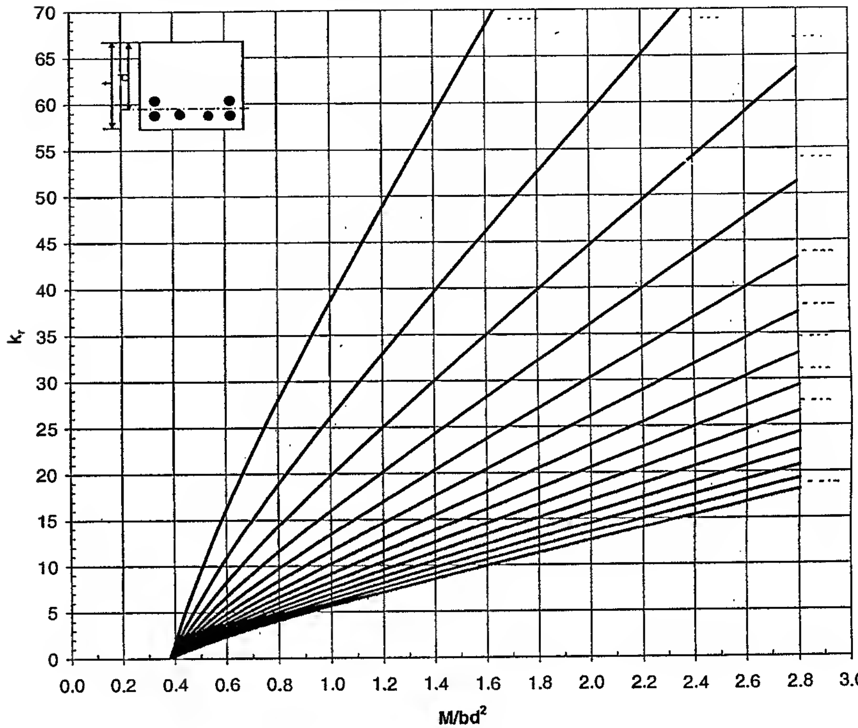
Values of S_m

μ	Bar Diameter									
	10	12	14	16	18	20	22	25	28	32
0.001	425	500	575	650	725	800	875	987	1100	1250
0.002	238	275	313	350	388	425	463	519	575	650
0.003	175	200	225	250	275	300	325	363	400	450
0.004	144	163	181	200	219	238	256	284	313	350
0.005	125	140	155	170	185	200	215	238	260	290
0.006	113	125	138	150	163	175	188	206	225	250
0.007	104	114	125	136	146	157	168	184	200	221
0.008	97	106	116	125	134	144	153	167	181	200
0.009	92	100	108	117	125	133	142	154	167	183
0.010	88	95	103	110	118	125	133	144	155	170
0.011	84	91	98	105	111	118	125	135	145	159
0.012	81	88	94	100	106	113	119	128	138	150
0.013	79	85	90	96	102	108	113	122	131	142
0.014	77	82	88	93	98	104	109	117	125	136
0.015	75	80	85	90	95	100	105	113	120	130

$$w_k = S_m \times k_r \times 10^{-4}$$

w_k factor for sections subjected to bending only

f_{cu}=30 N/mm², t/d=1.05, ribbed bars, n=10



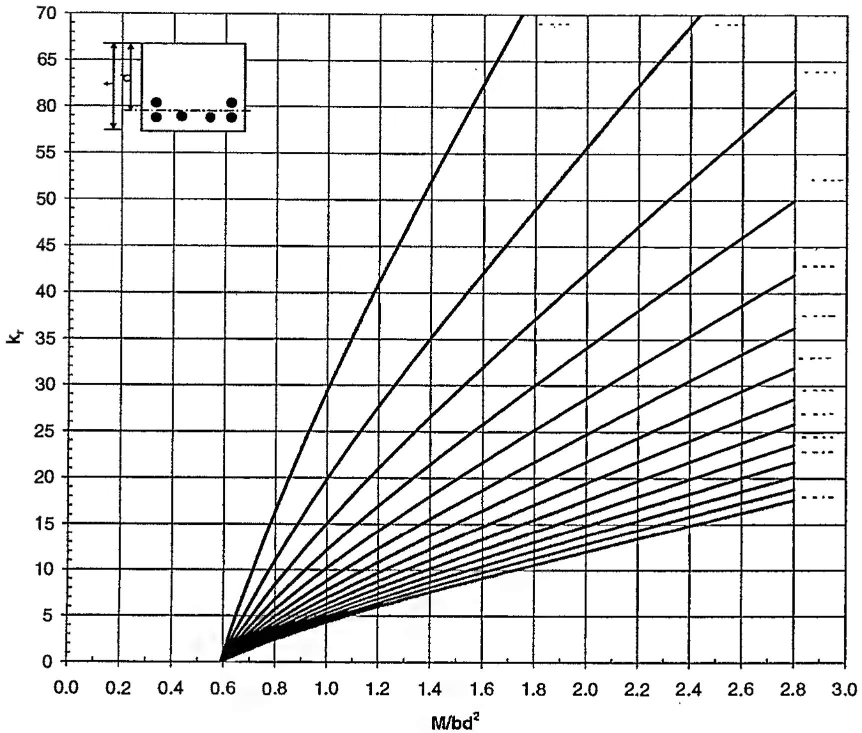
Values of S_m

μ	Bar Diameter									
	10	12	14	16	18	20	22	25	28	32
0.001	175	200	225	250	275	300	325	363	400	450
0.002	113	125	138	150	163	175	188	206	225	250
0.003	92	100	108	117	125	133	142	154	167	183
0.004	81	88	94	100	106	113	119	128	138	150
0.005	75	80	85	90	95	100	105	113	120	130
0.006	71	75	79	83	88	92	96	102	108	117
0.007	68	71	75	79	82	86	89	95	100	107
0.008	66	69	72	75	78	81	84	89	94	100
0.009	64	67	69	72	75	78	81	85	89	94
0.010	63	65	68	70	73	75	78	81	85	90
0.011	61	64	66	68	70	73	75	78	82	86
0.012	60	63	65	67	69	71	73	76	79	83
0.013	60	62	63	65	67	69	71	74	77	81
0.014	59	61	63	64	66	68	70	72	75	79
0.015	58	60	62	63	65	67	68	71	73	77

$$w_k = S_m \times k_r \times 10^{-4}$$

w_k factor for sections subjected to bending only

f_{cu}=50 N/mm^2, t/d=1.15, ribbed bars, n=10



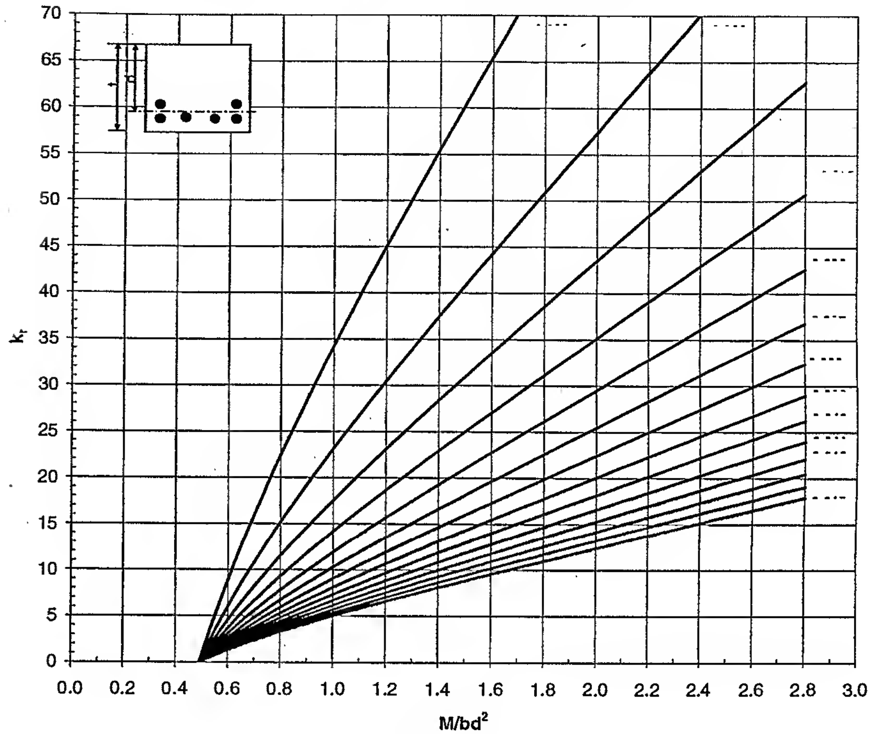
Values of S_m

μ	Bar Diameter 10	12	14	16	18	20	22	25	28	32
0.001	425	500	575	650	725	800	875	987	1100	1250
0.002	238	275	313	350	388	425	463	519	575	650
0.003	175	200	225	250	275	300	325	363	400	450
0.004	144	163	181	200	219	238	256	284	313	350
0.005	125	140	155	170	185	200	215	238	260	290
0.006	113	125	138	150	163	175	188	206	225	250
0.007	104	114	125	136	146	157	168	184	200	221
0.008	97	106	116	125	134	144	153	167	181	200
0.009	92	100	108	117	125	133	142	154	167	183
0.010	88	95	103	110	118	125	133	144	155	170
0.011	84	91	98	105	111	118	125	135	145	159
0.012	81	88	94	100	106	113	119	128	138	150
0.013	79	85	90	96	102	108	113	122	131	142
0.014	77	82	88	93	98	104	109	117	125	136
0.015	75	80	85	90	95	100	105	113	120	130

$$w_k = S_m \times k_r \times 10^{-4}$$

w_k factor for sections subjected to bending only

f_{cu}=50 N/mm^2, t/d=1.05, ribbed bars, n=10



Values of S_m

μ	Bar Diameter 10	12	14	16	18	20	22	25	28	32
0.001	175	200	225	250	275	300	325	363	400	450
0.002	113	125	138	150	163	175	188	206	225	250
0.003	92	100	108	117	125	133	142	154	167	183
0.004	81	88	94	100	106	113	119	128	138	150
0.005	75	80	85	90	95	100	105	113	120	130
0.006	71	75	79	83	88	92	96	102	108	117
0.007	68	71	75	79	82	86	89	95	100	107
0.008	66	69	72	75	78	81	84	89	94	100
0.009	64	67	69	72	75	78	81	85	89	94
0.010	63	65	68	70	73	75	78	81	85	90
0.011	61	64	66	68	70	73	75	78	82	86
0.012	60	63	65	67	69	71	73	76	79	83
0.013	60	62	63	65	67	69	71	74	77	81
0.014	59	61	63	64	66	68	70	72	75	79
0.015	58	60	62	63	65	67	68	71	73	77

$$w_k = S_m \times k_r \times 10^{-4}$$

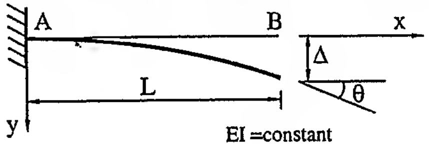
APPENDIX C

Slope and Deflection Equations

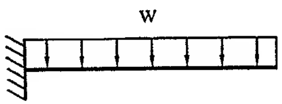
APPENDIX C

Deflections and Slopes of Beams

A-DEFLECTION AND SLOPES OF CANTILEVER BEAMS

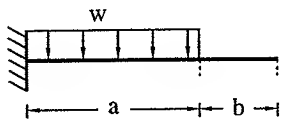


y= deflection at any point
Δ=deflection at certain point
θ= slope at certain point



$$y=\frac{w\,x^2}{24\,EI}(6L^2-4Lx+x^2)$$

$$\Delta_B=\frac{w\,L^4}{8\,EI} \qquad \theta_B=\frac{w\,L^3}{6\,EI}$$

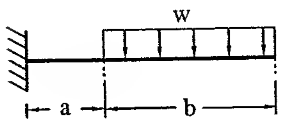


$$y=\frac{w\,x^2}{24\,EI}(6a^2-4ax+x^2) \qquad 0\le x\le a$$

$$y=\frac{w\,a^3}{24\,EI}(4x-a) \qquad a\le x\le L$$

$$\Delta_A=\frac{w\,a^4}{8\,EI} \qquad \theta_A=\frac{w\,a^3}{6\,EI}$$

$$\Delta_B=\frac{w\,a^3}{24\,EI}(4L-a) \qquad \theta_B=\frac{w\,a^3}{6\,EI}$$



$$y=\frac{w\,b\,x^2}{12\,EI}(3L+3\,a-2\,x) \qquad 0\le x\le a$$

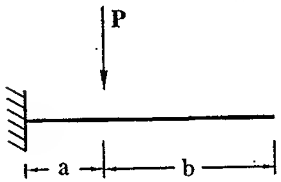
$$y=\frac{w}{24\,EI}(x^4-4Lx^3+6L^2x^2-4a^3x+a^4) \qquad a\le x\le L$$

$$\Delta_A=\frac{w\,a^2\,b}{12\,EI}(3L+a) \qquad \theta_A=\frac{w\,a\,b\,L}{2\,EI}$$

$$\Delta_B=\frac{w}{24\,EI}(3L^4-4a^3L+a^4) \qquad \theta_B=\frac{w}{6\,EI}(L^3-a^3)$$

$$y=\frac{P\,x^2}{6\,EI}(3L-x)$$

$$\Delta_B=\frac{P\,L^3}{3\,EI} \qquad \theta_B=\frac{P\,L^2}{2\,EI}$$

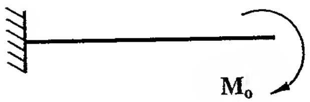


$$y=\frac{P\;x^2}{6\,EI}(3\,a-x) \qquad 0\le x\le a$$

$$y=\frac{P\,a^2}{6\,EI}(3\,x-a) \qquad a\le x\le L$$

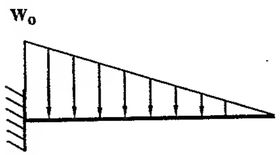
$$\Delta_A=\frac{P\,a^3}{3\,EI} \qquad \theta_A=\frac{P\,a^2}{2\,EI}$$

$$\Delta_B=\frac{P\,a^2}{6\,EI}(3L-a) \qquad \theta_B=\frac{P\,a^2}{2\,EI}$$



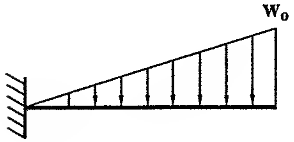
$$y=\frac{M_o\,x^2}{2\,EI}$$

$$\Delta_B=\frac{M_o\,L^2}{2\,EI} \qquad \theta_B=\frac{M_o\,L}{EI}$$



$$y=\frac{w_o\,x^2}{120\,L\,EI}(10L^3-10L^2x+5Lx^2-x^3)$$

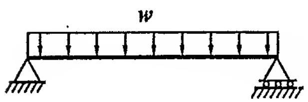
$$\Delta_B=\frac{w_o\,L^4}{30\,EI} \qquad \theta_B=\frac{w_o\,L^3}{24\,EI}$$



$$y = \frac{w_o x^2}{120\, L\, EI}(20L^3 - 10L^2 x + x^3)$$

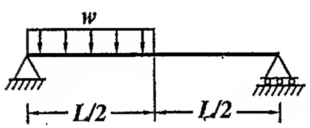
$$\Delta_B = \frac{11\, w_o L^4}{120\, EI} \qquad \theta_B = \frac{w_o L^3}{8\, EI}$$

B-DEFLECTION AND SLOPES OF SIMPLE BEAMS



$$y = \frac{w\, x}{24\, E\, I}(L^3 - 2Lx^2 + x^3)$$

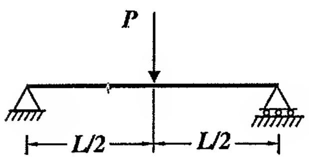
$$\Delta_c = \Delta_{max} = \frac{5\, w\, L^4}{384\, E\, I} \qquad \theta_A = \theta_B = \frac{w\, L^3}{24\, E\, I}$$



$$y = \frac{w\, x}{384\, E\, I}(9L^3 - 24Lx^2 + 16x^3) \qquad 0 \le x \le L/2$$

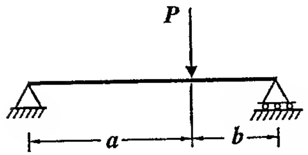
$$y = \frac{w\, x}{384\, E\, I}(8x^3 - 24Lx^2 + +17L^2 x - L^3) \qquad L/2 \le x \le L$$

$$\Delta_c = \frac{5\, w\, L^4}{768\, E\, I} \qquad \theta_A = \frac{3\, w\, L^3}{128\, E\, I} \qquad \theta_B = \frac{7\, w\, L^3}{384\, E\, I}$$



$$y = \frac{P\, x}{48\, E\, I}(3\, L^2 - 4x^2)$$

$$\Delta_c = \Delta_{max} = \frac{P\, L^3}{48\, E\, I} \qquad \theta_A = \theta_B = \frac{P\, L^2}{16\, E\, I}$$

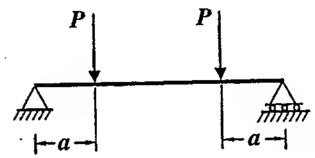


$$y = \frac{P\, b\, x}{6L\, E\, I}(L^2 - b^2 - x^2) \qquad 0 \le x \le a$$

$$\theta_A = \frac{P\, a\, b\, (L+b)}{6\, L\, E\, I} \qquad \theta_B = \frac{P\, a\, b\, (L+a)}{6\, L\, E\, I}$$

$$if\, a \ge b, \quad \Delta_c = \frac{P\, b\, (3L^2 - 4b^2)}{48\, E\, I}$$

$$if\, a \ge b, \quad x_1 = \sqrt{\frac{L^2 - b^2}{3}} \quad and \quad \Delta_{max} = \frac{P\, b\, (L^2 - b^2)^{3/2}}{9\sqrt{3}\, L\, E\, I}$$

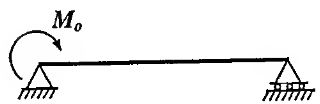


$$y = \frac{P\, x}{6\, E\, I}(3\, a\, L - 3a^2 - x^2) \qquad 0 \le x \le a$$

$$y = \frac{P\, a}{6\, E\, I}(3\, Lx - 3x^2 - a^2) \qquad a \le x \le L - a$$

$$\theta_A = \theta_B = \frac{P\, a\, (L-a)}{2\, E\, I}$$

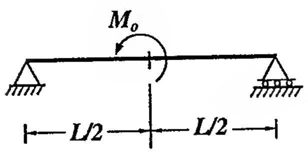
$$\Delta_c = \Delta_{max} = \frac{P\, a}{24\, E\, I}(3L^2 - 4a^2)$$



$$y = \frac{M_o\, x}{6\, L\, E\, I}(2\, L^2 - 3\, Lx + x^2)$$

$$\theta_A = \frac{M_o L}{3\, E\, I} \qquad \theta_B = \frac{M_o L}{6\, E\, I} \qquad \Delta_c = \frac{M_o L^2}{16\, E\, I}$$

$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \quad and \quad \Delta_{max} = \frac{M_o L^2}{9\, \sqrt{3}\, E\, I}$$

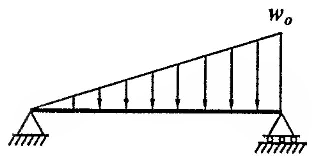


$$y = \frac{M_o\, x}{24\, L\, E\, I}(L^2 - 4\, x^2) \qquad 0 \le x \le L/2$$

$$\theta_A = \frac{M_o L}{24\, E\, I} \qquad \theta_B = -\frac{M_o L}{24\, E\, I} \qquad \Delta_c = 0$$

$$y=\frac{M_o\,x}{2EI}(L-x)$$

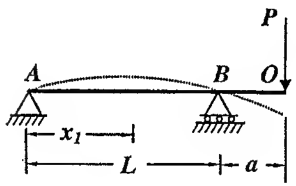
$$\theta_A=\theta_B=\frac{M_oL}{2EI}\qquad \Delta_c=\Delta_{\max}=\frac{M_oL^2}{8EI}$$



$$y=\frac{w_o\,x}{360\,L\,EI}(7L^4-10L^2x^2+3x^4)$$

$$\theta_A=\frac{7\,w_o\,L^3}{360\,EI}\qquad \theta_B=\frac{w_o\,L^3}{45\,EI}$$

$$\Delta_c=\frac{5\,w_o\,L^4}{768\,EI},\ x_1=0.5193\,L\quad and\quad \Delta_{\max}=0.00652\frac{w_o\,L^4}{EI}$$

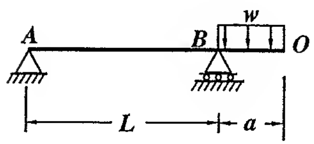


$$y=\frac{Pa\,x}{6\,L\,EI}(L^2-x^2)\qquad 0\le x\le L$$

$$y=\frac{P\,(x-L)}{6\,EI}[(x-L)^2-a\,(3x-L)]\qquad L\le x\le L+a$$

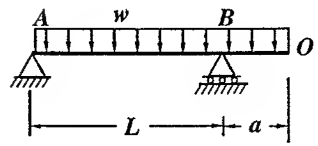
$$\Delta_O=\frac{Pa^2}{3\,EI}(L+a)$$

$$x_1=0.577L\quad and\quad \Delta_{x1}=\frac{a}{9\sqrt{3}}\frac{P\,L^2}{EI}$$



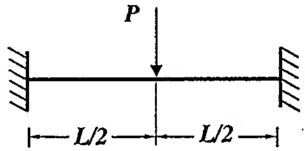
$$\Delta_c=\frac{w\,a^2}{18\sqrt{3}}\frac{L^2}{EI}$$

$$\Delta_O=\frac{w\,a^3}{24\,EI}(4L+3a)$$



$$\Delta_c=\frac{w\,L^2}{384\,EI}(5L^2-12\,a^2)$$

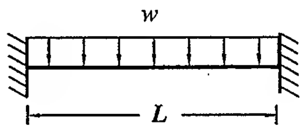
$$\Delta_O=\frac{w\,a}{24\,EI}\{a^2\,(4L+3a)-L^3\}$$



$$M_A=M_B=\frac{-P\,L}{8}$$

$$M_c=\frac{P\,L}{8}$$

$$\Delta_c=\frac{PL^3}{192\,EI}$$



$$M_A=M_b=-\frac{w\,L^2}{12}$$

$$M_C=\frac{w\,L^2}{24}$$

$$\Delta_c=\frac{wL^4}{8\,EI}$$

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Units Conversion Table

To transform from SI-units	To French –units	Multiply by factor
Concentrated loads		
1N	kg	0.1
1 kN	kg	100
1 kN	ton	0.1
Linear Loads /m′		
1 kN/m′	t/m′	0.1
Uniform Loads /m^2		
kN/m^2	t/m^2	0.1
N/m^2	kg/m^2	0.1
kN/m^2	kg/m^2	100
Stress		
N/mm^2 (=1 MPa)	kg/cm^2	10
kN/m^2	kg/cm^2	0.01
kN/m^2	ton/m^2	0.1
Density		
N/m^3	kg/m^3	0.1
kN/m^3	ton/m^3	0.1
kN/m^3	kg/m^3	100
Moment		
kN.m	ton.m	0.1
N.mm	kg.cm	0.01
Area		
m^2	cm^2	10000
mm^2	cm^2	0.01

رقم الإيداع بدار الكتب المصرية: ٢٠٠٦/١٩٦٦٨

طبع بشركة البلاغ للطباعة والنشر والتوزيع

الاصدار الاول

الطبعةالاولى: سبتمبر ٢٠٠٦
الطبعة الثانية: سبتمبر ٢٠٠٧
طبع في جمهورية مصر العربية

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DESIGN OF REINFORCED CONCRETE STRUCTURES

Prof. Mashhour Ghoneim   Dr. Mahmoud El-Mihilmy

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- Good theoretical background for each topic with code provisions.
- Extensive examples in each chapter utilizing SI units.
- All examples are worked out step-by-step ranging from simple to advanced.
- Full reinforcement details for every example.
- Numerous design charts.

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- Design of Deep Beams and Corbels
- Deflections of Reinforced Concrete Members
- Crack Control of Reinforced Concrete Members
- Design of Shallow Foundations and Pile Caps
- Design of Raft Foundations
- Strut-and-Tie Model for Reinforced Concrete Members
- Fundamentals of Prestressed Concrete
- Flexural Design of Prestressed Concrete Members
- Shear and Torsion in Prestressed Concrete
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